Delft University of Technology

MSc in Systems and Control

Robust control assignment: Part 1

(SC42145)

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Contents

1 System Analysis

The aim of the first part of the project is to design a SISO controller to pitch the blades of the wind turbine in order to regulate its rotational velocity and acquire a new desired generator speed.

Since the system under study is a MIMO system, the first step followed was to extract the SISO model describing the connection between the pitch angle and the rotational velocity, which are the SISO system input and output, respectively.

Since the SISO transfer function has negative gain (observed by transforming the system into zero-pole-gain form), it requires a controller with also negative gain. However, it was decided to multiply the SISO sytem by a negative sign in order to be able to design the controller with a negative feedback, which is more intuitive. Nevertheless, the controller gain sign will be switched once the synthesis is finished.

The pole-zero map of the SISO system, shown in Figure 1, gives relevant information on the system dynamics and the achievable bandwidth.

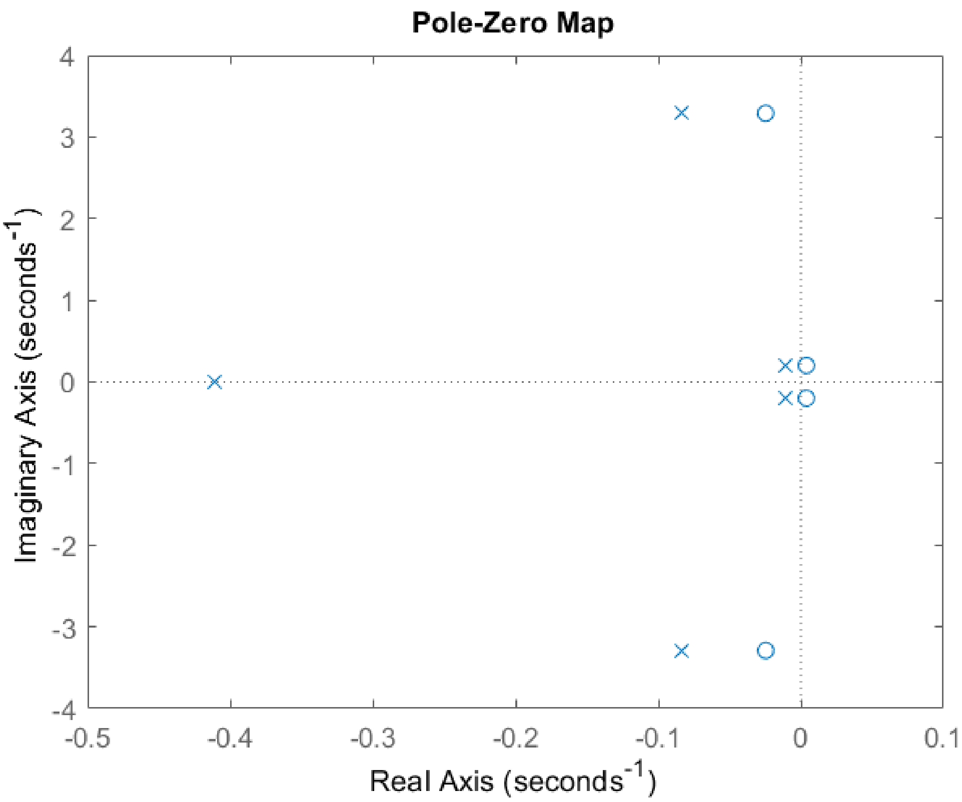

Figure 1: Pole-zero map of the SISO system

It can be observed that the system has a pair of complex conjugate zeros at the right half plane (RHP), at approximately 0.03 Hz. Since they can not be cancelled by the controller, they will imply a limitation on the achievable bandwidth. In order to analyze this in greater detail, the Bode plot is used.

The bode plot of the SISO system, presented in 2, shows the behaviour of the system to inputs with different frequencies, as well as the influence of each pole and zero in the system response.

It can be observed that the pair of complex conjugate zeros at the right hand side plane generate a decrease in phase of 180 degrees. In addition to this, it has been taken into account that, in order to have 0 steady-state error, an integrator term will be needed in the system transfer function. This will imply a pole at 0 frequency and thus the phase plot will start at $-90[deg]$.

The Bode stability criterion states that if the phase-shift of the system is larger than 180 degrees at the crossover frequency, the system becomes unstable. Due to this fact, the system's cross-over frequency has to be placed at frequencies lower than 0.03 Hz.

Therefore, it can be be concluded that the achieved bandwidth can not be higher than $0.1[Hz]$, because in that case, the decrease in phase introduced by the pair of complex conjugate zeros at the right half plane would make the system unstable.

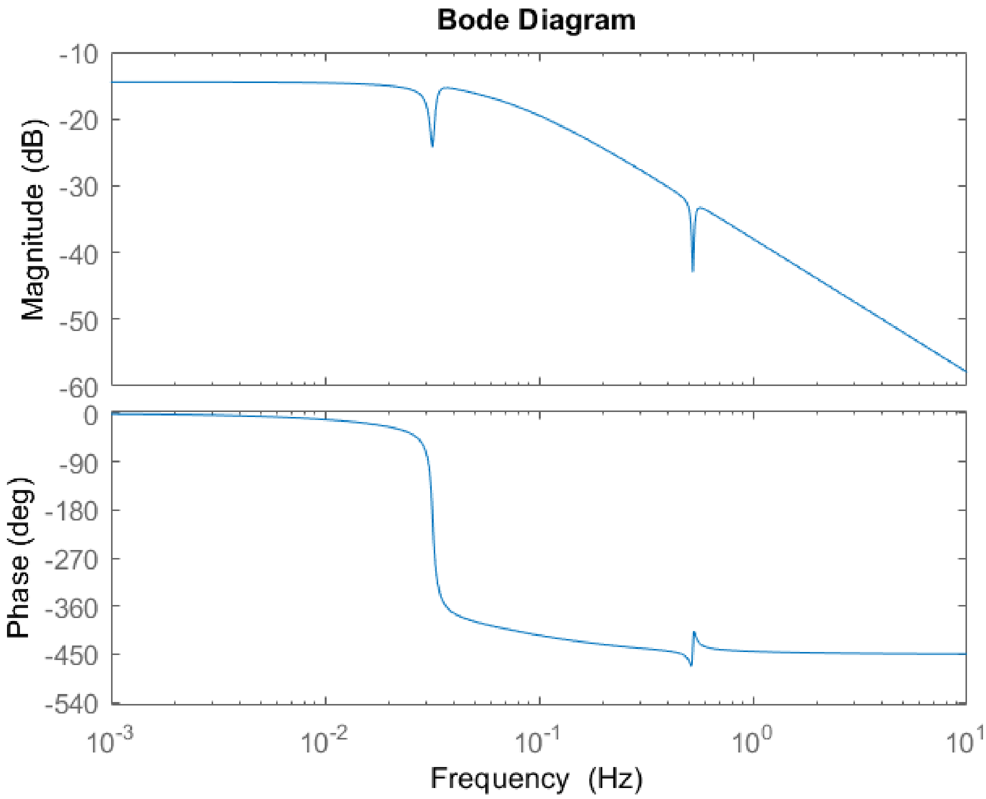

Figure 2: Bode Plot of the SISO system

2 System Requirements

The requirements for the controlled system include limitations in the system's transient response, this is, a maximum settling time and overshoot, as well as in the steady state response, for which the output should match the reference input.

In order to design a controller that achieves these requirements, the complementary Sensitivity Function T is analysed. This transfer function represents how the reference input of a controlled system affects the system's output.

$$T = \frac{K(s)G(s)}{1 + K(s)G(s)} \tag{1}$$

where G(s) and K(s) are the system and controller transfer functions, respectively.

It is required that the controlled system has zero steady-state error, what implies that the system output shall match the reference input for inputs with frequencies inside the bandwidth. This can be directly achieved if the Complementary Sensitivity Function is close to one in magnitude for said frequencies, which implies $K(s)G(s) >> 1$ for frequencies lower than the bandwidth.

The behaviour of $K(s)G(s)$ can be studied using the open loop bode diagram of the system. Since the system bandwidth can not be seen in this diagram, the cross-over frequency will be used as reference (since it corresponds to a lower frequency than the bandwidth and thus it is more restrictive). Moreover, the open loop bode diagram also provides the gain and phase margins, which give relevant intuition on the controlled system stability, overshoot and settling time.

Therefore, the followed approach is tuning $K(s)$ to maximize $K(s)G(s)$ for frequencies lower than the cross-over frequency. Meanwhile it is important to keep the gain and phase margins within values that generate a system response that fulfills the requirements.

Moreover, the cross-over frequency will be kept as large as possible (while bearing in mind the limitations induced by the complex conjugate RHP zeros) so that the system is able to rapidly converge to the reference input.

Finally, it is required that the transfer function decreases rapidly after the cross-over frequency so that the high-frequency input signals (disturbance) are not amplified by the system. Nevertheless, as shown in Figure 2, the system already behaves in that way, and thus no tuning is needed regarding that aspect.

The tools used within this approach are the open loop bode diagram and the closed-loop system step response.

As it can be observed in Figure 2, the current open loop system's gain for low frequencies is approximately $-14.5[dB]$. In order to increase it, a preliminary pure integral controller structure $K1(s)$ is selected. In this way, the synthesis will start with a simple controller, and other controller actions, such as proportional or derivative, will be included if the requirements can not be fulfilled with the current structure.

It has been stated that the open loop system's gain has to be increased, but it has yet to be specified at which frequencies it is desired to be increased. It is known that at frequency $0.03[Hz]$, the complex conjugate RHP zeros introduce a phase shift which would make the system unstable. Therefore, the system gain will be made larger at frequencies lower than $0.03[Hz]$, which corresponds to $0.2[rad/s]$.

A system with transfer function $0.2/s$ would have a cross-over frequency of $0.2[rad/s]$, this is, would exactly increase the gain until the frequency $0.2[rad/s]$, as desired in this case. Although the analyzed system is more complex than $0.2/s$, the integral constant of K1(s) is chosen in first place as $0.2[rad/s]$, in order to check the actual system cross-over frequency and performance.

$$K_1(s) = \frac{0.2}{s} \tag{2}$$

The influence of $K1(s)$ on the open loop system response is shown in Figure 3. If compared to Figure 2, it can be observed that the gain has increased as desired, although the cross-over frequency lies at $0.04[rad/s]$ approximately. Consequently, the controller can be further improved to increase the system cross-over frequency.

The performance of this controller is tested with the step response of the closed-loop system, which is presented in Figure 4. The settling time is $89.8[s]$ and the overshoot is $0.35\%[s]$, and thus the requirements are satisfied with $K_1(s)$. Nevertheless, a further iteration will be performed to optimize the close-loop response as much as possible.

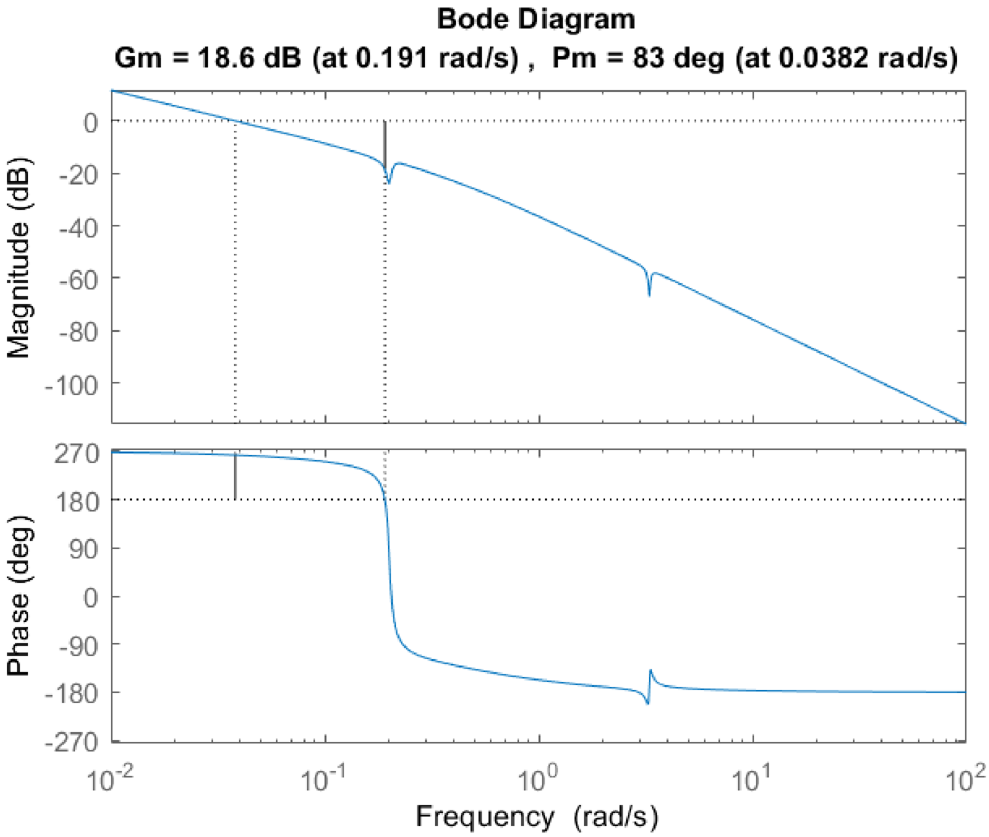

Figure 3: Bode Plot of the closed-loop system with K1(s)

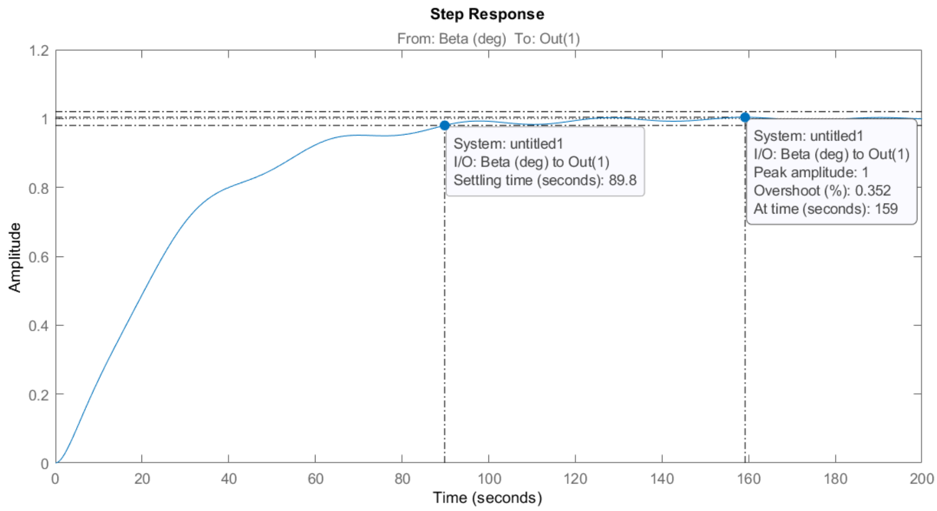

Figure 4: Step response of the closed-loop system with $K_1(s)$

In order to achieve a larger gain of the system, the integral constant has been increased until arriving to a solution that maximizes the cross-over frequency while satisfying the requirements. The chosen constant is at $0.26[rad/s]$, while the selected controller is as follows:

$$K(s) = \frac{0.26}{s} \tag{3}$$

The performance of the closed-loop system with the controller $K(s)$ is deeply addressed in the following chapter.

3 Time-domain simulation of the closed loop system

For creating the closed loop system the sign of the controller gain was changed to match the original system.

The system simulation for a step in the reference input signal is shown on Figure 5. The overshoot is 0.919% $< 1\%$, so it satisfies the requirement. The response of the system is generally slow, the settling time is $\approx 83.7[s]$.

From the Bode diagram of the closed loop system the achieved bandwidth can be seen (where the curve crosses the -3 dB line from above): c.a. $0.01[Hz]$ (Figure 6). It was previously described why the RHP zeros and the integral controller limit the maximal achievable bandwidth. It is certainly less than $0.03[Hz]$ with the integral controller so further tuning on the integral time constant will not result in significant improvement.

The phase and gain margins are shown on Figure 7, these are $16.5[dB]$ and $81.1[deg]$ respectively. For smaller gain margin the frequency components close to the -180 phase shift are not suppressed enough so that the overshoot will be too high.

Table 1 contains the quantitative characteristics of the system described previously.

Settling time	Overshoot	Bandwidth	Gain margin	Phase margin
$83.7[s]$	$0.919[\%]$	$0.01[Hz]$	$16.5[dB]$	$81.1[deg]$

Table 1: Summary of the system behaviour

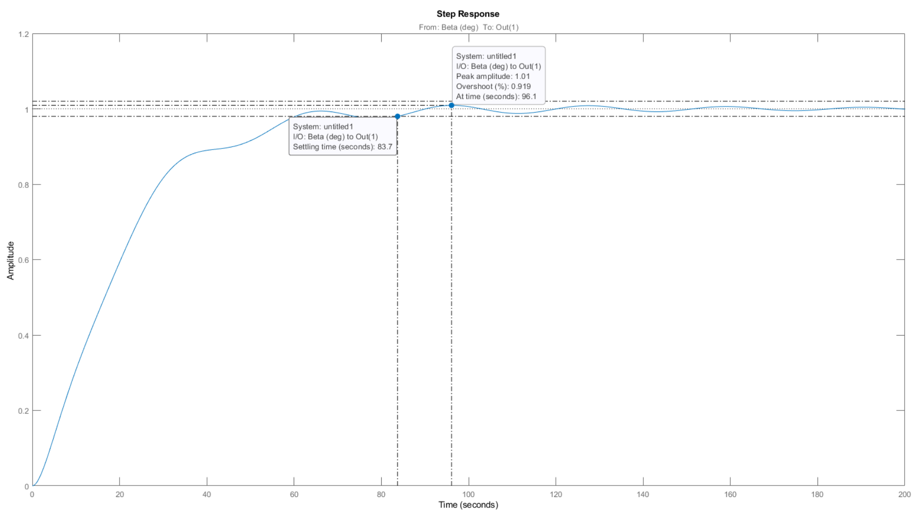

Figure 5: Step response of the closed loop system

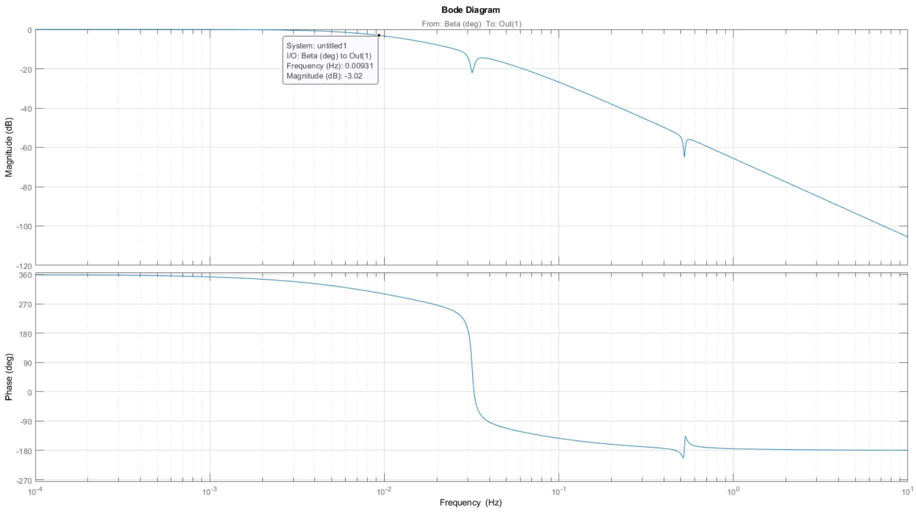

Figure 6: Bandwidth of the closed loop system

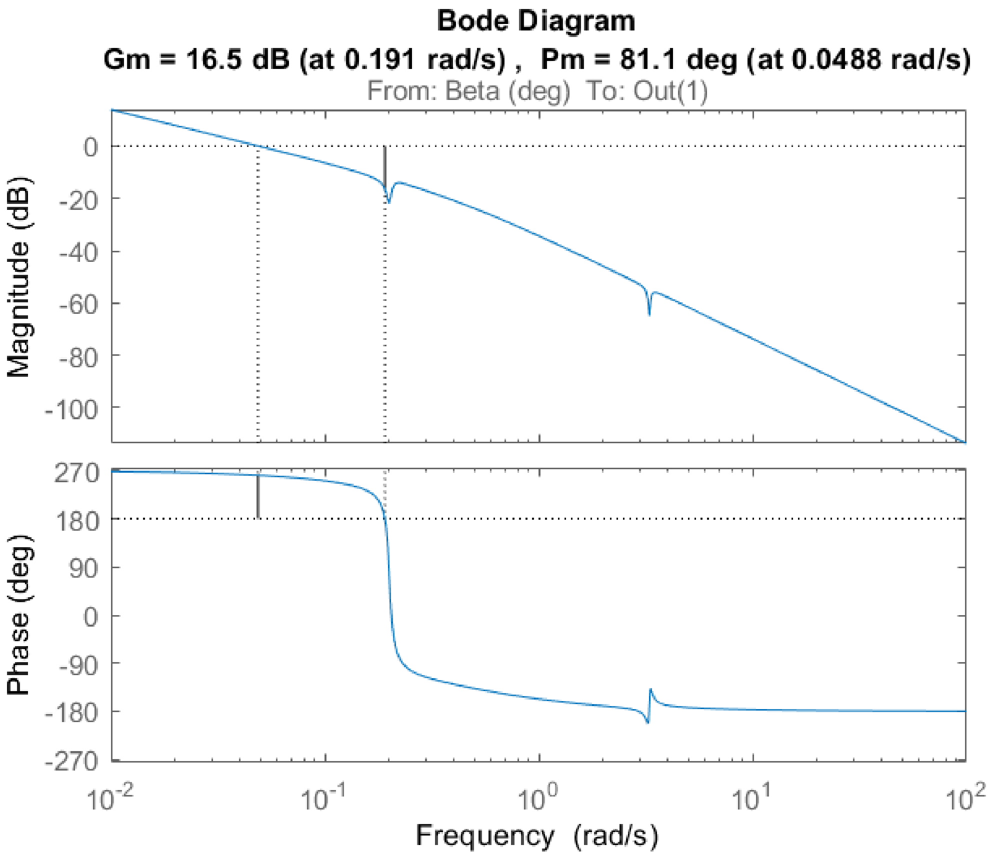

Figure 7: Designed phase and gain margin

4 Disturbance rejection of the closed loop system

To analyse to disturbance rejection of the created closed loop system the transfer function between the disturbance input and the generator speed output should be determined including the closed loop system with the designed controller.

Figure 8 shows the block diagram of the control loop. This can be easily rearranged to the form presented on Figure 9. G_d can be extracted from the original state-space model in Matlab. The transfer function of the feedback part is easily obtained with the 'feedback' command. The transfer function between the disturbance input and the generator speed is the multiplication of the previous 2 transfer functions.

$$G_{d2omega} = G_d(s)\frac{1}{1 + K(s)G(s)} \tag{4}$$

The simulated step-response is shown of Figure 10. The settling time is high (168[s]), in a real-life scenario multiple changes in the wind can happen during that time (the changes in the wind speed are modelled by step functions). Moreover, the error on the output is $0.4[rad/s]$ in peak.

In conclusion, this controller is not suitable for disturbance rejection purposes while keeping the performance specifications. By increasing the performance for reference tracking there is a trade-off in disturbance rejection. Multi-variable controller synthesis approaches might provide a better solution.

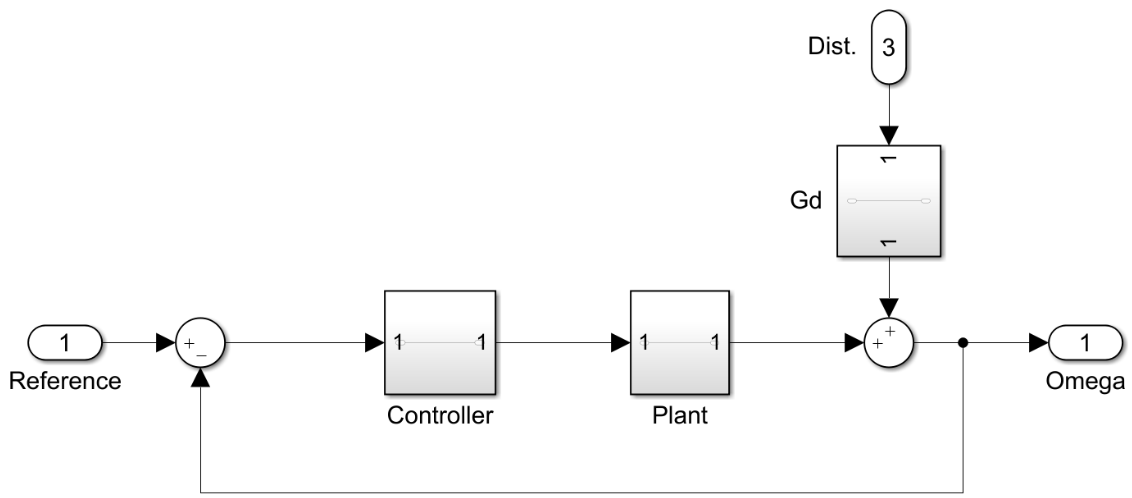

Figure 8: Block diagram of the control loop

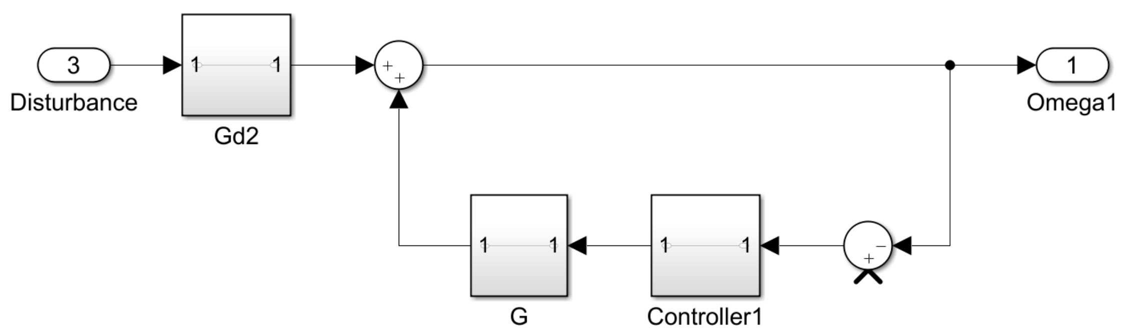

Figure 9: Block diagram from disturbance to the output

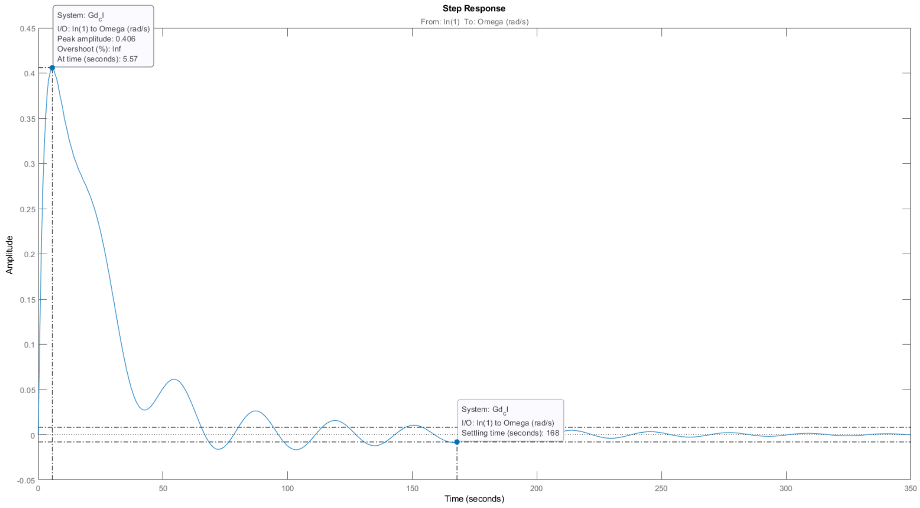

Figure 10: System response for step input on the disturbance channel