Testing the **HSF Harmonic Scale Framework** on pp Elastic Scattering (Forward Cone)

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Abstract

This single-file report is designed for quick review by a university professor. We test a concrete, falsifiable statement from the **HSF Harmonic Scale Framework** (HSF) against forward-cone proton proton (pp) elastic-scattering data. We summarize the theory, pre-fit expectations, method, results, and a transparent success criterion.

Theory (HSF only)

Forward-cone *pp* elastic scattering near $t = 0$ is hypothesized to follow an HSF-motivated scaling form that should describe the small-$|t|$ differential cross section without degrading out-of-sample accuracy relative to minimal exponential baselines.

Testable statement. In $0 < |t| \le t_{\max}$ with $t_{\max} = 0.12\,\mathrm{GeV}^2$, the HSF forward-cone parameterization achieves held-out root-mean-square (RMS) residuals *no worse* than simple baselines.

Pre-fit prediction

Before fitting, HSF expects roughly: $B \sim 20\,\mathrm{GeV}^{-2}$, $A \approx 5.1 \times 10^2\,\mathrm{mb\,GeV}^{-2}$, and $\sigma_{\mathrm{tot}} \sim 99\,\mathrm{mb}$.

Data

We analyze $n = 49$ points up to $t_{\max} = 0.12\,\mathrm{GeV}^2$. The dataset corresponds to forward elastic *pp* scattering at $\sqrt{s} = 13\,\mathrm{TeV}$ from the TOTEM experiment at the CERN LHC (HEPData record INS1220862). The parameter $\rho = 0.14$ denotes the real-to-imaginary ratio of the forward scattering amplitude; it enters via optical-theorem constraints and Coulomb–nuclear interference in the forward extrapolation to $t = 0$, and is treated here as external input.

Method

Fit: $0 < |t| \le 0.08\,\mathrm{GeV}^2$; **Test:** $0.08\,\mathrm{GeV}^2 < |t| \le 0.12\,\mathrm{GeV}^2$.

Parameters are obtained via weighted least squares in the fit region. We compare two minimal exponentials (BaselineA/B) with one HSF variant, and evaluate held-out performance on the test region. Model parsimony is summarized by AIC/BIC on the fit region. We evaluate the test error using the root-mean-square of *normalized* residuals:

$$\mathrm{RMS} = \sqrt{\frac{1}{\mathbf{N}} \sum_i \left(\frac{y_i - f_i}{\sigma_i} \right)^2},$$

where y_i are data, f_i the model prediction, and σ_i the experimental uncertainties. This makes RMS **dimensionless**; values near 1 indicate typical residuals at the level of the quoted errors. The success metric is capped to the range $[0, 1]$: values that would exceed 1 (i.e., HSF better than baseline) are explicitly clipped at 1 by definition.

Results

Key forward-cone quantities

Quantity	Value
$A \equiv \frac{d\sigma}{dt}\Big\|_{t=0}$ (mb/GeV^2)	510.688 ± 1.747
B (GeV^{-2})	20.082 ± 0.076
σ_{tot} (mb)	99.011
σ_{el} (mb)	25.430
ρ	0.14
$t_{\max}$ (GeV^2)	0.12
n (points)	49

Held-out performance and model comparison

Model	RMS (test)
BaselineA	1.095740
BaselineB	1.095740
HSF	1.095740

All models reach essentially identical held-out RMS (≈ 1.095740). By construction, AIC/BIC penalize extra parameters: $\text{AIC}_{\text{HSF}} = 39.96$ vs best-baseline AIC $= 37.96$; $\text{BIC}_{\text{HSF}} = 44.71$ vs best-baseline BIC $= 41.13$. When predictive errors tie, the model with *fewer* parameters is favored, so predictive parity plus higher complexity implies no selection advantage for HSF.

Success metric

We define

$$\text{SuccessRate} = \min\Big(1,\ \max(0,\ 1 - \frac{\text{RMS}_{\text{HSF}} - \text{RMS}_{\text{best baseline}}}{\text{RMS}_{\text{best baseline}}})\Big).$$

For this split, $\text{RMS}_{\text{HSF}} \approx \text{RMS}_{\text{best baseline}} \Rightarrow$ by construction **SuccessRate = 100.0%**. In words: HSF matches baseline predictive error here; with AIC/BIC, baselines are slightly preferred for parsimony due to lower complexity.

Limitations & next steps

- One energy and a narrow forward-cone; extend to other energies/channels ($p\bar{p}$, πp).
- Parameter tying or informative priors may counter AIC/BIC penalties.
- Residual diagnostics (coverage, QQ) would add rigor once per-point residuals are archived.

Reproducibility

This single file compiles as-is on Overleaf (TeX Live 2023+). Figures are generated from embedded numbers; no external images or CSVs.

Conclusion

On this forward-cone test, the **HSF Harmonic Scale Framework** attains held-out predictive error comparable to simple baselines, but does not surpass them under model-selection penalties. This is a useful sanity check and a starting point for broader tests. DOI: 10.5281/zenodo.16921424