Navigating by Falling Stars: Monetary Policy with Fiscally Driven Natural Rates*

Rodolfo G. Campos[1] Jesús Fernández-Villaverde[2] Galo Nuño[3] Peter Paz[4]

[1]Banco de España
[2]University of Pennsylvania, NBER, CEPR
[3]Banco de España, CEMFI, CEPR
[4]Bank of England

June 12, 2025

Abstract

We study the monetary-fiscal interaction in a heterogeneous-agent New Keynesian model with a fiscal block. Due to household heterogeneity, the stock of public debt affects the natural interest rate, prompting the central bank to adjust its monetary policy rule in response to the fiscal stance, ensuring that inflation remains at its target. There is, however, a minimum level of debt below which the steady-state inflation deviates from its target due to the zero lower bound on nominal rates. We analyze the response to a debt-financed fiscal expansion and quantify the impact of different timings in adapting the monetary policy rule, as well as the performance of alternative monetary policy rules that do not require an assessment of the natural rates. We validate our findings with a series of empirical estimates.

JEL Classification: E32, E58, E63.
Keywords: HANK models, natural rates, fiscal shocks.

*We thank our discussants Christian Bayer, Fernando Cirelli, Daisuke Ikeda, Alessandro Lin, Ralph Luetticke, and Gastón Navarro for their insightful discussions and comments. We are very grateful to Peter Hördahl for sharing his data on term premia and to Mario Alloza for his comments. We are grateful for the comments we received at various seminars and conferences. The views expressed in this manuscript are those of the authors and do not necessarily represent the views of the Bank of England, Banco de España, or the Eurosystem. All remaining errors are ours.

1 Introduction

Following the collapse of the Bretton Woods system in 1971, a broad consensus emerged among academics and policymakers: central banks would be responsible for controlling inflation, while treasuries would ensure debt sustainability. This separation of mandates was formalized in the Maastricht Treaty of 1992. Member states of the European Union established the European Central Bank (ECB) with a primary mandate of maintaining price stability. Simultaneously, national treasuries became subject to a set of fiscal rules designed to prevent debt and deficits from exceeding specified thresholds.

Economic theory lent strong intellectual support to this institutional arrangement. In the canonical representative-agent New Keynesian (RANK) model (Woodford, 2003; Galí, 2008), the central bank can achieve its price stability mandate provided three conditions are met. First, the treasury must ensure that public debt remains bounded. Second, the central bank must respond aggressively enough to inflation deviations —i.e., it must follow the so-called Taylor principle (or a close variant). Third, the central bank must track the natural rate of interest, r^*, in the long run, and this natural rate must be sufficiently high to prevent the zero lower bound (ZLB) on nominal interest rates from becoming binding.[1]

The key result in the benchmark RANK framework is that the natural rate depends solely on parameters that describe preferences and technology, such as the household discount factor or productivity growth. Many economists are confident in assuming that such parameters remain constant or evolve slowly in accordance with secular trends.

Interestingly, this key result breaks down once we deviate from the complete-markets representative-agent framework. Instead, consider a heterogeneous-agent New Keynesian (HANK) model, as popularized by Kaplan et al. (2018) and Auclert et al. (2024), among many others. These models incorporate a continuum of atomistic households subject to idiosyncratic risk that can save only by using non-state-contingent instruments. One important feature of HANK models is that the natural rate depends on the stock of public debt, as originally pointed out by Aiyagari and McGrattan (1998) and more recently by Rachel and Summers (2019). The intuition is straightforward: given market incompleteness, the stock of public debt determines how much households can self-insure against negative idiosyncratic shocks, and therefore the interest rate at which the savings market clears.

As pointed out by Kaplan et al. (2018), the link between public debt and natural rates

[1] In this paper, the natural rate refers to the real interest rate in the deterministic steady state of the economy. This concept differs from —though is often conflated with— the neutral interest rate, which is the real rate that would prevail in a counterfactual economy with fully flexible prices. The neutral rate evolves stochastically in response to economic shocks, while the natural rate is a long-run object. See Platzer et al. (2022) for a detailed comparison between the natural and neutral rates. The authors refer to the natural rate as the long-run r^* and the neutral rate as the short-run r^*.

opens the door to a form of monetary-fiscal interaction. This point has been explored further by Bayer et al. (2023) and Kaplan et al. (2023). If the treasury changes the long-run stock of public debt, this decision moves the natural rate. The central bank should then either incorporate the new natural rate into its Taylor rule or risk biasing long-term inflation away from its target. In this paper, we examine this form of monetary-fiscal interaction in both the long and short run using a quantitative HANK model calibrated to replicate key features of the U.S. economy.

This argument goes beyond HANK models. Overlapping generations (OLG) models, for instance, share the same feature, as shown in Eggertsson et al. (2019), Challe and Matvieiev (2024), and Aguiar et al. (2023). The latter authors introduce a tractable New Keynesian OLG model to analyze the global equilibrium dynamics of inflation, interest rates, and labor earnings in response to changes in the stock of public debt. Furthermore, the empirical literature has documented the significance of this channel in the data; see, for example, Rachel and Summers (2019) and Ferreira and Shousha (2023).

Fiscally driven natural rates vs. the FTPL. The reader should note that the interaction we focus on is conceptually different from the one analyzed under the fiscal theory of the price level (FTPL). The core insight of the FTPL, dating back to Sargent and Wallace (1981), is that if the treasury is not committed to guaranteeing debt sustainability, the central bank may be forced to accommodate fiscal expansions to prevent debt from exploding. In these circumstances, inflation is determined by the need to stabilize public debt. This is the logic behind the Maastricht Treaty discussed above.

In contrast, in our HANK model, the treasury collects taxes, spends, and funds deficits by issuing debt in accordance with a fiscal rule that stabilizes the long-run real stock of debt (a particular case of a passive fiscal policy). Fiscal policy affects the conduct of monetary policy not by refusing to stabilize debt, but by shifting the natural rate.

Our analysis. We propose a two-asset HANK model featuring a central bank that sets nominal interest rates according to a standard Taylor rule, and a treasury that conducts fiscal policy. The model includes nominal wage and price rigidity, long-term government debt, capital accumulation, and idiosyncratic household income risk. The economy has a unique deterministic steady state in which a temporary increase in government spending permanently raises the debt-to-GDP ratio, which in turn affects the natural rate of interest: the higher the resulting debt level, the higher the natural rate.

Steady-state inflation deviates from the central bank's target in proportion to the gap between the natural rate and the intercept of the Taylor rule. Thus, to ensure price stability, the central bank must adjust the intercept of its rule in response to fiscally induced changes in the natural rate.

There is, however, a situation in which the central bank in a HANK framework cannot fulfill its long-run mandate, even if it is willing to adapt its rule to the fiscal position. If the debt level is sufficiently low, the natural interest rate becomes negative. Should the sum of the natural rate and the inflation target fall below zero, the ZLB becomes binding in the long run, and inflation converges to the negative of the natural rate. Therefore, a minimum debt level exists that is compatible with the inflation target. For any debt level below this threshold, the central bank fails to deliver on its mandate.

We begin by examining the long-run effects of government debt on interest rates and inflation (a situation where nominal rigidities are irrelevant). We show that debt influences the natural rate and constrains the ability of monetary policy to achieve price stability in the steady state. The central bank faces a trade-off: it can either adjust the intercept of its Taylor rule to match the natural rate or tolerate persistent deviations of inflation from its long-run target. For example, if the central bank does not accommodate an increase in the natural rate caused by higher debt, steady-state inflation will rise. Moreover, our model generates a semi-elasticity of real interest rates with respect to government debt that aligns closely with both our empirical estimates and existing findings in the literature.[2]

After studying the long-run equilibrium, we analyze the short-run dynamics of monetary-fiscal interactions. We quantify the effects of a debt-financed fiscal expansion as the economy transitions from the initial steady state to a new steady state with a permanently higher debt level. Specifically, we consider an experiment in which government consumption increases temporarily, resulting in a higher debt-to-GDP ratio. We examine the resulting inflation dynamics and how they depend on the central bank's response. We also consider alternative scenarios in which tax rates or transfers adjust to mitigate the increase in public debt. In all cases, the fiscal expansion generates significant short-term deviations in inflation, even if the central bank adjusts its policy rule to ensure that inflation returns to its target in the long run.

We further analyze alternative monetary policy rules in the presence of fiscally driven changes in the natural rate. In particular, we consider the differential rule proposed by Orphanides and Williams (2002), which does not rely on real-time knowledge of the natural rate. This rule links changes in the nominal interest rate to deviations of inflation from its target. We demonstrate that such a rule yields superior stabilization outcomes compared to the standard Taylor rule, underscoring the benefits of monetary frameworks that do not necessitate the explicit estimation of r^* when fiscal policy alters the natural rate.

In a later section of the paper, we investigate the implications of policy timing. We analyze

[2]As one referee noted, many factors, such as demographics or changing risk aversion, can influence the natural rate. We simply argue that the mechanism highlighted in this paper is a potentially important factor in that evolution and that the central bank should take it into account.

how the timing of monetary policy adjustments in response to an announced fiscal expansion affects macroeconomic dynamics. Our results indicate that early adjustments to the monetary policy rule help anchor inflation expectations, whereas delayed responses lead to larger and more persistent fluctuations in inflation.

Finally, we explore the empirical relationship between natural rates and the central bank's reaction function. Using market-based measures of long-term nominal interest rates and inflation expectations, we identify periods of significant misalignment between the natural rate and monetary policy. We find evidence of a persistent gap between the natural rate and the intercept in central banks' monetary policy rules, particularly in the post-pandemic period.

Interpreting these findings through the lens of our model —which links the level of government debt to the natural rate— we suggest that the widening of this gap in recent years has been driven, at least in part, by expectations of higher long-run public debt levels.

Literature review. This paper is related to the large literature on monetary-fiscal interactions, particularly under the FTPL (see Leeper, 1991; Sims, 1994; Cochrane, 1999; Woodford, 1995; Schmitt-Grohe and Uribe, 2000, among others). Bhandari et al. (2017) study competitive equilibria with public debt in a general framework with heterogeneous agents. Mian et al. (2022) examine a model with bonds in the utility function and identify a minimum level of debt consistent with the ZLB, below which inflation falls short of target. Bianchi et al. (2022) analyze an environment in which a monetary-led and a fiscally led policy mix coexist, as the central bank accommodates unfunded fiscal shocks, resulting in persistent movements in inflation. Bigio et al. (2023) study expectations around a monetary-fiscal reform, in which monetary policy is temporarily tasked with generating inflation to help the treasury stabilize debt. After the reform, debt and inflation converge to a new steady state.

This paper also contributes to the literature analyzing fiscal and monetary policy in HANK models, including Oh and Reis (2012), Kaplan et al. (2018), Auclert et al. (2024), Hagedorn et al. (2019), McKay and Reis (2021), Wolf (2021), Ferriere and Navarro (2018), and Bilbiie (2024). A closely related contribution is Bayer et al. (2023), who show that a permanent increase in the debt-to-GDP ratio raises the real yield on public bonds in the long run, and that a two-asset HANK model can replicate the empirical semi-elasticity of real interest rates with respect to public debt. Our paper provides a more detailed examination of the interaction between fiscal and monetary policy in a quantitative environment similar to theirs.

Theoretical work on fiscal-monetary interaction in HANK models includes Hagedorn (2016), who demonstrates that prices and inflation are jointly and uniquely determined by fiscal and monetary policy in a HANK model with nominal debt. Kaplan et al. (2023) analyze the FTPL in a heterogeneous-agent model with flexible prices. In their setting, a permanently higher deficit lowers the steady-state real interest rate and reduces real public debt while raising long-run inflation for a given monetary stance. Relatedly, Angeletos et al. (2024) study a sticky-price version of the FTPL in a heterogeneous-agent model and find that fiscal deficits are only about half as inflationary as predicted by standard FTPL arithmetic. Independently of our work, Hänsel (2024) also examines how government debt expansions raise the natural rate and generate inflation in a two-asset HANK economy.

This paper is further connected to the literature on estimating the natural rate and its implications for monetary policy. Schmitt-Grohé and Uribe (2025) study how exogenous shifts in the permanent component of real interest rates affect output and inflation. Several methods are used to estimate natural rates, including semi-parametric approaches (e.g., Laubach and Williams, 2003; Holston et al., 2017), nonstructural time series methods (e.g., Lubik and Matthes, 2015), and those based on bond market expectations of long-run real rates (e.g., Christensen and Rudebusch, 2019; Davis et al., 2023). Because these methods often yield divergent results, there is significant uncertainty around natural rate estimates. Such uncertainty can lead to monetary policy misperceptions, as emphasized by Ajello et al. (2020). Chortareas et al. (2023) estimate a time-varying Taylor rule for the U.S. and document episodes in which the Federal Reserve appears to have misread the natural rate of interest. Bocola et al. (2024) measure shifts in the systematic conduct of monetary policy using bond market data. Daudignon and Tristani (2023) analyze the optimal monetary policy response to stochastic variation in the natural rate in a New Keynesian framework. Finally, Bauer and Rudebusch (2020) demonstrate the importance of accounting for time variation in natural rates for understanding the dynamics of the yield curve.

The rest of the paper is organized as follows. Section 2 introduces our HANK model. Section 3 discusses its calibration and computation. Section 4 explores long-run monetary-fiscal interactions. Section 5 analyzes their short-run dynamics. Section 6 studies how different monetary policy rules shape short-run inflation and real outcomes. Section 7 examines how the timing of the central bank's response to fiscal expansions affects inflation and welfare. Section 8 investigates the empirical relationship between the natural rate and monetary policy using a general econometric framework. Section 9 concludes.

2 The model

To study monetary policy when natural interest rates are influenced by fiscal policy, we develop a discrete-time HANK model. Time is indexed by t.

Households. Households are indexed by $i \in [0, 1]$ and derive utility from consumption, $c_{i,t}$, while experiencing disutility from labor, $n_{i,t}$. They smooth consumption by saving in liquid and illiquid accounts, following Kaplan et al. (2018) and Alves et al. (2020).

Given a discount factor β, each household solves the following intertemporal problem:

$$V(a_{i,t}, b_{i,t}, z_{i,t}) = \max_{c_{i,t}, a_{i,t+1}, b_{i,t+1}} u(c_{i,t}) - v(n_{i,t}) + \beta \mathbb{E}_t[V(a_{i,t+1}, b_{i,t+1}, z_{i,t+1})]$$

$$\text{s.t.}$$

$$c_{i,t} + a_{i,t+1} + b_{i,t+1} = (1 + r_t^a)a_{i,t} + (1 + r_t^b)b_{i,t} + (1 - \tau_n)\frac{W_t}{P_t} z_{i,t} n_{i,t} + T_t - \Phi_t(a_{it}, a_{it-1}),$$

$$a_{i,t+1} \geq 0, \qquad b_{i,t+1} \geq 0.$$

where $a_{i,t}$ is the household's real illiquid asset position at the start of the period, and $b_{i,t}$ is the real liquid asset position at the same point in time. The term $z_{i,t}$ denotes idiosyncratic labor productivity, while r_t^a and r_t^b represent the ex-post real returns on illiquid and liquid assets, respectively. Adjusting illiquid asset holdings incurs a quadratic cost, $\Phi_t(a_{i,t}, a_{i,t-1})$, reflecting their relative illiquidity compared to bonds. The nominal wage is denoted by W_t, and the price level is denoted by P_t. Labor income is taxed at a constant rate τ_n, and households receive real net lump-sum transfers, T_t, from the treasury.

At time t, each household i supplies $n_{i,t}$ hours of labor, chosen on its behalf by a union. Since each hour provides $z_{i,t}$ units of effective labor, aggregate hours are given by $N_t = \int_0^1 z_{i,t} n_{i,t} di$. The idiosyncratic productivity shock $z_{i,t}$ follows a Markov chain with mean $\mathbb{E}_t z_{i,t+1} = 1$. The nominal wage W_t is determined through union bargaining, as described below.

Unions. We adopt a standard formulation for sticky wages with heterogeneous agents, following Erceg et al. (2000), Auclert et al. (2021b), and Auclert et al. (2024). In this formulation, a union aggregates heterogeneous labor tasks into a homogeneous labor service (see Appendix A for details). The union employs all households for the same number of hours, $n_{i,t} = N_t$, and sets nominal wages to maximize average household welfare, subject to a penalty term defined over deviations of the nominal wage change from the central bank's inflation target, $\overline{\pi}$.

Solving this problem leads to a wage Phillips curve:

$$\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w \left[-\frac{\epsilon_w - 1}{\epsilon_w}(1-\tau_n)\frac{W_t}{P_t}\int u'(c_{i,t}) z_{i,t} di + v'(N_t)\right] N_t + \beta \mathbb{E}_t\left[\log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right)\right], \tag{1}$$

where ϵ_w is the elasticity of substitution between different labor tasks, κ_w is the slope of the Phillips curve (itself a nonlinear function of other parameters of the model), and $\pi_t^w \equiv \frac{W_t}{W_{t-1}} - 1$ is the nominal wage inflation rate.[3]

[3] Appendix G.1 also analyzes a flexible-wage version of the model. To approximate flexibility without

Final goods producer. A competitive final goods firm aggregates a continuum of intermediate goods, indexed by $j \in [0,1]$, with a constant elasticity of substitution $\epsilon_p > 1$ to produce the final output:

$$Y_t = \left(\int y_{j,t}^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}.$$

Given the intermediate goods prices $p_{j,i}$ and the final good price P_t, standard results yield an expression for the price level: $P_t = \left(\int_0^1 p_{j,t}^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}$.

Intermediate goods producers. There is a continuum of identical producers of intermediate goods operating under monopolistic competition. Because each producer produces only one good, we also use the index j for the intermediate goods producers. These intermediate goods producers have access to a constant returns-to-scale technology $y_{j,t} = \Theta k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}$, where $n_{j,t}$ is the labor input employed in production and $k_{j,t-1}$ is the capital input. The constant parameter Θ denotes the total factor productivity.

Intermediate goods producers own the capital stock and make forward-looking investment decisions. Capital depreciates at a rate δ. Investment entails a real cost:

$$\zeta(k_{j,t}/k_{j,t-1}) = k_{j,t}/k_{j,t-1} - (1-\delta) + \frac{1}{2\delta\epsilon_I}(k_{j,t}/k_{j,t-1} - 1)^2.$$

The equations governing investment are detailed in Appendix C.[4]

Intermediate goods producers set the price subject to a quadratic adjustment cost scaled with respect to the target inflation $\overline{\pi}$ and proportional to aggregate final output:

$$\xi(p_{j,t}, p_{j,t-1}) = \frac{\epsilon_p}{2\kappa_p} \left(\log(p_{j,t}) - \log((1+\overline{\pi})p_{j,t-1}) \right)^2 Y_t.$$

From this point forward, we apply the standard result that, in a symmetric equilibrium, all intermediate goods producers make identical decisions regarding production, investment, and pricing.

Thus, solving the pricing decision of intermediate goods producers yields the standard price Phillips curve:

$$\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right) = \kappa_p \left(mc_t - \frac{1}{\mu_p} \right) + \mathbb{E}_t \left[\frac{1}{1+r_{t+1}^a} \frac{Y_{t+1}}{Y_t} \log\left(\frac{1+\pi_{t+1}}{1+\overline{\pi}} \right) \right],$$

where $\mu_p \equiv \epsilon_p/(\epsilon_p - 1)$ is a constant markup over marginal costs charged by intermediate

altering the model's structure, we assume the union's wage adjustment cost is near zero (or equivalently, that κ_w is very large). This approach closely replicates the baseline output and inflation dynamics.

[4]Investment can also be expressed as the sum of investment without adjustment costs, $k_{j,t} + (1-\delta)k_{j,t-1}$, plus an adjustment cost term $\phi(k_{j,t}/k_{j,t-1})k_{j,t-1} \equiv \frac{1}{2\delta\epsilon_I}(k_{j,t}/k_{j,t-1} - 1)^2 k_{j,t-1}$.

goods producers. The marginal cost can be expressed as a function of aggregate variables. It is given by $mc_t = \frac{W_t/P_t}{(1-\alpha)Y_t/N_t}$, that is, the ratio of the real wage and the marginal product of labor. See Appendix B for details.

Due to monopolistic competition, intermediate goods producers earn profits. We denote aggregate profits by Π_t. These profits are taxed at a constant rate τ_d and the proceeds are paid to the treasury. Thus, the after-tax aggregate dividends distributed to investors by intermediate goods producers are given by:

$$d_t = (1-\tau_d)\Pi_t = (1-\tau_d)\left(Y_t - \frac{W_t}{P_t}N_t - I_t - \xi(P_t, P_{t-1})\right).$$

where the gross aggregate investment $I_t = \zeta(K_t/K_{t-1})K_{t-1} = K_t - (1-\delta)K_{t-1} + \phi(K_t/K_{t-1})K_{t-1}$ includes the adjustment cost of capital.

Monetary policy. The central bank sets the nominal interest rate, i_t, on bonds according to a standard monetary policy rule that responds to inflation and is subject to a ZLB:

$$i_t = \max\left\{(1+i_{t-1})^{\rho_i}\left[(1+\overline{r})(1+\overline{\pi})\left(\frac{1+\pi_t}{1+\overline{\pi}}\right)^{\phi_\pi}\right]^{1-\rho_i} - 1,\ 0\right\}, \tag{2}$$

where $\rho_i \in [0,1)$ is a smoothing parameter, $\phi_\pi \geq 1$ determines the strength of the reaction to deviations of inflation, and $\overline{\pi}$ is the inflation target. We give the parameter $\overline{r}$ the name of policy rule intercept.

Fiscal policy. The treasury finances real government consumption, G_t, and real lump-sum transfers to households, T_t, using revenue from taxes on labor paid by households and taxes on profits paid by firms. A negative transfer T_t is equivalent to a lump-sum tax. To finance deficits, the treasury issues long-term nominal debt, which households hold in their liquid accounts.

Government consumption and transfers follow an exogenous process. In contrast, tax revenue is endogenous, as it consists of receipts from taxes on labor and profits, which depend on other endogenous variables:

$$\mathcal{T}_t = \int_0^1 \tau_n \frac{W_t}{P_t} z_{i,t} n_{i,t} di + \tau_d \Pi_t.$$

Debt is nominal, and its long-term nature is modeled as in Woodford (2001). Long-term bonds B pay a nominal dividend of 1 in the first period, δ_B in the second period, δ_B^2 in the third period, and so on, with $0 \leq \delta_B < 1$.[5] In the special case where $\delta_B = 0$, long-term

[5]The flow of payments of a bond that starts paying today equals one plus δ times the flow of payments of a bond that starts paying tomorrow because $1 + \delta + \delta^2 + \cdots = 1 + \delta(1 + \delta + \delta^2 + \dots)$. Hence, there is a simple recursive representation of the debt accumulation equation.

debt effectively becomes one-period debt. The relative price of debt at date t in terms of consumption is denoted by Q_t.

The equation for government debt accumulation is:

$$B_t = (1 + r_t^b)B_{t-1} + (G_t + T_t - \mathcal{T}_t),$$

where B_t is the real market value of government debt.[6] The face value of government debt in real terms at each date t can be obtained as the ratio B_t/Q_t. The ex-post real return on bonds in the above equation is defined by:

$$1 + r_t^b \equiv \frac{(1 + \delta_B Q_t)}{Q_{t-1}} \frac{P_{t-1}}{P_t}.$$

We define the short-term ex-ante nominal interest rate, i_t, as the expected nominal return of holding a bond for one period. It is determined by:

$$1 + i_t = \frac{1 + \delta_B \mathbb{E}_t Q_{t+1}}{Q_t}.$$

The ex-ante real return on government bonds depends on the expected nominal interest rate and expected inflation, and is given by:

$$\mathbb{E}_t[1 + r_{t+1}^b] = \mathbb{E}_t \left[\frac{1 + i_t}{1 + \pi_{t+1}} \right].$$

The ex-post real return on bonds may deviate from the ex-ante real return due to unexpected shocks. This discrepancy arises from two factors: (i) a sudden change in the price level at time t, and (ii) adjustments in Q_t, leading to a gap between the expected and realized ex-post nominal return.

Illiquid assets. Households invest their illiquid assets in a mutual fund, which holds equity in intermediate producers. Let p_t denote the real value of equity. Since the mutual fund is fully invested in equity, its expected real return depends solely on dividends and equity price fluctuations.

We assume the mutual fund operates in a competitive market and therefore does not earn any profits. Consequently, the return on the illiquid asset equals the expected return on

[6] As shown in Appendix D, the real market value is the product of the bond price in terms of consumption, Q_t, and the real stock of long-term debt at the end of period t. Denoting the nominal stock of bonds as $B_t^{\$}$, we have $B_t \equiv Q_t(B_t^{\$}/P_t)$.

equity:

$$\mathbb{E}_t[1 + r^a_{t+1}] = \frac{\mathbb{E}_t[d_{t+1} + p_{t+1}]}{p_t}.$$

Aggregation and market clearing. In equilibrium, the labor market, liquid asset market, illiquid asset market, and goods markets clear:

$$\begin{aligned}
N_t &= \int_0^1 z_{i,t} n_{i,t} di, \\
B_t &= \int_0^1 b_{i,t+1} di, \\
p_t &= \int_0^1 a_{i,t+1} di, \\
C_t &= \int_0^1 c_{i,t} di,
\end{aligned}$$

and the aggregate resource constraint holds: $G_t + C_t + I_t + \xi_t + \Phi_t = Y_t$.

3 Calibration and computation

We calibrate the model at a quarterly frequency, using standard parameter values from the literature and aligning them with U.S. economic data. Table 1 summarizes the calibration.

Preferences. Following Kaplan et al. (2018), we assume log utility over consumption, $u(c) = \log(c)$, and set the annual discount rate to 5.1%, implying a quarterly discount factor of $\beta = 0.988$. Disutility from labor follows a function with a constant Frisch elasticity: $v(n) = \nu_\varphi n^{1+\frac{1}{\varphi}}/(1 + \frac{1}{\varphi})$. We adopt a unitary Frisch elasticity, φ, consistent with Kaplan et al. (2018). The preference shifter, $\nu_\varphi = 0.612$, is calibrated internally, as detailed below.

Income process and borrowing limit. We adopt the persistence and standard deviation of income shocks from Auclert et al. (2021a). The income process has a persistence of 0.966 and an innovation standard deviation of 0.92. We approximate this process using a Markov chain with three discrete states, computed following Rouwenhorst (1995).

Production. We normalize steady-state quarterly output and hours worked to be equal, implying a total factor productivity parameter of $\Theta = 0.57$ (this normalization is inconsequential for the quantitative results). The elasticity between labor tasks is set to $\epsilon_w = 10$, a standard value in the literature (e.g., Wolf, 2021), which yields a wage markup of $\mu_w = \epsilon_w/(\epsilon_w - 1) = 1.1$. The wage Phillips curve slope is $\kappa_w = 0.1$, following Aggarwal et al. (2023). The elasticity of substitution between varieties is $\epsilon_p = 10$, implying a price markup of $\mu_p = \epsilon_p/(\epsilon_p - 1) = 1.1$. The price Phillips curve slope is set to $\kappa_p = 0.1$, consistent with

Table 1: Calibration parameters, Baseline model

	Parameter	Value	Target/Sources
	Preferences		
σ	CRRA	1	Kaplan et al. (2018)
β	Quarterly discount factor	0.988	Kaplan et al. (2018)
φ	Frisch elasticity of labor supply	1	Kaplan et al. (2018)
ν_φ	Disutility of labor parameter	0.612	Internally calibrated
	Income process		
ρ_e	Persistence income process (quarterly)	0.966	Auclert et al. (2021a)
σ_e	Std. dev. idiosyncratic shock (quarterly)	0.92	Auclert et al. (2021a)
	Production		
Θ	Constant level of TFP	0.57	$N_{ss} = Y_{ss}$
κ_w	Slope of the wage Phillips curve	0.1	Standard
κ_p	Slope of the price Phillips curve	0.1	Standard
μ_w	Wage markup	1.1	Standard
μ_p	Price markup	1.1	Standard
α	Capital share	0.27	$K_{ss} = 2.25Y_{ss}$
K_{ss}	Capital stock	$2.25Y_{ss}$	Standard
δ	Quarterly depreciation rate	0.02	Standard
ϵ_I	Quadratic adjustment cost parameter	4	Auclert et al. (2021a)
	Fiscal policy		
r^a_{ss}	Illiquid real interest rate (annual)	0.032	Internally calibrated
r^b_{ss}	Liquid real interest rate (annual)	0.01	Baseline case
B	Real government debt	2.8	Debt-to-GDP 70%
G	Real government consumption	0.2	Spending-to-GDP 20%
τ_n	Tax rate on labor income	0.3	Kaplan et al. (2018)
T	Real net transfers	0.065	B constant in DSS
τ_d	Tax rate on profits	0.465	$A_{ss} + B_{ss} = 3.5Y_{ss}$
δ_B	Coupon rate of long bonds	0.95	Duration in Doepke and Schneider (2006)
	Monetary policy		
ϕ_π	Taylor rule coefficient	1.25	Standard
$\overline{\pi}$	Inflation target (annual)	0.02	Standard
ρ_i	interest rate smoothing	0.8	Clarida et al. (1999)
	Adjustment cost function		
χ_0	adjustment cost parameters	3.78	Internally calibrated
χ_1	adjustment cost parameters	37.43	Internally calibrated

recent evidence of a flat Phillips curve (e.g., Hazell et al., 2022).

Fiscal policy. We analyze steady states with varying levels of public debt. As a benchmark, we set public debt at 70% of annual GDP, with an annualized real interest rate on debt (the liquid asset) of $r^b_{ss} = 1\%$.

We calibrate steady-state government consumption to 20% of GDP, aligning with U.S. data and common values in the literature (e.g., Auclert et al., 2024; Bayer et al., 2023). The labor tax rate is set at 0.3, a standard choice (e.g., Kaplan et al., 2018; Bayer et al., 2023). The profit tax is calibrated to 0.465, ensuring total wealth (the sum of liquid and illiquid assets) equals 350% of GDP. Net transfers are set to balance the budget in the DSS, covering government spending, transfers, and interest payments on public debt. This implies that transfers amount to 6.5% of GDP. We set $\delta = 0.95$ so that the steady-state bond duration is 18 quarters (4.5 years), consistent with U.S. asset and liability durations estimated by Doepke and Schneider (2006).[7]

Monetary policy. We parameterize the interest rate rule using standard values from the literature. Following Clarida et al. (1999), we set the interest rate smoothing parameter $\rho_i = 0.8$. The Taylor rule coefficient on inflation, $\phi_\pi = 1.25$, follows Kaplan et al. (2018).[8]

Adjustment costs. Adjusting the stock of illiquid assets incurs a real cost:

$$\Phi_t(a_{i,t}, a_{i,t-1}) \equiv \frac{\chi_1}{2}\left(\frac{a_{i,t} - (1 + r_t^a)a_{i,t-1}}{\tilde{a}}\right)^2 \tilde{a},$$
$$\tilde{a} = \max\{(1 + r_t^a)a_{i,t-1}, \chi_0\}.$$

This adjustment cost function is a quadratic discrete-time adaptation of the cost introduced by Alves et al. (2020) and later used by Auclert et al. (2021a) and Faia et al. (2022). The parameters χ_0 and χ_1 are calibrated internally, as explained below.

Internally calibrated parameters. We calibrate four parameters internally: χ_0 and χ_1 in the adjustment cost function for reallocating funds between liquid and illiquid accounts, the disutility of labor ν_φ, and the steady-state real interest rate on illiquid assets r_{ss}^a.

The values of χ_0, χ_1, and r_{ss}^a are chosen to match three targets. First, we normalize the steady-state liquid interest rate r_{ss}^b to 1%. Second, we target a steady-state illiquid asset level of 2.8 times output, computed as total assets minus public debt.[9] Third, we match the U.S. hand-to-mouth household share, estimated at 41% by Kaplan and Violante (2022) using 2019 SCF data. Given these values, we calibrate ν_φ to normalize steady-state output to one.

The resulting values are presented in Table 1. The adjustment cost parameters are $\chi_0 = 3.78$ and $\chi_1 = 37.43$. The disutility of labor is calibrated to $\nu_\varphi = 0.612$, and the steady-state annual real interest rate on illiquid assets is 3.17%. The complete calibration

[7] The duration is given by $D_{ss} = (1 + i_{ss})/(1 + i_{ss} - \delta)$.

[8] As one referee pointed out to us, the value of ϕ_π is quantitatively important: the strength of the inflation response in the Taylor rule will influence the size and persistence of deviations of inflation from target. For this reason, we set $\phi_\pi = 1.25$, a value that lies well within the range of consensus estimates in the literature.

[9] This is consistent with Kaplan et al. (2018). Setting total assets to 3.5 times annual output aligns with Auclert et al. (2021a) and Bayer et al. (2023).

yields an annual marginal propensity to consume (MPC) of 0.396, which —although not directly targeted— aligns well with empirical estimates for the United States, as reviewed by Kaplan and Violante (2022).

Computation. Given our objective of analyzing the effects of unanticipated long-term fiscal policy changes within our HANK model, we adopt the sequence-space method to compute impulse responses to one-time, unanticipated aggregate shocks. Specifically, we utilize the toolkit developed by Auclert et al. (2021a) for both solving the model and decomposing impulse responses.

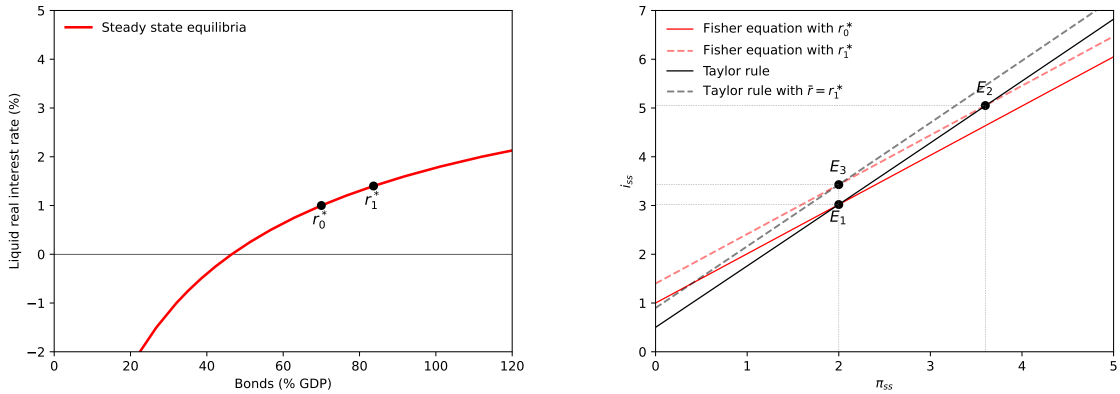
4 Monetary-fiscal interactions in the long run

In this section, we examine the long-run effects of government debt on interest rates and inflation. Section 4.1 explores how government debt influences the natural rate and constrains monetary policy. If the central bank does not adjust its policy rule to accommodate the higher natural rate induced by increased debt, steady-state inflation will rise. Additionally, we show that the ZLB imposes a minimum debt threshold below which the central bank cannot achieve its inflation target. In Section 4.2, starting from the deterministic steady state (DSS) of our baseline calibration in Section 3, we quantify these effects by comparing steady states at different debt levels. Our model produces a semi-elasticity of real interest rates with respect to government debt that aligns closely with both our empirical estimates and the findings of Rachel and Summers (2019) and Bayer et al. (2023).

4.1 The natural interest rate and long-run inflation

We begin by analyzing the DSS of the model. In the DSS, aggregate shocks are absent, but households continue to experience idiosyncratic shocks. We denote all steady-state variables with the subscript "ss," except for the steady-state liquid real interest rate. Instead of r^b_{ss}, we use the more conventional notation r^*, as this variable corresponds to what is commonly referred to as the long-run natural rate. Additionally, we retain the time subscript t for variables related to household choices, as households still face idiosyncratic shocks.

Panel (a) of Figure 1 shows the real interest rate on liquid assets across steady states, varying the market value of government debt. The point labeled r^*_0 marks the baseline steady state in our calibrated model, where the debt-to-GDP ratio is 70% and the real interest rate on liquid assets is 1%. Unlike in standard representative agent models, where the interest rate is set by the household discount factor, in heterogeneous agent models, r^* is determined by the interaction of bond demand and supply.



(a) Determination of r^* (b) Steady-state Taylor rule and Fisher equation

Note: Bonds (%GDP) denotes the real market value of government debt relative to annual GDP, expressed in percentage points.

Figure 1: Determination of the real interest rate r^* in steady state and monetary policy options

Kaplan et al. (2023) show how households' precautionary saving motives create an upward-sloping demand curve for government bonds, while the government's budget constraint sets bond supply.[10] These steady-state constraints link real debt to real interest rates. We denote the equilibrium function connecting the debt stock to the natural rate as $r^*(B_{ss})$. Building on their results on existence and uniqueness, we examine the implications of debt changes for monetary policy.

Long-term inflation. We approximate the monetary policy rule (2) in the DSS by $i_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}$. Combining this rule with the long-run Fisher equation, $i_{ss} = r^* + \pi_{ss}$, yields:

$$r^* + \pi_{ss} \approx \max\{\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), 0\}. \tag{3}$$

Equation (3) has two solutions (Benhabib et al., 2002). The first, a *non-binding ZLB* scenario, arises when $\overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}) > 0$. In this case:

$$r^* + \pi_{ss} \approx \overline{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}), \tag{4}$$

which simplifies to:

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \overline{r}}{\phi_\pi - 1}. \tag{5}$$

That is, steady-state inflation equals the central bank's target plus a term reflecting the gap between the Taylor rule intercept and the steady-state natural rate. To keep long-run

[10]This mechanism builds on Aiyagari (1994), who showed how uninsured idiosyncratic risk leads to an upward-sloping asset demand curve, and Aiyagari and McGrattan (1998), who studied optimal government debt in incomplete market economies with precautionary saving.

inflation at target, the central bank must set the Taylor rule intercept equal to the natural rate— the standard New Keynesian prescription.

The second solution, a *binding ZLB* scenario, occurs when $\overline{r}+\overline{\pi}+\phi_\pi(\pi_{ss}-\overline{\pi}) \leq 0$, making the right-hand side zero. In this case, the nominal interest rate is zero and:

$$\pi_{ss} = -r^*. \tag{6}$$

Higher inflation if the central bank does not react. Figure 1 illustrates how changes in government debt affect steady-state inflation. Panel (a) compares two debt levels, r_1^* and r_0^*, where higher debt raises the natural interest rate, so $r_1^* > r_0^*$. Panel (b) shows the interaction between the Taylor rule and the Fisher equation, both evaluated at steady-state values.[11]

A higher natural rate shifts the Fisher equation upward (from the solid to the dashed red line), as a higher real rate implies a higher nominal rate for any given steady-state inflation. The size of this shift is $r_1^* - r_0^* > 0$. If the central bank leaves its Taylor rule unchanged (solid black line), the economy moves from point E_1 to E_2, resulting in higher steady-state inflation.

Alternatively, the central bank can adjust the Taylor rule intercept. To keep inflation at target, it must shift the Taylor rule curve (to the dashed black line) by the same amount as the Fisher equation. This adjustment delivers a new steady state at E_3, with the original 2% inflation target and a higher natural rate, r_1^*.

The minimum debt level compatible with an inflation target. We now consider the case in which the central bank adjusts its monetary policy rule in response to changes in the natural interest rate. As discussed above, when the ZLB does not bind, the central bank can set the intercept of the Taylor rule equal to the new natural rate to keep inflation on target. That is, $\overline{r} = r^*(B_{ss})$, which ensures $\pi_{ss}(B_{ss}) = \overline{\pi}$. This outcome can always be achieved if the natural interest rate is sufficiently high. However, when the natural rate is too low, the ZLB may become a binding constraint.

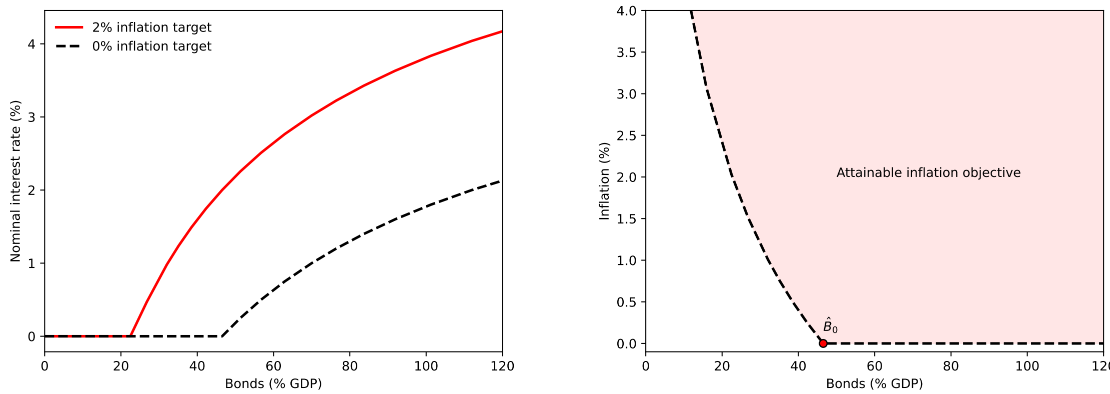
There exists a threshold level of debt, $\hat{B}_{\overline{\pi}}$, at which the nominal interest rate hits zero:

$$r^*(\hat{B}_{\overline{\pi}}) + \overline{\pi} = 0.$$

For debt levels below this threshold, $B_{ss} < \hat{B}_{\overline{\pi}}$, the central bank cannot meet its inflation target, as nominal interest rates cannot fall below zero. In this case, the ZLB binds, $i_{ss} = 0$, and steady-state inflation is given by $\pi_{ss} = -r^*(B_{ss})$. We refer to $\hat{B}_{\overline{\pi}}$ as the *minimum debt level compatible with the inflation target* $\overline{\pi}$, since the central bank can only achieve its target in the DSS if $B_{ss} \geq \hat{B}_{\overline{\pi}}$.

We illustrate this result in Figure 2. Panel (a) shows the nominal interest rate for two

[11]We thank Daisuke Ikeda for suggesting such a figure to illustrate the link between a higher natural rate and long-run inflation.



(a) Steady-state nominal interest rate (b) Steady-state inflation

Figure 2: Steady-state nominal interest rate and inflation for different inflation targets
Note: The graphs use the baseline calibration but allow B_{ss} to vary. In Panel (a), the dashed line shows results for a 0% inflation target, and the solid red line for 2%. The shaded area in Panel (b) represents the set of non-negative inflation targets that can be achieved in equilibrium at various debt levels. The debt level $\hat{B}_0$ at the kink is the lowest level consistent with an inflation target of zero.

inflation targets: 2% (solid red) and 0% (dashed black). For a 2% target, the minimum debt level $\hat{B}_{2\%}$ is 22.7% of GDP. For a 0% target, it rises to 46.8%.[12] Panel (b) plots a frontier (dashed line) showing combinations of steady-state inflation and minimum debt levels for positive inflation targets. The shaded area above the frontier represents the set of (non-negative) inflation targets that can be achieved in equilibrium at different debt levels. The value $\hat{B}_0$ shown in the figure is the lowest debt level consistent with an inflation target of zero.

4.2 Long-run quantitative effects of the fiscal expansion

Next, we compare the long-run effects of a fiscal expansion by analyzing two steady states: a baseline with government debt at 70% of GDP, and an alternative with debt at 80% of initial GDP—that is, 10 pp higher.

In the new DSS, the higher debt level must be financed through higher taxes, lower government consumption, lower transfers, or some combination of these. Figure 10 in the appendix shows the resulting values of r^* under each option. The effects on r^* are quantitatively similar across cases. For this reason, we focus on the scenario in which government consumption adjusts to balance the budget in the new DSS.[13]

[12]In these simulations, government consumption adjusts to ensure that the treasury satisfies its budget constraint.

[13]While steady-state outcomes are similar across funding methods, the transition paths differ substantially.

	Initial steady state	New steady state	Difference
Bonds (% GDP)	70.000	80.000	10.000
Illiquid real interest rate (%)	3.170	3.197	0.027
Liquid real interest rate (%)	1.000	1.307	0.307
Liquid nominal interest rate (%)	3.020	3.333	0.313
Output (% GDP)	100.000	99.742	-0.258
Investment (% GDP)	18.000	17.911	-0.089
Consumption (% GDP)	60.292	60.549	0.257
Govt. consumption (% GDP)	20.000	19.606	-0.394
Portfolio costs (% GDP)	1.708	1.676	-0.032
Total Tax revenue (% GDP)	27.125	27.075	-0.051
Primary surplus (% GDP)	0.697	1.043	0.346

Table 2: DSS in the baseline HANK model

Note: All variables expressed as a percentage of GDP refer to the level of GDP in the initial steady state. The nominal interest rate in the initial DSS is 3.02%, not 3.00%, because it satisfies the non-linear Fisher equation: $i_{ss} = 1.01 \times 1.02 - 1 \approx 3.02\%$.

Table 2 reports the values of key variables that differ between the initial and final DSS. As shown in the third row, the natural interest rate r^* increases by 31 basis points (bps) in the new DSS. To balance the budget, government consumption must fall. This reduction reflects not only the additional debt servicing required at the initial interest rate, but also the higher interest rate itself, leading to a total decrease in G_{ss} of 39 bps.

As shown earlier, maintaining price stability requires the central bank to raise the intercept in its monetary policy rule, $\bar{r}$, by the same amount as the increase in r^*. Otherwise, steady-state inflation would rise to 3.2 pp—exceeding the inflation target by 1.2 pp—as implied by equation (5) under our calibration of the Taylor rule with $\phi_\pi = 1.25$.

The increase in steady-state debt, B_{ss}, affects several real variables. In the new DSS, households receive higher interest payments and experience a positive wealth effect, leading to a 26 bps increase in consumption. In line with this rise in consumption, households reduce their labor supply, resulting in a 26 bps decline in output. This channel —where changes in government spending affect labor supply through wealth effects— has been recognized since Christiano and Eichenbaum (1992).

Semi-elasticity. Our model produces a semi-elasticity of the real return on bonds with respect to government debt that is consistent with the empirical evidence. Following Bayer et al. (2023), we define the semi-elasticity of the steady-state real return on bonds with respect

We therefore consider taxes, transfers, and government consumption separately when analyzing the short-run effects of a fiscal expansion.

to the stock of government debt as:

$$\eta_B \equiv \frac{\partial r^b_{ss}(B_{ss})}{\partial \ln B_{ss}} \approx \frac{\Delta r^b_{ss}}{\Delta \ln B_{ss}}.$$

Comparing the two steady states under consideration, a fiscal expansion that increases government debt by 10 pp of GDP (from 70% to 80%) implies a semi-elasticity:

$$\eta_B \equiv \frac{\Delta r^b_{ss}}{\Delta \ln B_{ss}} = \frac{0.307}{\ln(80/70)} \approx 2.3\%.$$

This result is in line with the estimate reported by Bayer et al. (2023), who find $\eta_B = 2.5\%$.[14] As noted by Bayer et al. (2023), this magnitude of semi-elasticity implies that debt financing becomes meaningfully more expensive for the government.

Empirical evidence on the response of r^*. We conduct an empirical analysis to quantify how the natural interest rate responds to changes in government debt. Specifically, we estimate the response of r^* to an increase in the debt-to-GDP ratio. As our measure of the natural rate, we use the estimates from Lubik and Matthes (2015), based on a time-varying parameter vector autoregressive (TVP-VAR) model.[15]

To estimate the impulse response function (IRF), we use local projections (LP; Jordà, 2005). The estimating equation is specified as:

$$r^*_{t+h} = \alpha_h + \beta_h \left(\tfrac{B}{Y}\right)_{t-1} + \mathbf{X}_t \boldsymbol{\gamma}_h + u_{t+h},$$

for $h = 0, \ldots, H$, where r^*_{t+h} is the value of the natural rate h periods ahead, and $\left(\tfrac{B}{Y}\right)_{t-1}$ denotes the lagged debt-to-GDP ratio. The coefficient β_h traces the IRF of r^* to a debt shock. The vector $\mathbf{X}_t$ includes additional controls: lags of r^*, the debt-to-GDP ratio, the federal funds rate, inflation, and the unemployment rate. The sample covers the period from 1967:Q1 to 2023:Q2.

Figure 3 displays the IRF of r^* to a 1 percentage point increase in the public debt-to-GDP ratio. The average response of r^* across horizons is 3.7 basis points per 1 pp increase in debt. This aligns closely with the summary provided by Rachel and Summers (2019, Table 2), who report an average effect of 3.5 basis points based on prior empirical studies.

Scaling our estimate to a 10 pp fiscal expansion implies an increase in r^* of 37 bps –remarkably close to the 31 bps rise generated by our model for the same debt increase

[14] Bayer et al. (2023) compute η_B based on a simulation that raises public debt by 6 pp of GDP. However, since the semi-elasticity is defined as a ratio, it is unit-free and this difference should matter only in the presence of strong non-linearities —which do not appear to be present in this context.

[15] A continuously updated dataset is available at `https://www.richmondfed.org/research/national_economy/natural_rate_interest`.

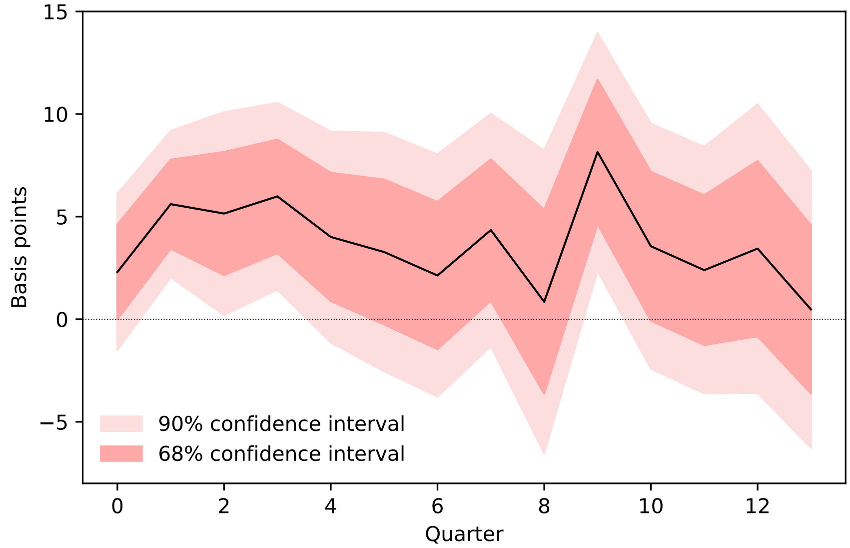


Figure 3: IRF of r^* to a 1 percentage point increase in the public debt-to-GDP ratio, LP
Note: We estimate a local projection using the specification $r^*_{t+h} = \alpha_h + \beta_h (B/Y)_{t-1} + \mathbf{X}_t \boldsymbol{\gamma}_h + u_{t+h}$, and plot the regression coefficient β_h (solid line) associated with the lagged public debt-to-GDP ratio, $(B/Y)_{t-1}$. The control vector $\mathbf{X}_t$ includes four lags of the change in r^*, three additional lags of the debt-to-GDP ratio, and four lags of the federal funds rate, the GDP deflator, and the unemployment rate. The shaded regions show the 68% and 90% confidence intervals, computed using Eicker–Huber–White standard errors, following Montiel Olea and Plagborg-Møller (2021).

(Table 2). We conclude that our model delivers a response of r^* that falls well within the empirically estimated range. In Appendix E, we present complementary results based on a structural vector autoregression (SVAR), which yields similar estimates.

5 Monetary-fiscal interactions in the short run

5.1 Short-run quantitative effects of a fiscal expansion

We now examine the short-run dynamics as the economy transitions from the initial steady state to the new steady state described in the previous section, in which government debt is permanently higher. To accumulate this additional debt, the fiscal authority must temporarily increase government consumption or transfers, reduce taxes, or implement some combination of these three options.

In all three cases, the economy eventually converges to a new steady state with higher debt, though the transition paths differ depending on the fiscal instrument used. These differences highlight the importance of considering the specific policy tool when assessing the effects of debt-financed expansions.

We consider a common path for the stock of government debt in real terms that leads the economy from its initial to its new steady state. To construct this path, we first analyze a scenario in which the fiscal expansion is driven entirely by changes in government consumption. We then examine alternative paths that achieve equivalent debt accumulation through changes in taxes or net transfers.

A fiscal expansion driven by government consumption. In the first scenario, we implement a simple feedback rule that gradually increases government debt toward its new steady-state level. Specifically, we assume:

$$G_t = G_{ss} - \phi_G(B_{t-1} - B_{ss}), \tag{7}$$

where G_{ss} and B_{ss} denote the steady-state values of real government consumption and the real market value of government debt, respectively. The parameter $\phi_G \in (0, 1)$ governs the speed of adjustment when debt deviates from its steady-state level.[16] This rule is analyzed in Auclert et al. (2020, Section 5.3), and is closely related to the fiscal rules studied by Auclert and Rognlie (2018).[17]

In this scenario, the economy begins in the initial DSS. At $t = 0$, the treasury announces a new path for government consumption, $\{G_t\}_{t=0}^{\infty}$, that follows the rule in (7) and converges to the new steady state. The corresponding values of G_{ss} and B_{ss} are reported in Table 2. At the same time, the central bank permanently adjusts the intercept of the Taylor rule to match the new steady-state natural rate, r^*, also reported in Table 2. Agents fully internalize the announced path for government consumption and the change in monetary policy, and do not anticipate any further changes in the environment.

The solid red lines in Figure 4 illustrate the transition dynamics in this first experiment. The path of government consumption is shown in panel (b). It peaks at approximately 5 pp of initial GDP and then gradually declines over more than six years, eventually settling at a permanently lower level to service the higher debt burden.

The temporary increase in government consumption generates an output expansion of about 4 pp at its peak (panel e), driven primarily by the direct fiscal impulse. However, this expansion is accompanied by crowding out of private spending. Private consumption declines by roughly 1.5 pp (panel f), as households face higher real interest rates, while investment falls by about 0.4 pp (panel g). These effects reflect the medium-term rise in the real interest rate, shown in panel (i).

The composition of aggregate demand shifts significantly, as the expansion in government consumption more than offsets the contraction in private spending. This reallocation generates

[16] We set $\phi_G = 0.1$. With this choice, the rule in (7) converges to the target level of debt under our calibration and for the range of debt levels considered.

[17] Kaplan et al. (2023), by contrast, consider a fiscal rule in which the deficit adjusts directly, rather than government consumption.

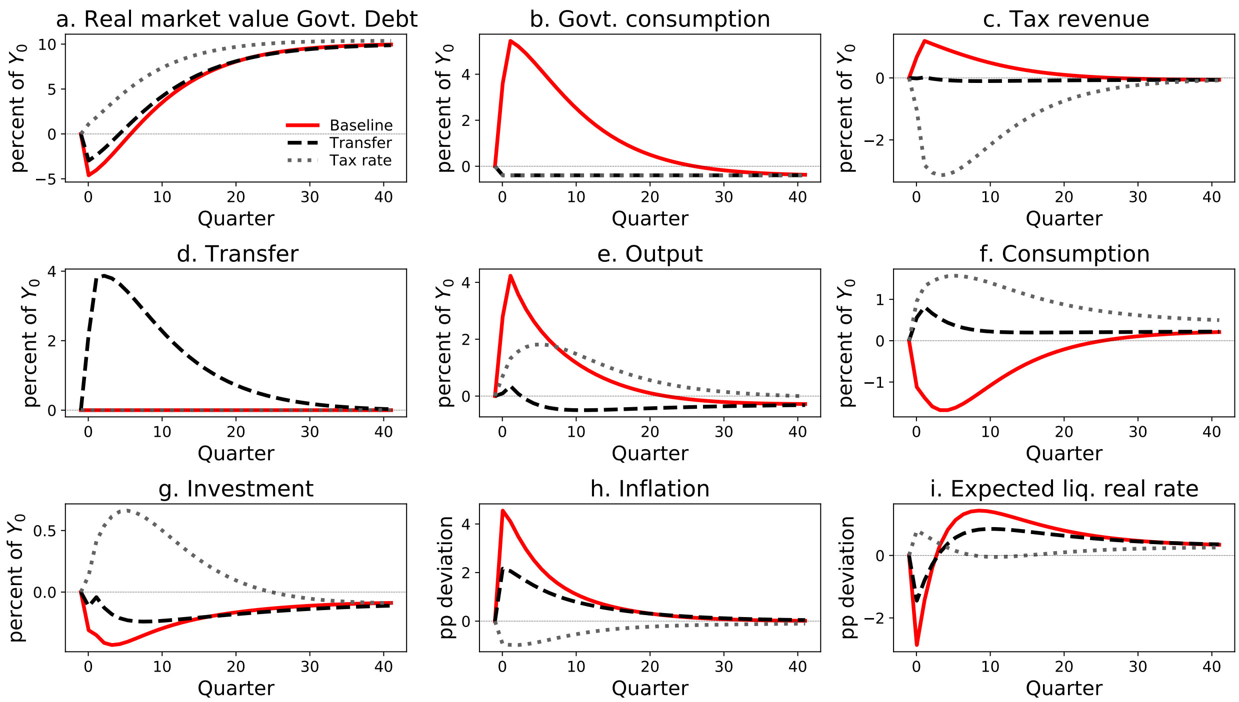


Figure 4: Comparison of different dynamics after a surprise debt-financed fiscal expansion

an inflationary spike of about 4 pp above the steady state (panel h), driven by both demand-side pressures from fiscal stimulus and supply-side constraints as the economy operates above potential. These short-run inflation dynamics highlight how fiscal expansions can generate price pressures even when private demand is partially crowded out.

To contain inflation, the central bank raises the nominal interest rate for an extended period. As a result, the real interest rate on liquid assets increases by about 1 pp in the medium term (panel i). The initial decline in the ex-ante real interest rate reflects the inertia built into the Taylor rule.

As shown in panel (a), the transition to the new DSS unfolds over approximately 40 quarters. In the short run, the real value of government debt declines due to an unexpected spike in inflation, which lowers the market value of outstanding nominal liabilities. As the short-run dynamics dissipate, the economy gradually converges to a DSS with higher government debt, lower government consumption, higher private consumption, and elevated real interest rates.

A fiscal expansion driven by changes in the tax rate. We now consider a second scenario that begins from the same initial DSS, converges to the same final DSS, and follows exactly the same path for the real stock of government debt. However, in this case, the fiscal adjustment is implemented entirely through changes in labor taxation, leading to different implications for inflation.

We fix government consumption at its new DSS level, i.e., $G_t = G_{ss}$ for all $t \geq 0$, and compute the sequence of labor tax rates $\{\tau_{n,t}\}_{t=0}^{\infty}$ that generates the same trajectory for real debt issuance. At time $t = 0$, the treasury announces this path of labor taxes and the permanent shift in government consumption to the value reported in Table 2. As in the previous experiment, the central bank adjusts the intercept in the Taylor rule to the new DSS value of r^* (also shown in Table 2). Agents take the new paths for government consumption, tax rates, and the monetary policy intercept as given, and do not expect any further changes to the environment.[18]

Compared to the case with a temporary increase in government consumption, the tax-based fiscal expansion produces markedly different short-run dynamics. These are illustrated by the gray dotted line in Figure 4. The tax reduction stimulates both consumption and investment (panels f and g). Despite the increase in private demand, inflation declines in the short run (panel h). This disinflation reflects the interaction between aggregate demand and supply.

On the supply side, the lower labor tax rate raises after-tax wages, which incentivizes greater labor supply and boosts investment (panel g). On the demand side, the tax cut increases disposable income, stimulating private consumption (panel f). However, the reduction in labor taxes dampens wage inflation by more than the increase in consumption-driven labor demand. This results in downward pressure on marginal costs and, consequently, on inflation (panel h). This disinflationary effect influences monetary policy: lower inflation leads to a decline in real interest rates over the medium term.

A fiscal expansion driven by changes in net transfers. In this third scenario, we proceed analogously to the previous case, but instead back out the sequence of net lump-sum transfers $\{T_t\}_{t=0}^{\infty}$ rather than labor tax rates. We compute this path period by period to ensure that the real stock of government debt follows the same trajectory as in the previous two scenarios. At time $t = 0$, the treasury announces the entire transfer sequence $\{T_t\}_{t=0}^{\infty}$.[19]

As before, the treasury simultaneously announces that government consumption G moves permanently to its new DSS level. The central bank adjusts the intercept of the Taylor rule to the new steady-state value of r^*. Agents take as given the announced path of transfers, the new level of government consumption, and the revised monetary policy intercept. No further changes to the environment are anticipated.

The transfer-based fiscal expansion is shown in Figure 4 with dashed black lines. While this scenario delivers a path for the real market value of debt similar to the baseline case

[18] Additional figures showing the dynamics of the real stock of government debt, the Taylor rule intercept, and the illiquid interest rate are provided in Appendix G.2, Figure 12. These confirm that while market values may differ, the path of real government debt is identical across scenarios.

[19] Tax revenues remain endogenous, as they are the product of the tax rate and output. While the tax rate remains constant, output fluctuates over time.

(panel a), it operates through distinct transmission channels.

In a standard representative agent model, debt-financed transfers are subject to a Ricardian effect, whereby households save the transfers in anticipation of future tax payments. In contrast, our heterogeneous agent framework with long-term debt generates meaningful real effects through two key mechanisms.

First, there is a wealth redistribution effect. The increase in inflation (panel h) reduces the real value of outstanding long-term bonds via the Fisher effect, transferring wealth from bondholders to recipients of transfers. This redistribution is non-neutral because households differ in their marginal propensities to consume due to incomplete markets and borrowing constraints.

Second, a precautionary savings channel is at work. The higher supply of government bonds improves households' ability to self-insure against idiosyncratic risk. This enhanced risk-sharing boosts consumption (panel f), even as output and investment decline (panels e and g). The fall in labor supply reflects an income effect from higher transfers, while the rise in real interest rates (panel i) discourages capital accumulation.

These dynamics highlight how heterogeneity and market incompleteness alter the transmission of fiscal policy relative to representative agent benchmarks.

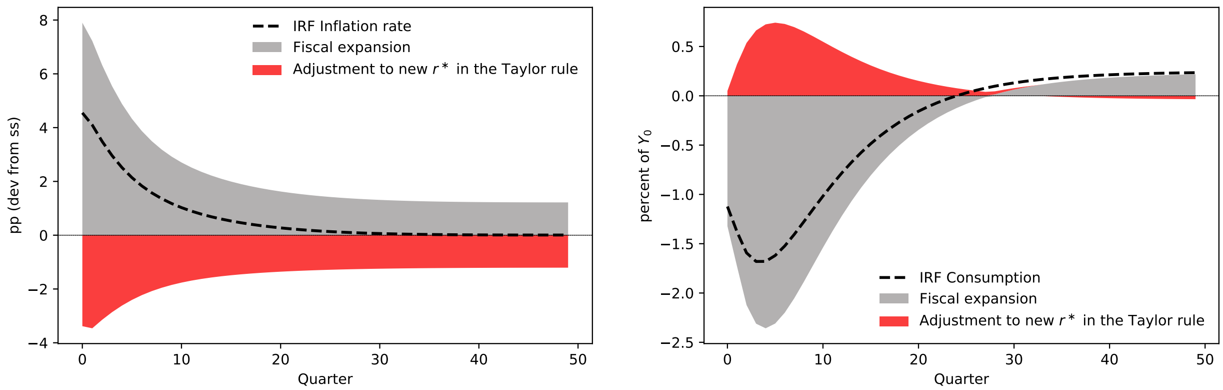
Short-term debt. In Appendix G.3, we compare our baseline model with an alternative specification in which government debt is entirely short term. As shown in Figure 13, the initial erosion of real government debt is more pronounced in the baseline model with long-term debt. This difference arises because long-term debt makes the market value of government liabilities more sensitive to changes in interest rates and inflation. As a result, the surprise inflation triggered by the fiscal expansion has a stronger impact on the real value of debt when its maturity is longer.

This Fisher effect amplifies the macroeconomic responses: with long-term debt, we observe larger movements in output, consumption, and inflation. The amplification occurs because the erosion in the real value of debt effectively redistributes resources from bondholders to the government, enabling a more forceful fiscal expansion.

5.2 The short-run impact of adjusting the Taylor rule

During the transition, both fiscal and monetary policy adjust. Fiscal policy moves temporarily to increase government debt, while monetary policy raises the intercept in the Taylor rule, $\bar{r}$, in response to the higher natural rate, r^*. To isolate the role of each policy change, we focus on the first scenario —where a temporary increase in government consumption drives the fiscal expansion — and decompose the impulse responses of inflation and consumption into their underlying components.

Following Auclert et al. (2021a), we apply a sequence-space representation to express the general equilibrium response of each variable as the sum of contributions from distinct policy channels. This decomposition allows us to identify how specific elements of the policy shift drive the aggregate responses.



(a) Inflation (b) Consumption

Figure 5: Decomposition of the IRFs of inflation and consumption

Figure 5 presents this decomposition for inflation (panel a) and consumption (panel b). For inflation, the gray area depicts the path that inflation would have followed in response to the fiscal expansion alone, assuming no change in the monetary policy rule. The red area captures the offsetting effect of raising the intercept of the Taylor rule to the new value of r^*. By adapting the intercept to the higher natural rate, the central bank helps contain inflation in both the short and long run. This stabilizing effect reaches approximately -1.2 pp, consistent with the long-run prediction from equation (5).

The IRF for consumption dynamics (panel b) highlights a key interaction between monetary and fiscal policy. Absent any monetary policy adjustment (gray area), the fiscal expansion generates a notable decline in consumption, driven by standard crowding-out effects. However, the adjustment in monetary policy (red area) partially offsets this decline. This effect operates through the "Fisher channel": by containing inflation more effectively, the monetary policy adjustment prevents the erosion of households' real wealth, thereby supporting consumption relative to a scenario in which the policy rule remains unchanged.

These results highlight a central trade-off in monetary policy, as fiscal expansion increases public debt and the natural rate. The central bank can adjust its policy rule to track the higher r^*, as in our baseline, which helps anchor inflation near the target but dampens the fiscal multiplier. Alternatively, it can maintain the original rule without adjusting the intercept, thereby allowing stronger fiscal transmission at the cost of persistently higher inflation and nominal interest rates, as the central bank reacts more aggressively to inflation deviations.

Figure 14 in Appendix G.4 illustrates this mechanism through a counterfactual analysis. In the absence of an intercept adjustment in the Taylor rule, inflation rises persistently, and the nominal interest rate increases in response. In the short run, both private consumption and investment decline more sharply. However, output increases relative to the baseline due to the larger expansion in government consumption enabled by higher inflation.

In essence, by anchoring long-run inflation expectations through the intercept adjustment, monetary policy constrains the government's fiscal space, thereby reducing the degree of crowding out in private consumption.

6 Alternative monetary policy rules

Our analysis so far has focused on a central bank that adjusts the intercept of its Taylor rule to accommodate changes in the natural rate resulting from fiscal policy. In practice, however, such adjustments may be difficult to implement, as they require timely and accurate estimates of r^*. This raises the question of whether alternative monetary policy rules can better address fiscal-driven fluctuations in the natural rate.

A particularly appealing alternative is a policy rule that does not rely on knowledge of the natural rate at all. One such rule, originally proposed by Orphanides and Williams (2002) and tracing back to early work by Phillips (1954), links changes in the nominal interest rate to deviations of inflation from its target. The rule takes the form:

$$\log(1+i_t) = \log(1+i_{t-1}) + \phi_\pi^{OW} \log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right), \tag{8}$$

where the nominal interest rate i_t adjusts gradually in response to deviations of inflation π_t from the target $\overline{\pi}$, without requiring explicit tracking of r^*.

The main advantage of rule (8) is that it does not require knowledge of the natural rate, r^*, as it does not enter the rule explicitly. As a result, the rule is automatically robust to fluctuations in the natural rate of interest, and it could be argued that it is a better empirical description of central banks' actual behavior. However, this feature may also be a limitation in settings where movements in r^* reflect valuable information about underlying economic conditions not fully captured by inflation dynamics.

For guidance on choosing the parameter ϕ_π^{OW} in rule (8), we follow Orphanides and Williams (2002) and set $\phi_\pi^{OW} = 0.35$. This corresponds to their "robust rule" specification, which performed well across a wide range of model environments in their analysis (see Table 5 in their paper).

Figure 6 compares the impulse response functions under three different monetary policy rules: our baseline Taylor rule (solid red line), the Orphanides-Williams rule (dashed black

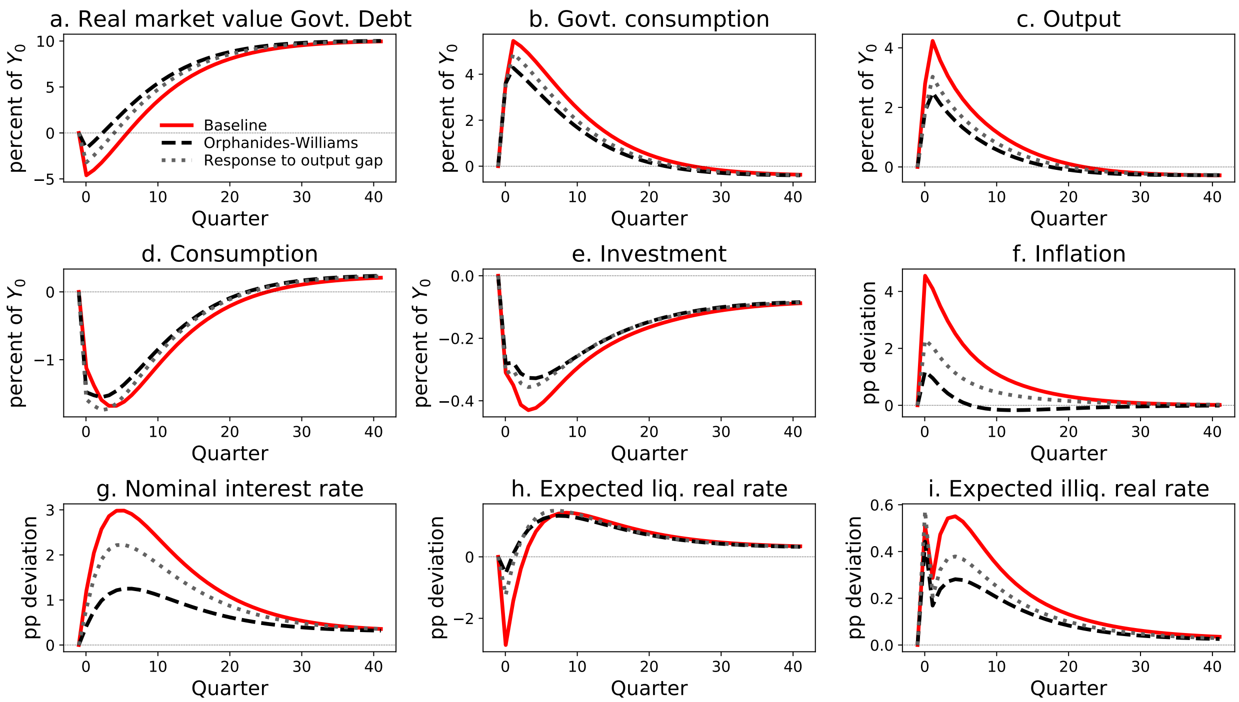


Figure 6: Comparison of dynamics after a surprise debt-financed fiscal expansion under different monetary rules

line), and a Taylor rule augmented with a response to the output gap (dotted gray line). In the latter case, the Taylor rule takes the form:

$$1 + i_t = (1 + i_{t-1})^{\rho_i} \left[(1 + \overline{r})\,(1 + \overline{\pi}) \left(\frac{1 + \pi_t}{1 + \overline{\pi}} \right)^{\phi_\pi} \left(\frac{Y_t}{Y_{ss}} \right)^{\phi_y} \right]^{1-\rho_i},$$

where the interest rate responds not only to inflation but also to the output gap. Following Orphanides and Wieland (1998), we set the output coefficient to $\phi_y = 0.1$.

All three simulations consider a debt-financed fiscal expansion in which the real market value of government debt rises by 10 pp of initial GDP. As in earlier experiments, the path of government consumption $\{G_t\}_{t=0}^{\infty}$ is determined by the fiscal rule in equation (7).

The Orphanides-Williams rule produces notably different dynamics compared to the baseline Taylor rule. The key distinction lies in how it affects nominal variables. Under this rule, inflation exhibits substantially less volatility (panel f), accompanied by a more gradual adjustment in the nominal interest rate (panel g). This smoother policy response also leads to distinct outcomes for real variables: consumption recovers earlier (panel d), and the decline in investment is milder (panel e). These improved stabilization outcomes can be attributed to the rule's implicit history dependence.

To better understand these results, we can express the Orphanides-Williams rule as:

$$\log(1+i_t) = \log(1+i_0) + \phi_\pi^{OW} \sum_{i=0}^{t-1} \log\left(\frac{1+\pi_{t-i}}{1+\overline{\pi}}\right),$$

where the initial nominal rate is given by $\log(1+i_0) = \log[(1+\bar{r})(1+\overline{\pi})]$, and $\bar{r}$ is the intercept of the Taylor rule prior to the fiscal announcement—that is, the natural rate in the initial DSS.

The implied ex-post real interest rate on liquid assets is then:

$$\log\left(1+r_t^b\right) = \log(1+\bar{r}) + (\phi_\pi^{OW} - 1)\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right) + \phi_\pi^{OW} \sum_{i=1}^{t-1} \log\left(\frac{1+\pi_{t-i}}{1+\overline{\pi}}\right),$$

while under the baseline Taylor rule, it takes the simpler form:

$$\log\left(1+r_t^b\right) = \log(1+r^*) + (\phi_\pi - 1)\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right),$$

where we have used the fact that the central bank updates the Taylor rule intercept to r^* in response to the fiscal shock.

The key to understanding these differences lies in how each rule processes information. The standard Taylor rule responds only to current inflation deviations, while the Orphanides-Williams rule effectively accumulates past deviations. This makes the real rate under the Orphanides-Williams rule a more persistent, slow-moving variable (panel h). Even without explicitly targeting the new natural rate, the central bank gradually "learns" it through the accumulation of inflation deviations.

Figure 6 shows that the Taylor rule augmented with an output gap response delivers intermediate levels of inflation and output volatility, but does not match the stabilization performance of the Orphanides-Williams rule. This suggests that history dependence –rather than the inclusion of additional target variables– may be central to robust monetary policy in the face of fiscal-driven shifts in the natural rate.[20]

These results highlight an important policy implication: when fiscal policy shifts the natural rate, monetary frameworks that do not rely on explicit estimation of r^* may deliver superior stabilization outcomes.

[20] Additional analysis in Appendix G.5 compares our baseline case, which includes interest rate smoothing, with a specification that omits it. Without smoothing, the economy exhibits larger fluctuations in both nominal and real variables. This comparison highlights how gradual policy adjustment through smoothing helps stabilize the response to fiscal expansion.

7 Anticipated effects

So far, we have considered a case in which the central bank updates its monetary policy rule immediately upon learning of a fiscal shock. We now examine how the timing of this update affects macroeconomic dynamics, including scenarios in which the central bank receives advance notice of an upcoming fiscal expansion.

The economy begins in the initial DSS. At time $t = 0$, the treasury announces that it will implement a debt-financed fiscal expansion starting in 12 quarters. Specifically, it commits to adopting the fiscal rule in (7), with a delayed activation. During the first 12 quarters, G_{ss} and B_{ss} remain at their initial steady-state values. Beginning in quarter $t = 12$, these parameters shift to their new DSS levels, as reported in Table 2.

7.1 Three scenarios

We consider three possible responses by the central bank. In the first case, the central bank responds immediately to the announcement of the future fiscal expansion, adjusting the intercept of the Taylor rule, $\bar{r}$, to the new value of r^* at time $t = 0$. In the second case, the central bank waits until the fiscal rule parameters change and adjusts $\bar{r}$ at $t = 12$. In the third scenario, the central bank further delays this adjustment and updates $\bar{r}$ only at $t = 24$. Agents have perfect foresight in all cases.

Figure 7 compares the short-run dynamics under these three monetary policy responses. We show the case of immediate adjustment upon announcement (forward-looking, solid red line), adjustment at the time of fiscal implementation (coincident, dashed black line), and delayed adjustment 12 quarters after implementation (dotted-dashed grey line).

In the forward-looking case, where the central bank adjusts the Taylor rule intercept immediately upon announcement, significant anticipation effects arise. Ahead of the fiscal change at $t = 12$, households increase savings in response to expected higher real interest rates, leading to a decline in consumption (panel e). This rise in savings reduces aggregate demand, generating deflationary pressure as firms lower prices, causing inflation to fall (panel g).

In line with this disinflation, the nominal interest rate initially declines (panel h), then rises sharply as the fiscal expansion nears. Real interest rates spike after the announcement due to lower inflation (panel i), but then gradually fall as inflation recovers. The temporary drop in real rates eases the fiscal burden, allowing for a short-term decline in public debt and an increase in government consumption (panels a and c) between the announcement and the start of the fiscal expansion.

Next, we examine the IRFs when the central bank maintains its original policy rule until the fiscal change takes effect at $t = 12$ (dashed black line). While dynamics converge across

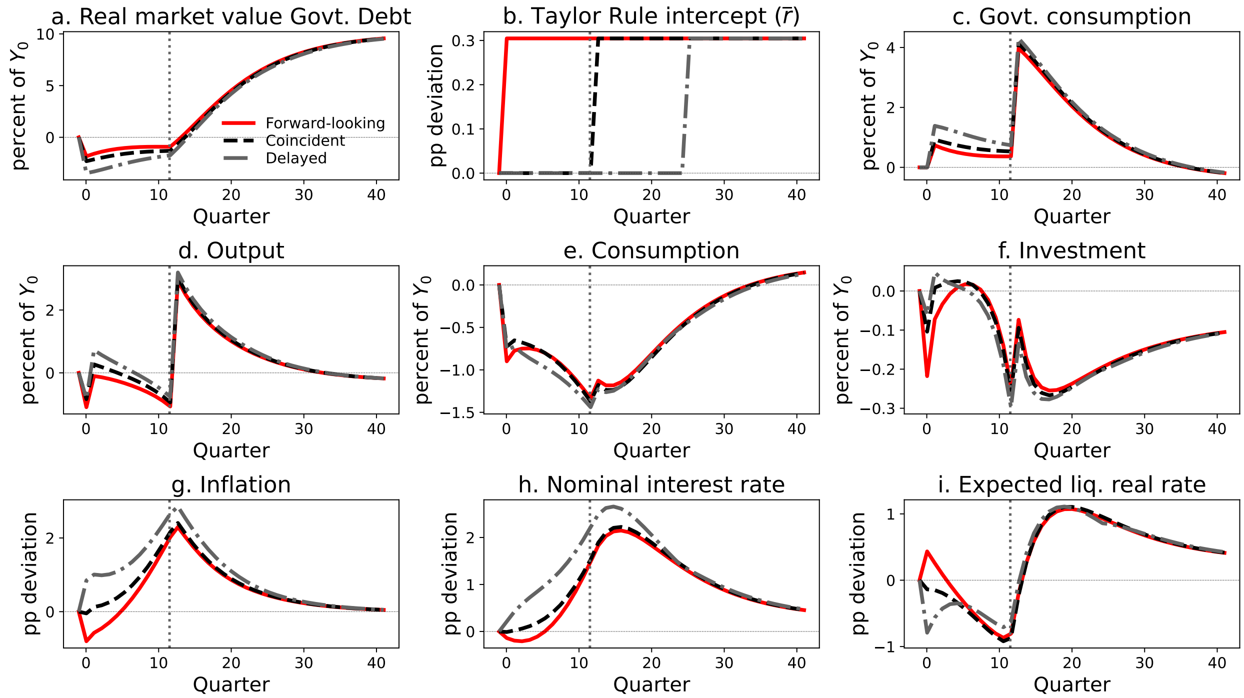


Figure 7: Dynamics of an anticipated debt-financed fiscal expansion

Note: Deviation from the initial DSS following a shock that announces the implementation of a new fiscal rule 12 quarters ahead. Solid red line: the central bank adjusts the Taylor rule intercept immediately upon announcement; dashed black line: the central bank adjusts at the time the fiscal rule is implemented ($t = 12$); dotted-dashed grey line: the central bank delays its response by an additional 12 quarters ($t = 24$).

regimes after the fiscal expansion begins, notable differences arise in the pre-implementation period ($t = 0 - 11$). Without anticipatory monetary policy, the economy experiences positive inflation, unlike the forward-looking case (panel g). As a result, the nominal interest rate rises initially, rather than falling as in the forward-looking scenario (panel h).

These nominal differences occur despite a lower Taylor rule intercept (panel b), highlighting the role of policy timing in shaping inflation expectations. In contrast, the paths of real variables remain broadly similar to those in the forward-looking case, as shown in panels (d), (e), and (f).

In the case of a delayed response (dot-dashed gray line), where the monetary policy adjustment occurs 12 quarters after the fiscal change ($t = 24$), nominal effects are more pronounced. This scenario results in higher inflation and requires a stronger nominal interest rate response during the transition, with the policy rate reaching a higher peak (panels g and h). Once the Taylor rule is updated at $t = 24$, dynamics converge toward those in the earlier scenarios. Despite these larger nominal differences, the paths of real variables (panels d through f) and the real interest rate (panel i) remain broadly similar to the other two cases.

Three insights emerge from this analysis. First, delaying the intercept adjustment in the Taylor rule results in progressively higher inflation during the transition, with the delayed case exhibiting the largest impact. Second, all scenarios display similar dynamics after the fiscal policy takes effect, highlighting the importance of eventual policy alignment. Third, while the timing of monetary adjustment significantly alters the paths of nominal variables (inflation and the nominal interest rate, panels g and h), real variables (output, consumption, and investment, panels d–f) respond similarly across all timing cases.

These results underscore the importance of monetary policy timing for price stability while also highlighting the robustness of real activity to different adjustment schedules. They illustrate how anticipation effects and coordination between fiscal and monetary policy shape the smoothness of macroeconomic transitions.

7.2 Welfare analysis of monetary policy timing

Next, we analyze the welfare gains and losses associated with different timings of monetary policy responses to changes in r^*. We compute welfare in consumption-equivalent terms along the transition paths for the three timing scenarios described above.

Let λ denote the welfare cost of transitioning to the new steady state under policy regime $\mathcal{A}$, relative to remaining in the initial deterministic steady state. Specifically, λ measures the fraction of steady-state consumption that a household would require—or be willing to forgo—to be indifferent between the two regimes. Formally, λ is implicitly defined by:

$$\mathbb{W}\left(\{c^{\mathcal{A}}_{i,t}, n^{\mathcal{A}}_{i,t}\}_{i\in[0,1],t\geq 0}\right) = \mathbb{W}\left(\{(1+\lambda)c_{i,ss}, n_{i,ss}\}_{i\in[0,1],t\geq 0}\right), \tag{9}$$

where $\{c_{i,ss}, n_{i,ss}\}$ denotes consumption and labor in the initial steady state, and $\{c^{\mathcal{A}}_{i,t}, n^{\mathcal{A}}_{i,t}\}$ refers to the transition path under regime $\mathcal{A}$. The aggregator $\mathbb{W} : [0,1] \to \mathbb{R}$ maps individual utility levels into aggregate welfare using a utilitarian criterion with equal weights. A positive value of λ indicates a welfare gain under the alternative scenario.

Table 3: Consumption compensation analysis of monetary policy timing

Policy Rule	Welfare Gain/Loss (%)
Forward-looking	0.01
Coincident	-0.06
Delayed	-0.16

Note: Values represent welfare gains/losses during transition from 70% to 80% debt-to-GDP ratio relative to remaining at the initial deterministic steady state.

Table 3 presents the consumption compensation required under different monetary policy timing scenarios. In the forward-looking case, where monetary policy adjusts immediately upon announcement, the required compensation is positive—0.01%—implying that households could sustain a small consumption reduction while maintaining the same level of welfare.

When the monetary adjustment coincides with the fiscal implementation ($t = 12$), the required compensation remains modest at 0.06%. However, delaying the monetary response by an additional 12 quarters increases the required compensation to 0.16%, reflecting higher welfare costs from postponing the adjustment.

While all paths eventually converge, earlier monetary policy responses reduce the consumption adjustment needed to preserve household welfare during the transition to a higher debt-to-GDP ratio.

8 Natural rates and the central bank's reaction function

In this section, we explore the empirical relationship between r^* and monetary policy within a general econometric framework. We begin with a flexible representation of the central bank's policy rule:

$$i_t = \sum_{k=1}^{K} \rho_k i_{t-k} + \left(1 - \sum_{k=1}^{K} \rho_k\right) \Phi\left(\pi_t, \mathbb{E}_t[\pi_{t+1}], \dots, \mathbb{E}_t[\pi_{t+N}], \mathbb{E}_t[\tilde{y}_{t+1}], \dots, \mathbb{E}_t[\tilde{y}_{t+N}]\right),$$

where the autoregressive term $\sum_{k=1}^{K} \rho_k i_{t-k}$ captures interest rate smoothing, and $\Phi(\cdot)$ is a general nonlinear function that maps current and expected inflation and output gaps to the nominal interest rate. This formulation is intended to flexibly reflect central bank behavior in practice.

To linearize the rule, we take a first-order Taylor expansion around the inflation target $\overline{\pi}$ and a zero output gap. The resulting approximation is:

$$i_t \approx \sum_{k=1}^{K} \rho_k i_{t-k} + \Phi_0 + \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t[\pi_{t+n}]}\right) (\mathbb{E}_t[\pi_{t+n}] - \overline{\pi}) + \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t[\tilde{y}_{t+n}]}\right) \mathbb{E}_t[\tilde{y}_{t+n}].$$

In a DSS, we have $i_{t-k} = i_{ss}$ for all k, $\mathbb{E}_t[\pi_{t+n}] = \pi_{ss}$, and $\mathbb{E}_t[\tilde{y}_{t+n}] = 0$ for all n. Substituting these conditions into the linearized policy rule yields:

$$i_{ss} \approx \bar{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}),$$

where $\phi_\pi = \sum_{n=0}^{N} \left(\frac{\partial \Phi}{\partial \mathbb{E}_t[\pi_{t+n}]}\right)$ is the inflation response coefficient, and the intercept $\bar{r} = \Phi_0 - \overline{\pi}$. Using the Fisher equation to substitute for the DSS nominal interest rate, we obtain (outside the ZLB):

$$r^* + \pi_{ss} \approx \bar{r} + \overline{\pi} + \phi_\pi(\pi_{ss} - \overline{\pi}).$$

We can then re-derive equation (5):

$$\pi_{ss} \approx \overline{\pi} + \frac{r^* - \bar{r}}{\phi_\pi - 1},$$

which links the deviation of long-run inflation from the target, $\pi_{ss} - \overline{\pi}$, to the *policy gap*, defined as the difference between the natural rate and the intercept in the central bank's reaction function, $r^* - \bar{r}$.

If the DSS objects inherit the time-varying nature of fiscal shocks, they can be treated as random variables rather than fixed parameters. In particular, if the intercept $\bar{r}$ perfectly tracks the natural rate r^*, the policy gap is always zero and steady-state inflation remains anchored at the target, $\pi_{ss} = \overline{\pi}$.

Alternatively, if the Taylor rule is stable –meaning its intercept does not co-move with long-term inflation ($cov(\bar{r}, \pi_{ss}) = 0$)– we can express the variance of steady-state inflation as:

$$var(\pi_{ss}) \approx \frac{cov(r^*, \pi_{ss}) - cov(\bar{r}, \pi_{ss})}{\phi_\pi - 1} = \frac{cov(r^*, \pi_{ss})}{\phi_\pi - 1},$$

assuming a constant ϕ_π. In this case, the policy gap can be written as:

$$r^* - \bar{r} = \frac{cov(r^*, \pi_{ss})}{var(\pi_{ss})} \left(\pi_{ss} - \overline{\pi}\right). \tag{10}$$

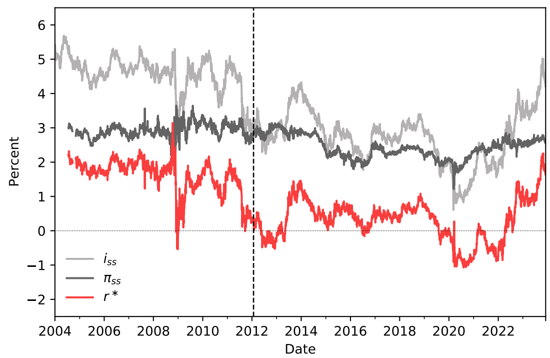
Equation (10) provides an empirical measure of the policy gap that can be estimated using market data.

To estimate the policy gap empirically, we require observable proxies for the (potentially time-varying) DSS values of real interest rates and inflation. A natural approach is to use market-based measures of long-term nominal rates and inflation expectations for the United States. We do not rely on the natural rate estimates of Lubik and Matthes (2015), as we also need a consistent proxy for long-term inflation. Market data provide a unified and forward-looking source for both variables.

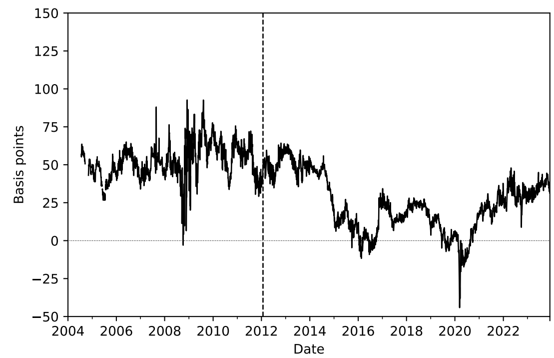
Specifically, we use daily data on the 5-year 5-year (5y5y) forward nominal yield, a standard measure of the 5-year interest rate expected five years ahead, and a common proxy for long-term nominal rates. For long-term inflation expectations, we employ 5y5y inflation-linked swaps (ILS), which reflect the fixed rate that compensates for expected inflation in swap contracts. The long-term real interest rate is then computed as the difference between the 5y5y nominal rate and the 5y5y ILS.[21]

[21] For robustness, we also use 5y5y TIPS yields as a proxy for long-term real rates and derive break-even

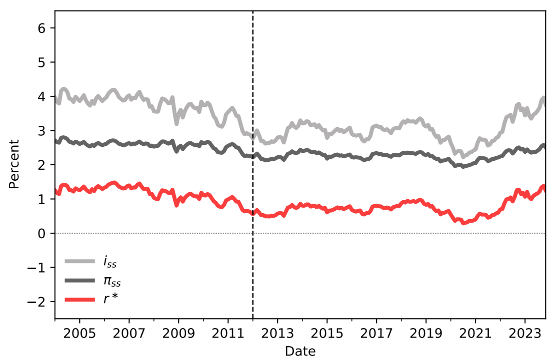
We set the value of the inflation target $\overline{\pi}$ to 2%. Although the Federal Reserve formally adopted this target in January 2012, it was widely viewed as the de facto target well before then, consistent with the practices of other major central banks such as the ECB and the Bank of England. To remain conservative, however, we compute $\frac{cov(r^*, \pi_{ss})}{var(\pi_{ss})}$ using data starting from January 2012. For this sample period, the estimated value is 0.56, implying a Taylor rule coefficient of $\phi_\pi = 1.56$—a value broadly in line with standard calibrations in the literature.



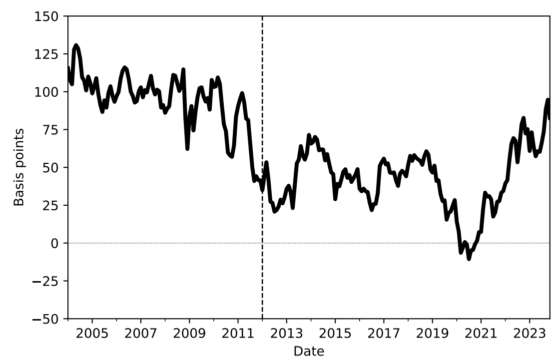
(a) Long-term nominal and real rates and inflation.



(b) Policy gap $r^* - \overline{r}$



(c) Data adjusted for term premia



(d) Policy gap $r^* - \overline{r}$ (adj. data)

Figure 8: Policy gaps

Note: Panel (a): i_{ss} is the 5y5y forward nominal rate derived from the zero-coupon U.S. yield curve; π_{ss} is the 5y5y inflation-linked swap (ILS) rate. The natural rate r^* is computed as $r^* = i_{ss} - \pi_{ss}$. Panel (b): the policy gap is calculated using the data from Panel (a). Panel (c): term premia are removed from the series in Panel (a) using the methodology of Hördahl and Tristani (2014). Panel (d): the policy gap is computed using the adjusted data from Panel (c). The dashed vertical line marks the announcement of the 2% inflation target (January 24, 2012).

Panel (a) of Figure 8 plots the three time series starting from their earliest availability (2004). Two key patterns emerge. First, market expectations of long-term nominal and real rates, as well as inflation, are neither constant nor driven solely by low-frequency secular

inflation as the difference between nominal and real yields. Results are quantitatively similar.

trends—they exhibit substantial high- and medium-term volatility. Second, nominal and real interest rates display greater volatility than inflation.

Panel (b) plots the estimated policy gap. This gap was clearly positive before 2014, narrowed between 2015 and 2020, and widened again following the large fiscal expansion after the COVID-19 pandemic. These patterns suggest that market participants perceive the Federal Reserve's reaction function as not consistently aligned with the natural rate, contributing to fluctuations in long-term inflation.

However, the series above may not reflect true expectations, as they are influenced by term premia—compensation demanded by risk-averse investors for bearing interest rate and inflation risk. As a robustness check, we adjust for term premia using the methodology of Hördahl and Tristani (2014). The adjusted series appear in Panel (c), and the recalculated policy gap in Panel (d). The resulting series closely tracks the original, especially after 2012. The main difference lies in the higher implied Taylor coefficient, $\phi_\pi = 2.63$.

9 Conclusion

This paper has analyzed monetary-fiscal interactions in a HANK model with an explicit fiscal block. In our framework, the stock of public debt affects the natural interest rate, necessitating that the central bank adjust its monetary policy rule in response to the fiscal stance to maintain price stability. We demonstrate that a minimum threshold of debt exists, below which steady-state inflation deviates from its target as the ZLB becomes binding.

We also examine the effects of a debt-financed fiscal expansion and quantify how different timings in adapting the monetary policy rule influence inflation dynamics. In addition, we evaluate the performance of alternative monetary policy rules that do not rely on explicit estimates of the natural rate.

The empirical evidence supports the key mechanisms in our model —namely, the response of natural rates to fiscal shocks and the subsequent adjustment of monetary policy.

A key question we leave open is how these new monetary-fiscal interactions should inform the optimal conduct of monetary policy. We leave this issue for future research.

References

Aggarwal, R., A. Auclert, M. Rognlie, and L. Straub (2023): "Excess Savings and Twin Deficits: The Transmission of Fiscal Stimulus in Open Economies," *NBER Macroeconomics Annual*, 37, 325–412.

Aguiar, M. A., M. Amador, and C. Arellano (2023): "Pareto Improving Fiscal and Monetary Policies: Samuelson in the New Keynesian Model," Working Paper 31297, National Bureau of Economic Research.

Aiyagari, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109, 659–684.

Aiyagari, S. R. and E. R. McGrattan (1998): "The Optimum Quantity of Debt," *Journal of Monetary Economics*, 42, 447–469.

Ajello, A., I. Cairó, V. Curdia, T. A. Lubik, and A. Queraltó (2020): "Monetary Policy Tradeoffs and the Federal Reserve's Dual Mandate," Finance and Economics Discussion Series 2020-066, Board of Governors of the Federal Reserve System.

Alves, F., G. Kaplan, B. Moll, and G. L. Violante (2020): "A Further Look at the Propagation of Monetary Policy Shocks in HANK," *Journal of Money, Credit and Banking*, 52, 521–559.

Angeletos, G.-M., C. Lian, and C. K. Wolf (2024): "Deficits and Inflation: HANK meets FTPL," Working Paper 33102, National Bureau of Economic Research.

Auclert, A., B. Bardóczy, M. Rognlie, and L. Straub (2021a): "Using the Sequence-Space Jacobian to Solve and Estimate Heterogeneous-Agent Models," *Econometrica*, 89, 2375–2408.

Auclert, A. and M. Rognlie (2018): "Inequality and Aggregate Demand," Working Paper 24280, National Bureau of Economic Research.

Auclert, A., M. Rognlie, M. Souchier, and L. Straub (2021b): "Exchange Rates and Monetary Policy with Heterogeneous Agents: Sizing up the Real Income Channel," Working Paper 28872, National Bureau of Economic Research.

Auclert, A., M. Rognlie, and L. Straub (2020): "Micro Jumps, Macro Humps: Monetary Policy and Business Cycles in an Estimated HANK Model," Working Paper 26647, National Bureau of Economic Research.

——— (2024): "The Intertemporal Keynesian Cross," *Journal of Political Economy*, 132, 4068–4121.

Bauer, M. D. and G. D. Rudebusch (2020): "Interest Rates Under Falling Stars," *American Economic Review*, 110, 1316–1354.

Bayer, C., B. Born, and R. Luetticke (2023): "The Liquidity Channel of Fiscal Policy," *Journal of Monetary Economics*, 134, 86–117.

Benhabib, J., S. Schmitt-Grohé, and M. Uribe (2002): "Avoiding Liquidity Traps," *Journal of Political Economy*, 110, 535–563.

Bhandari, A., D. Evans, M. Golosov, and T. J. Sargent (2017): "Public debt in economies with heterogeneous agents," *Journal of Monetary Economics*, 91, 39–51.

Bianchi, F., R. Faccini, and L. Melosi (2022): "A Fiscal Theory of Persistent Inflation," Working Paper 30727, National Bureau of Economic Research.

Bigio, S., N. Caramp, and D. Silva (2023): "Sticky Inflation," Working paper, UCLA.

Bilbiie, F. O. (2024): "Monetary Policy and Heterogeneity: An Analytical Framework," *Review of Economic Studies*, rdae066.

Bocola, L., A. Dovis, K. Jørgensen, and R. Kirpalani (2024): "Bond Market Views of the Fed," Working Paper 32620, National Bureau of Economic Research.

Challe, E. and M. Matvieiev (2024): "On Natural Interest Rate Volatility," *European Economic Review*, 167, 104796.

Chortareas, G., G. Kapetanios, and O. Kaykhusraw (2023): "Equilibrium Interest Rates and Monetary Policy (Mis)perceptions," Tech. rep., King's College London.

Christensen, J. H. E. and G. D. Rudebusch (2019): "A New Normal for Interest Rates? Evidence from Inflation-Indexed Debt," *Review of Economics and Statistics*, 101, 933–949.

Christiano, L. J. and M. Eichenbaum (1992): "Current Real-Business-Cycle Theories and Aggregate Labor-Market Fluctuations," *American Economic Review*, 82, 430–450.

Clarida, R., J. Gali, and M. Gertler (1999): "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, 37, 1661–1707.

Cochrane, J. H. (1999): "A Frictionless View of US Inflation," *NBER Macroeconomics Annual 1998*, 13, 323–421.

Daudignon, S. and O. Tristani (2023): "Monetary Policy and the Drifting Natural Rate of Interest," Working Paper Series 2788, European Central Bank.

Davis, J., C. Fuenzalida, L. Huetsch, B. Mills, and A. M. Taylor (2023): "Global Natural Rates in the Long Run: Postwar Macro Trends and the Market-Implied r* in 10 Advanced Economies," Working Paper 31787, NBER.

Doepke, M. and M. Schneider (2006): "Inflation and the Redistribution of Nominal Wealth," *Journal of Political Economy*, 114, 1069–1097.

Eggertsson, G. B., N. R. Mehrotra, and J. A. Robbins (2019): "A Model of Secular Stagnation: Theory and Quantitative Evaluation," *American Economic Journal: Macroeconomics*, 11, 1–48.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46, 281–313.

Faia, E., E. Shabalina, and D. Wiczer (2022): "Heterogeneous Effects of Monetary Policy across Income and Race: The Labour Mobility Channel," Discussion Paper 17741, CEPR.

Ferreira, T. R. and S. Shousha (2023): "Determinants of global neutral interest rates," *Journal of International Economics*, 145, 103833.

Ferriere, A. and G. Navarro (2018): "The Heterogeneous Effects of Government Spending: It's All About Taxes," Tech. Rep. 1237, FRB International Finance Discussion Paper.

Galí, J. (2008): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*, Princeton University Press.

Hagedorn, M. (2016): "A Demand Theory of the Price Level," Discussion Paper 11364, CEPR.

Hagedorn, M., I. Manovskii, and K. Mitman (2019): "The Fiscal Multiplier," Working Paper 25571, NBER.

Hänsel, M. (2024): "Idiosyncratic Risk, Government Debt and Inflation," Tech. Rep. 2403.00471, arXiv.

Hazell, J., J. Herreño, E. Nakamura, and J. Steinsson (2022): "The Slope of the Phillips Curve: Evidence from U.S. States," *The Quarterly Journal of Economics*, 137, 1299–1344.

Holston, K., T. Laubach, and J. C. Williams (2017): "Measuring the Natural Rate of Interest: International Trends and Determinants," *Journal of International Economics*, 108, 59–75.

Hördahl, P. and O. Tristani (2014): "Inflation Risk Premia in the Euro Area and the United States," *International Journal of Central Banking*, 10, 1–47.

Jordà, Ò. (2005): "Estimation and Inference of Impulse Responses by Local Projections," *American Economic Review*, 95, 161–182.

Kaplan, G., B. Moll, and G. L. Violante (2018): "Monetary Policy According to HANK," *American Economic Review*, 108, 697–743.

Kaplan, G., G. Nikolakoudis, and G. L. Violante (2023): "Price Level and Inflation Dynamics in Heterogeneous Agent Economies," Working Paper 31433, National Bureau of Economic Research.

Kaplan, G. and G. L. Violante (2022): "The Marginal Propensity to Consume in Heterogeneous Agent Models," *Annual Review of Economics*, 14, 747–775.

Laubach, T. and J. C. Williams (2003): "Measuring the Natural Rate of Interest," *Review of Economics and Statistics*, 85, 1063–1070.

Leeper, E. M. (1991): "Equilibria Under 'Active' and 'Passive' Monetary and Fiscal Policies," *Journal of Monetary Economics*, 27, 129–147.

Lubik, T. A. and C. Matthes (2015): "Calculating the Natural Rate of Interest: A Comparison of Two Alternative Approaches," *Richmond Fed Economic Brief*, EB 15-10.

McKay, A. and R. Reis (2021): "Optimal Automatic Stabilizers," *Review of Economic Studies*, 88, 2375–2406.

Mian, A. R., L. Straub, and A. Sufi (2022): "A Goldilocks Theory of Fiscal Deficits," Working Paper 29707, National Bureau of Economic Research.

Montiel Olea, J. L. and M. Plagborg-Møller (2021): "Local Projection Inference is Simpler and More Robust Than You Think," *Econometrica*, 89, 1789–1823.

Oh, H. and R. Reis (2012): "Targeted Transfers and the Fiscal Response to the Great Recession," *Journal of Monetary Economics*, 59, 50–64.

ORPHANIDES, A. AND V. W. WIELAND (1998): "Price Stability and Monetary Policy Effectiveness when Nominal Interest rRates are Bounded at Zero," Finance and Economics Discussion Series 1998-35, Board of Governors of the Federal Reserve System (U.S.).

ORPHANIDES, A. AND J. C. WILLIAMS (2002): "Robust Monetary Policy Rules with Unknown Natural Rates," *Brookings Papers on Economic Activity*, 2002, 63–118.

PHILLIPS, A. W. (1954): "Stabilisation Policy in a Closed Economy," *Economic Journal*, 64, 290–323.

PLATZER, J., R. TIETZ, AND J. LINDÉ (2022): "Natural versus Neutral Rate of Interest: Parsing Disagreement about Future Short-Term Interest Rates," VoxEU Column, July, 26, 2022.

RACHEL, L. AND L. H. SUMMERS (2019): "On Secular Stagnation in the Industrialized World," *Brookings Papers on Economic Activity*, 50, 1–76.

ROTEMBERG, J. (1982): "Monopolistic Price Adjustment and Aggregate Output," *Review of Economic Studies*, 49, 517–531.

ROUWENHORST, K. G. (1995): "Asset Pricing Implications of Equilibrium Business Cycle Models," in *Frontiers of Business Cycle Research*, ed. by T. F. Cooley, Princeton: Princeton University Press, 294–330.

SARGENT, T. J. AND N. WALLACE (1981): "Some Unpleasant Monetarist Arithmetic," *Federal Reserve Bank of Minneapolis Quarterly Review*, 5.

SCHMITT-GROHE, S. AND M. URIBE (2000): "Price Level Determinacy and Monetary Policy under a Balanced-Budget Requirement," *Journal of Monetary Economics*, 45, 211–246.

SCHMITT-GROHÉ, S. AND M. URIBE (2025): "The Macroeconomic Consequences of Natural Rate Shocks," *Review of Economics and Statistics*, 1–45.

SIMS, C. A. (1994): "A Simple Model for Study of the Determination of the Price Level and the Interaction of Monetary and Fiscal Policy," *Economic Theory*, 4, 381–399.

WOLF, C. K. (2021): "Interest Rate Cuts vs. Stimulus Payments: An Equivalence Result," Working Paper 29193, National Bureau of Economic Research.

WOODFORD, M. (1995): "Price-Level Determinacy without Control of a Monetary Aggregate," *Carnegie-Rochester Conference Series on Public Policy*, 43, 1–46.

——— (2001): "Fiscal Requirements for Price Stability," *Journal of Money, Credit and Banking*, 33, 669–728.

——— (2003): *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press.

Online Appendix

A Derivation of the nonlinear wage Phillips curve with trend inflation

Households are indexed by i and time is denoted by t. The labor supply of a household at date t is denoted by $n_{i,t}$. This labor supply is itself an aggregation over a continuum of labor services, which are imperfect substitutes. A separate union exists for each labor service. We index different labor services (and unions) by $k \in [0,1]$ and denote a particular labor service of type k provided by household i at date t by $n_{i,k,t}$, with $n_{it} = \int_0^1 n_{i,k,t} dk$. Each union k aggregates the efficient labor units of its members into a union-specific task:

$$N_{k,t} = \int z_{i,t} n_{i,k,t} di.$$

Unions set a common wage per efficiency unit, $W_{k,t}$, and their members must supply labor as demanded at that wage. A competitive labor packer then combines these tasks into aggregate labor services using a constant-elasticity-of-substitution technology:

$$N_t = \left(\int N_{k,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} dk \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \epsilon_w > 1,$$

and sells these packaged labor services to intermediate goods firms in a competitive market at a nominal price W_t.

Unions face a quadratic utility cost for adjusting the nominal wage $W_{k,t}$:

$$\frac{\psi}{2} \log \left(\frac{W_{k,t}}{W_{k,t-1}(1+\overline{\pi})} \right)^2.$$

The problem solved by the union at each date t is:

$$\max_{\{W_{k,t+\tau}\}} \mathbb{E}_0 \sum_{\tau \geq 0} \beta^{t+\tau} \left[\int \left[u(c_{i,t}) - v(n_{i,t}) \right] di - \frac{\psi}{2} \log \left(\frac{W_{k,t+\tau}}{W_{k,t+\tau-1}(1+\overline{\pi})} \right)^2 \right],$$

subject to the demand curve $N_{k,t} = \left(\frac{W_{k,t}}{W_t} \right)^{-\epsilon_w} N_t$, where $W_t^{1-\epsilon_w} = \int W_{k,t}^{1-\epsilon_w} dk$. It can be shown (see Auclert et al. 2024 for the details) that the solution is a wage Phillips curve of the form:

$$\log\left(\frac{1+\pi_t^w}{1+\overline{\pi}}\right) = \kappa_w\left[-\frac{(\epsilon_w-1)}{\epsilon_w}(1-\tau_n)\frac{W_t}{P_t}\int u'(c_{i,t})z_{i,t}di + v'(N_t)\right]N_t + \beta\mathbb{E}_t\left[\log\left(\frac{1+\pi_{t+1}^w}{1+\overline{\pi}}\right)\right],$$

with slope $\kappa_w \equiv \frac{\epsilon_w}{\psi}$.

In this setup, all unions set the same wage, $W_{k,t} = W_t$, at time t, and all households work the same number of hours and also the same number of hours on each task, so that $n_{i,k,t} = n_{i,t}$ is constant across houdeholds. Because unions employ a representative sample from the population, and idiosyncratic productivities average to one, labor per task also equals aggregate union labor services and $n_{i,k,t} = N_{k,t}$. This also implies that $N_t = n_{i,t}$ given that, from the definition of aggregate labor supply,

$$\begin{aligned}
N_t &= \left(\int_0^1 \left(\int z_{i,t}n_{i,k,t}di\right)^{\frac{\epsilon_w-1}{\epsilon_w}} dk\right)^{\frac{\epsilon_w}{\epsilon_w-1}} \\
&= \left(\int_0^1 N_{k,t}^{\frac{\epsilon_w-1}{\epsilon_w}} \left(\int z_{i,t}di\right)^{\frac{\epsilon_w-1}{\epsilon_w}} dk\right)^{\frac{\epsilon_w}{\epsilon_w-1}} \\
&= \left(\int_0^1 n_{i,t}^{\frac{\epsilon_w-1}{\epsilon_w}} (1)^{\frac{\epsilon_w-1}{\epsilon_w}} dk\right)^{\frac{\epsilon_w}{\epsilon_w-1}} = n_{i,t}.
\end{aligned}$$

B Derivation of the price Phillips curve

Intermediate goods producers are indexed by j. They produce their goods using a production function $F(k_{j,t-1}, n_{j,t}) = \Theta k_{j,t-1}^{\alpha} n_{j,t}^{1-\alpha}$. They own the capital stock, $k_{j,t-1}$, used in their production process and hire labor services, $n_{j,t}$, in a competitive market. The capital stock is predetermined, as it must be chosen one period in advance, whereas labor can be hired in each period t. We will consider an equilibrium in which all firms act symmetrically given the path of aggregate variables. To ensure that this is an equilibrium, we assume that all firms start with the same amount of capital $k_{j,-1} = K_{-1} > 0$.

Adjustments to the capital stock face an adjustment cost given by the following expression:

$$\phi\left(\frac{k_{j,t}}{k_{j,t-1}}\right) \equiv \frac{1}{2\delta\epsilon_I}\left(\frac{k_{j,t}}{k_{j,t-1}} - 1\right)^2$$

with first derivative:

$$\phi'\left(\frac{k_{j,t}}{k_{j,t-1}}\right) = \frac{1}{\delta\epsilon_I}\left(\frac{k_{j,t}}{k_{j,t-1}} - 1\right).$$

When intermediate goods producers change prices, they are subject to an adjustment cost function à la Rotemberg (1982):

$$\xi(p_{j,t}, p_{j,t-1}) = \underbrace{\frac{\epsilon_p}{2\kappa_p}\left[\log\left(\frac{p_{j,t}}{\tilde{\pi}p_{j,t-1}}\right)\right]^2}_{\lambda(p_{j,t},p_{j,t-1})} Y_t,$$

where we have defined the gross inflation rate $\tilde{\pi} \equiv 1 + \overline{\pi}$ to economize on notation. Function $\lambda(p_{j,t}, p_{j,t-1})$ has partial derivatives that satisfy:

$$\lambda_1(p_{j,t}, p_{j,t-1})p_{j,t} = \frac{\epsilon_p}{\kappa_p}\left[\log\left(\frac{p_{j,t}}{\tilde{\pi}p_{j,t-1}}\right)\right]$$
$$\lambda_2(p_{j,t+1}, p_{j,t})p_{j,t} = -\frac{\epsilon_p}{\kappa_p}\left[\log\left(\frac{p_{j,t+1}}{\tilde{\pi}p_{j,t}}\right)\right].$$

Because of these adjustment costs, each intermediate goods producer's state variables include the previous period's price, $p_{j,t-1}$, and the capital stock, $k_{j,t-1}$. The intermediate goods producer also takes the demand for its differentiated good $F(k_{j,t-1}, n_{j,t}) = Y_t\left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon_p}$ as given and discounts future payoffs using the date-t expected real interest rate on illiquid assets, $\mathbb{E}_t[r^a_{t+1}]$.

The Bellman equation of the decision problem of a typical intermediate producer is:

$$\begin{aligned}\mathcal{J}_t(p_{j,t-1}, k_{j,t-1}) = \max_{p_{j,t},k_{j,t},n_{j,t}} \Bigg\{ & \frac{p_{j,t}}{P_t}F(k_{j,t-1}, n_{j,t}) - \frac{W_t}{P_t}n_{j,t} - (k_{j,t} - (1-\delta)k_{j,t-1}) \\ & - \phi\left(\frac{k_{j,t}}{k_{j,t-1}}\right)k_{j,t-1} - \lambda(p_{j,t}, p_{j,t-1})Y_t \\ & + \mathbb{E}_t\left[\frac{1}{1+r^a_{t+1}}\mathcal{J}_{t+1}(p_{j,t}, k_{j,t})\right]\Bigg\},\end{aligned} \tag{11}$$

subject to:

$$F(k_{j,t-1}, n_{j,t}) = Y_t\left(\frac{p_{j,t}}{P_t}\right)^{-\epsilon_p}$$

The constraint can also be rewritten in a more convenient form as:

$$\left(\frac{F(k_{j,t-1}, n_{j,t})}{Y_t}\right)^{-\frac{1}{\epsilon_p}} Y_t = \frac{p_{j,t}}{P_t}Y_t.$$

Let η_t denote the Lagrange multiplier on this production constraint. Thus, the first-order condition for the labor choice $n_{j,t}$ is:

$$\left(\frac{p_{j,t}}{P_t} - \eta_t\left(\frac{1}{\epsilon_p}\right)\left(\frac{F(k_{j,t-1}, n_{j,t})}{Y_t}\right)^{-\frac{1}{\epsilon_p}-1}\right) F_n(k_{j,t-1}, n_{j,t}) = \frac{W_t}{P_t}.$$

The term in parentheses on the left-hand side of this equation can be identified as the real marginal cost, given that $mc_t \equiv d\mathrm{Cost}/dy_t = (d\mathrm{Cost}/dn_t)/(dy_t/dn_t) = (W_t/P_t)/F_n$.

In equilibrium, all intermediate goods producers choose the same price, so that $p_{j,t} = P_t$ and also choose to hire the same level of employment. This satisfies the first-order conditions provided that intermediate producers also choose the same level of capital, so that output and the marginal product of labor are the same for all producers. In the next section, we verify that choosing the same levels of capital is indeed optimal, given that all other choices are symmetric. This implies:

$$mc_t = 1 - \eta_t\left(\frac{1}{\epsilon_p}\right).$$

The first-order condition for the price $p_{j,t}$ is:

$$\frac{1}{P_t}F(k_{j,t-1}, n_{j,t}) - \lambda_1(P_t, P_{t-1})Y_t + \mathbb{E}_t\left[\frac{1}{1+r^a_{t+1}}\frac{\partial \mathcal{J}_{t+1}(P_t, k_t)}{\partial P_t}\right] - \frac{\eta_t}{P_t}Y_t = 0.$$

The envelope condition yields:

$$\frac{\partial \mathcal{J}_t(p_{j,t-1}, k_{t-1})}{\partial p_{j,t-1}} = -\lambda_2(p_{j,t}, p_{j,t-1})Y_t.$$

Combining the previous equations and multiplying by $p_{j,t}$:

$$\left(\frac{p_{j,t}}{P_t}(1-\eta_t)\right) Y_t = \lambda_1(p_{j,t}, p_{j,t-1})p_{j,t}Y_t + \mathbb{E}_t\left[\frac{1}{1+r^a_{t+1}}\lambda_2(p_{j,t+1}, p_{j,t})p_{j,t}Y_{t+1}\right].$$

Using the partial derivatives of the Rotemberg adjustment cost function and the definition of gross inflation, $1+\pi_t \equiv \frac{P_t}{P_{t-1}}$:

$$\log\left(\frac{P_t}{P_{t-1}\tilde{\pi}}\right) = \frac{\kappa_p}{\epsilon_p}(1-\eta_t) + \mathbb{E}_t\left[\frac{1}{1+r^a_{t+1}}\left(\log\left(\frac{P_{t+1}}{P_t\tilde{\pi}}\right)\right)\frac{Y_{t+1}}{Y_t}\right].$$

Furthermore, from the relationship between the marginal cost and the Lagrange multiplier, it follows that:

$$\frac{1-\eta_t}{\epsilon_p} = \frac{1}{\epsilon_p} - (1 - mc_t),$$

and this leads to the price Phillips curve in the main text:

$$\log\left(\frac{1+\pi_t}{1+\overline{\pi}}\right) = \kappa_p\left(mc_t - \frac{1}{\mu_p}\right) + \mathbb{E}_t\left[\frac{1}{1+r^a_{t+1}}\frac{Y_{t+1}}{Y_t}\log\left(\frac{1+\pi_{t+1}}{1+\overline{\pi}}\right)\right].$$

where $\mu_p = \epsilon_p/(\epsilon_p - 1)$.

C Derivation of the equation for aggregate investment

From the Bellman equation (11), we derive the first-order condition for $k_{j,t}$:

$$1 + \phi'\left(\frac{k_{j,t}}{k_{j,t-1}}\right) = \mathbb{E}_t\left[\frac{1}{1+r^a_{t+1}}\frac{\partial \mathcal{J}_{t+1}}{\partial k_{j,t}}\right] \equiv Q^k_t.$$

Here, we have written Q^k_t without dependence on j, as we are considering a symmetric equilibrium in which firms make symmetric choices in future periods. We can already see that this implies that the ratio $k_{j,t}/k_{j,t-1}$ must be constant across intermediate producers. Coupled with the assumption that all intermediate producers start out with the same initial stock of capital, this implies that the path of the capital stock must be the same for all firms.

The envelope condition is:

$$\frac{\partial \mathcal{J}_t}{\partial k_{j,t-1}} = mc_t \cdot F_k(k_{j,t-1}, n_{j,t}) + (1-\delta) - \phi\left(\frac{k_{j,t}}{k_{j,t-1}}\right) + \phi'\left(\frac{k_{j,t}}{k_{j,t-1}}\right)\frac{k_{j,t}}{k_{j,t-1}}.$$

This equation is also satisfied in a symmetric equilibrium. From here on we replace $k_{j,t}$ and $k_{j,t-1}$ with their aggregate counterparts K_t and K_{t-1}.

Rewriting the first-order condition for $k_{j,t}$ using the derivative obtained before for the investment adjustment cost function, and replacing the aggregate capital stock:

$$\underbrace{\frac{1}{\delta\epsilon_I}\left(\frac{K_t}{K_{t-1}} - 1\right)}_{\phi'\left(\frac{k_t}{k_{t-1}}\right)} = Q^k_t - 1. \tag{12}$$

This equation can be multiplied by $\frac{K_t}{K_{t-1}}$ on both sides to obtain

$$\phi'\left(\frac{K_t}{K_{t-1}}\right)\frac{K_t}{K_{t-1}} = (Q^k_t - 1)\frac{K_t}{K_{t-1}}.$$

Using this expression in the envelope condition leads to

$$(1+r^a_t)Q^k_{t-1} = mc_t \cdot F_k(K_{t-1}, n_t) - \left(\frac{K_t}{K_{t-1}} - (1-\delta)\right) - \phi\left(\frac{K_t}{K_{t-1}}\right) + \frac{K_t}{K_{t-1}}Q^k_t,$$

where $F_k(K_{t-1}, N_t) = \alpha \frac{Y_t}{K_{t-1}}$.

This equation determines the evolution of the value of investing, Q_t^k. Together with the first-order condition for capital (12), it governs the evolution of the aggregate capital stock.

D Derivation of the equation for government debt accumulation

The budget constraint of the treasury, expressed in nominal terms, is given by:

$$Q_t B_t^\$ = (1 + \delta_B Q_t) B_{t-1}^\$ + P_t(G_t + T_t - \mathcal{T}_t), \tag{13}$$

where $B_t^\$$ is the stock of government bonds at the end of period t, measured in dollars, and Q_t is the market price of one unit of these bonds.

The left-hand side of equation (13) represents the nominal market value of government bonds at the end of period t. On the right-hand side, there are two terms. The first term, $(1+\delta_B Q_t)B_{t-1}^\$$, relates to past debt issuance. It includes a nominal dividend of \$1 (representing the first coupon payment from debt carried over from period $t-1$) plus the market value of this debt after the coupon payment. Due to the geometric decay of coupons, this market value—equal to the present value of future coupon payments, $\delta_B + \delta_B^2 + \ldots$—can be expressed as a fraction, δ_B, of the prior debt amount.[22] The sum of the current coupon and the remaining debt price is then multiplied by the number of bonds carried over from the previous period, $B_{t-1}^\$$. The second term, $P_t(G_t + T_t - \mathcal{T}_t)$, represents the government's primary deficit, which includes government spending, tax revenue, and transfers, all expressed in nominal terms.

To express this equation in real terms, it is necessary to divide both sides by the price level P_t:

$$Q_t \frac{B_t^\$}{P_t} = (1 + \delta_B Q_t) \frac{P_{t-1}}{P_t} \frac{B_{t-1}^\$}{P_{t-1}} + (G_t + T_t - \mathcal{T}_t).$$

Let us define $\tilde{B}_t \equiv \frac{B_t^\$}{P_t}$, the amount of debt in circulation at the end of period t, expressed in real terms. Thus:

$$Q_t \tilde{B}_t = (1 + \delta_B Q_t) \frac{P_{t-1}}{P_t} \tilde{B}_{t-1} + (G_t + T_t - \mathcal{T}_t).$$

Multiply and divide the first term on the right-hand side by Q_{t-1} to get:

$$Q_t \tilde{B}_t = \frac{(1 + \delta_B Q_t)}{Q_{t-1}} \frac{P_{t-1}}{P_t} Q_{t-1} \tilde{B}_{t-1} + (G_t + T_t - \mathcal{T}_t).$$

[22] This follows from $\delta_B + \delta_B^2 + \delta_B^3 + \cdots = \delta_B(1 + \delta_B + \delta_B^2 + \ldots)$ for $\delta_B < 1$.

Finally, to reach the equation in the main text, we define the real market value of debt at the end of period t by $B_t \equiv Q_t \tilde{B}_t = Q_t \frac{B_t^\$}{P_t}$. Then:

$$B_t = \frac{(1+\delta_B Q_t)}{Q_{t-1}} \frac{P_{t-1}}{P_t} B_{t-1} + (G_t + T_t - \mathcal{T}_t),$$

which can also be expressed as

$$B_t = (1+r_t^b) B_{t-1} + (G_t + T_t - \mathcal{T}_t).$$

Recall from the main text that we have defined the ex-post real return on bonds as $1 + r_t^b \equiv \frac{(1+\delta_B Q_t)}{Q_{t-1}} \frac{P_{t-1}}{P_t}$.

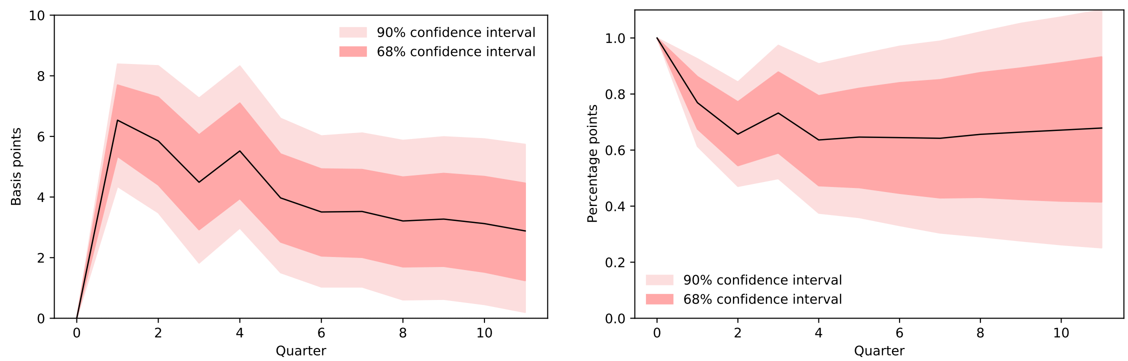
E Validating evidence

In this section, we provide empirical support for the response of the natural rate to an increase in public debt.

E.1 Response of the natural rate to an increase in public debt

We estimate a structural vector autoregression (SVAR) model including r^*, the debt-to-GDP ratio, the federal funds rate, inflation, and the unemployment rate. The identification of a structural shock follows a recursive (Cholesky) scheme, assuming that r^* responds to the lagged value of the debt-to-GDP ratio. This is consistent with our local projections specification and reflects the idea that the central bank responds quickly —but not instantaneously— to fiscal shocks, possibly due to delays in the reporting of fiscal data.

Figure 9 plots the impulse responses to a 1 percentage point increase in the debt-to-GDP ratio. The results indicate that r^* rises by approximately 3 to 6 bps. In addition, the increase in debt is highly persistent. These findings are broadly consistent with those generated by our model. While the model may slightly understate the total effect of fiscal shocks on r^*, its predictions remain within the empirical confidence range—and stand in stark contrast to the RANK model, where r^* remains unchanged.



(a) Response of r^* (b) Response of debt as a percentage of GDP

Figure 9: IRFs of r^* and public debt to a 1 pp increase in the debt-to-GDP ratio, SVAR

Note: Impulse response functions (IRFs) of r^* and the debt-to-GDP ratio to a 1 percentage point shock in public debt. The SVAR is identified using a Cholesky decomposition with the ordering: debt-to-GDP ratio, r^*, inflation, unemployment rate, and federal funds rate. Lag length: $p = 4$. Solid lines represent point estimates; shaded areas denote 68% and 90% confidence intervals from a wild bootstrap with 10,000 replications.

F Steady states under alternative fiscal regimes

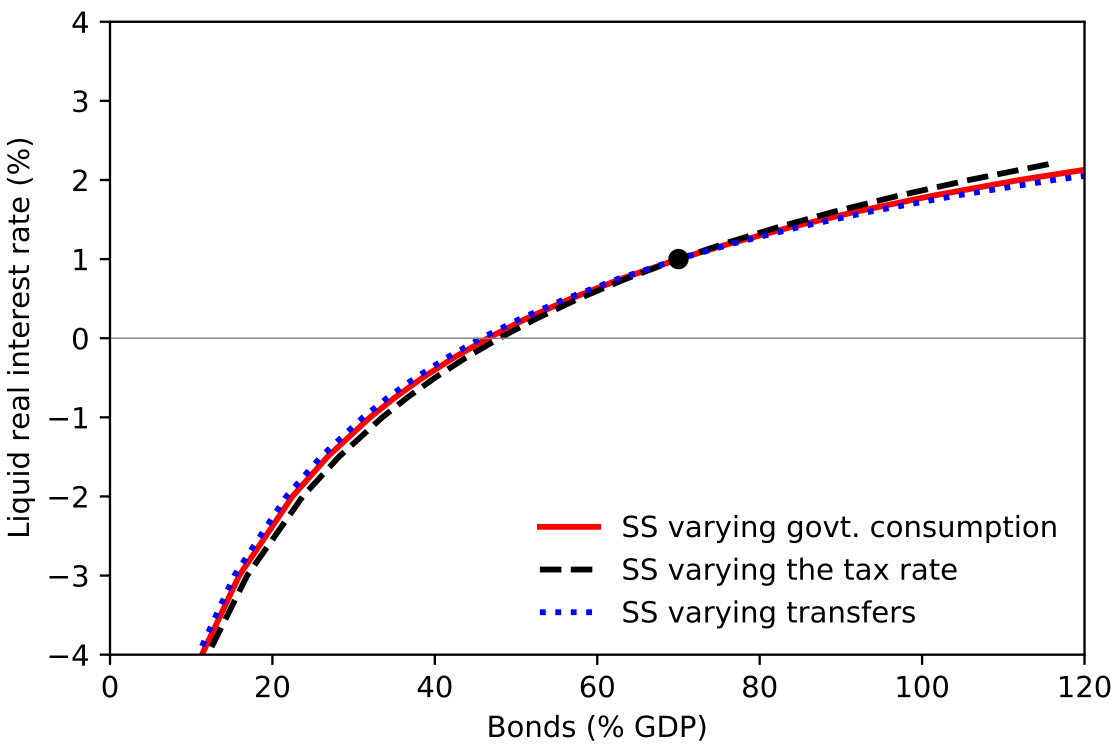


Figure 10: Steady states under alternative fiscal regimes

Figure 10 plots the steady-state values of r^* when we change G, the tax rate, or transfers in budget-equivalent ways. The main finding is that the results are nearly identical.

G Transitions for special cases

G.1 Short-run quantitative effects of fiscal expansion with flexible wages

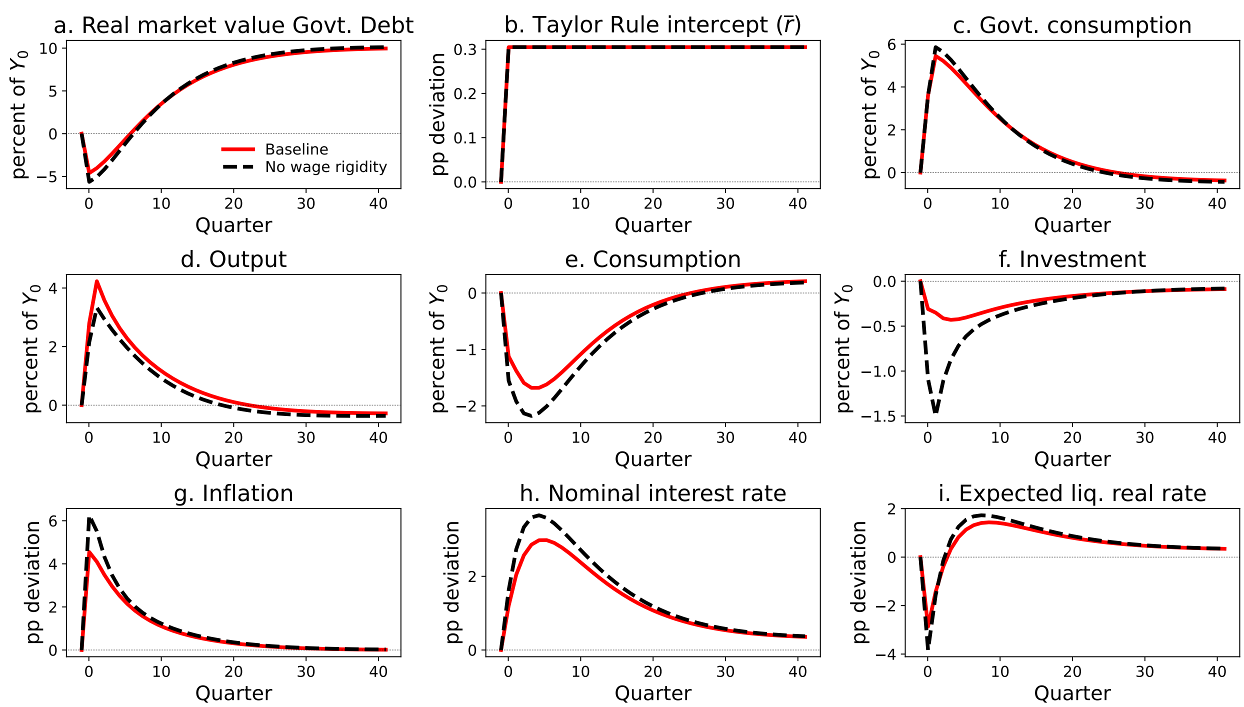


Figure 11: Dynamics after a surprise debt-financed fiscal expansion with flexible wages

Figure 11 displays the IRFs for a variant of the model calibrated to an environment with flexible wages. We implement this by setting $\kappa_w = 1{,}000{,}000$. Given $\epsilon_w = 10$, this implies a wage adjustment penalty of $\psi = 10^{-5}$, since $\kappa_w \equiv \frac{\epsilon_w}{\psi}$, where ϵ_w is the elasticity of substitution across labor tasks and ψ is the wage adjustment cost parameter.

G.2 Short-run quantitative effects of alternative fiscal expansions

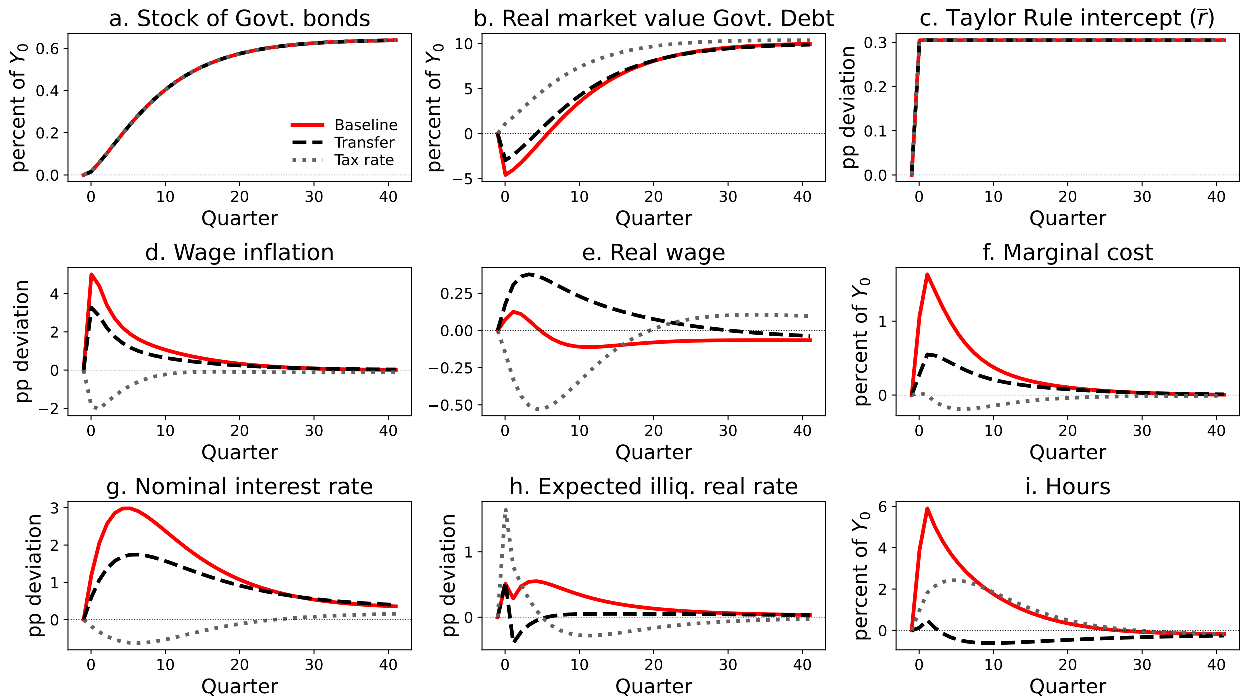


Figure 12: Dynamics after a surprise debt-financed fiscal expansion (additional variables)

G.3 Simulation of the baseline model with short-term debt

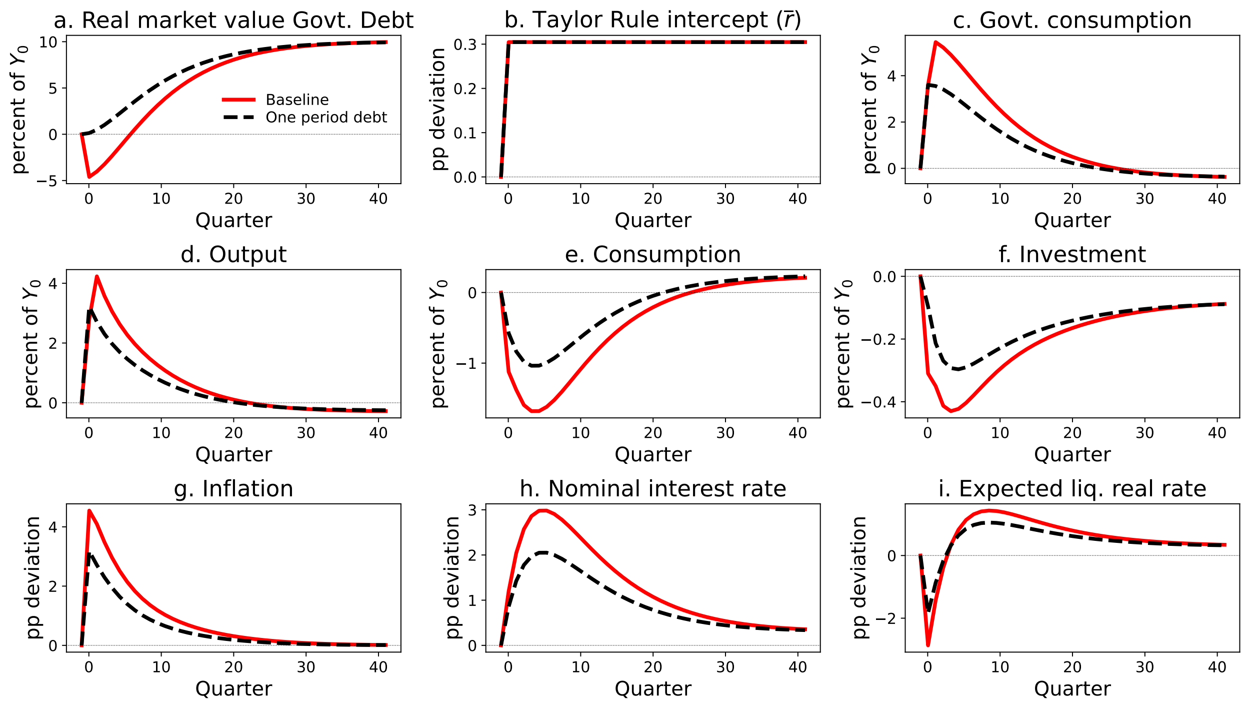


Figure 13: Dynamics after a surprise debt-financed fiscal expansion: Model with short-term debt

G.4 Simulation of the baseline model without a monetary policy adjustment

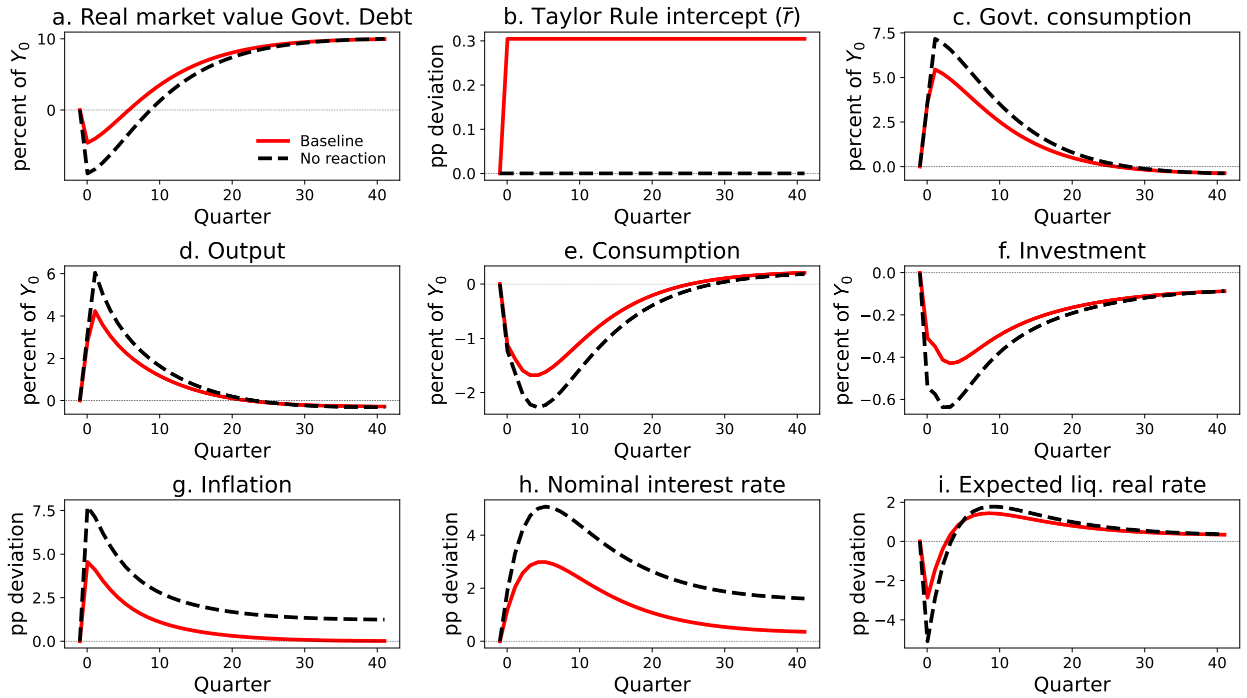


Figure 14: Dynamics after a surprise debt-financed fiscal expansion: Baseline vs. no monetary policy adjustment

G.5 A Taylor rule without interest rate smoothing

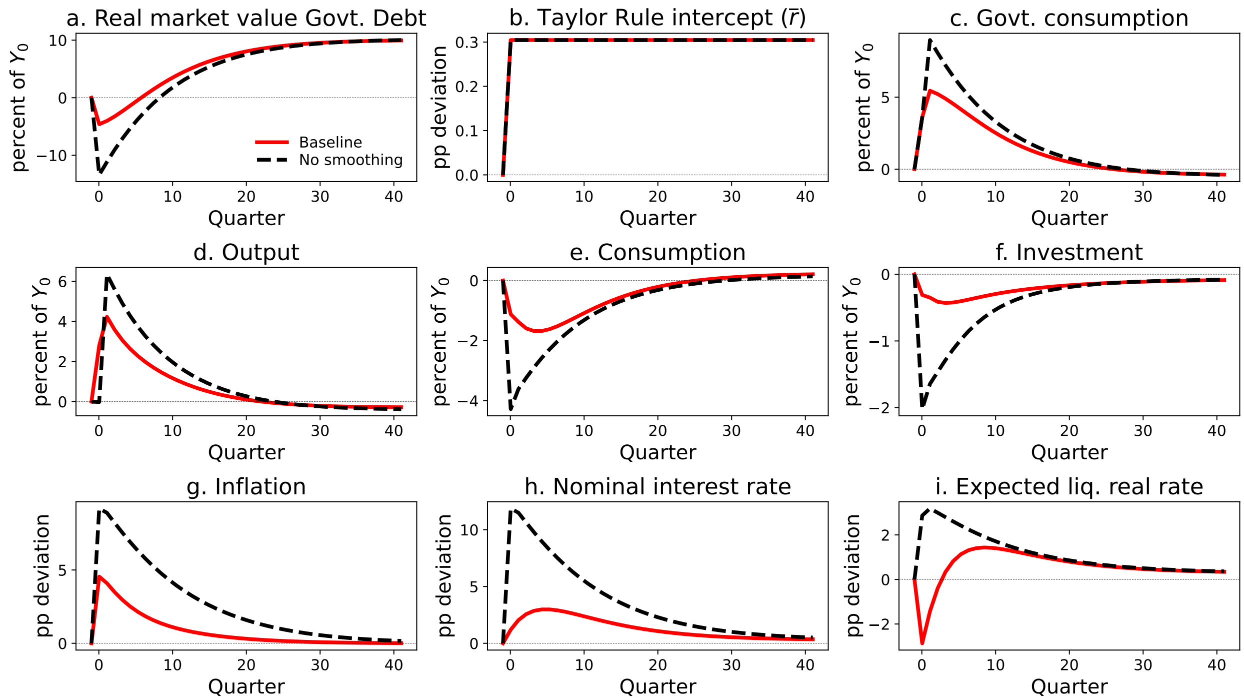


Figure 15: Dynamics after a surprise debt-financed fiscal expansion: No interest rate smoothing in the Taylor rule

G.6 Evaluation of the transitional dynamics for alternative monetary rules: Utilitarian welfare criterion

We evaluate the welfare implications of alternative monetary policy rules in the context of a permanent increase in public debt from 70% to 80% of GDP. Our analysis considers both the total welfare cost of higher debt and how this cost varies across monetary regimes.

Let λ denote the welfare cost of transitioning to the new steady state under policy regime $\mathcal{A}$, relative to remaining in the initial deterministic steady state. The parameter λ captures the fraction of steady-state consumption that a household would be willing to forgo—or would need to receive –to be indifferent between the baseline and the alternative regime. Formally, λ is implicitly defined by:

$$\mathbb{W}\left(\{c_{i,t}^{\mathcal{A}}, n_{i,t}^{\mathcal{A}}\}_{i\in[0,1],t\geq 0}\right) = \mathbb{W}\left(\{(1+\lambda)c_{i,ss}, n_{i,ss}\}_{i\in[0,1],t\geq 0}\right), \tag{14}$$

where $\{c_{i,ss}, n_{i,ss}\}$ denote consumption and labor in the initial steady state, and $\{c_{i,t}^{\mathcal{A}}, n_{i,t}^{\mathcal{A}}\}$ represent the transition path under policy regime $\mathcal{A}$. The aggregator $\mathbb{W} : [0,1] \to \mathbb{R}$ maps individual utility levels into aggregate welfare using a utilitarian criterion with equal weights.

A positive value of λ indicates a welfare gain under the alternative regime.

Table 4: Welfare Analysis of Alternative Monetary Rules

Policy Rule	Welfare Gain/Loss (%)
Baseline Taylor ($\phi_\pi = 1.25$)	-0.27
No reaction	-0.66
No smoothing	-0.92
With Output Gap ($\phi_y = 0.1$)	-0.06
Orphanides-Williams	0.07

Note: Values represent welfare gains/losses during transition from 70% to 80% debt-to-GDP ratio relative to remaining at the initial deterministic steady state.

Table 4 shows that the Orphanides-Williams rule yields the highest welfare, with a gain of 0.07% relative to the deterministic steady state (and 0.34% relative to the baseline Taylor rule). In contrast, failing to adjust the intercept ("No reaction") or eliminating interest rate smoothing results in significantly larger welfare losses, underscoring the importance of both responsiveness and gradualism in monetary policy.

The Taylor rule with an output gap response also performs well, producing only a small welfare loss of 0.06%. This suggests that incorporating real-side indicators can improve outcomes even in the absence of full history dependence. Overall, these findings underscore the importance of monetary policy frameworks that either incorporate persistent inflation responses or provide sufficient flexibility to adapt to fiscal-driven shifts in the natural rate.