Moral Hazard versus Liquidity and the Optimal Timing of Unemployment Benefits*

Rodolfo G. Campos† J. Ignacio García-Pérez‡ Iliana Reggio§

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Abstract

This paper shows that an unemployment insurance scheme in which unemployment benefits decrease over the unemployment spell allows to separately estimate the liquidity and moral hazard effects of unemployment insurance. We empirically estimate these effects using Spanish administrative data in a Regression Kink Design (RKD) that exploits two kinks in the schedule of unemployment benefits with respect to prior labor income. We derive a "sufficient statistics" formula for the optimal level of unemployment benefits that generalizes results by Chetty (2008) for the case in which unemployment benefits are allowed to vary over the unemployment spell. In our empirical application, we find that moral hazard effects dominate liquidity effects in Spain, and that the benefits of unemployment insurance are low relative to the costs, both at the beginning of the unemployment spell and after the sixth month in unemployment, when replacement rates are 10 percentage points lower. Our calibration implies that over the period 1992–2012 benefit levels in Spain were set too high and that a reduction would be welfare improving.

JEL classification: J64, H11.

Keywords: unemployment duration, liquidity, moral hazard, optimal unemployment insurance, sufficient statistics, regression kink design.

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†Banco de España, Alcalá 48, 28014, Madrid, Spain; email: rodolfo.campos@bde.es

‡Departamento de Economía, Universidad Pablo de Olavide, 41013 Sevilla, Spain. Phone +34954977975, Fax +34954349339; email: jigarper@upo.es

§Universidad Autónoma de Madrid, 28049 Cantoblanco (Madrid), Spain; email: iliana.reggio@uam.es

1 Introduction

The main rationale in favor of unemployment insurance is that it allows unemployed workers to smooth their consumption while unemployed. At the same time, unemployment insurance distorts the relative price of leisure and consumption, and therefore reduces the marginal incentive to search for a new job. From a normative point of view, unemployment insurance needs to trade off the consumption-smoothing benefits with the moral hazard costs induced by the provision of insurance.

Chetty (2008) shows that only part of the response of job search intensity to unemployment insurance is due to moral hazard, and that the remainder is a "liquidity effect": if financial markets are incomplete and unemployed workers are unable to borrow, then the unemployed will have a strong incentive to search for a job. If this is the case, then raising the level of unemployment benefits alleviates the incompleteness of financial markets by providing additional liquidity to unemployed workers. As shown by Chetty (2008), the relative size of the moral hazard and liquidity effects determines the optimal level of unemployment insurance.

The analysis by Chetty (2008) focuses only on unemployment benefits that are constant during the unemployment spell. However, in many real-world unemployment insurance schemes benefits do not remain constant during the unemployment spell. This is the case in Spain, where benefits decrease after an initial six-month period. A vast theoretical literature (e.g., Hopenhayn and Nicolini, 1997) shows that the timing of benefits matters, and finds that time-varying unemployment benefits are usually optimal. In contrast, in more recent work, Shimer and Werning (2008) show that when workers can borrow and save, then economic theory implies that a constant or nearly constant scheme is optimal.

In this paper we empirically address the optimality of unemployment benefits that vary over time. We use economic theory and the institutional details of unemployment insurance in Spain to show that the evolution of liquidity and moral hazard effects over the unemployment spell are identified given appropriate data. We then empirically estimate liquidity and moral hazard effects using a Regression Kink Design (RKD) that exploits kinks in the schedule of unemployment benefits with respect to prior labor income. Armed with the resulting estimates, we calculate optimal unemployment insurance levels for Spain using a "sufficient statistics" formula that generalizes that of Chetty (2008) to an environment in which unemployment benefits are allowed to vary over the unemployment spell.

The work closest to the objective of this paper is Kolsrud et al. (2018), which studies the dynamic aspect of unemployment insurance and the optimal timing of unemployment benefits both theoretically and empirically, using administrative data for Sweden. However, that paper does not attempt to separate moral hazard and liquidity effects à la Chetty (2008). Rather, it identifies the dynamic welfare benefits of unemployment insurance using consumption data (which they calculate as a residual). From a theoretical standpoint, the paper by Kolsrud et al. (2018) follows the line of Chetty (2006), who uses consumption expenditure data, rather than Chetty (2008), who uses labor market data exclusively. One of the drawbacks of using consumption expenditure is that it is necessary to assume a functional form for the utility function, including the level of risk aversion, and to restrict the ways in which utility differs between employed and unemployed workers.[1]

The main theoretical contribution of our paper is to provide a novel identification result of moral hazard and liquidity effects of unemployment insurance which relies exclusively on variation of unemployment benefits during an unemployment spell. This differs from the identification of the effects by Chetty (2008) and Landais (2015) whose applications are tailored to the US, where unemployment benefits are flat during the unemployment spell. With flat benefits the response of hazard rates to unemployment benefits does not contain enough information to separate moral hazard and liquidity effects; Chetty (2008) resorts to the use of lump-sum severance payments to approximate the liquidity effect and calculates the moral hazard effect as a residual whereas Landais (2015) uses changes in the length of the unemployment coverage periods as an approximation to changes in benefit levels in order to disentangle moral hazard and liquidity effects.

In contrast, in this paper we prove that when unemployment benefits vary over the unemployment spell, then moral hazard and liquidity effects can be identified directly from how hazard rates respond to unemployment benefits. Starting from the environment modeled by Kolsrud et al. (2018), in which benefits change over time, and exploiting intratemporal and intertemporal first order conditions, we show that moral hazard and liquidity effects respond differently to payments that occur at different times during an unemployment spell. The intuition behind this result is that, as time progresses, it becomes more likely that a worker will have exited unemployment, and therefore the moral hazard effect wears off and diminishes with respect to the liquidity effect. Benefits that vary over the unemployment spell are therefore composed of differing shares of moral hazard and liquidity components, and this different composition can be used to back out the unobservable moral hazard and liquidity effects that lead to an agent's optimal behavior.

The moral hazard and liquidity effects of unemployment insurance are identified from two estimates: the effect on the beginning-of-spell hazard rate of raising benefits in the first 6 months of the spell and the effect on this same hazard rate of raising benefits in the following 18 months of the spell.[2] An ideal experimental setting would have benefits increase in each of the two sub-periods for a random sample of the population and identify the effect by comparing treated with untreated workers. In lieu of this experimental design, we use a Regression Kink

[1] Another issue that arises when relying on consumption is that the theoretical model from which the sufficient statistics optimality result is derived is formulated for an individual agent whereas consumption expenditure is usually measured at the household level and difficult to assign to a particular household member.

[2] The model assumes that agents behave in a forward-looking manner. Forward-looking behavior has been documented for various optimal choices by Spanish households by Barceló and Villanueva (2016) and Campos and Reggio (2015), and by Rebollo-Sanz and Rodríguez-Planas (2020) for unemployment durations.

Design (RKD). In Spain, unemployment benefits are tied to labor income over the 180 working days prior to the onset of unemployment but are capped above and below at an amount that is a multiple of an index called IPREM. These caps induce kinks in the relationship between income and benefits. Using these kinks, and the methodology described by Card et al. (2015) and Nielsen et al. (2010), and also used by Landais (2015), we estimate the parameters of interest using a sample of Spanish administrative Social Security data (*Muestra Continua de Vidas Laborales*, MCVL).

Our estimates imply that the moral hazard effect is stronger than the liquidity effect in both periods. In our preferred specification, the response of the probability of finding a job to an increase in unemployment benefits in the first 6-month period is composed of 89% moral hazard effect and 11% liquidity effect. For the subsequent 18-months in which unemployment benefits are at a lower level, the moral hazard effect amounts to 79% and the liquidity effect to 21% of the impact on the hazard rate. When these numbers are compared to the cost of providing additional insurance according to the sufficient statistics formula, our estimates imply that benefits are too high in both periods.

The paper proceeds as follows. In Section 2 we set up the model, and derive our main identification result and the formula for optimal benefits. In Section 3 we describe our empirical strategy, and describe the context and the data used for our estimations. In Section 4 we report estimation results and apply our formula for optimal unemployment insurance for Spain. We conclude in Section 5.

2 Theory

The environment of our model is essentially that of Kolsrud et al. (2018) and differs from the model by Chetty (2008) and Landais (2015) because unemployment benefits are allowed to vary over time.

2.1 Environment

Choices, constraints and preferences

In the model time is discrete and indexed by $t = 0, 1, \ldots, T$. There is a continuum (mass 1) of agents indexed by i. In each period t, an agent i can be in one of two mutually exclusive states: employed or unemployed. At $t = 0$ the agent is initially unemployed. Starting at $t = 0$, the worker transitions into employment in the next period with a probability $h_{i,t+1} \in [0, 1]$ that depends on an individual choice of search effort $s_{i,t+1} \geq 0$. We assume that the function that maps search effort into the probability of becoming employed satisfies $h'_{i,t+1}(s) > 0$ and

$h''_{i,t+1}(s) \leq 0$. Employment is an absorbing state, so that—once employed—the probability of transitioning back into unemployment is zero. For later use, we denote the probability of being unemployed (the individual survival rate) at date $t \geq 1$ by $S_{i,t} = \prod_{j=1}^{t}(1 - h_{i,j}(s_{i,j}))$. The population survival rate is given by $S_t = \int S_{i,t}\, di$. Because all agents start out unemployed, $S_0 = S_{i,0} = 1$.

Agents have time-separable preferences with common discount factor $\beta \in (0, 1]$. They choose consumption (in the employed and in the unemployed state) and search effort (in the unemployed state). A random variable $\omega_{i,t}$ collects the history of all information relevant for the agent's decision problem up to time t. The initial condition $\omega_{i,0}$ is taken as given by agents in their optimization problem and may, in principle, differ across agents.[3] We use the notation $\mathbb{E}_t$ to denote the expectation of variables over possible values taken by the random variable ω_{t+1} conditional on information available at time t.

Unemployed agents have a period utility function given by $v_i^u(c, s)$, which depends positively on the level of consumption c and negatively on search effort s. We assume that this function is strictly concave in consumption and strictly convex in search effort. Like Chetty (2008), we assume that this function is separable in consumption and search effort, although not all the results that we report depend on that.[4] Employed agents have a period utility function $v_i^e(c)$ that is increasing and concave in consumption.[5]

Agents receive a wage w_t and pay lump sum taxes τ in periods in which they are employed. When unemployed, agents do not earn a wage and receive unemployment benefits $b_t \geq 0$ instead. Unemployment benefits depend on the length of the unemployment spell and become zero once unemployment benefits are exhausted. Agents, both employed or unemployed, borrow or save using assets $a_{i,t}$ that yield a net return r in the next period, which is constant and known beforehand. They face a borrowing limit: $a_{i,t} \geq \bar{a}$ and may also receive non-labor income y_t, which is independent of the employment state of the agent. The possibility of receiving non-labor income allows for a clean way of expressing moral hazard and liquidity effects, disentangling them from the intertemporal smoothing choice. At the cost of including an additional variable, we will show that the decomposition into liquidity and moral hazard effects continues to hold more generally than in the environment considered by Chetty (2008).[6]

Formally, the agent's problem can be expressed as a recursive problem using two value functions:

[3] For some or our results (Proposition 2) we will assume an ex-ante homogeneous population, with $\omega_{i,0} = \omega_0$.

[4] The result in Lemma 1, for example, does not require this assumption.

[5] Notice that our setup allows for the case in which the period utility function when employed is the same function as when unemployed evaluated at $s = 0$, but also the case where it is a completely different function.

[6] Notice that the model does not impose any particular value for y_t, or even that it is different from zero. The model just adds the *possibility* of receiving unconditional payments to the environment of Chetty (2008).

one for the unemployed state,

$$
\begin{aligned}
V^u_{i,t}(\omega_{i,t}) = \max v^u_i(c^u_{i,t}(\omega_{i,t}), s_{i,t+1}(\omega_{i,t})) \\
&+ h_{i,t+1}(s_{i,t+1}(\omega_{i,t}))\beta \mathbb{E}_t V^e_{i,t+1}(\omega_{i,t+1}) \\
&+ (1 - h_{i,t+1}(s_{i,t+1}(\omega_{i,t})))\beta \mathbb{E}_t V^u_{i,t+1}(\omega_{i,t+1})
\end{aligned} \tag{1}
$$

subject to

$$
a_{i,t+1}(\omega_{i,t}) = (1+r)a_{i,t}(\omega_{i,t-1}) + b_t - c^u_{i,t}(\omega_{i,t}) + y_t, \tag{2}
$$

$$
a_{i,t+1}(\omega_{i,t}) \geq \bar{a}, \tag{3}
$$

and one for the employed state,

$$
V^e_{i,t}(\omega_{i,t}) = \max v^e_i(c^e_{i,t}(\omega_{i,t})) + \beta \mathbb{E}_t V^e_{i,t+1}(\omega_{i,t+1}) \tag{4}
$$

subject to

$$
a_{i,t+1}(\omega_{i,t}) = (1+r)a_{i,t}(\omega_{i,t-1}) + w_t - \tau - c^e_{i,t}(\omega_{i,t}) + y_t, \tag{5}
$$

$$
a_{i,t+1}(\omega_{i,t}) \geq \bar{a}. \tag{6}
$$

Each agent i chooses the sequences $\{c^u_{i,t}(\omega_{i,t}), c^e_{i,t}(\omega_{i,t}), s_{i,t+1}(\omega_{i,t}), a_{i,t+1}(\omega_{i,t})\}$, with initial conditions $\omega_{i,0}$ and $a_{i,0}$, taking as given the parameters $\{w_t, b_t, y_t\}$, τ, r, $\bar{a}$.

Unemployment insurance scheme and the planner's problem

Unemployed agents receive benefits starting at $t = 1$.[7] They are set at $\bar{b}_1 > 0$ for the first B_1 periods in which the worker is unemployed and at $\bar{b}_2 > 0$ for the next B_2 periods; after that they revert to zero.[8] Therefore, unemployment insurance payments cover $B \equiv B_1 + B_2$ periods and the stream of duration-dependent unemployment benefits is described by the following T-dimensional vector:

$$
\mathbf{b} = (\underbrace{\bar{b}_1, \ldots, \bar{b}_1}_{1,\ldots,B_1}, \underbrace{\bar{b}_2, \ldots, \bar{b}_2}_{B_1+1,\ldots,B_1+B_2}, \underbrace{0, \ldots, 0}_{B_1+B_2+1,\ldots,T}). \tag{7}
$$

Following Kolsrud et al. (2018), and given the two-part benefit structure, the planner's budget constraint can be expressed in terms of population survival rates and the exogenous interest

[7] Agents start out unemployed at $t = 0$ and it takes at least a period to become employed. Therefore, benefits obtained at $t = 0$ would not have any incentive effect on search behavior in that period, which is why we assume that unemployment benefits start at $t = 1$.

[8] The model can be easily extended to benefits that take more than two possible positive values.

rate r as follows:

$$G(\mathbf{b},\tau) \equiv \tau \sum_{t=1}^{T}(1+r)^{-t}(1-S_t) - \bar{b}_1 \sum_{t=1}^{B_1}(1+r)^{-t}S_t - \bar{b}_2 \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}S_t = \bar{G}, \tag{8}$$

where $\bar{G}$ is an exogenous budget target.

The planner chooses unemployment insurance parameters $(\mathbf{b},\tau)$ to maximize the integral over expected utilities of agents at date 0 subject to the intertemporal budget constraint. Formally, the objective function of the planner is given by:

$$V^P(\mathbf{b},\tau) = \int V^u_{i,0}(\omega_{i,0})\, di + \lambda \left(G(\mathbf{b},\tau) - \bar{G}\right). \tag{9}$$

2.2 Liquidity and moral hazard

The response of agents to changes in the level of unemployment benefits in any future period can be decomposed into a liquidity and a moral hazard component. The following lemma proves that the standard decomposition by Chetty (2008, eq. 8) holds in our model.

Lemma 1 *The effect of increasing current or future unemployment benefits on the probability of exiting unemployment at date $t=1$ can be decomposed into a liquidity effect and a moral hazard component:*

$$\frac{\partial h_{i,1}(s_{i,1})}{\partial b_j} = \frac{\partial h_{i,1}(s_{i,1})}{\partial y_j} - \frac{\partial h_{i,1}(s_{i,1})}{\partial w_j}, \quad j \geq 1 \tag{10}$$

An increase in the unemployment benefit level in any (current or future) period lowers *current* search intensity through two channels. The first channel is the liquidity component. Raising b_j raises income and relaxes the budget constraint. This allows the agent to maintain a higher level of consumption while unemployed, and therefore reduces the urgency of finding a new job. The agent rationally lowers search effort for this reason, and the likelihood of finding a job decreases. The liquidity component is captured in (10) by the expression $\frac{\partial h_{i,1}}{\partial y_j} \leq 0$, which measures the effect of an unconditional payment y_j on the probability of exiting unemployment. The second component, $-\frac{\partial h_{i,1}}{\partial w_j} < 0$, is due to a standard moral hazard response; a higher benefit reduces the incentive to search for a job because it raises the value of being unemployed relative to that of working.

The decomposition in Lemma 1 does not require the assumption that utility is separable in consumption and search effort. An intuitive way of seeing why this must happen, is to take a perspective that is usual in the finance literature. Consider the question of how an agent responds to receiving an extra dollar j periods in the future when this dollar can be paid through instruments that pay off in different states of the world. Non-labor income y_j pays off in all possible states of the world; the unemployment benefit b_j pays off only in the unemployed state, and the wage w_j pays off only in the employed state. Because employment and unemployment exhaust all possibilities, increasing b_j by one dollar is equivalent to increasing y_j by one dollar while simultaneously reducing w_j by one dollar. Therefore, a forward-looking agent responds by choosing search effort in the same way when faced with either of these two alternatives.

In our formula, we express the liquidity component in terms of changes in the flow variable y instead of assets a, as is the case in the model of Chetty (2008). This shows that the decomposition is essentially an intratemporal relationship, in the sense that it is not directly affected by details on how consumption can be smoothed intertemporally. For example, it does not depend directly on the value of the interest rate r or the discount factor β. However, as the time horizon under consideration varies, an agent's optimal intertemporal choices imply that the magnitudes of the liquidity and moral hazard components follow a certain pattern. Indeed, as shown in the next lemma, an agent's optimal behavior implies that the moral hazard component wears off more rapidly than the liquidity component. This is a direct consequence of the fact that survival rates are decreasing. Lemma 2 proves that a result derived in the Online Appendix of Landais (2015) for the environment of Chetty (2008) also holds in our model.

Lemma 2 *If the borrowing constraint is slack in periods $t = 1, \ldots, 1 + j$, then*

$$\frac{\partial h_{i,1}}{\partial y_{j+1}} = \frac{\partial h_{i,1}}{\partial y_1}(1+r)^{-j}, \quad j \geq 0 \tag{11}$$

and

$$\frac{\partial h_{i,1}}{\partial w_{j+1}} = \frac{\partial h_{i,1}}{\partial w_1}\frac{S_{i,j+1}}{S_{i,1}}(1+r)^{-j}, \quad j \geq 0. \tag{12}$$

The result in (11) states that, if the borrowing constraint does not bind, then raising non-labor income y now, or j periods from now, impacts optimal search behavior in exactly the same way, if this income has the same present value. The agent's ability to smooth consumption intertemporally implies that the agent's choices cannot be improved upon by reallocating resources purely across periods. A key fact behind the result in (11) is that the payment of y is unconditional, i.e., does not depend on the realization of a particular state of the world.

In contrast, the relationship in (12) on the moral hazard component depends on an additional term related to the survival rates because wages are not an unconditional payment. An extra dollar in wages leads to a payment only in the states of the world in which the agent is employed. Therefore, the receipt of this dollar earlier or later is not a purely intertemporal reallocation. Raising the wage after j periods distorts the consumption-search choice only to the extent that the agent expects to still be unemployed. Because the probability of being unemployed decreases with the horizon j, as the agent has had more opportunities to reach the employed state, the moral hazard component weakens for periods that lie further in the future. The rate at which it weakens is given by $\frac{S_{i,j+1}}{S_{i,1}}$. This ratio equals period j's probability of being unemployed conditional on being unemployed in the period under consideration (period 1 in our case), as measured by the individual survival rates.

Whereas Lemma 1 decomposes the full impact of a change in benefits into liquidity and moral hazard components for a single period, in the two-level unemployment scheme we are considering benefits change in multiple periods simultaneously. By summing (10) over several periods, the liquidity and moral hazard *effects* of changes in $\bar{b}_1$ and $\bar{b}_2$ can therefore be expressed as sums of the per-period liquidity and moral hazard components:[9]

$$\frac{\partial h_{i,1}}{\partial \bar{b}_1} = \sum_{t=1}^{B_1} \frac{\partial h_{i,1}}{\partial b_t} = \underbrace{\sum_{t=1}^{B_1} \frac{\partial h_{i,1}}{\partial y_t}}_{LIQ_{i,1}} - \underbrace{\sum_{t=1}^{B_1} \frac{\partial h_{i,1}}{\partial w_t}}_{MH_{i,1}}, \tag{13}$$

$$\frac{\partial h_{i,1}}{\partial \bar{b}_2} = \sum_{t=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial b_t} = \underbrace{\sum_{t=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial y_t}}_{LIQ_{i,2}} - \underbrace{\sum_{t=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial w_t}}_{MH_{i,2}}. \tag{14}$$

Lemma 2, which states that the moral hazard component wears off more rapidly than the liquidity component, implies that the relative importance of the moral hazard effect will be larger when considering a raise of benefits at the start of the unemployment spell ($\bar{b}_1$), rather than later in the unemployment spell ($\bar{b}_2$). Because of this, it is reasonable to expect that liquidity and moral hazard effects can be disentangled in unemployment insurance schemes where unemployment benefits vary over time. We formally prove this result and derive the formulas for liquidity and moral hazard effects in Proposition 1. The proof is in the appendix. Following Chetty (2008) and Kolsrud et al. (2018), from now on, for simplicity, we focus on the special case in which $r = 0$ and denote durations $D_i = \sum_{t=1}^{T} S_{i,t}$, $D_{i,1} = \sum_{t=1}^{B_1} S_{i,t}$, $D_{i,2} = \sum_{t=B_1+1}^{B_1+B2} S_{i,t}$.[10]

Proposition 1 *If the borrowing constraint does not bind, then the expressions for the liquidity*

[9] We distinguish liquidity and moral hazard *components*, which refer to a single period, from liquidity and moral hazard *effects*, which refer to the decomposition related to changes in $\bar{b}_1$ and $\bar{b}_2$, affecting various periods simultaneously.

[10] The expressions for the general case with $r > 0$ are derived in the proof of the proposition as well.

and moral hazard effects are given by:

$$
\begin{aligned}
LIQ_{i,1} &= \sum_{t=1}^{B_1} \frac{\partial h_{i,1}}{\partial y_t} = \frac{B_1}{B_2 D_{i,1} - B_1 D_{i,2}} \left(D_{i,1} \frac{\partial h_{i,1}}{\partial \bar{b}_2} - D_{i,2} \frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\
MH_{i,1} &= \sum_{t=1}^{B_1} \frac{\partial h_{i,1}}{\partial w_t} = \frac{D_{i,1}}{B_2 D_{i,1} - B_1 D_{i,2}} \left(B_1 \frac{\partial h_{i,1}}{\partial \bar{b}_2} - B_2 \frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\
LIQ_{i,2} &= \sum_{t=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial y_t} = \frac{B_2}{B_2 D_{i,1} - B_1 D_{i,2}} \left(D_{i,1} \frac{\partial h_{i,1}}{\partial \bar{b}_2} - D_{i,2} \frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\
MH_{i,2} &= \sum_{t=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial w_t} = \frac{D_{i,2}}{B_2 D_{i,1} - B_1 D_{i,2}} \left(B_1 \frac{\partial h_{i,1}}{\partial \bar{b}_2} - B_2 \frac{\partial h_{i,1}}{\partial \bar{b}_1} \right),
\end{aligned}
\tag{15}
$$

The importance of Proposition 1 is that it proves that with two levels of unemployment insurance the unobservable liquidity and moral hazard effects can be identified solely from data on unemployment spells.[11] Given estimates of $\frac{\partial h_1}{\partial \bar{b}_1}$ and $\frac{\partial h_1}{\partial \bar{b}_2}$, an agent's liquidity and moral hazard effect can be computed from observable durations $D_{i,1}$ and $D_{i,2}$ and the known entitlement periods B_1 and B_2. We will describe how to estimate $\frac{\partial h_1}{\partial \bar{b}_1}$ and $\frac{\partial h_1}{\partial \bar{b}_2}$ in Section 3 but first we show how liquidity and moral hazard effects characterize optimal unemployment benefit levels.

2.3 Optimal unemployment benefits

We now turn to the normative implications of the model. The liquidity and moral hazard effects identified in the previous section characterize how changing unemployment benefit levels affects welfare of an individual worker and, coupled with information on the costs of providing unemployment insurance, they determine optimal unemployment insurance in a sufficient statistics formula. In the spirit of the model by Chetty (2008), we assume that agents are ex-ante homogeneous. This means that they are homogeneous (also in their preferences) at date $t = 0$ but experience idiosyncratic realizations of the individual random variable $\{\omega_{i,t}\}_{t=1}^{T}$. Because liquidity and moral hazard effects are determined at the start of an unemployment spell (they decompose the impact of future variables on the job-finding rate at the start of the unemployment spell), when all agents are identical, ex-ante homogeneity implies that liquidity and moral hazard effects are equal across agents. We can therefore drop the i subindex and express liquidity and moral hazard effects as LIQ_1 and MH_1, and LIQ_2 and MH_2. Population averages of individual durations are computed as $D = \int D_i \, di$, $D_1 = \int D_{1,i} \, di$, and

[11] As pointed out by an anonymous referee, almost all countries can be thought of as having a dual system of unemployment insurance once welfare programs, often called unemployment assistance, supporting long-term unemployed who exhausted their benefits, are taken into account. Therefore, another way of reading the paper is that it offers a way of jointly designing the unemployment insurance and assistance programs.

$D_2 = \int D_{i,2}\, di$.[12]

With this notation, the following proposition states our normative theoretical result on the optimality of unemployment benefits.

Proposition 2 *If agents are ex-ante homogeneous and $r = 0$, then optimality of $\bar{b}_1$ and $\bar{b}_2$ requires*

$$R_1 \equiv -\frac{LIQ_1}{MH_1} = \varepsilon_{D_1,\bar{b}_1} + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1}\varepsilon_{D_2,\bar{b}_1} + \frac{\tau D}{\bar{b}_1 D_1}\varepsilon_{D,\bar{b}_1}, \tag{16}$$

$$R_2 \equiv -\frac{LIQ_2}{MH_2} = \varepsilon_{D_2,\bar{b}_2} + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2}\varepsilon_{D_1,\bar{b}_2} + \frac{\tau D}{\bar{b}_2 D_2}\varepsilon_{D,\bar{b}_2}, \tag{17}$$

where $\varepsilon_{x,b}$ denotes the elasticity of a variable x with respect to b.

The formula in Proposition 2 combines the liquidity and moral hazard effects with high-level elasticities in order to empirically assess whether unemployment benefits are at their optimal levels in a "sufficient statistics" framework.[13] The liquidity and moral hazard effects are identified off the beginning-of-spell hazard rates using Proposition 1 whereas the elasticities can be estimated from unemployment durations. The ratios $R_k = -\frac{LIQ_k}{MH_k}$, for $k = 1, 2$, on the left hand side of the optimality conditions, capture the social marginal benefit of increasing unemployment insurance. If liquidity effects predominate over moral hazard effects, then insurance is more valuable. The right hand side captures social marginal costs of raising the level of unemployment insurance because of the ensuing increase in unemployment durations.[14] At the optimum, marginal benefits and marginal costs coincide.

The formula generalizes the result that Chetty (2008) derived for the case of a single benefit level, and a balanced budget. A single benefit level is obtained in our environment by setting $\bar{b}_1 = \bar{b}$ and $\bar{b}_2 = 0$. A balanced budget implies that $G(\mathbf{b}, \tau) = \bar{G} = 0$. Imposing these two conditions simplifies the condition for optimality to

$$R = -\frac{LIQ}{MH} = \varepsilon_{D_B,\bar{b}} + \frac{D}{T-D}\varepsilon_{D,\bar{b}}. \tag{18}$$

Chetty further assumes for simplicity that $\varepsilon_{D_B,\bar{b}} = \varepsilon_{D,\bar{b}}$. Imposing this additional equality leads to the result by Chetty (2008, eq. 14), according to which benefits are at their optimal level if and only if

$$R = -\frac{LIQ}{MH} = \frac{T}{T-D}\varepsilon_{D,\bar{b}}. \tag{19}$$

[12]Notice that ex-ante homogeneity also implies that all agents share the same ex-ante expected durations D_i, $D_{i,1}$, and $D_{i,2}$.

[13]The conditions in the proposition are necessary for an optimum. Sufficiency is obtained by requiring that the budget constraint $G(\mathbf{b}, \tau) = \bar{G}$ holds.

[14]As shown by Kolsrud et al. (2018), when there are multiple levels of unemployment benefits, the costs of raising unemployment benefits contain cross-period elasticities of the form $\frac{\bar{b}_{k'} D_{k'}}{\bar{b}_k D_k}\varepsilon_{D_{k'},\bar{b}_k}$ ($k' \neq k$), as is the case here.

2.4 Discussion on heterogeneity

The theoretical decomposition in Proposition 1 applies to each individual agent i and therefore holds for arbitrary degrees of heterogeneity in the population. Proposition 2, on the other hand, allows only for ex-post heterogeneity, as it assumes that agents are homogeneous from an ex-ante perspective. The assumption of ex-ante homogeneity may not seem too strong at first sight, as it allows for widely different final outcomes across the population, and also that the agents' dislike for search effort differs in all but the initial period. However, it does impose requirements that may not be immediately obvious, and which may not hold in practice; for example, the requirement that all agents start from the same initial conditions implies that they start with the same level of initial assets. This leads to the question of whether the assumption of ex-ante homogeneity can be relaxed in certain special cases.

An interesting special case in which the ex-ante homogeneity assumption can be dispensed with is an environment like that of Chetty (2008), with preferences that are separable in consumption and search effort and a deterministic linear relationship between search effort and the probability of finding a job. In such an environment, as we state in the following proposition, a quadratic specification of the disutility of search effort yields the same optimality conditions as Proposition 2 without requiring that all agents start out with the same initial conditions.

Proposition 3 *Assume that liquidity and moral hazard effects* $\{LIQ_k, MH_k\}_{k\in\{1,2\}}$ *are known,* $r = 0$*, and that* $\forall i$*:* $h_{i,t}(s) = s$*,* $v_i^e(c) = \hat{v}^e(c)$*,* $v_i^u(c,s) = \hat{v}^u(c) - \frac{1}{2}\psi s^2$*,* $\psi > 0$*. Then, as in Proposition 2, optimality of* $\bar{b}_1$ *and* $\bar{b}_2$ *implies* (16) *and* (17)*.*

There may also be other configurations of the environment and preferences that lead to the same optimality conditions, or configurations involving more heterogeneity that lead to different but still tractable expressions. For example, if agents differ in first-period job search disutility or have different search effort productivities, then the population-wide liquidity and moral hazard terms that are required for Proposition 2 would be calculated as the weighted average over their individual counterparts, where the weights would reflect the cross-sectional distribution of the parameters that govern this additional heterogeneity. Because the details depend on the type of heterogeneity that is assumed, we do not pursue this further, as it is beyond the scope of this paper.

As we now turn to the estimation of the objects of interest, the takeaway from this discussion is that our normative theoretical result concerning the characterization of optimal unemployment insurance is less general, as it depends on a certain degree of homogeneity in the population, than the positive result regarding the decomposition of benefit changes into liquidity and moral hazard effects.

3 Empirical implementation: strategy, context and data

3.1 Empirical objects of interest

We estimate the effect of variation in unemployment benefit levels on a number of outcome variables. To separate liquidity from moral hazard effects (Proposition 1), the variables of interest are $\frac{\partial h_1}{\partial \bar{b}_1}$ and $\frac{\partial h_1}{\partial \bar{b}_2}$: the effect of changing unemployment benefit levels in each period on the hazard of exiting unemployment at the beginning of an unemployment spell. To evaluate the normative theoretical result (Proposition 2), it is necessary to obtain estimates of the effect of $\bar{b}_1$ and $\bar{b}_2$ on D_1, D_2, and D: the expected unemployment duration while on benefits $\bar{b}_1$ and $\bar{b}_2$, and the total expected unemployment duration.

3.2 Empirical strategy

To estimate the effect of benefit levels on the variables of interest we exploit the piece-wise linear kinked relationship between pre-unemployment labor income and the level of unemployment benefits. We exploit two kinks that arise due to a change in replacement rates during the unemployment spell. This strategy, termed the Regression Kink Design (RKD), is a close relative of a regression discontinuity design, and has been used in the context of unemployment benefits by Landais (2015) for the US, and by Card et al. (2015) for Austria. One of the advantages of the RKD is that the source of variation in unemployment benefits is time-invariant. In contrast, empirical strategies that use changes in legislation over time, face the potential pitfall that changes in legislation might be endogenous to labor market conditions.

Regression kink design

In the RKD, Y is an outcome of interest, V is an observed "running variable" (labor income prior to the unemployment spell in our case) that affects Y through a smooth function $g(V)$, and $b(V)$ is the observed variable of interest (unemployment benefits), which is a deterministic and continuous function of V with a kink at $V = \bar{v}$. The relationship between these variables is described by a constant-effect additive model:

$$Y = \theta b(V) + g(V) + \epsilon. \tag{20}$$

The logic of the RKD is that, given smoothness of $g(V)$ and the kink in $b(V)$, if $b(V)$ affects Y, then there should also be a kink in the relationship between V and Y at the point $V = \bar{v}$. As shown by Card et al. (2015), the coefficient of interest θ can then be calculated from

$$\theta = \frac{\lim_{v_0 \to \bar{v}^+} \left.\frac{dE[Y|V=v]}{dv}\right|_{v=v_0} - \lim_{v_0 \to \bar{v}^-} \left.\frac{dE[Y|V=v]}{dv}\right|_{v=v_0}}{\lim_{v_0 \to \bar{v}^+} b'(v_0) - \lim_{v_0 \to \bar{v}^-} b'(v_0)}. \tag{21}$$

The expressions $v_0 \to \bar{v}^+$ and $v_0 \to \bar{v}^-$ respectively indicate that right-hand-side and left-hand-side limits are taken.

The numerator in (21) is the change in the slope in the conditional expectation function at the location of the kink and the denominator is the change in the slope of the deterministic function $b(V)$ at the kink. The value of the denominator does not need to be estimated; it is determined by the known administrative rule for calculating unemployment benefits as a function of prior labor income. The numerator, on the other hand, is estimated using a parametric specification of the form:

$$E[Y|V=v] = \alpha + \eta' X + \sum_{p=1}^{P} \gamma_p (v-\bar{v})^p + \sum_{p=1}^{P} \beta_p W (v-\bar{v})^p, \tag{22}$$

where Y and V are, as before, the outcome of interest and the running variable (always pre-unemployment labor income in our case), X stands for additional covariates, $\bar{v}$ is the level of the running variable at which the kink takes place, and W is a dummy variable that takes the value one for observations above the kink, and zero otherwise. This specification is estimated for observations with $|v-\bar{v}| \leq h$, where h is the bandwidth size. The numerator in (21) is captured by the coefficient β_1 in (22).

Unemployment benefits in Spain

In Spain, access to unemployment benefits requires that a worker has worked for at least 360 working days in the six-year period prior to becoming unemployed. Once unemployed, the worker is entitled to unemployment benefits for a period that ranges from 120 to 720 days, depending on the length of the worker's prior employment spells. To obtain the maximum entitlement of 720 days the worker must have worked during at least 2,160 days. These days are equivalent to six years in employment and they do not need to be consecutive. The level of benefits is based on labor earnings in the 180 working days (registered at the Social Security Administration) prior to the onset of unemployment. For the period we analyze, the level of benefits is set at 70% of prior labor income during the first six months in unemployment, and at 60% during the remainder of the period in which the worker is entitled to unemployment benefits.

Benefits are capped below and above by values b_{min} and b_{max} that depend on an index called IPREM, whose values are set by the government on a yearly basis. Minimum and maximum benefits are a function of IPREM and also of the worker having zero, one, or two or more dependents. A dependent is defined as someone who receives no income, lives with the person claiming unemployment, and is less than 26 years old, or older than 26 but with a disability degree greater than 33%. The level of unemployment benefits depends on prior labor income V according to:

$$b_k(V) = \begin{cases} b_{min} & \text{if } V \times r_k \leq b_{min} \\ V \times r_k & \text{if } b_{min} < V \times r_k \leq b_{max} \\ b_{max} & \text{if } V \times r_k > b_{max} \end{cases}, \qquad k = 1, 2, \tag{23}$$

where b_{min} and b_{max} depend on the calendar year and number of dependents, and $r_1 = 70\%$ and $r_2 = 60\%$ are the replacement ratios. As an example, in Figure 1 we plot unemployment benefits as a function of prior labor income V for an individual without dependents using the value of the IPREM in 2011.

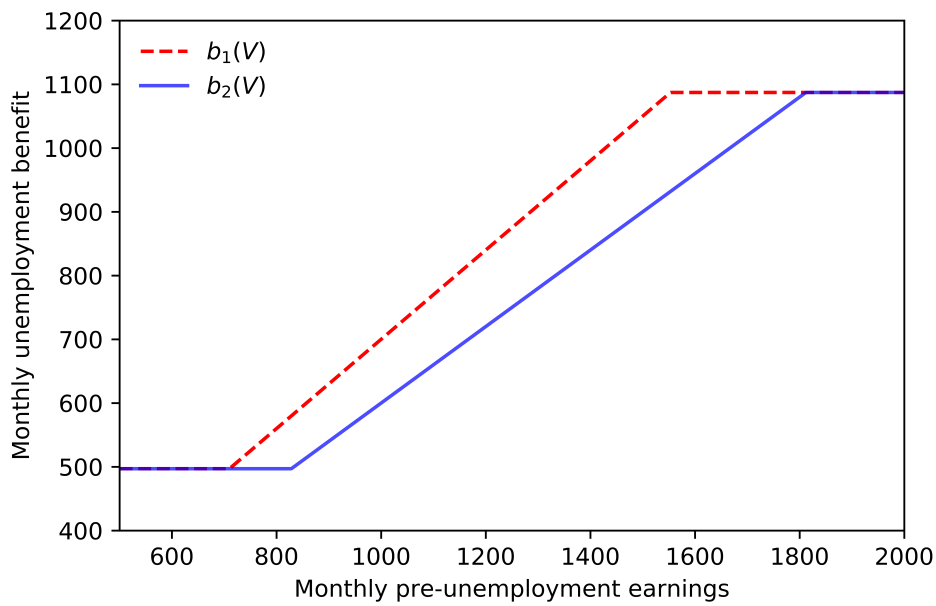

Figure 1: Unemployment Benefits as a function of pre-unemployment earnings in Spain

Note: We calculate unemployment benefits for an individual with no dependents using the value of the IPREM in 2011. The dashed red line corresponds to the level of unemployment benefits in the first six months of unemployment. The blue line corresponds to unemployment benefits in the remainder of the unemployment spell. Both unemployment benefits and pre-unemployment earnings are expressed in euros.

The figure shows that the horizontal difference at b_{max} is larger than at b_{min}. Also, the number of workers in the sample at or close to the maximum kink is larger than at the minimum kink. For these reasons, in our estimations we will focus only the kink at b_{max}.

Regression kink design in the presence of two kinks

Our application differs from a classical RKD design with just one kink. In our application, there are two variables of interest, $b_1(V)$ and $b_2(V)$, with kinks at two different points, $\bar{v}_1 = \frac{b_{max}}{0.7}$ and $\bar{v}_2 = \frac{b_{max}}{0.6}$, and therefore two parameters to be estimated: θ_1 and θ_2. The relationship between these variables and the outcome Y is described by

$$Y = \theta_1 b_1(V) + \theta_2 b_2(V) + g(V) + \epsilon. \tag{24}$$

Because there are two kinks, observations may fall into three separate cases. An observation is defined as an unemployment spell. Unemployment spells belong to one of three mutually exclusive groups. The first group contains unemployment spells with pre-unemployment earnings below the first kink, the second group those with pre-unemployment earnings above the first kink but below the second kink, and the third group those with pre-unemployment earnings above the second kink. Formally:

$$Y = \begin{cases} \theta_1 r_1 V + \theta_2 r_2 V + g(V) + \epsilon & \text{if } V \leq \frac{b_{max}}{r_1} \\ \\ \theta_1 b_{max} + \theta_2 r_2 V + g(V) + \epsilon & \text{if } \frac{b_{max}}{r_1} < V \leq \frac{b_{max}}{r_2} \\ \\ \theta_1 b_{max} + \theta_2 b_{max} + g(V) + \epsilon & \text{if } V > \frac{b_{max}}{r_2} \end{cases} \tag{25}$$

The parameters θ_1 and θ_2 can be recovered from comparing the slopes for different groups of unemployment spells. The derivative of the outcome variable with respect to the running variable for each group is:

$$\frac{\partial Y}{\partial V} = \begin{cases} \theta_1 r_1 + \theta_2 r_2 + g'(V) & \text{if } V \leq \frac{b_{max}}{r_1} \\ \theta_2 r_2 + g'(V) & \text{if } \frac{b_{max}}{r_1} < V \leq \frac{b_{max}}{r_2} \\ g'(V) & \text{if } V > \frac{b_{max}}{r_2} \end{cases} \tag{26}$$

Subtracting the expression for the slopes for neighbouring groups leads to expressions composed of each of the parameters of interest θ_k multiplied by the corresponding replacement ratio r_k:

$$\begin{aligned}
\left.\frac{\partial Y}{\partial V}\right|_{\text{below kink 1}} - \left.\frac{\partial Y}{\partial V}\right|_{\text{above kink 1, below kink 2}} &= \theta_1 r_1 \\
\left.\frac{\partial Y}{\partial V}\right|_{\text{above kink 1, below kink 2}} - \left.\frac{\partial Y}{\partial V}\right|_{\text{above kink 2}} &= \theta_2 r_2
\end{aligned} \tag{27}$$

The parametric specification adapted for our application with two kinks is

$$E[Y|V=v] = \alpha + \eta' X + \sum_{p=1}^{P} \gamma_p (v-k_1)^p + \sum_{j=1}^{2}\sum_{p=1}^{P} \beta_{jp} W_j (v-k_j)^p, \tag{28}$$

where Y and V are, as before, the outcome of interest and the running variable (pre-unemployment labor income), P is the order of the polynomial, X stands for additional covariates, and W_j is equal to 1 if pre-unemployment income is above kink j ($v \geq k_j$). The numerator in (21) corresponds to the coefficients β_{j1}.

Using the parametric specification, the parameters of interest are recovered from the coefficients β_{11} and β_{21} by first computing the differences between the slopes, and then equating the results to the expressions in (27).

Comparing the slopes leads to:

$$\begin{aligned}
\left.\frac{\partial Y}{\partial V}\right|_{\text{below kink 1}} - \left.\frac{\partial Y}{\partial V}\right|_{\text{above kink 1, below kink2}} &= -\beta_{11} \\
\left.\frac{\partial Y}{\partial V}\right|_{\text{above kink 1, below kink2}} - \left.\frac{\partial Y}{\partial V}\right|_{\text{above kink 2}} &= -\beta_{21}
\end{aligned} \tag{29}$$

The parameters of interest can then be found by combining (27) and (29):

$$\begin{aligned}
\theta_1 r_1 &= -\beta_{11} \\
\theta_2 r_2 &= -\beta_{21},
\end{aligned} \tag{30}$$

and solving these equations for known values $r_1 = 0.7$ and $r_2 = 0.6$.

As shown in (28) we estimate a single equation with two kinks whereas all the empirical RKD literature that we are aware of applies the methodology to a single kink. In a Monte Carlo exercise contained in the appendix, we study whether estimating each kink separately would also have been a valid strategy. Our results imply that estimating one RKD per kink in an environment with two kinks, or simultaneously estimating both kinks, as we do, leads to equally precise estimates for the parameters of interest.

The assumptions made by the RKD are that the direct effect of the running variable (pre-unemployment earnings) on the outcome of interest is smooth and that unobserved heterogeneity does not change discontinuously at the kink in the running variable. In our case, manipulation of the running variable would imply that the worker is able to control labor earnings in the 180 working days prior to becoming unemployed. If this manipulation occurs, then it would imply a concentration of workers around the kinks. In Section 4 we discuss the validity of the RKD design in our case in detail.

3.3 Data

We use data from the Continuous Sample of Working Histories (Muestra Continua de Vidas Laborales, MCVL). This is a dataset based on administrative records made available by the Spanish Social Security Administration. Each wave contains a random sample of 4% of all individuals who had some type of contact with the Social Security Administration, either by working or by receiving a contributory benefit (such as unemployment insurance, permanent disability insurance, old-age subsidies, etc.) during at least one day in the year the sample is selected.

The MCVL reconstructs the labor market histories of individuals in the sample back to 1967 (although data on earnings are available only starting in 1980). Moreover, this dataset has a yearly longitudinal structure, meaning that an individual who is present in any wave and remains registered with Social Security (registration with the Social Security Administration is required to receive unemployment benefits) stays in the sample in subsequent waves. In addition, in each wave the sample is refreshed with new entrants to guarantee that the sample is representative of the population. We use eleven waves (2005-2015) in our estimations. Starting in 2005 ensures that only workers who were not registered with the Social Security Administration in the period 2005-2015 would be excluded from this sample by design.

Information is available on the entire employment, non-employment and pension history of workers, including the exact duration of each employment, non-employment and disability or retirement pension spell. The data contain several variables that describe the characteristics of the job, such as the sector of activity, type of contract, number of hours, and qualification requirements. The data also contain information on some personal characteristics, such as age, sex, nationality, and level of education. Periods of non-employment are identified using information on the dates in which the firm does not pay social security contributions for the worker. Non-employment spells during which the worker receives unemployment benefits are clearly identified as unemployment spells. Given that, for each worker, the dataset contains all social security payments made by firms, we compute the exact entitlement to unemployment benefits for each unemployment spell and the level of unemployment benefits also for workers who switched jobs before becoming unemployed.

In our estimations we restrict the sample to unemployment spells starting between January 1, 1992 and July 14, 2012, because the calculation of unemployment benefits changes after that latter date. For unemployment spells during this period, the institutional framework for calculating unemployment benefits remained constant with a replacement ratio of 70% in the first six months and 60% during the subsequent 18 months of an unemployment spell. We restrict our attention to complete unemployment spells after full-time employment and only for individuals who had most jobs in the general regime. We further restrict the sample to individuals who are aged between 30 and 50.[15]

Table 1 contains descriptive statistics of the unemployment spells that we use in the regressions. There are 61,795 unemployment spells in our base sample. We report the mean and standard deviation of duration-related variables, earnings, and some additional variables used as covariates in the estimations. On average, non-employment spells in the sample last about 299 days, 130 days in the first six-month period of unemployment, and an extra 162 days in the second 18-month period. In addition to observing long durations of unemployment, we also observe an important fraction of unemployed individuals exhausting their benefits (25% of the spells last the maximum possible duration). Around 46% of the unemployed exit unemployment in the first six months. Average pre-unemployment earnings are EUR 1,819 and average unemployment benefits hover around 1,138 in the first period and 1,040 in the second period.[16] The average age is 39 years, 69% of the sample consists of males, and in 52% of the unemployment spells, workers held a permanent contract in their prior job.

4 Results

4.1 Graphical evidence

The key identifying assumption in an RKD is the existence of a smooth relationship between the running variable and the dependent variable. This assumption is less likely to hold if there is a discontinuity in the density of the running variable around the kink locations. In Figure 2 we plot the probability density function of pre-unemployment earnings at both kink points. The plot suggests no manipulation of earnings at the kink points and a smooth relationship. We normalize earnings so that in both graphs the kink is located at one. Both figures also include the results of the usual test for discontinuities proposed by McCrary (2008). The test results suggest that the assumption of no manipulation of the running variable at the kinks is satisfied.[17]

The second testable assumption is that the conditional distribution function of predetermined variables is smooth at the kinks. We plot the relationships between pre-unemployment earnings

[15] We exclude workers older than 50 because in Spain they are eligible for subsidies that provide incentives to stay out of the labor force until they can legally retire.

[16] All monetary values are expressed in 2011 constant euros.

[17] The implementation of this test is based on the tests of manipulation of the running variable for RD designs by McCrary (2008), which is implemented also for the case of the RKD by Landais (2015).

Table 1: Descriptive statistics: spells in regression sample

	Mean	SD
Duration		
Entitlement	662.85	(60.51)
Total NE duration	299.24	(257.47)
Duration Period 1	129.96	(63.46)
Duration Period 2	162.16	(203.73)
Exhaustion	0.25	(0.43)
Exit during period 1	0.46	(0.50)
Earnings		
UB period 1	1,138.13	(133.06)
UB period 2	1,039.48	(161.80)
Pre-unemployment Earnings	1,818.89	(382.40)
Covariates		
Age	38.46	(5.87)
Male	0.69	(0.46)
Highest job qualification	0.10	(0.30)
Lowest job qualification	0.22	(0.41)
Permanent Contract	0.52	(0.50)
Obs.	61,795	

Note: Entitlement is the number of days that a worker is entitled to receive unemployment benefits. Total NE duration is the number of days in non-employment. Duration Period 1 corresponds to the first six months of the unemployment spell, and Period 2 to the subsequent 18 months. Exhaustion is a dummy variable that takes the value one if the worker exhausts unemployment benefits. Exit during period 1 is a dummy variable that takes the value one if the unemployment spell ends during the first six months. UB denotes unemployment benefits. Pre-unemployment earnings are defined as average monthly earnings in the previous 180 working days. Age is computed at the beginning of the unemployment spell. All monetary values are expressed in real terms in constant 2011 euros.

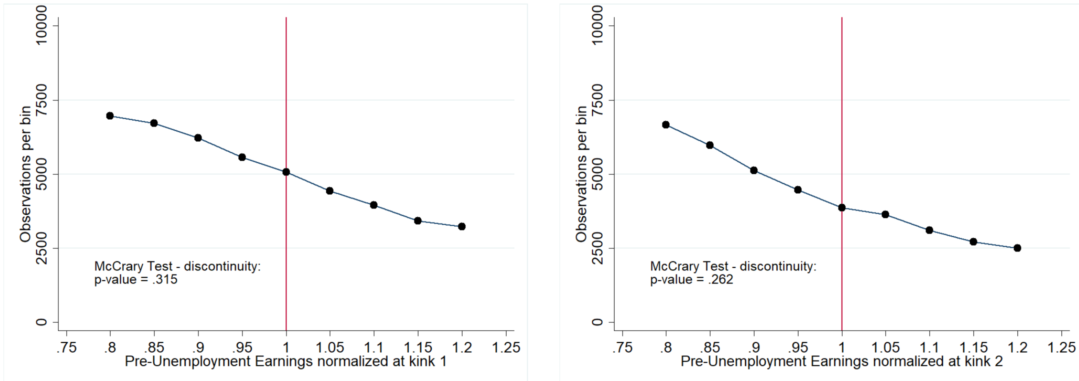

Figure 2: Frequency distribution of pre-unemployment earnings around the kinks

Notes: The figures show the frequency distribution of pre-unemployment earnings normalized at each of the kinks. These figures graphically show the smoothness of the distribution of the running variable. We include the p-value for a McCrary test. The null hypothesis of no discontinuity is not rejected at the usual confidence levels.

and a set of predetermined variables: age at the time of unemployment, being male, living in a region with a high unemployment rate, and having a job that requires low qualification. The graphs in Figure 3 show average values of each predetermined characteristic for each bin of the running variable, and provide graphical evidence on the smoothness in the relationship between these covariates and pre-unemployment earnings, with no jumps at any of the kinks.[18]

We also formally test for changes in the slope of these relationships. The predetermined variables on which we run tests are the following: age at time of unemployment, being male, living in a large province (with a population of more than 2 million), having a job that requires only low qualification, and having a job that requires high qualification. In Table 2 we present the p-values for the null hypothesis that there is no change in the slopes around the kinks. There is no evidence of significant changes in the slope at the kinks for the relationships between pre-unemployment earnings and the predetermined variables.

[18]The dataset is not very rich in terms of predetermined variables. We graph only some variables for brevity, although the picture is similar when we use different geographical or qualification variables.

Table 2: RKD estimations on several predetermined variables

	Age	Male	Large Regions	Low Qualification	High Qualification
			Bandwidth 250		
Kink 1	0.708	0.059	0.422	0.543	0.932
Kink 2	0.769	0.954	0.426	0.466	0.502
			Bandwidth 350		
Kink 1	0.412	0.250	0.229	0.335	0.527
Kink 2	0.161	0.295	0.425	0.619	0.703
			Bandwidth 450		
Kink 1	0.544	0.133	0.721	0.582	0.791
Kink 2	0.083	0.581	0.646	0.819	0.448

Note: Using each predetermined variable as the dependent variable and for different bandwidths, we estimate the baseline equation and test whether there is a change in the slope of the relationship between each variable and pre-unemployment earnings for both kinks. Each value in the table shows the p-value corresponding to the null hypothesis of no change in the slope.

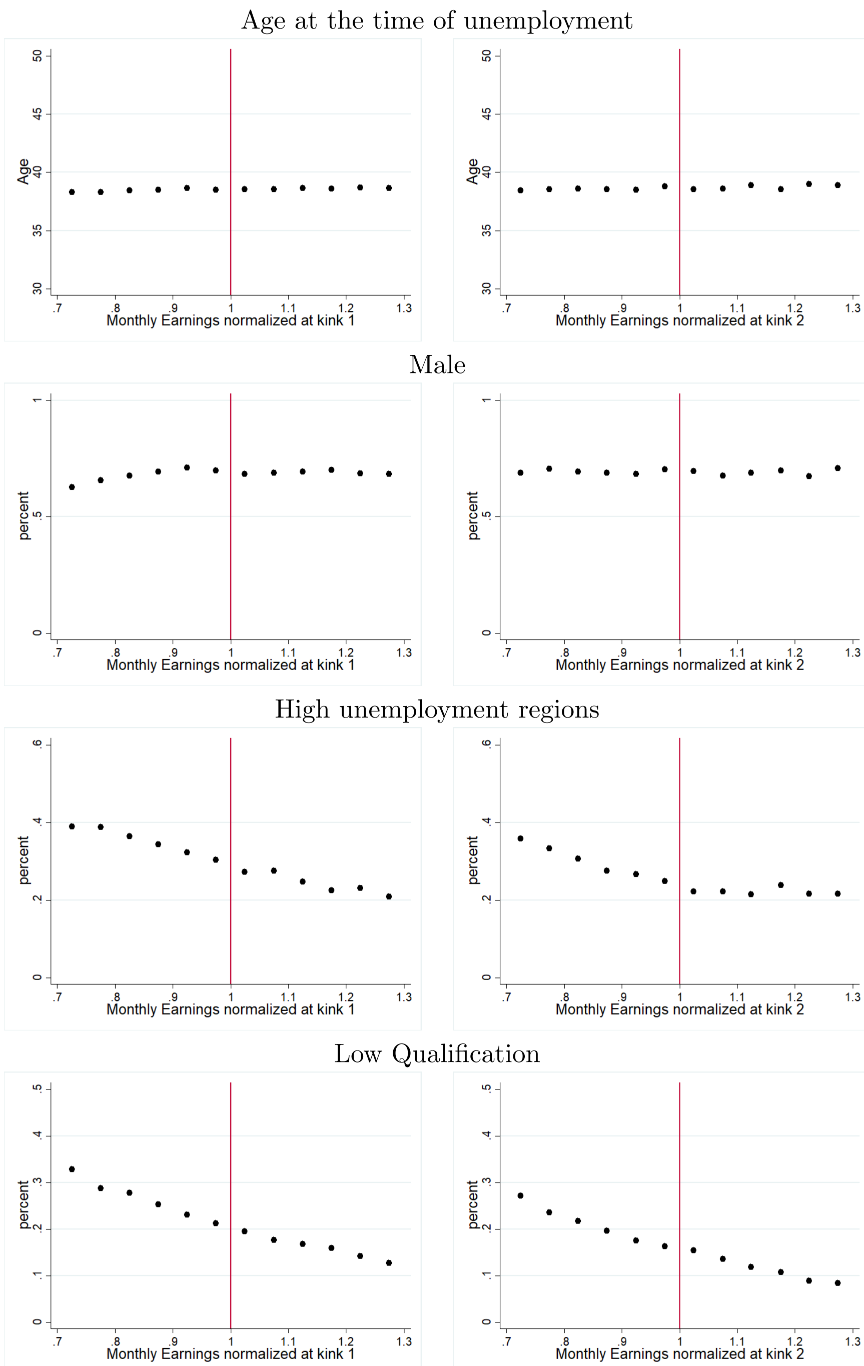

Figure 3: Pre-unemployment earnings and predetermined variables

Note: Each figure shows the mean values of the corresponding variable in each bin of pre-unemployment earnings normalized at each kink. These graphs give a visual validation of the assumption of smoothness of predetermined variables around the kinks.

Finally, we provide preliminary and visual evidence of changes in the slopes in the relationship between unemployment duration and pre-unemployment earnings in Figure 4. The figure contains four graphs, one for each outcome variable of interest: the probability of leaving unemployment in the first period, unemployment duration in the first period, unemployment duration in the second period, and total non-employment duration. We show the fit of a linear regression between each outcome and pre-unemployment earnings. These graphs provide visual evidence of different slopes in the relationship between earnings and each outcome variable around the kinks. Although the magnitude of these changes appears to be small, the differences in slopes go in the expected direction.

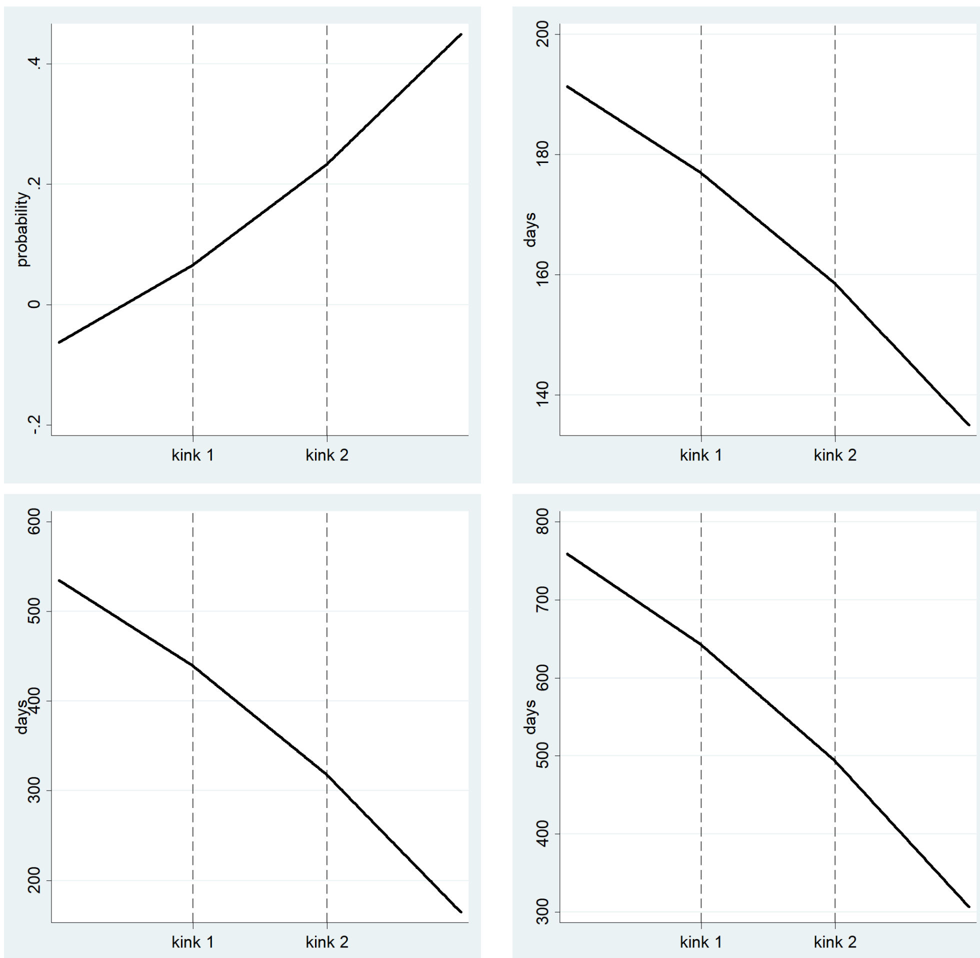

Figure 4: Unemployment duration and pre-unemployment earnings around the kinks

Note: Figures shows predicted values from linear regressions of each outcome variable on pre-unemployment earnings.

4.2 Estimation results

We present estimates of the baseline specification in (28) for a quadratic choice of the polynomial $g(\cdot)$ and for a bandwidth $h = 450$.[19] Control variables consist of year and month dummies, age at the time of becoming unemployed, and this age squared, a dummy variable for being male, having a permanent contract in the previous job, the number of prior unemployment spells, the number of the current spell, qualifications of the job, and regions. Results for the variables of interest are presented in Table 3. We transform the coefficients obtained in the regressions into the marginal impact of increasing benefits in each one of the two periods on each outcome according to the formula in (30): θ_1 represents the impact on any dependent variable of increasing benefits in the first six-months period, and θ_2 the impact of increasing benefits in the second period.[20]

The probability of exiting unemployment in the first six-months (our measure for h_1) decreases with higher unemployment benefits. Coefficients in first column are multiplied by 100, so that an EUR 100 increase in $\bar{b}_1$, the level of unemployment benefits in the first period, implies a decrease of around 1.4 percentage points in h_1. In turn, an EUR 100 increase in $\bar{b}_2$ implies a decrease of around 2.0 percentage points in h_1. The second column in Table 3 shows that unemployment duration in the first six months also increases with unemployment benefits: D_1 increases on average by 1.4 days per EUR 100 increase in $\bar{b}_1$ and by 2.1 days per EUR 100 increase in $\bar{b}_2$. Unemployment duration in the second period, D_2, increases on average by about 10 and 14 additional days per EUR 100 increase in $\bar{b}_1$ and $\bar{b}_2$. Finally, total non-employment duration D increases by around 12 and 17 additional days per EUR 100 increase in unemployment benefits in periods 1 and 2.

[19]In Appendix B we provide robustness checks for our main results. We analyze the robustness to other bandwidth choices, to the choice of the polynomial order, and to the exclusion of covariates. Main results do not vary with the bandwidth choice, and as expected, precision increases with the size of the bandwidth. In particular, the relative importance of liquidity and moral hazard effects remains remarkably constant. We also include placebo and permutation tests that support the validity of the RKD in our case

[20]Note that the dependent variable in the first column in Table 3 is in both cases a dummy variable that takes the value one if the worker exits unemployment in the first six months (Period 1) and zero otherwise. Both estimations use all workers in the sample around each kink regardless the actual duration of the unemployment spell. Therefore these estimates are not affected by selection bias.

Table 3: RKD estimates on several outcomes

	(1) Exit in period 1 h_1	(2) Duration period 1 D_1	(3) Duration period 2 D_2	(4) Non-employment duration D	(5) MH	(6) Optimal
$\frac{\partial}{\partial \bar{b}_1}$	-0.014**	0.014*	0.096***	0.121***	89%	Too high
	(0.007)	(0.009)	(0.028)	(0.035)		
$\frac{\partial}{\partial \bar{b}_2}$	-0.020**	0.021*	0.142***	0.168***	79%	Too high
	(0.009)	(0.011)	(0.036)	(0.045)		
Observations	61,795	61,795	61,795	61,795		

Note: All estimations include controls for year and month dummies, age (at the time of becoming unemployed) and age squared, a dummy variable for being male, a dummy for having a permanent contract in the previous job, dummies for the qualifications of the job, for the number of the unemployment spell, and dummies for regions. Duration in each period is measured as days in spent unemployed in each period. Total duration is days in non-employment. Coefficients are transformed in order to obtain the values of interest. The value for MH represents the relative importance of the moral hazard effect, and Optimal shows if unemployment benefits are too high or too low with respect to optimal levels. The period is 1992–2012 and workers are between 30 and 50 years old.

4.3 Liquidity and moral hazard effects in the estimation sample

The results reported in Table 3 yield an estimate of $\frac{\partial h_1}{\partial \bar{b}_1} = -0.014$ and $\frac{\partial h_1}{\partial \bar{b}_2} = -0.020$. Using the formulas in Proposition 1, these effects can be separated into a liquidity effect and a moral hazard effect.[21] We define a period as lasting six months (180 days). With this convention, $B_1 = 1$ (180 days) and $B_2 = 2.67$ (using a weighted average of entitlements in the sample).[22] We calculate the durations D_1 and D_2 for the sample of spells used in the main regression and set $D_1 = \frac{131.5}{180} = 0.73$ and $D_2 = \frac{163.5}{180} = 0.91$. Plugging these values into the formulas of Proposition 1, we find that

$$\frac{\partial h_1}{\partial \bar{b}_1} = -0.014 = \underbrace{-0.0016}_{LIQ_1} - \underbrace{0.0128}_{MH_1} \tag{31}$$

and

$$\frac{\partial h_1}{\partial \bar{b}_2} = -0.020 = \underbrace{-0.004}_{LIQ_2} - \underbrace{0.016}_{MH_2} \tag{32}$$

Over the first six-month period after the onset if unemployment, the liquidity effect accounts for 11% of the total effect whereas the moral hazard effect accounts for 89%. Over the period during which unemployment benefits are at $\bar{b}_2$, liquidity and moral hazard effects respectively account for 21% and 79% of the total response. In consequence, the ratios of liquidity to moral hazard effects, which are needed for the normative results of Proposition 2, are estimated at $R_1 = \frac{11\%}{89\%} = 0.13$ and $R_2 = \frac{21\%}{79\%} = 0.27$.

In comparison, Chetty (2008) finds that the liquidity effect accounts for 60% of the total effect. His estimated ratio of liquidity to moral hazard effects is therefore $R = \frac{60\%}{40\%} = 1.5$. Landais (2015) reports a lower ratio of liquidity to moral hazard effects of $R = 0.9$, implying that approximately 47% of the total effect corresponds to the liquidity effect. Our estimates imply larger moral hazard effects for the case of Spain than for the United States. Our estimates for R_1 and R_2 are very close to the consumption-based value of unemployment insurance estimated for Spain by Campos and Reggio (2020), who report values ranging from 0.163 to 0.237 for coefficients of relative risk aversion between 2 and 3.

The results in Table 3 also yield estimates on the fiscal cost of raising unemployment benefits. The fiscal externalities of raising $\bar{b}_1$ are determined by $\varepsilon_{D_1,\bar{b}_1} = 0.14$, $\varepsilon_{D_2,\bar{b}_1} = 0.76$, and

[21] To assess the validity of the assumption that the budget constraint is not binding, we conduct the slackness test proposed by Landais (2015). Using the RKD design, we estimate the effect of unemployment benefits in period 2 on the probability of exiting unemployment in the 180 days after exhaustion of unemployment benefits conditional on still being unemployed. The estimated coefficient on second-period benefits is of the expected sign and significantly different from zero: -0.009** (0.004). This suggests that agents are not hand-to-mouth in the last period and that they can still transfer resources across periods.

[22] For a maximum entitlement of 720 days, $B_1 + B_2 = 180$ days/180 days + 540 days/180 days = 720 days/180 days = 3.

$\varepsilon_{D,\bar{b}_1} = 0.42$. In turn, the fiscal externalities of raising $\bar{b}_2$ are related to the estimated elasticities $\varepsilon_{D_1,\bar{b}_2} = 0.21$, $\varepsilon_{D_2,\bar{b}_2} = 1.13$, and $\varepsilon_{D,\bar{b}_2} = 0.58$. The elasticities for total duration in Spain are in line with the value of $\varepsilon_{D,\bar{b}} = 0.5$ commonly assumed for the United States based on the survey by Krueger and Meyer (2002). For Spain, Rebollo-Sanz and Rodríguez-Planas (2020) find an elasticity of unemployment duration to the replacement rate ($\varepsilon_{D,r}$) of 0.86, although for a different period. For Sweden, Kolsrud et al. (2018) estimate $\varepsilon_{D,\bar{b}} = 1.53$, $\varepsilon_{D_1,\bar{b}} = 1.32$, and $\varepsilon_{D_2,\bar{b}} = 1.62$ for a joint increase in $\bar{b}_1$ and $\bar{b}_1$ and, using 2001 data, $\varepsilon_{D,\bar{b}_2} = 0.68$, $\varepsilon_{D_1,\bar{b}_2} = 0.60$, and $\varepsilon_{D_2,\bar{b}_2} = 0.59$. Although there are differences in context and time, the comparison with these other studies suggests that our estimates for the elasticities of durations are in a plausible range.

4.4 Optimal unemployment insurance: calibration for Spain

Armed with our estimates we now attempt to shed light on whether $\bar{b}_1$ and $\bar{b}_2$ are set at their optimal levels. Results for hazard rates yielded the estimates required for R_1 and R_2. These numbers need to be compared to the right hand side of the expression in Proposition 2. As Chetty (2008), we consider the case in which the budget is balanced. Substituting $\tau D = \bar{b}_1 D_1 + \bar{b}_2 D_2$ into (16) and (17) yields the following expressions:

$$R_1 = -\frac{LIQ_1}{MH_1} = \varepsilon_{D_1,\bar{b}_1} + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1}\varepsilon_{D_2,\bar{b}_1} + \frac{D}{T-D}\left(1 + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1}\right)\varepsilon_{D,\bar{b}_1}, \tag{33}$$

$$R_2 = -\frac{LIQ_2}{MH_2} = \varepsilon_{D_2,\bar{b}_2} + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2}\varepsilon_{D_1,\bar{b}_2} + \frac{D}{T-D}\left(1 + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2}\right)\varepsilon_{D,\bar{b}_2}. \tag{34}$$

Assuming that in the long term the ratio of time spent in unemployment to time spent working is $\frac{D}{T-D} = 0.10$, and given the estimated elasticities, the right hand side for the first period is calculated at 1.09, of which 1.00 is due to the rise in expected benefit payments and, the remainder, 0.09 is the drop in expected revenue arising from an increase in $\bar{b}_1$. For the second period, the expected marginal cost of raising unemployment benefits is estimated at 1.42, of which 1.31 is due to the expected rise in payments and 0.11 is due to the fall in revenues:

$$R_1 = 0.13 < 1.09 = \underbrace{\varepsilon_{D_1,\bar{b}_1} + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1}\varepsilon_{D_2,\bar{b}_1}}_{1.00} + \underbrace{\frac{D}{T-D}\left(1 + \frac{\bar{b}_2 D_2}{\bar{b}_1 D_1}\right)\varepsilon_{D,\bar{b}_1}}_{0.09} \tag{35}$$

and

$$R_2 = 0.27 < 1.42 = \underbrace{\varepsilon_{D_2,\bar{b}_2} + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2}\varepsilon_{D_1,\bar{b}_2}}_{1.31} + \underbrace{\frac{D}{T-D}\left(1 + \frac{\bar{b}_1 D_1}{\bar{b}_2 D_2}\right)\varepsilon_{D,\bar{b}_2}}_{0.11}. \tag{36}$$

Given these point estimates for Spain, marginal costs appear to clearly exceed the marginal benefit of raising unemployment benefits, both for $\bar{b}_1$ and $\bar{b}_2$.

The ratios used in our analysis are constructed using the point estimates in Table 3. In order to incorporate the uncertainty from those estimations, we bootstrap standard errors using 5,000 replications to obtain the empirical distribution for R_1 and R_2. Using these empirical distributions, we test the hypothesis that R_k is equal to the right hand side of the expression in Proposition 2, against the alternative that R_k is lower (implying that optimal $\bar{b}_k$ is lower). After allowing for this uncertainty, our calibration for Spain implies that over the period 1992–2012 benefit levels were set too high for both periods and that a reduction would be welfare improving. This finding agrees qualitatively with the consumption-based results for Spain reported by Campos and Reggio (2020).

A statistical extrapolation

To assess the approximate distance between actual and optimal unemployment benefits we perform a statistical extrapolation of the optimality conditions in (33) and (34) by assuming that the estimated parameters $\frac{\partial h_1}{\partial \bar{b}_k}$, $\frac{\partial D_1}{\partial \bar{b}_k}$, $\frac{\partial D_2}{\partial \bar{b}_k}$ remain constant over the range of possible benefit levels $\bar{b}_1$ and $\bar{b}_2$. In other words, the statistical extrapolation substitutes the possibly non-linear relationship between benefit levels the variables of interest (first-period hazard rates and unemployment durations) by a first-order (linear) approximation.

The sufficient statistics approach is specifically tailored to assessing whether certain policy is locally optimal, rather than to exploring the effects of counterfactual policy alternatives. Statistical extrapolations are sometimes used to give a rough indication of the distance that separates policies in place from the optimal ones (e.g., Gruber, 1997). A major caveat of this type of exercise is that a statistical extrapolation is not immune to the Lucas Critique and is less reliable than methods that use an economic model to explicitly model how behavior changes with policies.[23]

The results of our extrapolation exercise are shown in Figure 5.[24] To ease the interpretation, we express the figure in terms of replacement rates instead of benefit levels. The left panel shows an intermediate solution, in which we update only the left hand side of the equation (the R_k terms) and the right panel shows the complete solution, where the right hand side (the fiscal cost of unemployment benefits) is also updated.

Because there are two instruments, optimality for any given period can be achieved by a continuum of combinations of replacement rates. Globally optimal replacement rates are located at the intersection of the two lines in the graph on the right, which shows that there is a unique point at which both optimality conditions are satisfied simultaneously. It is essential

[23] This point and the relative merits of statistical approximations and structural estimation is discussed in a review article by Chetty (2009).

[24] We perform a grid search over 1,000 equally-spaced grid-points in each interval $[0, \bar{b}_k]$. The total number of points in the two-dimensional grid is therefore $10^3 \times 10^3 = 10^6$ and the distance between contiguous grid points along any dimension implies differences in monthly benefit levels below EUR 1.50.

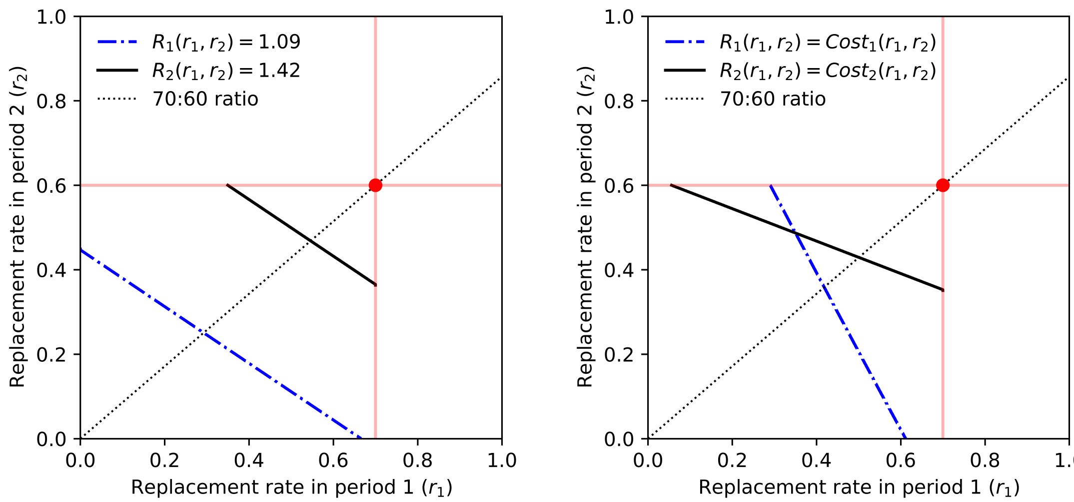

Figure 5: Optimality according to a statistical extrapolation

Notes: the dot situated at (0.7, 0.6) corresponds to current replacement rates. The locus of replacement rates that lead to an optimal value of R_1 is depicted by a solid line and the locus of replacement rates that lead to an optimal value of R_2 by a dot-dashed line. The dotted line indicates the set of replacement rates that satisfy the ratio $r_1/r_2 = 0.7/0.6$.

for obtaining a unique optimum that changes in benefits induce changes in fiscal costs. This can be seen by comparing the graph on the left with the one on the right. In the graph on the left, which does not take into account changes in costs, the two lines depicting optimality are almost parallel and do not intersect. This is resolved by the rotation in opposite directions of the locus of replacement rates that are optimal for each period in the graph on the right, once the impact on costs is taken into account. This difference in slopes allows r_1 and r_2 to fulfill their separate roles in achieving optimality.[25]

At the optimal point, replacement rates are $r_1^* = 0.35$, $r_2^* = 0.49$. Compared to replacement rates that were in place, the optimal replacement rate is 50% lower in the first period and 20% lower in the second period of an unemployment spell. Taking these results at face value, the change in benefits in the labor market reform in 2012, which made the benefit schedule steeper by decreasing replacement rates over the second period from 60% to 50% achieved almost the optimal level for that period.[26] The reform in 2012 kept the replacement rate for the first six months unchanged, which is too high according to the statistical extrapolation. Because of the elevated moral hazard effect in the first period, the benefits of maintaining such a high replacement rate are not enough to compensate the expected fiscal costs that arise in the first six months of an unemployment spell.

4.5 Extensions

Model-based liquidity and moral hazard effects of workers with different entitlements

In our baseline estimation in Section 4.3 we restrict the sample to workers who are entitled to at least 540 days of unemployment benefits, in an attempt to use a group of unemployed workers that is plausibly more homogeneous. In this subsection we use the model to obtain estimates of R_2 for a wider population by incorporating workers with shorter entitlement periods. In principle, the separation into liquidity and moral hazard using Proposition 1 can be performed for any homogeneous group of workers entitled to shorter periods of unemployment benefits provided that $\frac{\partial h_1}{\partial \bar{b}_1}$ and $\frac{\partial h_1}{\partial \bar{b}_2}$ can be estimated. Unfortunately, in the case of Spain, sample sizes to estimate $\frac{\partial h_1}{\partial \bar{b}_1}$ and $\frac{\partial h_1}{\partial \bar{b}_2}$ are too small to obtain precise estimates except for the pool of workers who are entitled to long lengths of unemployment benefits.

However, it turns out that the model implies a relationship between liquidity and moral hazard effects across periods 1 and 2 that can be exploited. In Spain, entitlement periods increase by

[25] Because $\bar{b}_1$ and $\bar{b}_2$ enter the right hand side of (33) and (34) in opposing ways, the slopes of the right hand side of the optimality equations are approximately reciprocal to each other. Moreover, because this difference in slopes is not directly tied to the linear approximation, it holds more generally.

[26] Unfortunately, the data currently available is not sufficient to test this result on the more recent post-reform period.

brackets of 60 days until reaching the maximum of 720 days. We index workers by their total entitlement period and define types $e = 240, 300, \ldots 720$.[27]

The theory implies a specific relationship between liquidity effects and moral hazard effects across periods. Dividing first and second period liquidity and moral hazard effects, the expressions in Proposition 1 imply that for any entitlement type e:

$$LIQ_2^e = \frac{B_2^e}{B_1^e} LIQ_1^e \tag{37}$$

and

$$MH_2^e = \frac{D_2^e}{D_1^e} MH_1^e. \tag{38}$$

Because durations are observable in the data, the second-period liquidity and moral hazard effects for any type of worker can be obtained from the first-period counterparts, if they are known.

We will use an approximation based on the fact that it is likely that types are more similar in terms of their first-period than in terms of their second-period liquidity and moral hazard effects. To use a concrete example, consider the two types with the highest entitlements: type $e = 720$ and $e = 660$. These two types share a first-period entitlement $B_1 = 180$ but the higher type has a B_2^{720} of 540 days and the lower type a B_2^{660} of 480 days, i.e., the two types differ only in that one of them is entitled to an additional 60 days of unemployment benefits in the second period. It is therefore likely that both types will behave similarly when facing the hypothetical question of how much search effort to exert at the start of an unemployment spell (h_1) in response to changes in the wage (w) or non-labor income (y) in the first period. In comparison, their response to a change in the wage or non-labor income in the second period will reasonably differ because the second period has a different length for each type. This implies that the two types in question will have liquidity and moral hazard effects that are approximately equal in period 1, i.e., $LIQ_1^{660} \approx LIQ_1^{720}$ and $MH_1^{660} \approx MH_1^{720}$ but their period 2 liquidity and moral hazard effects will differ.

Using this argument, and the relationships derived from Proposition 1, for any type e we approximate the second-period liquidity and moral hazard effects by

$$LIQ_2^e \approx \frac{B_2^e}{B_1^e} LIQ_1^* \tag{39}$$

and

$$MH_2^e \approx \frac{D_2^e}{D_1^e} MH_1^*, \tag{40}$$

[27] The first type ($e = 240$) corresponds to $B_1 = 180/180$ and $B_2 = 60/180$, the second type ($e = 300$) to $B_1 = 180/180$ and $B_2 = 120/180$, and so on until type $e = 720$, for whom $B_1 = 180/180$ and $B_2 = 540/180$. These types are listed in the first column of Table 4.

where LIQ_1^* and MH_1^* are the liquidity and moral hazard effects obtained in Section 4.3 for the sample with $e \in [540, 720]$. We expect the approximation to be increasingly worse as we move to lower values of e, because the similarity with the decision problem faced by types with long benefit entitlements decreases as we move to lower types and the different lengths of B_2^e may start to spill over into first-period decisions.[28]

In Table 4 we compute R_2 for an increasingly larger subset of workers, ranging from entitlements of 720 days down to 240 days. The approximation derived from the theory becomes less reliable as lower types are included. We do not impose a specific cutoff of when the approximation stops to be credible and, instead, present the evidence for all possible cutoffs, allowing each reader to decide when approximations start to fail. Results in the column denoted "Cumulative R_2" are obtained in the following way: first, we calculate the liquidity effect (LIQ_2^e) and the moral hazard effect (MH_2^e) for each type using the approximations in (39) and (40). Second, we compute LIQ_2 and MH_2 as a weighted average of liquidity and moral hazard effects for all types 720 down to the type listed in Column 1. Third, we take the ratio of the average liquidity and moral hazard effects to obtain R_2. The cumulative R_2 is therefore our model-based estimate of a population-wide R_2 when the population is defined for an increasingly larger range of types.

Table 4: Ratio of liquidity to moral hazard effects in period 2 implied by the model for increasingly larger groups of the population

Type e	B_1	B_2	Cumulative R_2
720	180/180	540/180	0.247
660	180/180	480/180	0.266
600	180/180	420/180	0.270
540	180/180	360/180	0.272
480	180/180	300/180	0.274
420	180/180	240/180	0.275
360	180/180	180/180	0.274
300	180/180	120/180	0.273
240	180/180	60/180	0.270

Note: The table presents computations of R_2 for an increasingly larger subset of workers, ranging from entitlements of 720 days all the way down to 240 days. The cumulative R_2 column is our model-based estimate of R_2 when the population is defined for an increasingly larger range of types.

The striking result from the model-based results in Table 4 is that R_2 is always close to the value obtained for the estimation sample and the moral hazard effect is always between 78.5% and 80.2%. Although the proportions of liquidity and moral hazard effects vary with the entitlement

[28] Notice that because D_2^e and B_2^e are increasing in e, both the liquidity effect and the moral hazard effect necessarily become smaller for lower types.

period, the moral hazard effect always dominates the liquidity effect and the ratio stays roughly constant. These findings should not be taken as precise estimates of the liquidity and moral hazard effects of the total population. Rather, they are an approximate measure of the relative importance of liquidity and moral hazard effects in the overall population based on the theory developed in Section 2. According to this theory and the approximations, it seems that the importance of the liquidity effect relative to the moral hazard effect may be common to a wider set of the population.

Additional moment conditions for the moral hazard effect

The difference in entitlements in the population and the clean thresholds at which they occur allow for the use of additional moment conditions to identify the moral hazard effect. For this, we take advantage of the theoretical results by Landais (2015), who shows that the response of hazard rates to changes in the entitlement period can be useful in identifying the moral hazard effect. His result adapted to our setting and notation relates the moral-hazard effect to the following linear combination of derivatives:

$$-MH_2\left(1-\frac{B_2 S_{B_1+B_2}}{D_{i,2}}\right)=\frac{\partial h_1}{\partial \bar{b}_2}-\frac{B_2}{\bar{b}_2}\frac{\partial h_1}{\partial B_2}, \tag{41}$$

where $S_{B_2+B_2}$ is the survival rate at the time when employment benefits expire and $\frac{\partial h_1}{\partial B_2}$ is the change in the first-period hazard rate induced by a change in the length of the entitlement to unemployment benefits.[29]

In the case of Spain, entitlements are determined by thresholds in the number of days worked before the unemployment spell. The derivative $\frac{\partial h_1}{\partial B_2}$ can therefore be estimated using a regression discontinuity design (RDD) using the number of days worked as the running variable. Because of our prior result that liquidity effects are relatively small, we expect a small coefficient for this derivative, which is directly related to the liquidity effect.

In the first column of Table 5 we show the estimate of this derivative using all entitlement thresholds in our estimation sample simultaneously. The point estimate is of the expected sign and small. It is not significantly different from zero. We experimented with estimating this coefficient separately for each threshold and for various bandwidth choices and found small and insignificant estimates in all cases.

At first glance, this result seems to corroborate our finding that moral hazard effects dominate over liquidity effects. When we combine this new moment with the moments estimated in our main results, we find evidence for even smaller liquidity effects, which are completely eclipsed by moral hazard effects. To combine the various moment conditions, we estimate a system that

[29] We show how to derive this result for our environment in the appendix. A subtle point when this equation is taken to the data is that benefits $\bar{b}_2$ have to be expressed in terms of the same time unit as B_2.

imposes the cross-equation restrictions implied by the theory using a GMM estimator.[30] The results are shown in the second column of Table 5.

Table 5: Estimates using an additional moment

	(1) RDD	(2) GMM
$\frac{\partial h_1}{\partial \bar{b}_1}$		-0.018***
		(0.005)
$\frac{\partial h_1}{\partial \bar{b}_2}$		-0.010***
		(0.003)
$\frac{\partial h_1}{\partial \overline{B}_2}$	-0.006	
	(0.068)	
Observations	6,935	64,949

Note: Estimations include controls for year and month dummies, age (at the time of becoming unemployed) and age squared, a dummy variable for being male, a dummy for having a permanent contract in the previous job, dummies for the qualifications of the job, for the number of the unemployment spell, and dummies for regions.

The GMM estimation yields a stronger estimated effect for $\bar{b}_1$ and a weaker effect for $\bar{b}_2$. The effect of $\overline{B}_2$ is omitted because we substituted it to obtain the cross-equation restrictions. Plugging these estimates into the equation that solves for the moral hazard and liquidity effect yields an even larger moral hazard effect than the one obtained in our main results. In fact, these estimated coefficients would imply the second period moral hazard effect is 100% of the total, and would lead to the conclusion that unemployment benefits in the second period are too high, regardless of the magnitude estimated for the fiscal cost (as long as it is strictly positive). This statement does not imply that *any* positive level of unemployment benefits is too high and that they should therefore be set to zero. As discussed in the statistical extrapolation exercise, the estimates have a local nature and do not impose restrictions on the ratio of liquidity to moral hazard effects for benefit levels that are distant from those observed in practice. A valid takeaway from this section is that taking into account moment conditions based on the length of unemployment benefit coverage appears to reinforce the conclusion that unemployment benefits are too high in the second period of the unemployment spell.

[30] The details of the specification used for the GMM estimation are shown in the appendix.

5 Conclusion

In this paper we study unemployment insurance schemes with time-varying benefits. We make two theoretical contributions. Our first theoretical contribution is to show that an insurance scheme in which unemployment benefits vary during the unemployment spell, as is the case in Spain, where benefits are higher during the first six months of unemployment, and lower afterwards, provides the necessary variation in the data to separately identify the moral hazard and liquidity effects of Chetty (2008). Our second theoretical contribution is to derive a "sufficient statistics" formula which, using the separation into liquidity and moral hazard effects, allows us to verify whether the benefits at each of their time-varying levels are set at their optimal level.

We use administrative data from Spain (the MCVL) and a Regression Kink Design to obtain the estimates needed to disentangle liquidity and moral hazard effects. We then feed these estimates into the formula for the optimal benefit level. Our findings indicate that moral hazard effects dominate and that the marginal benefits of unemployment insurance are low relative to the costs. This finding implies that unemployment benefits are above the optimal level, even in the second period of the unemployment spell, when the replacement rate in the Spanish system is a bit lower.

Our model is admittedly stylized, assumes a certain degree of homogeneity in the population, and does not include general equilibrium effects. This calls for caution when using it for public policy. However, we hope that the ease of applying the formula using data just on unemployment spells and the novel identification of liquidity and moral hazard effects will earn it a place among the numerous tools in the arsenal of policymakers.

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Appendices

A Theoretical appendix

A.1 Proofs

Lemma 1. The first order condition of the problem of an unemployed agent with respect to search effort $s_{i,t+1}(\omega_{i,t})$ is, for all $\omega_{i,t}$:

$$-\frac{\partial v_i^u(c_{i,t}^u(\omega_{i,t}), s_{i,t+1}(\omega_{i,t}))}{\partial s_{i,t+1}(\omega_{i,t})} = \beta\left[\mathbb{E}_t V_{i,t+1}^e(\omega_{i,t+1}) - \mathbb{E}_t V_{i,t+1}^u(\omega_{i,t+1})\right] h_{i,t+1}'(s_{i,t+1}(\omega_{i,t})). \quad (42)$$

For $t = 0$, this first order condition is

$$-\frac{\partial v_i^u(c_{i,0}^u(\omega_{i,0}), s_{i,1}(\omega_{i,0}))}{\partial s_{i,1}(\omega_{i,0})} = \beta\left[\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right] h_{i,1}'(s_{i,1}(\omega_{i,0})). \quad (43)$$

Notice that the expectations in the expression are conditional on information available at date $t = 0$ and are therefore functions of $\omega_{i,0}$, the exogenous initial condition. Taking the derivative of the first order condition with respect to $x \in \{b_j, w_j, y_j\}$, $j \geq 1$, produces:

$$\begin{aligned}
-\frac{\partial^2 v_i^u(\cdot)}{\partial s \partial c^u}\frac{\partial c_{i,0}^u(\omega_{i,0})}{\partial x} - \frac{\partial^2 v_i^u(\cdot)}{\partial s^2}\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x}& \\
= \beta\left[\frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right] & h_{i,1}'(s_{i,1}(\omega_{i,0})) \\
+ \beta\left[\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right] & h_{i,1}''(s_{i,1}(\omega_{i,0}))\frac{\partial s_{i,1}(\omega_{i,0}))}{\partial x}. \quad (44)
\end{aligned}$$

This expression can be rearranged as

$$\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x} = \Lambda(\omega_{i,0})\left[\frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^u(\omega_{i,1}) + \frac{\partial^2 v_i^u(\cdot)}{\partial s \partial c^u}\frac{\partial c_{i,0}^u(\omega_{i,0})}{\partial x}\right], \quad (45)$$

where

$$\Lambda(\omega_{i,0}) \equiv \frac{\beta h'_{i,1}(s_{i,1}(\omega_{i,0}))}{-\frac{\partial^2 v_i^u(\cdot)}{\partial s^2} + \frac{\partial v_i^u(\cdot)}{\partial s}\frac{h''_{i,1}(s_{i,1}(\omega_{i,0}))}{h'_{i,1}(s_{i,1}(\omega_{i,0}))}} > 0. \tag{46}$$

If the utility function is separable in consumption and search effort, then (47) simplifies to:

$$\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x} = \Lambda(\omega_{i,0})\left[\frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^e(\omega_{i,1}) - \frac{\partial}{\partial x}\mathbb{E}_0 V_{i,1}^u(\omega_{i,1})\right]. \tag{47}$$

However, we do not impose the assumption of separable utility in this Lemma.

Using the Envelope condition for the maximized functions V^e and V^u, the effect on the expected value functions in period t of raising benefits, wages, or non-labor income in period $t+j$, $j \geq 1$, for any fixed $\omega_{i,t}$ is:

$$\begin{aligned} \frac{\partial \mathbb{E}_t V_{i,t+1}^e(\omega_{i,t+1})}{\partial b_{t+j}} &= 0, \quad j \geq 1 \\ \frac{\partial \mathbb{E}_t V_{i,t+1}^u(\omega_{i,t+1})}{\partial b_{t+j}} &= \beta^j S_{i,t+j}\mathbb{E}_t\left[v_{i,1}^u(c_{i,t+j}^u(\omega_{i,t+j}), s_{i,t+j+1}(\omega_{i,t+j}))|U\right], \quad j \geq 1 \end{aligned} \tag{48}$$

$$\begin{aligned} \frac{\partial \mathbb{E}_t V_{i,t+1}^e(\omega_{i,t+1})}{\partial w_{t+j}} &= \beta^j \mathbb{E}_t\left[v_{i,1}^e(c_{i,t+j}^e(\omega_{i,t+j}))|E\right], \quad j \geq 1 \\ \frac{\partial \mathbb{E}_t V_{i,t+1}^u(\omega_{i,t+1})}{\partial w_{t+j}} &= \beta^j (1 - S_{i,t+j})\mathbb{E}_t\left[v_{i,1}^e(c_{i,t+j}^e(\omega_{i,t+j}))|E\right], \quad j \geq 1 \end{aligned} \tag{49}$$

$$\begin{aligned} \frac{\partial \mathbb{E}_t V_{i,t+1}^e(\omega_{i,t+1})}{\partial y_{t+j}} &= \beta^j \mathbb{E}_t\left[v_{i,1}^e(c_{i,t+j}^e(\omega_{i,t+j}))|E\right], \quad j \geq 1 \\ \frac{\partial \mathbb{E}_t V_{i,t+1}^u(\omega_{i,t+1})}{\partial y_{t+j}} &= \beta^j (1 - S_{i,t+j})\mathbb{E}_t\left[v_{i,1}^e(c_{i,t+j}^e(\omega_{i,t+j}))|E\right] \\ &\quad + \beta^j S_{i,t+j}\mathbb{E}_t\left[v_{i,1}^u(c_{i,t+j}^u(\omega_{i,t+j}), s_{i,t+j+1}(\omega_{i,t+j}))|U\right], \quad j \geq 1 \end{aligned} \tag{50}$$

For $t = 0$, these results imply that, for all $j \geq 1$,

$$\frac{\partial \mathbb{E}_0 V_{i,1}^e(\omega_{i,1})}{\partial b_j} = \frac{\partial \mathbb{E}_0 V_{i,1}^e(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V_{i,1}^e(\omega_{i,1})}{\partial w_j} \tag{51}$$

and

$$\frac{\partial \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})}{\partial b_j} = \frac{\partial \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})}{\partial w_j}, \tag{52}$$

and, after subtracting these two equations,

$$\begin{aligned} \left(\frac{\partial \mathbb{E}_0 V_{i,1}^e(\omega_{i,1})}{\partial b_j} - \frac{\partial \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})}{\partial b_j}\right) &= \left(\frac{\partial \mathbb{E}_0 V_{i,1}^e(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})}{\partial y_j}\right) \\ &\quad - \left(\frac{\partial \mathbb{E}_0 V_{i,1}^e(\omega_{i,1})}{\partial w_j} - \frac{\partial \mathbb{E}_0 V_{i,1}^u(\omega_{i,1})}{\partial w_j}\right). \end{aligned} \tag{53}$$

Inspection of the agent's problem reveals that y_j, b_j, w_j only appear in budget constraints and that an increase in y_j compensated by a simultaneous decrease in b_j and w_j leaves all budget constraints unchanged. Therefore, optimal consumption must satisfy:

$$\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial y_j} - \frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial b_j} - \frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial w_j} = 0. \tag{54}$$

Using this fact, (53) can also be expressed in the following way (by adding a term that equals zero to the equation):

$$\begin{aligned}
&\left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial b_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial b_j} + \frac{\partial^2 v^u_i(\cdot)}{\partial s \partial c^u}\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial b_j}\right)\\
&\quad = \left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial y_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial y_j} + \frac{\partial^2 v^u_i(\cdot)}{\partial s \partial c^u}\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial y_j}\right)\\
&\quad\quad - \left(\frac{\partial \mathbb{E}_0 V^e_{i,1}(\omega_{i,1})}{\partial w_j} - \frac{\partial \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})}{\partial w_j} + \frac{\partial^2 v^u_i(\cdot)}{\partial s \partial c^u}\frac{\partial c^u_{i,0}(\omega_{i,0})}{\partial w_j}\right).
\end{aligned} \tag{55}$$

By multiplying both sides by $\Lambda(\omega_{i,0}) \neq 0$, and comparing with (47), it follows that

$$\frac{\partial s_{i,1}(\omega_{i,0})}{\partial b_j} = \frac{\partial s_{i,1}(\omega_{i,0})}{\partial y_j} - \frac{\partial s_{i,1}(\omega_{i,0})}{\partial w_j}, \quad j \geq 1. \tag{56}$$

The final step is to multiply both sides of this equation by $h'_{i,1}(s_{i,1}(\omega_{i,0})) \neq 0$ and to notice that, by the chain-rule,

$$\frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial x} = h'_{i,1}(s_{i,1}(\omega_{i,0}))\frac{\partial s_{i,1}(\omega_{i,0})}{\partial x},$$

for variables $x \in \{b_j, y_j, w_j\}$. This leads to expression (10) in the Lemma. Q.E.D.

Lemma 2. From the expressions derived in the proof of Lemma 1, for separable utility:

$$\begin{aligned}
\frac{1}{\Lambda(\omega_{i,t})}\frac{\partial s_{i,t+1}(\omega_{i,t})}{\partial y_{t+j}} &= \frac{\partial}{\partial y_{t+j}}\mathbb{E}_t V^e_{i,t+1}(\omega_{t+1}) - \frac{\partial}{\partial y_{t+j}}\mathbb{E}_t V^u_{i,t+1}(\omega_{t+1})\\
&= \beta^j S_{i,t+j}\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j}))|E\right]\\
&\quad - \beta^j S_{i,t+j}\mathbb{E}_t\left[v^u_{i,1}(c^u_{i,t+j}(\omega_{i,t+j}), s_{i,t+j+1}(\omega_{i,t+j}))|U\right].
\end{aligned} \tag{57}$$

Taking the ratio of this equation evaluated in two consecutive periods $t+j$ and $t+j+1$ yields:

$$\frac{\frac{\partial s_{i,t+1}}{\partial y_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial y_{t+j}}} = \beta\frac{S_{i,t+j+1}}{S_{i,t+j}}\frac{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j+1})|E\right] - \mathbb{E}_t\left[v^u_{i,1}(c^u_{i,t+j+1}, s_{i,t+j+2})|U\right]}{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j})|E\right] - \mathbb{E}_t\left[v^u_{i,1}(c^u_{i,t+j}, s_{i,t+j+1})|U\right]} \tag{58}$$

When the borrowing constraint does not bind, the Euler equation of an unemployed worker between two consecutive periods is

$$\mathbb{E}_t v^e_{i,1}(c_{i,t+j}) = (1+r)\beta\mathbb{E}_t v^e_{i,1}(c_{i,t+j+1}) \tag{59}$$

and the Euler equation of an unemployed worker is

$$\mathbb{E}_t v^u_{i,1}(c_{i,t+j},\cdot) = (1+r)\beta[(1-h_{i,t+j+1})\mathbb{E}_t v^u_{i,1}(c_{i,t+j+1},\cdot) + h_{i,t+j+1}\mathbb{E}_t v^e_{i,1}(c_{i,t+j+1},\cdot)]. \tag{60}$$

Subtracting these two Euler equations and rearranging yields:

$$\frac{\mathbb{E}_t v^e_{i,1}(c_{i,t+j+1}) - \mathbb{E}_t v^u_{i,1}(c_{i,t+j+1},\cdot)}{\mathbb{E}_t v^e_{i,1}(c_{i,t+j}) - \mathbb{E}_t v^u_{i,1}(c_{i,t+j},\cdot)} = \frac{1}{(1+r)\beta(1-h_{i,t+j+1})} \tag{61}$$

Substituting this expression into (58) simplifies to

$$\frac{\frac{\partial s_{i,t+1}}{\partial y_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial y_{t+j}}} = \frac{S_{i,t+j+1}}{S_{i,t+j}(1-h_{i,t+j+1})}\frac{1}{1+r} = \frac{1}{1+r} \tag{62}$$

This implies that

$$\frac{\partial s_{i,t+1}}{\partial y_{t+j+1}} = \frac{\partial s_{i,t+1}}{\partial y_{t+1}}(1+r)^{-j} \tag{63}$$

Multiplying both sides by $h'(s_{t+1}) \neq 0$ and noticing that, by the Chain rule, $\frac{\partial h_{i,t+1}}{\partial x} = h'(s_{t+1})\frac{\partial s_{i,t+1}}{\partial x}$ produces:

$$\frac{\partial h_{i,t+1}}{\partial y_{t+j+1}} = \frac{\partial h_{i,t+1}}{\partial y_{t+1}}(1+r)^{-j}. \tag{64}$$

This is (11), the first equation in the Lemma.

The impact of the wage rate on search effort is given by the following relationship:

$$\begin{aligned}\frac{1}{\Lambda(\omega_{i,t})}\frac{\partial s_{i,t+1}(\omega_{i,t})}{\partial w_{t+j}} &= \frac{\partial}{\partial w_{t+j}}\mathbb{E}_t V^e_{i,t+1}(\omega_{t+1}) - \frac{\partial}{\partial w_{t+j}}\mathbb{E}_t V^u_{i,t+1}(\omega_{t+1}) \\ &= \beta^j S_{i,t+j}\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j}(\omega_{i,t+j}))|E\right].\end{aligned}$$

Taking the ratio between two consecutive periods:

$$\frac{\frac{\partial s_{i,t+1}}{\partial w_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial w_{t+j}}} = \beta\frac{S_{i,t+j+1}}{S_{i,t+j}}\frac{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j+1})|E\right]}{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j})|E\right]} \tag{65}$$

Notice that the Euler equation for an employed worker implies that

$$\beta\frac{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j+1})|E\right]}{\mathbb{E}_t\left[v^e_{i,1}(c^e_{i,t+j})|E\right]} = \frac{1}{1+r} \tag{66}$$

Therefore,

$$\frac{\frac{\partial s_{i,t+1}}{\partial w_{t+j+1}}}{\frac{\partial s_{i,t+1}}{\partial w_{t+j}}} = \frac{S_{i,t+j+1}}{S_{i,t+j}}\frac{1}{1+r} \tag{67}$$

and

$$\frac{\partial s_{i,t+1}}{\partial w_{t+j+1}} = \frac{\partial s_{i,t+1}}{\partial w_{t+1}}\frac{S_{i,t+j+1}}{S_{i,t+1}}(1+r)^{-j} \tag{68}$$

Multiplying both sides by $h'(s_{t+1})$ and invoking the Chain Rule again yields (12), the second equation in the Lemma. Q.E.D.

Proposition 1. Start from the decomposition in (10) written in a slightly different form (the only difference is shifting the index j by one, so that it starts at zero):

$$\frac{\partial h_{i,1}}{\partial b_{j+1}} = \frac{\partial h_{i,1}}{\partial y_{j+1}} - \frac{\partial h_{i,1}}{\partial w_{j+1}}, \quad j \geq 0. \tag{69}$$

Substitute the results from Lemma 2 into this equation to obtain:

$$\frac{\partial h_{i,1}}{\partial b_{j+1}} = \frac{\partial h_{i,1}}{\partial y_1}(1+r)^{-j} - \frac{\partial h_{i,1}}{\partial w_1}\frac{S_{i,j+1}}{S_{i,1}}(1+r)^{-j}, \quad j \geq 0 \tag{70}$$

Sum this equation over $j = 0, \ldots, B_1 - 1$ to obtain $\frac{\partial h_{i,1}}{\partial \bar{b}_1}$:

$$\begin{aligned}
\frac{\partial h_{i,1}}{\partial \bar{b}_1} &= \sum_{j=0}^{B_1-1} \frac{\partial h_{i,1}}{\partial b_{j+1}} = \left(\sum_{j=0}^{B_1-1} \frac{1}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=0}^{B_1-1} \frac{S_{i,j+1}}{(1+r)^{j+1}}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
&= \sum_{j=1}^{B_1} \frac{\partial h_{i,1}}{\partial b_j} = \left(\sum_{j=1}^{B_1} \frac{1+r}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=1}^{B_1} \frac{S_{i,j}}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
&\equiv \underbrace{(1+r)\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial y_1}}_{LIQ_{i,1}} - \underbrace{\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial w_1}}_{MH_{i,1}},
\end{aligned} \tag{71}$$

where $\tilde{S}_{i,1} = (1+r)^{-1}S_{i,1}$, $\tilde{B}_1 = \sum_{t=1}^{B_1}(1+r)^{-t}$ and $\tilde{D}_{i,1} = \sum_{t=1}^{B_1}(1+r)^{-t}S_{i,t}$.

Sum also over $j = B_1, \ldots, B_1 + B_2 - 1$ to obtain $\frac{\partial h_{i,1}}{\partial \bar{b}_2}$:

$$\begin{aligned}
\frac{\partial h_{i,1}}{\partial \bar{b}_2} &= \sum_{j=B_1}^{B_1+B_2-1} \frac{\partial h_{i,1}}{\partial b_{j+1}} = \left(\sum_{j=B_1}^{B_1+B_2-1} \frac{1}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=B_1}^{B_1+B_2-1} \frac{S_{i,j+1}}{(1+r)^{j+1}}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
&= \sum_{j=B_1+1}^{B_1+B_2} \frac{\partial h_{i,1}}{\partial b_j} = \left(\sum_{j=B_1+1}^{B_1+B_2} \frac{1+r}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial y_1} - \frac{1}{\tilde{S}_{i,1}}\left(\sum_{j=B_1+1}^{B_1+B_2} \frac{S_{i,j}}{(1+r)^j}\right)\frac{\partial h_{i,1}}{\partial w_1} \\
&\equiv \underbrace{(1+r)\tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial y_1}}_{LIQ_{i,2}} - \underbrace{\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial w_1}}_{MH_{i,2}},
\end{aligned} \tag{72}$$

where $\tilde{B}_2 = \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}$ and $\tilde{D}_{i,2} = \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}S_{i,t}$. Notice that, when $r = 0$, the terms simplify, so that $\tilde{B}_1(0) = B_1$, $\tilde{B}_2(0) = B_2$, $\tilde{D}_{i,1}(0) = D_{i,1}$, and $\tilde{D}_{i,2}(0) = D_{i,2}$.

The two equations (71) and (72) can be collected in matrix form as follows:

$$\begin{bmatrix} \frac{\partial h_{i,1}}{\partial \bar{b}_1} \\ \frac{\partial h_{i,1}}{\partial \bar{b}_2} \end{bmatrix} = \begin{bmatrix} (1+r)\tilde{B}_1(r) & -\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,1} \\ (1+r)\tilde{B}_2(r) & -\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,2} \end{bmatrix} \begin{bmatrix} \frac{\partial h_{i,1}}{\partial y_1} \\ \frac{\partial h_{i,1}}{\partial w_1} \end{bmatrix} \tag{73}$$

This matrix admits an inverse if $\frac{\tilde{D}_{i,1}(r)}{\tilde{B}_1(r)} \neq \frac{\tilde{D}_{i,2}(r)}{\tilde{B}_2}(r)$, which for $r = 0$ turns into $\frac{D_{i,1}}{B_1} \neq \frac{D_{i,2}}{B_2}$. Because $S_{i,t}$ is non-increasing in t, the condition is satisfied with $\frac{D_{i,1}}{B_1} > \frac{D_{i,2}}{B_2}$ if $h_{i,t} > 0$ at least once for $1 < t < B_1 + B_2$. Computing the inverse and pre-multiplying both sides of the equation with this inverse yields:

$$\begin{bmatrix} \frac{\partial h_{i,1}}{\partial y_1} \\ \frac{\partial h_{i,1}}{\partial w_1} \end{bmatrix} = \frac{1}{\tilde{B}_2(r)\frac{1+r}{\tilde{S}_{i,1}}\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\frac{1+r}{\tilde{S}_{i,1}}\tilde{D}_{i,2}(r)} \begin{bmatrix} -\frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,2}(r) & \frac{1}{\tilde{S}_{i,1}}\tilde{D}_{i,1}(r) \\ -(1+r)\tilde{B}_2(r) & (1+r)\tilde{B}_1(r) \end{bmatrix} \begin{bmatrix} \frac{\partial h_{i,1}}{\partial \bar{b}_1} \\ \frac{\partial h_{i,1}}{\partial \bar{b}_2} \end{bmatrix} \tag{74}$$

Therefore,

$$\begin{aligned} \frac{\partial h_{i,1}}{\partial y_1} &= \frac{(1+r)^{-1}}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\ \frac{\partial h_{i,1}}{\partial w_1} &= \frac{\tilde{S}_{i,1}}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right). \end{aligned} \tag{75}$$

Finally, substituting these results into (71) and (72) yields:

$$\begin{aligned} LIQ_{i,1}(r) &= \left.\frac{\partial h_{i,1}}{\partial y}\right|_{B_1} = \frac{\tilde{B}_1(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\ MH_{i,1}(r) &= \left.\frac{\partial h_{i,1}}{\partial w}\right|_{B_1} = \frac{\tilde{D}_{i,1}(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\ LIQ_{i,2}(r) &= \left.\frac{\partial h_{i,1}}{\partial y}\right|_{B_2} = \frac{\tilde{B}_2(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{D}_{i,1}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{D}_{i,2}(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \\ MH_{i,2}(r) &= \left.\frac{\partial h_{i,1}}{\partial w}\right|_{B_2} = \frac{\tilde{D}_{i,2}(r)}{\tilde{B}_2(r)\tilde{D}_{i,1}(r) - \tilde{B}_1(r)\tilde{D}_{i,2}(r)} \left(\tilde{B}_1(r)\frac{\partial h_{i,1}}{\partial \bar{b}_2} - \tilde{B}_2(r)\frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \end{aligned} \tag{76}$$

Setting $r = 0$ leads to the expressions in (15) in the Proposition. Q.E.D.

Proposition 2. The planner solves the following problem:

$$V^P(\mathbf{b}, \tau) = \max \int V_{i,0}(\omega_{i,0})\, di + \lambda \left(G(\mathbf{b}, \tau) - \bar{G} \right), \tag{77}$$

where

$$\begin{aligned} G(\mathbf{b}, \tau) &= \tau \sum_{t=1}^{T} (1+r)^{-t}(1 - S_t) - \bar{b}_1 \sum_{t=1}^{B_1} (1+r)^{-t} S_t - \bar{b}_2 \sum_{t=B_1+1}^{B_1+B_2} (1+r)^{-t} S_t \\ &= \tau \sum_{t=1}^{T} (1+r)^{-t} - \tau \tilde{D}(r) - \bar{b}_1 \tilde{D}_1(r) - \bar{b}_2 \tilde{D}_2(r), \end{aligned} \tag{78}$$

where we have used $\tilde{D}_1(r) = \sum_{t=1}^{B_1}(1+r)^{-t}S_t$, $\tilde{D}_2(r) = \sum_{t=B_1+1}^{B_1+B_2}(1+r)^{-t}S_t$, and $\tilde{D}(r) = \sum_{t=1}^{T}(1+r)^{-t}$.

For any agent i:

$$\begin{aligned}\frac{\partial}{\partial b_t}V_{i,0}(\omega_{i,0}) &= \frac{\partial}{\partial b_t}\mathbb{E}_0 V_{i,1}(\omega_{i,1})\\ &= -\frac{1}{\Lambda(\omega_{i,0})}\frac{\partial s_{i,1}(\omega_{i,0})}{\partial b_t}\\ &= -\frac{1}{h'_{i,1}(s_{i,1}(\omega_{i,0}))\Lambda(\omega_{i,0})}\frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial b_t}\end{aligned}\tag{79}$$

Assuming that agents are ex-ante homogeneous, so that $\forall i : h_{i,1}(s) = h_1(s)$ and $\forall i : \omega_{i,0} = \omega_0$,

$$\begin{aligned}\frac{\partial}{\partial b_t}\int V_{i,0}(\omega_{i,0})\,di &= \int \frac{\partial}{\partial b_t}V_{i,0}(\omega_{i,0})\,di\\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\int \frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial b_t}\,di\\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial b_t}\end{aligned}\tag{80}$$

and

$$\begin{aligned}\frac{\partial}{\partial \bar{b}_1}\int V_{i,0}(\omega_{i,0})\,di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\sum_{t=1}^{B_1}\frac{\partial h_1(s_1(\omega_0))}{\partial b_t}\\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}[LIQ_1(r) - MH_1(r)]\end{aligned}\tag{81}$$

Equivalently,

$$\begin{aligned}\frac{\partial}{\partial \bar{b}_2}\int V_{i,0}(\omega_{i,0})\,di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\sum_{t=B_1+1}^{B_1+B_2}\frac{\partial h_1(s_1(\omega_0))}{\partial b_t}\\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}[LIQ_2(r) - MH_2(r)]\end{aligned}\tag{82}$$

Consider first the impact of changing τ in only one period, i.e., a movement in τ_t:

$$\begin{aligned}\frac{\partial}{\partial \tau_t}V_{i,0}(\omega_{i,0}) &= -\frac{\partial}{\partial w_t}V_{i,0}(\omega_{i,0})\\ &= -\beta^t(1 - S_{i,t})\mathbb{E}_0^e\left[v^e_{i,1}(c^e_{i,t})\right]\\ &= -\frac{1 - S_{i,t}}{S_{i,t}}\frac{\partial}{\partial w_t}[\mathbb{E}_0 V^e_{i,1}(\omega_{i,1}) - \mathbb{E}_0 V^u_{i,1}(\omega_{i,1})]\\ &= -\frac{1 - S_{i,t}}{S_{i,t}}\frac{1}{\Lambda(\omega_{i,0})}\frac{\partial s_{i,1}(\omega_{i,0})}{\partial w_t}\\ &= -\frac{1 - S_{i,t}}{S_{i,t}}\frac{1}{h'_{i,1}(s_{i,1}(\omega_{i,0}))\Lambda(\omega_{i,0})}\frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial w_t}\end{aligned}\tag{83}$$

Using ex-ante homogeneity

$$\begin{aligned}\frac{\partial}{\partial \tau_t}\int V_{i,0}(\omega_{i,0})\,di &= \int \frac{\partial}{\partial \tau_t}V_{i,0}(\omega_{i,0})\,di \\ &= -\frac{1-S_t}{S_t}\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\int \frac{\partial h_{i,1}(s_{i,1}(\omega_{i,0}))}{\partial w_t}\,di \\ &= -\frac{1-S_t}{S_t}\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial w_t} \end{aligned} \tag{84}$$

Therefore,

$$\begin{aligned}\frac{\partial}{\partial \tau}\int V_{i,0}(\omega_{i,0})\,di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\sum_{t=1}^{T}\frac{1-S_t}{S_t}\frac{\partial h_1(s_1(\omega_0))}{\partial w_t} \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\sum_{t=1}^{T}\frac{1-S_t}{S_t}\frac{S_t}{S_1(1+r)^{t-1}}\frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial w_1}\sum_{t=1}^{T}\frac{1-S_t}{S_1(1+r)^{t-1}} \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial w_1}\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{S}_1(r)}, \end{aligned} \tag{85}$$

where $\tilde{T}(r)=\sum_{t=1}^{T}(1+r)^{-t}$ and $\tilde{D}(r)$ and $\tilde{S}_1(r)$ are defined as in the proof of Proposition 1. Notice that, from the proof of Proposition 1,

$$\begin{aligned} MH_1(r) &= \frac{1}{\tilde{S}_1}\tilde{D}_1(r)\frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \\ MH_2(r) &= \frac{1}{\tilde{S}_1}\tilde{D}_2(r)\frac{\partial h_1(s_1(\omega_0))}{\partial w_1} \end{aligned} \tag{86}$$

Using these relationships, the derivative with respect to τ can be written in terms of MH_1 and MH_2:

$$\begin{aligned}\frac{\partial}{\partial \tau}\int V_{i,0}(\omega_{i,0})\,di &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\partial h_1(s_1(\omega_0))}{\partial w_1}\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{S}_1(r)} \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}MH_1(r) \qquad (87) \\ &= -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)}MH_2(r) \qquad (88) \end{aligned}$$

At an interior optimum

$$\begin{aligned}\frac{\partial}{\partial \bar{b}_1}\int V_{i,0}(\omega_{i,0})\,di &= -\lambda\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} \\ -\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}[LIQ_1(r)-MH_1(r)] &= -\lambda\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1} \end{aligned} \tag{89}$$

and

$$\frac{\partial}{\partial\tau}\int V_{i,0}(\omega_{i,0})\,di = -\lambda\frac{\partial G(\mathbf{b},\tau)}{\partial\tau}$$
$$-\frac{1}{h'(s_1(\omega_0))\Lambda(\omega_0)}\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}MH_1(r) = -\lambda\frac{\partial G(\mathbf{b},\tau)}{\partial\tau} \tag{90}$$

Taking the ratio:

$$-\frac{LIQ_1(r)-MH_1(r)}{\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}MH_1(r)} = -\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_1}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau}$$
$$-\frac{LIQ_1(r)-MH_1(r)}{MH_1(r)} = -\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_1}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau}$$
$$-\frac{LIQ_1(r)}{MH_1(r)} = -\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_1}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau} - 1 \tag{91}$$

Following similar steps,

$$-\frac{LIQ_2(r)-MH_2(r)}{\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)}MH_2(r)} = -\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_2}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau}$$
$$-\frac{LIQ_2(r)-MH_2(r)}{MH_2(r)} = -\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)}\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_2}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau}$$
$$-\frac{LIQ_2(r)}{MH_2(r)} = -\frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)}\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_2}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau} - 1 \tag{92}$$

Notice that, by the Implicit Function Theorem,

$$-\frac{\partial G(\mathbf{b},\tau)}{\partial\bar{b}_k}/\frac{\partial G(\mathbf{b},\tau)}{\partial\tau} = \frac{\partial\tau}{\partial\bar{b}_k}|_{G(\mathbf{b},\tau)=\bar{G}} \tag{93}$$

$$G(\mathbf{b},\tau)=\bar{G} \Leftrightarrow \tau(\tilde{T}(r)-\tilde{D}(r)) = \overline{G} + \bar{b}_1\tilde{D}_1(r) + \bar{b}_2\tilde{D}_2(r) \tag{94}$$

Therefore,

$$\frac{\partial\tau}{\partial\bar{b}_1}(\tilde{T}(r)-\tilde{D}(r)) - \tau\frac{\partial\tilde{D}(r)}{\partial\bar{b}_1} = \tilde{D}_1(r) + \bar{b}_1\frac{\partial\tilde{D}_1(r)}{\partial\bar{b}_1} + \bar{b}_2\frac{\partial\tilde{D}_2(r)}{\partial\bar{b}_1} \tag{95}$$

and

$$\frac{\partial\tau}{\partial\bar{b}_1} = \frac{1}{\tilde{T}(r)-\tilde{D}(r)}\left[\tilde{D}_1(r) + \bar{b}_1\frac{\partial\tilde{D}_1(r)}{\partial\bar{b}_1} + \bar{b}_2\frac{\partial\tilde{D}_2(r)}{\partial\bar{b}_1} + \tau\frac{\partial\tilde{D}(r)}{\partial\bar{b}_1}\right]$$
$$= \frac{\tilde{D}_1(r)}{\tilde{T}(r)-\tilde{D}(r)}\left[1 + \varepsilon_{\tilde{D}_1,\bar{b}_1} + \frac{\bar{b}_2\tilde{D}_2(r)}{\bar{b}_1\tilde{D}_1(r)}\varepsilon_{\tilde{D}_2,\bar{b}_1} + \frac{\tau\tilde{D}(r)}{\bar{b}_1\tilde{D}_1(r)}\varepsilon_{\tilde{D},\bar{b}_1}\right] \tag{96}$$

Substituting into (91):

$$
\begin{aligned}
-\frac{LIQ_1(r)}{MH_1(r)} &= \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}\left(-\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_1}/\frac{\partial G(\mathbf{b},\tau)}{\partial \tau}\right)-1 \\
&= \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_1(r)}\frac{\partial \tau}{\partial \bar{b}_1}|_{G(\mathbf{b},\tau)=\bar{G}}-1 \\
&= \varepsilon_{\tilde{D}_1,\bar{b}_1}+\frac{\bar{b}_2\tilde{D}_2(r)}{\bar{b}_1\tilde{D}_1(r)}\varepsilon_{\tilde{D}_2,\bar{b}_1}+\frac{\tau\tilde{D}(r)}{\bar{b}_1\tilde{D}_1(r)}\varepsilon_{\tilde{D},\bar{b}_1} \qquad (97)
\end{aligned}
$$

Setting $r=0$ yields the first expression in the proposition.

Analogously,

$$
\begin{aligned}
\frac{\partial \tau}{\partial \bar{b}_2} &= \frac{1}{\tilde{T}(r)-\tilde{D}(r)}\left[\tilde{D}_2(r)+\bar{b}_2\frac{\partial \tilde{D}_2(r)}{\partial \bar{b}_1}+\bar{b}_1\frac{\partial \tilde{D}_1(r)}{\partial \bar{b}_2}+\tau\frac{\partial \tilde{D}(r)}{\partial \bar{b}_2}\right] \\
&= \frac{\tilde{D}_2(r)}{\tilde{T}(r)-\tilde{D}(r)}\left[1+\varepsilon_{\tilde{D}_2,\bar{b}_2}+\frac{\bar{b}_1\tilde{D}_1(r)}{\bar{b}_2\tilde{D}_2(r)}\varepsilon_{\tilde{D}_1,\bar{b}_2}+\frac{\tau\tilde{D}(r)}{\bar{b}_2\tilde{D}_2(r)}\varepsilon_{\tilde{D},\bar{b}_2}\right] \qquad (98)
\end{aligned}
$$

and

$$
\begin{aligned}
-\frac{LIQ_2(r)}{MH_2(r)} &= \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)}\left(-\frac{\partial G(\mathbf{b},\tau)}{\partial \bar{b}_2}/\frac{\partial G(\mathbf{b},\tau)}{\partial \tau}\right)-1 \\
&= \frac{\tilde{T}(r)-\tilde{D}(r)}{\tilde{D}_2(r)}\frac{\partial \tau}{\partial \bar{b}_2}|_{G(\mathbf{b},\tau)=\bar{G}}-1 \\
&= \varepsilon_{\tilde{D}_2,\bar{b}_2}+\frac{\bar{b}_1\tilde{D}_1(r)}{\bar{b}_2\tilde{D}_2(r)}\varepsilon_{\tilde{D}_1,\bar{b}_2}+\frac{\tau\tilde{D}(r)}{\bar{b}_2\tilde{D}_2(r)}\varepsilon_{\tilde{D},\bar{b}_2} \qquad (99)
\end{aligned}
$$

Setting $r=0$ yields the second expression in the proposition. Q.E.D.

Proposition 3. The quadratic specification implies that $\Lambda(\omega_{i,0})=\frac{1}{\psi}>0$ does not depend on the state of the world. In addition, the linear deterministic relationship between search effort and $h_{i,t}$ implies that $h'_{i,1}(s(\omega_{i,0}))=1$ does also not depend in the state of the world. Therefore, $\frac{1}{h'_{i,1}(s(\omega_{i,0}))\Lambda(\omega_{i,0})}=\psi$ can be taken out of the expectations over i without assuming that agents share the same $\omega_{i,0}$. Thus, the exact same steps as in the proof of Proposition 2 can be retraced leading to the same result in the end. Q.E.D.

A.2 Extension: additional moments for the moral hazard effect

Adapting the key insight behind Proposition 1 by Landais (2015) to our environment, we obtain:

$$
\frac{\partial h_{i,1}}{\partial B_2}\approx \bar{b}_2\frac{\partial h_{i,1}}{\partial b_{B_1+B_2+\Delta}}=\bar{b}_2\left(\frac{\partial h_{i,1}}{\partial y_1}-\frac{S_{i,B_1+B_2+\Delta}}{S_{i,1}}\frac{\partial h_{i,1}}{\partial w_1}\right), \qquad (100)
$$

where $\Delta\geq 0$ is the length of a time-step. The approximation will be better for smaller time-steps. The last equality follows directly from (70). Solving this equation for $\frac{\partial h_{i,1}}{\partial y_1}$ yields

$$
\frac{\partial h_{i,1}}{\partial y_1}\approx \frac{1}{\bar{b}_2}\frac{\partial h_{i,1}}{\partial B_2}+\frac{S_{i,B_1+B_2+\Delta}}{S_{i,1}}\frac{\partial h_{i,1}}{\partial w_1} \qquad (101)
$$

The decomposition of a change in $\bar{b}_2$ into a liquidity and moral hazard effect is given by (72):

$$\frac{\partial h_{i,1}}{\partial \bar{b}_2} = \underbrace{B_2 \frac{\partial h_{i,1}}{\partial y_1}}_{LIQ_{i,2}} - \underbrace{\frac{1}{S_{i,1}} D_{i,2} \frac{\partial h_{i,1}}{\partial w_1}}_{MH_{i,2}}, \tag{102}$$

Substituting $\frac{\partial h_{i,1}}{\partial y_1}$ into this equation,

$$\frac{\partial h_{i,1}}{\partial \bar{b}_2} \approx B_2 \left(\frac{1}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} + \frac{S_{i,B_1+B_2+\Delta}}{S_{i,1}} \frac{\partial h_{i,1}}{\partial w_1} \right) - MH_{i,2}. \tag{103}$$

After rearranging and expressing $\frac{\partial h_{i,1}}{\partial w_1}$ in terms of $MH_{i,2}$, we obtain:

$$-MH_{i,2} \left(1 - \frac{B_2 S_{i,B_1+B_2+\Delta}}{D_{i,2}} \right) \approx \frac{\partial h_{i,1}}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} \tag{104}$$

and setting $\Delta = 0$:

$$-MH_{i,2} \left(1 - \frac{B_2 S_{i,B_1+B_2}}{D_{i,2}} \right) \approx \frac{\partial h_{i,1}}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2}. \tag{105}$$

GMM estimation

The moral hazard effect can be eliminated using the last equation in (15).

$$-\frac{D_{i,2}}{B_2 D_{i,1} - B_1 D_{i,2}} \left(B_1 \frac{\partial h_{i,1}}{\partial \bar{b}_2} - B_2 \frac{\partial h_{i,1}}{\partial \bar{b}_1} \right) \left(1 - \frac{B_2 S_{i,B_1+B_2}}{D_{i,2}} \right) = \frac{\partial h_{i,1}}{\partial \bar{b}_2} - \frac{B_2}{\bar{b}_2} \frac{\partial h_{i,1}}{\partial B_2} \tag{106}$$

Solving this equation for $\frac{\partial h_{i,1}}{\partial B_2}$:

$$\frac{\partial h_{i,1}}{\partial B_2} = -\bar{b}_2 \Xi \frac{\partial h_{i,1}}{\partial \bar{b}_1} + \frac{\bar{b}_2}{B_2} (\Xi B_1 - 1) \frac{\partial h_{i,1}}{\partial \bar{b}_2}, \tag{107}$$

where $\Xi \equiv \left(1 - \frac{B_2 S_{i,B_1+B_2}}{D_{i,2}} \right) \frac{D_{i,2}}{B_2 D_{i,1} - B_1 D_{i,2}}$.

Using this theoretical relationship, we define two error terms in terms of the variables of interest:

$$\begin{aligned} \epsilon_{i,1}(\theta_1, \theta_2) &\equiv y_i - \left(\alpha + x_i' \eta + \gamma (v_i - k_1) - \sum_{j=1}^{2} \frac{\theta_j}{r_j} W_{i,j} (v_i - k_j) \right), \\ \epsilon_{i,2}(\theta_1, \theta_2) &\equiv y_i - \left(\tilde{\alpha} + x_i' \tilde{\eta} + \tilde{\gamma} (d_i - \bar{d}) - \bar{b}_2 \left(\Xi \theta_1 - \frac{\Xi B_1 - 1}{B_2} \theta_2 \right) \tilde{W}_i (d_i - \bar{d}) \right). \end{aligned} \tag{108}$$

The first error expresses the RKD specification in (22) in terms of the variables of interest, θ_1 and θ_2. The second error is a RDD specification with running variable d_i (days in prior jobs eligible for unemployment benefits in the current spell) and threshold $\bar{d}$, the number of days at which observation i switches from one entitlement period to the next. For ease of notation, we have expressed these equations using only first-degree polynomials. They can be generalized to higher-degree polynomials by adding the appropriate terms. Each of these equations has its own bandwidth parameter, which governs which observations are included in the estimation. Using these error terms in the moment contributions, the parameters θ_1 and θ_2 can be estimated via GMM using standard methods.

B Robustness checks

B.1 Sensitivity to bandwidth choice, polynomial order, and covariates

Our estimates are robust to the use of different bandwidths. As noted by other authors, for instance Landais (2015), a regression kink design is more demanding in terms of bandwidth size than a regression discontinuity design. In Figure 6 we plot the point estimates for the probability of exiting unemployment in the first period for different bandwidths along with 95% confidence intervals. We find that main results vary very little with the bandwidth choice, with less precise estimates at the second kink for small bandwidth sizes. In particular, the relative importance of liquidity and moral hazard effects remains remarkably constant.

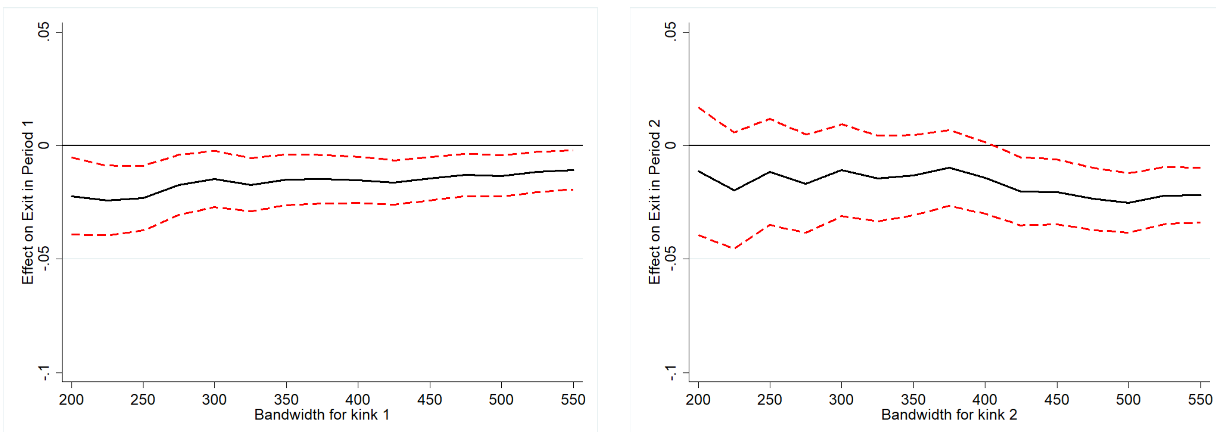

Figure 6: Estimates on the probability of exiting unemployment in the first period for different bandwidths, with 95% confidence intervals.

Regarding the polynomial order choice, even though point estimates change, moral hazard continues to dominate in both periods. More importantly, in both cases benefits of unemployment insurance are low relative to the costs. The Akaike Criterion, whose results are presented in the last column of Table 6, selects the quadratic specification as the preferred specification, although all values are very similar.

Finally, in Table 7 we present the results from estimating our baseline equation when no covariates are included. Results remain practically identical.

In order to detect whether the inclusion of the long-term unemployed is affecting the results, we repeat the regression in the first column of Table 7 on a reduced sample, excluding observations in which the unemployment spell lasts for the full period of coverage or longer. The expected sign of the effect of this sample restriction is ex-ante ambiguous because a change in the sample will also lead to a shift in the polynomial that approximates the smooth relationship between the running variable and the dependent variable. We calculate the coverage period using the administrative rule that tranforms the number of days in prior jobs to entitlement periods. We estimate the coefficient for θ_1 at -0.011* (0.006) and that of θ_2 at -0.027*** (0.009). The

Table 6: Summary of results using linear or quadratic polynomials.

	(1) MH Period 1	(2) Optimal Period 1	(3) MH Period 2	(4) Optimal Period 2	(5) AIC
Linear	89%	Too high	80%	Too high	84335.880
Quadratic	89%	Too high	79%	Too high	84328.684
Cubic	88%	Too high	77%	Too high	84331.395

Note: The table summarizes results from using a linear, a quadratic, and a cubic specification. The value for MH represents the relative importance of the moral hazard effect, Optimal denotes if unemployment benefits are too high or too low with respect to optimal levels, and AIC denotes Akaike's Information Criterion.

Table 7: RKD estimations on several outcomes: Period 1992 - 2012, workers between 30 and 50 years old

	(1) Exit in period 1	(2) Duration period 1	(3) Duration period 2	(4) Non-employment duration	(5) MH	(6) Optimal
θ_1	-0.018***	0.019**	0.124***	0.155***	84%	Too high
	(0.007)	(0.009)	(0.029)	(0.036)		
θ_2	-0.027***	0.029**	0.177***	0.212***	71%	Too high
	(0.009)	(0.011)	(0.037)	(0.047)		
Observations	61,939	61,939	61,939	61,939		

Note: All estimates are from specifications with no covariates, for the quadratic case. Duration in each period is measured as days in unemployment in each period. Total duration is days in non-employment. Coefficients are transformed in order to obtain the values of interest: the impact of increasing benefits in each period on each outcome. The value for MH represents the relative importance of the moral hazard effect, and Optimal shows if unemployment benefits are too high or too low with respect to optimal levels.

magnitudes of the point estimates are lower than in the baseline in the first case and similar in the second case, although the precision in the estimates does not lead to a rejection of the hypothesis that they are equal to the baseline at the usual confidence levels.

B.2 Placebo kinks and Permutation Tests

Following Landais (2015) and Kolsrud et al. (2018) we add a test aimed at detecting the kinks assuming that the actual location of the kinks is not known. We estimate our baseline equation for a range of placebo kinks and compare the R-squared obtained in each estimation. The placebo kinks are placed in EUR 25 increments from the true location of the kinks (we move both kinks outward or inward at the same time). We cannot use a wide range for the placebo kinks because both actual kinks are relatively close. The location in which the R-squared is maximized is situated at a EUR 25 difference from the real kink points. We present in Figure 7 the evolution of the R-squared for different locations of the kinks. We observe that R-squared is similar in a range of EUR 25, and that it drops when we move farther away.

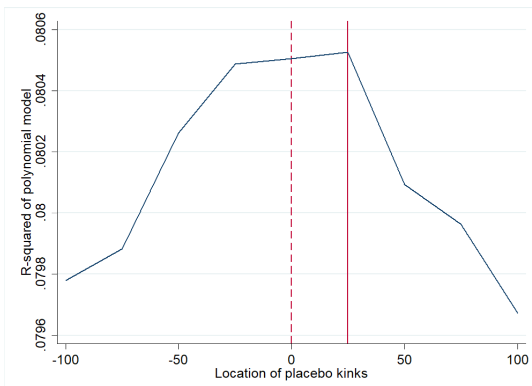

Figure 7: R-squared for different locations of the kinks.

Note: We plot the R-squared corresponding to the baseline regression computed using placebo kinks, represented in euros relative to the actual kinks, set at 0. The solid line shows the kink value at which the R-squared is maximized.

We also use placebo kinks in a different way, based on the permutation procedure suggested by Ganong and Jaeger (2018) . This strategy implies testing the significance of our parameters of interest using a range of placebo kinks instead of the actual location of each kink. The main argument, adapted to our problem, is that if the true relationship between the probability of leaving unemployment and pre-unemployment earnings is highly non-linear, many placebo kinks will show significant and large estimates.

The permutation procedure by Ganong and Jaeger (2018) assesses whether the true coefficient estimate is larger than those at placebo kinks placed away from the true kink. This procedure allows to compute 95% confidence intervals for the parameters of interest.[31] We transform these

[31] We base our computations on the Stata codes made available by Kolsrud et al. (2018).

confidence intervals and calculate the corresponding standard errors and show them in Table 8. In general, standard errors are similar to the robust standard errors in our baseline estimation.

Table 8: RKD estimations on several outcomes: Period 2005 - 2012, workers between 30 and 50 years old

	(1) Exit in period 1	(2) Duration period 1	(3) Duration period 2	(4) Non-employment duration
θ_1	-0.014**	0.014*	0.096***	0.121***
Robust s.e.	(0.007)	(0.009)	(0.028)	(0.035)
Perm. Test s.e.	(0.008)	(0.002)	(0.036)	(0.044)
θ_2	-0.020**	0.021*	0.142***	0.168***
Robust s.e.	(0.009)	(0.011)	(0.036)	(0.045)
Perm. Test s.e.	(0.010)	(0.004)	(0.056)	(0.060)

Note: We present the estimates from our baseline equation. We include robust standard errors from the baseline estimation and standard errors from the permutation test method by Ganong and Jaeger (2018). Coefficients are transformed in order to obtain the values of interest: the impact of increasing benefits in each period on each outcome.

C Monte Carlo exercise

In the empirical application in this paper, the unemployment benefit scheme exhibits two kinks in the relationship between pre-unemployment earnings and unemployment benefits. Most of the econometric tools developed in the RKD literature are designed for the case of only one kink.

We compare results from two alternative estimation strategies in the presence of two kinks using a Monte Carlo procedure. In a first strategy, we treat the kinks as independent, and perform an estimation separately at each kink disregarding the existence of the other. This strategy follows the procedure in the classical situation with one kink. In the second strategy—the one that we used in this paper—we estimate a single equation including both kinks simultaneously. We compare both strategies to evaluate their performance in a setting that emulates the main characteristics of the case analyzed in this paper and find that they both work equally well.

C.1 Setup

Our Monte Carlo procedure takes into account the characteristics of the unemployment insurance scheme in Spain in the period 1992-2012. The level of benefits is set at $r_1 = 70\%$ of prior labor income during the first six months in unemployment, and at $r_2 = 60\%$ during the remainder of the period in which the worker is entitled to unemployment benefits.

The data for the Monte Carlo simulation are generated using the following equation, which is linear in V:[32]

$$Y_i = 1 + 0.30V_i + 0.10b_{1i} + 0.15b_{2i} + u_i \quad i = 1, ..., N,$$

where u_i is sampled from a Normal$\,(0, 0.25)$ and $V_i = \exp(z_i)500 + 1000$, whit z_i sampled from a Normal$\,(0, 1)$. We consider a sample size of $N \in \{1,000; 2,000; 5,000\}$ in each simulation, and conduct $5,000$ replications. We use three different bandwidths $h \in \{200, 350, 500\}$.

First strategy: one equation per kink

In the first strategy we estimate one equation for each kink separately. Therefore, the estimates for θ_1 and θ_2 are obtained independently from each other from the following equations:

$$E[Y|V = v] = \alpha + \gamma_1(v - k_1) + \beta_{j1}W_j(v - k_j), \quad j = 1, 2, \tag{109}$$

where $W_j = 1$ for those observations above the corresponding kink. The equation is estimated for a bandwidth h, using observations such that $|V - k_j| < h$. Then, we compute $\hat{\theta}_j = -\hat{\beta}_{j1}/r_j$, $j = 1, 2$. This is the usual strategy if only one kink is present.

[32] We construct V using a distribution similar to that observed in our dataset. We set the values of the parameters close to the point estimates obtained in our empirical exercise for total duration. Conclusions about the relative performance of the strategies are unaffected by changes in these values.

Second strategy: two kinks in the same equation

In the second strategy, we estimate a single equation to obtain the two parameters of interest, as in the paper. The equation we estimate is:

$$E[Y|V=v] = \alpha + \gamma_1(v-k_1) + \sum_{j=1}^{2} \beta_{j1} W_j(v-k_j), \tag{110}$$

where W_j is equal to 1 if pre-unemployment earnings are above kink j ($v \geq k_j, j = 1, 2$).

C.2 Results

We show the main results of the analysis in Table 9 for θ_1 and in Table 10 for θ_2. We present the mean values, the standard deviation, and 95% confidence intervals for the corresponding estimates for $\hat{\theta}_j = -\hat{\beta}_j/r_j$, $j = 1, 2$ from each strategy for $N = 5,000$.

Table 9: Monte Carlo results for θ_1 (true value: $\theta_1 = 0.10$)

Strategy	h	Mean	Std. Dev.	95% Conf. Interval	
Two equations	200	.0999957	.0013465	.0977691	.1022019
Two equations	350	.1000027	.0005675	.0990646	.100933
Two equations	500	.099996	.0003993	.0993362	.1006439
One equation	200	.1000077	.0008388	.0986043	.1013617
One equation	350	.1000055	.0004742	.0992114	.1007861
One equation	500	.1000005	.000349	.0994151	.1005778

Note: Results from Monte Carlo simulation using 5,000 replications and 5,000 observations in each replication. We use three different bandwidths in each strategy.

Table 10: Monte Carlo results for θ_2 (true value: $\theta_2 = 0.15$)

Strategy	h	Mean	Std. Dev.	95% Conf. Interval	
Two equations	200	.1500265	.0020791	.146672	.1534579
Two equations	350	.1499993	.000895	.1485171	.1514795
Two equations	500	.1499934	.0006254	.1489811	.1510052
One equation	200	.1500283	.001264	.1479446	.1521159
One equation	350	.1500122	.0007248	.1488294	.1511856
One equation	500	.1499981	.0005347	.1491137	.1508795

Note: Results from Monte Carlo simulation using 5,000 replications and 5,000 observations in each replication. We use three different bandwidths in each strategy.

To complete the analysis, in Tables 11 and 12 we show the proportion of rejections of the null hypothesis $H_0 : \theta_1 = 0.10$ versus $H1 : \theta_1 \neq 0.10$ and $H_0 : \theta_2 = 0.15$ versus $H1 : \theta_2 \neq 0.15$. As expected, precision increases with sample size, but results are good even with small sample sizes. We observe that both null hypotheses are rejected in approximately the same proportion as the significance level (5%) in almost all cases.

Table 11: Monte Carlo results for the proportion of rejections of $H_0 : \theta_1 = 0.10$

	$N = 2,000$	$N = 5,000$	$N = 10,000$
Two Equations h=200	0.055	0.054	0.053
Two Equations h=350	0.050	0.052	0.057
Two Equations h=500	0.045	0.050	0.052
One Equation h=200	0.053	0.054	0.051
One Equation h=350	0.051	0.051	0.048
One Equation h=500	0.049	0.049	0.054

Note: Results from Monte Carlo simulation using 5,000 replications and 1,000, 2,000, and 5,000 observations in each replication. We show the proportion of rejections of the null that each θ_1 is equal to the true value used in the simulations, using a significance level of 5%.

Table 12: Monte Carlo results for the proportion of rejections of $H_0 : \theta_2 = 0.15$

	$N = 2,000$	$N = 5,000$	$N = 10,000$
Two Equations h=200	0.063	0.056	0.051
Two Equations h=350	0.060	0.054	0.056
Two Equations h=500	0.054	0.054	0.052
One Equation h=200	0.058	0.057	0.050
One Equation h=350	0.059	0.043	0.054
One Equation h=500	0.055	0.050	0.050

Note: Results from Monte Carlo simulation using 5,000 replications and 1,000, 2,000, and 5,000 observations in each replication. We show the proportion of rejections of the null that each θ_2 is equal to the true value used in the simulations, using a significance level of 5%.

Because precise estimates for the parameters of interest are obtained in both cases, we conclude that using two separate equations, one for each kink, is a valid strategy.