# Plasma Notes

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# 1 February

- Read about Cascades in Chapter 8 [2].
- Go through HM derivation by hand following notes.

## 1.1 How do Tokamaks work and also fusion?

For understanding this refer to [3]. Then write down thoughts here.

## 1.2 The types of drifts.

The $E \times B$ and polarization drift are described as,

$$\mathbf{v}_{E\times B} = -\frac{1}{B}\nabla\phi \times \hat{z}, \tag{1}$$

$$\mathbf{v}_P = -\frac{1}{\omega_{ci}B}\frac{d}{dt}\nabla\phi. \tag{2}$$

Where $d/dt$ is defined as,

$$\frac{d}{dt} = \frac{\partial}{\partial t} - \frac{1}{B}(\nabla\phi \times \hat{z}) \cdot \nabla + \dots \tag{3}$$

The $\dots$ refer to higher-order corrections.

## 1.3 Hasegawa-Mima derivation and origins.

Drift waves and their nonlinear interactions are one of the most fundamental elementary processes in magnetized inhomogenous plasmas. The simplest model equation that includes a fundamental nonlinear process is known as the Charney-Hasegara-Mima equation. The advective nonlinearity (Lagrange nonlinearity) associated with $E \times B$ motion plays a fundamental role in drift wave dynamics. This nonlinearity appears in the fluid description as well as in the Vlasov description of plasmas.

The simplest model equation is constructed for the inhomogenous (Opposite of homogenous, which means it is uniform without irregularities) slab plasma, which is magnetized by a strong magnetic field in the $z$-direction. There is also a density gradient in the $x$-direction. Plasma temperature is constant, and temperature pertubations are not considered. Ion temperature is assumed to be much smaller than that of electrons. The pertubations is manly propagating in the $(x, y)$ plane, and has a small wave number in the direction of the magnetic field $k_z \ll k_\perp$. The small but finite $k_z$ is essential, so that the drift wave turulence is a quasi-two-dimensional turbulence. The electrostatic pertubation $\tilde{\phi}$ is considered. Under these specifications, the dynamical equation of plasmas is investigated and the nonlinear equation is deduced.

First, the electron response is considered. The thermal velocity is taken to be much faster than the phase velocity of waves, $v_{Te} \gg \omega/|k_z|$, so that the pressure balance of electrons along the magnetic field line provides the Boltzmann response of electrons as,

$$\frac{\tilde{n_e}}{n_0} = \frac{e\tilde{\phi}}{T_e}, \tag{4}$$

where $n_0$ is the unpertrubed density and $T_e$ is the electron temperature. The ion dynamics is studied by employing the continuity equation,

$$\frac{\partial}{\partial t} n_i + \nabla \cdot (n_i v_\perp) = 0, \tag{5}$$

and the equation of motion,

$$m_i \frac{d}{dt} v_\perp = e(-\nabla\phi + v_\perp \times B). \tag{6}$$

Ions are immobile in the direction of the magnetic field line. Time scales are assumed to be much longer than the period of the ion cyclotron motion, and the equation of motion is solved by expansion with respect to $w_{ci}^{-1} d/dt$, where $w_{ci} = eB/m_i$ is an ion cyclotron frequency. Here the velocity is defined as,

$$v_\perp = -\frac{1}{B}\nabla\phi \times \hat{z} - \frac{1}{w_{ci}B}\frac{d}{dt}\nabla\phi + \ldots, \tag{7}$$

with,

$$\frac{d}{dt} = \frac{\partial}{\partial t} - \frac{1}{B}(\nabla\phi \times \hat{z}) \cdot \nabla + \cdots, \tag{8}$$

In addition to the smallness parameter $w_{ci}^{-1} d/dt$, the normalized perturbation amplitude $e\tilde{\phi}/T_e$ and the density gradient (normalized to the wavelength) $1/L_n k$ are also taken as smallness parameters. The ordering here is to assume that they are in the same order of magnitude, i.e.,

$$\frac{1}{w_{ci}}\left|\frac{d}{dt}\right| \sim \frac{e\left|\tilde{\phi}\right|}{T_e} \sim O\left(k^{-1}L_n^{-1}\right). \tag{9}$$

This assumption, $\tilde{n_e}/n_0 \sim O\left(k^{-1}L_n^{-1}\right)$, means that we consider that the turbulence amplitude is of the order of the mixing length estimate. With this ordering, the perpendicular velocity takes a form as,

$$v_\perp = -\frac{1}{B}\nabla\phi \times \hat{z} - \frac{1}{w_{ci}B}\left(\frac{\partial}{\partial t} - \frac{1}{B}(\nabla\phi \times \hat{z} \cdot \nabla\right)\nabla\phi, \tag{10}$$

where $\nabla$ indicated the dervative with respect to $(x, y)$. The mechanism is illustrated in Figure A1.2 in [2]. Consider there are two modes obliquely propagating. Owing to the electric perturbation of mode 1, ions are subject to the $E \times B$ motion so that they drift along the equi-potential surface of mode 1. This $E \times B$ motion induces a Doppler shift, so that the electric field of mode 2 has additional temporal oscillation. This temporal variation causes the polarization drift due to mode 2. Thus, this polarization drift turns to be a beat of two modes. Putting $n_i = n_0 + \tilde{n}_i$ into the continuity equation gives,

$$\frac{\partial}{\partial t}\frac{\tilde{n}_i}{n_0} + v_\perp \cdot \left(\frac{\nabla n_0}{n_0} + \frac{\nabla \tilde{n}_i}{n_0}\right) + \left(1 + \frac{\tilde{n}_i}{n_0}\right) \nabla \cdot v_\perp = 0. \tag{11}$$

Now, the charge neutrality condition,

$$\tilde{n}_i = \tilde{n}_e \tag{12}$$

is employed, so that $\tilde{n}_i/n_0$ in (11) is replaced by $e\tilde{\phi}/T_e$. Thus becomes,

$$\begin{aligned}\frac{\partial}{\partial t}\frac{e\tilde{\phi}}{T_e} - \rho_s c_s \left(\frac{e\nabla\tilde{\phi}}{T_e} \times \hat{z}\right) \cdot \left(\frac{\nabla n_0}{n_0} + \frac{e\nabla\tilde{\phi}}{T_e}\right) \\ + \left(1 + \frac{e\tilde{\phi}}{T_e}\right) \nabla \cdot -\rho_s c_s \left(\frac{e\nabla\tilde{\phi}}{T_e} \times \hat{z}\right) \\ -\rho_s^2 \left(\frac{\partial}{\partial t} - \rho_s c_s \left(\frac{e\nabla\tilde{\phi}}{T_e} \times \hat{z}\right) \cdot \nabla\right) \frac{e\nabla\tilde{\phi}}{T_e} = 0,\end{aligned} \tag{13}$$

where $c_s$ is the ion sound speed and $\rho_s$ is the ion cyclotron radius at sound speed. Several terms vanish due to being perpendicular to the $E \times B$ drift and others because they are higher-order smallness terms. Thus, the equation turns out to be,

$$\begin{aligned}\frac{\partial}{\partial t}\frac{e\tilde{\phi}}{T_e} + \rho_s c_s \left(\frac{\nabla n_0}{n_0} \times \hat{z}\right) \cdot \frac{e\tilde{\phi}}{T_e} - \rho_s^2 \frac{\partial}{\partial t}\frac{e\nabla^2\tilde{\phi}}{T_e} \\ +\rho_s^3 c_s \nabla \cdot \left(\left(\frac{e\tilde{\phi}}{T_e} \times \hat{z}\right) \cdot \nabla\right) \frac{e\nabla\tilde{\phi}}{T_e} = 0.\end{aligned} \tag{14}$$

The second term is a linear term with the derivative in the direction of electron diamagnetic drift, and yields the drift frequency. The third term denotes the effect of the polarization drift of ions. The fourth term is the nonlinear coupling term, which is orginated by the combination of the $E \times B$ drift by one mode and the polarization drift by another. With the normalization,

$$\frac{\rho_s}{L_n}\omega_{ci} t \to t, \tag{15}$$

$$\left(\frac{x}{\rho_s}, \frac{y}{\rho_s}\right) \to (x, y), \tag{16}$$

$$\frac{L_n}{\rho_s}\frac{e\tilde{\phi}}{T_e} \to \phi, \tag{17}$$

by which $\phi$ takes the value of the order of unity, the equation is therefore rewritten as,

$$\frac{\partial}{\partial t}\left(\phi - \nabla_{\perp}^{2}\phi\right) + \nabla_{y}\phi - \nabla_{\perp}\phi \times \hat{z} \cdot \nabla_{\perp}\nabla_{\perp}\phi = 0, \tag{18}$$

where the derivative on the $(x, y)$ plane is explicitly written. This equation is also written as,

$$\partial_t\left(\phi - \nabla_{\perp}^{2}\phi\right) + \nabla_{y}\phi - \left[\phi, \nabla_{\perp}^{2}\phi\right] = 0. \tag{19}$$

The derivation was followed using instructions from, [2]. Read more with regards to the Fourier Decomposition.

## 1.4 Work thus far with pseudo-spectral code.

## 1.5 What is multiple scale perturbation?

Multiple-scale analysis comprises techniques used to construct uniformly valid approximations to the solutions of perturbation problems, both for small as well as large values of the independent variables. This is done by introducing fast-scale and slow-scale variables for an independent variable, and subsequently treating these variables, fast and slow, as if they are independent.

Let's begin by constructing asymptotic expansions of periodic solutions of ODEs. The first example, Duffing's equation, is a Hamiltonian system with a family of periodic solutions. The second example, van der Pol's equation, has an isolated limit cycle.

### 1.5.1 *Duffing's* equation

Consider an undamped nonlinear oscillator described by Duffing's equation

$$y'' + y + \epsilon y^3 = 0 \tag{20}$$

where the prime denotes a dervative with respect to time t. We look for solutions $y(t, \epsilon)$ that satisfy the initial conditions

$$y(0, \epsilon) = 1, \tag{21}$$

$$y'(0, \epsilon) = 0. \tag{22}$$

We look for straightforward expansion of an asymptotic solution as $\epsilon \to 0$,

$$y(t, \epsilon) = y_0(t) + \epsilon y_1(t) + O(\epsilon^2). \tag{23}$$

The leading-order perturbation equations are

$$y_0'' + y_0 = 0, \tag{24}$$

$$y_0(0) = 1, \tag{25}$$

$$y_0'(0) = 0, \tag{26}$$

with the solution

$$y_0(t) = \cos t. \tag{27}$$

The next-order perturbation equations are

$$y_1'' + y_1 + y_0^3 = 0, \tag{28}$$

$$y_1(0) = 0, \tag{29}$$

$$y_1'(0) = 0, \tag{30}$$

with the solution

$$y_1(t) = \frac{1}{32}\left[\cos 3t - \cos t\right] - \frac{3}{8} t \sin t. \tag{31}$$

This solution contains a *secular term* that grows linearly in $t$. As a result, the expansion is not uniformly valid in t, and breaks down when $t = O(\epsilon)$ and $\epsilon y_1$ is no longer a small correction to $y_0$.

The solution is, in fact, a perodic function of $t$. The straightforward expansion breaks down because it does not account for the dependence of the period of the solution on $\epsilon$.

## 1.6 What is Vlasov description of plasmas?

## 1.7 What is an inverse cascade?

## 1.8 What is multiple-scale analysis?

## 1.9 What is Landau damping?

## 1.10 What is Shear Stress?

According to sources, the origin of turbulence comes from shear stress. Turbulent fluid motion is described as an irregular condition of flow in which the various quantities show a random variation with time and space coordinates, so that statistically distinct average values can be discerned (key feature: random fluctuations, which means non-periodic, or also secondary motion). Another part mentions that turbulence originates as an instability of laminar flows if Re becomes too large. Laminar flow (or streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. Reynolds number (Re) is described as the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities.

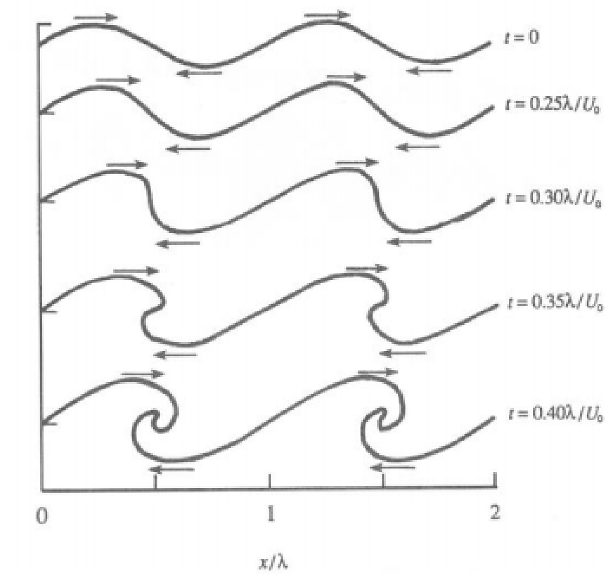


**Figure 1.1:** Nonlinear numerical calculation of the evolution of a vortex sheet that has been given a small sinusoidal displacement.

With respect to laminar and turbulent flow regimes:

- laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion;
- turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

The Reynolds number is defined as,

$$\mathrm{Re} = \frac{uL}{\nu}. \tag{32}$$

Final note, when shear stress is applied to the surface of the fluid, the fluid will continously deform, i.e. it will set up some kind of flow pattern inside the. Essentially shear stress is the opposite of the normal force that we are taught in undergraduate first year physics. Shear stress is a force that acts parallel to the body.

## 1.11 What is a thermal avalanche?

## 1.12 How does the D-11 Tokamak work at UCSD?

## 1.13 How does Turbuelnce affect fluid heat transport?

Read the following document: [13]

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