General instructions: Show complete and clear solutions.

1. Evaluate the following limits. Use ∞ or $-\infty$ when applicable.

   (a) $\displaystyle\lim_{x\to 2^-}\left(\frac{5}{x^2+4x-12}-\frac{2}{x-2}\right)$ **[10]**

   (b) $\displaystyle\lim_{x\to 0}\frac{\sin^2 3x}{x\sin 2x}$ **[10]**

   (c) $\displaystyle\lim_{x\to -\infty}\frac{\sqrt{25x^2+7}+11x}{7-2x}$ **[10]**

2. Find all constants a and b such that the function f is continuous on $[2,\infty)$. **[15]**

$$f(x)=\begin{cases} ax[\![x]\!] & \text{if } x<3\\ b(x-3)-2x^2 & \text{if } 3\le x\le 5\\ 3b\cos(\pi x) & \text{if } 5<x\end{cases}$$

3. Find the derivative of each function. Do not simplify.

   (a) $f(x)=\dfrac{x\cos x}{\left(x^2+\sqrt{\tan x}\right)^2}$ **[10]**

   (b) $g(x)=\tan\left(\dfrac{\sin^2 x}{x}\right)$ **[10]**

4. Prove using the precise definition of a limit that $\displaystyle\lim_{x\to 5}\left(2-\frac{x}{10}\right)=\frac{3}{2}$. **[10]**

5. Let $f(x)=x^4-16x^2$. Show that there is no tangent line to the graph of f that passes through the point $(0,22)$. **[12]**

6. Let n be a constant such that $0<n<3$. Show that the equation

$$x^3+9x^2+2x-2n=0$$

   has a positive root and a negative root. **[13]**