

Plasma Notes

Contents

1 February	1
1.1 How do Tokamaks work and also fusion?	1
1.2 The types of drifts.	1
1.3 Hasegawa-Mima derivation and origins.	1
1.4 Work thus far with pseudo-spectral code.	2
1.5 What is multiple scale perturbation?	2
1.5.1 <i>Duffing's</i> equation	2
1.6 What is Vlasov description of plasmas?	3
1.7 What is an inverse cascade?	3
1.8 What is multiple-scale analysis?	3
1.9 What is Landau damping?	3
1.10 What is Shear Stress?	3
1.11 What is a thermal avalanche?	5
1.12 How does the D-11 Tokamak work at UCSD?	5
1.13 How does Turbulence affect fluid heat transport?	5
References	6

List of Figures

1.1	Nonlinear numerical calculation of the evolution of a vortex sheet that has been given a small sinusoidal displacement.	4
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List of Tables

1 February

1.1 How do Tokamaks work and also fusion?

1.2 The types of drifts.

The $E \times B$ and polarization drift are described as,

$$\mathbf{v}_{E \times B} = -\frac{1}{B} \nabla \phi \times \hat{z}, \quad (1)$$

$$\mathbf{v}_P = -\frac{1}{\omega_{ci} B} \frac{d}{dt} \nabla \phi. \quad (2)$$

Where d/dt is defined as,

$$\frac{d}{dt} = \frac{\partial}{\partial t} - \frac{1}{B} (\nabla \phi \times \hat{z}) \cdot \nabla + \dots \quad (3)$$

The ... refer to higher-order corrections.

1.3 Hasegawa-Mima derivation and origins.

Drift waves and their nonlinear interactions are one of the most fundamental elementary processes in magnetized inhomogeneous plasmas. The simplest model equation that includes a fundamental nonlinear process is known as the Charney-Hasegawa-Mima equation. The advective nonlinearity (Lagrange nonlinearity) associated with $E \times B$ motion plays a fundamental role in drift wave dynamics. This nonlinearity appears in the fluid description as well as in the [Vlasov](#) description of plasmas.

The simplest model equation is constructed for the inhomogeneous (Opposite of homogeneous, which means it is uniform without irregularities) slab plasma, which is magnetized by a strong magnetic field in the z -direction. There is also a density gradient in the x -direction. Plasma temperature is constant, and temperature perturbations are not considered. Ion temperature is assumed to be much smaller than that of electrons. The perturbations are mainly propagating in the (x, y) plane, and has a small wave number in the direction of the magnetic field $k_z \ll k_\perp$. The small but finite k_z is essential, so that the drift wave turbulence is a quasi-two-dimensional turbulence. The electrostatic perturbation $\tilde{\phi}$ is considered. Under these specifications, the dynamical equation of plasmas is investigated and the nonlinear equation is deduced.

First, the electron response is considered. The thermal velocity is taken to be much faster than the phase velocity of waves, $v_{Te} \gg \omega/|k_z|$, so that the pressure balance of electrons along the

magnetic field line provides the Boltzmann response of electrons as,

$$\frac{\tilde{n}_e}{n_0} = \frac{e\tilde{\phi}}{T_e}, \quad (4)$$

where n_0 is the unpertrubed density and T_e is the electron temperature. The ion dynamics is studied by employing the continuity equation,

$$\frac{\partial}{\partial t} n_i + \nabla \cdot (n_i v_{\perp}) = 0, \quad (5)$$

and the equation of motion,

$$m_i \frac{d}{dt} v_{\perp} = e(-\nabla \phi + v_{\perp} \times B). \quad (6)$$

Ions are **immobile** in the direction of the magnetic field line. Time scales are assumed to be much longer than the period of the ion cyclotron motion, and the equation of motion is solved by expansion with respect to $w_{ci}^{-1} d/dt$, where $w_{ci} = eB/m_i$ is an ion cyclotron frequency.

To be continued [1]...

1.4 Work thus far with pseudo-spectral code.

1.5 What is multiple scale perturbation?

Multiple-scale analysis comprises techniques used to construct uniformly valid approximations to the solutions of perturbation problems, both for small as well as large values of the independent variables. This is done by introducing fast-scale and slow-scale variables for an independent variable, and subsequently treating these variables, fast and slow, as if they are independent.

Let's begin by constructing asymptotic expansions of periodic solutions of ODEs. The first example, Duffing's equation, is a Hamiltonian system with a family of periodic solutions. The second example, van der Pol's equation, has an isolated limit cycle.

1.5.1 *Duffing's equation*

Consider an undamped nonlinear oscillator described by Duffing's equation

$$y'' + y + \epsilon y^3 = 0 \quad (7)$$

where the prime denotes a dervative with respect to time t. We look for solutions $y(t, \epsilon)$ that satisfy the initial conditions

$$y(0, \epsilon) = 1, \quad (8)$$

$$y'(0, \epsilon) = 0. \quad (9)$$

We look for straightforward expansion of an asymptotic solution as $\epsilon \rightarrow 0$,

$$y(t, \epsilon) = y_0(t) + \epsilon y_1(t) + O(\epsilon^2). \quad (10)$$

The leading-order perturbation equations are

$$y_0'' + y_0 = 0, \quad (11)$$

$$y_0(0) = 1, \quad (12)$$

$$y_0'(0) = 0, \quad (13)$$

with the solution

$$y_0(t) = \cos t. \quad (14)$$

The next-order perturbation equations are

$$y_1'' + y_1 + y_0^3 = 0, \quad (15)$$

$$y_1(0) = 0, \quad (16)$$

$$y_1'(0) = 0, \quad (17)$$

with the solution

$$y_1(t) = \frac{1}{32} [\cos 3t - \cos t] - \frac{3}{8} t \sin t. \quad (18)$$

This solution contains a *secular term* that grows linearly in t . As a result, the expansion is not uniformly valid in t , and breaks down when $t = O(\epsilon)$ and ϵy_1 is no longer a small correction to y_0 .

The solution is, in fact, a periodic function of t . The straightforward expansion breaks down because it does not account for the dependence of the period of the solution on ϵ .

1.6 What is Vlasov description of plasmas?

1.7 What is an inverse cascade?

1.8 What is multiple-scale analysis?

1.9 What is Landau damping?

1.10 What is Shear Stress?

According to sources, the origin of turbulence comes from shear stress. Turbulent fluid motion is described as an irregular condition of flow in which the various quantities show a random variation

with time and space coordinates, so that statistically distinct average values can be discerned (key feature: random fluctuations, which means non-periodic, or also secondary motion). Another part mentions that turbulence originates as an instability of laminar flows if Re becomes too large. Laminar flow (or streamline flow) occurs when a fluid flows in parallel layers, with no disruption between the layers. At low velocities, the fluid tends to flow without lateral mixing, and adjacent layers slide past one another like playing cards. Reynolds number (Re) is described as the ratio of inertial forces to viscous forces within a fluid which is subjected to relative internal movement due to different fluid velocities.

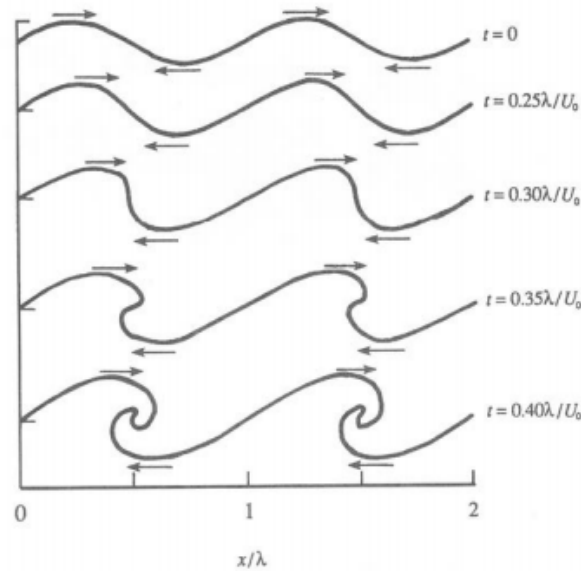


Figure 1.1: Nonlinear numerical calculation of the evolution of a vortex sheet that has been given a small sinusoidal displacement.

With respect to laminar and turbulent flow regimes:

- laminar flow occurs at low Reynolds numbers, where viscous forces are dominant, and is characterized by smooth, constant fluid motion;
- turbulent flow occurs at high Reynolds numbers and is dominated by inertial forces, which tend to produce chaotic eddies, vortices and other flow instabilities.

The Reynolds number is defined as,

$$Re = \frac{uL}{\nu}. \quad (19)$$

Final note, when shear stress is applied to the surface of the fluid, the fluid will continuously deform, i.e. it will set up some kind of flow pattern inside the. Essentially shear stress is the opposite of the normal force that we are taught in undergraduate first year physics. Shear stress is a force that acts parallel to the body.

1.11 What is a thermal avalanche?

1.12 How does the D-11 Tokamak work at UCSD?

1.13 How does Turbulence affect fluid heat transport?

Read the following document: [9]

References

- [1] Patrick H. Diamond, Sanae-I. Itoh, and Kimitaka Itoh. *Modern Plasma Physics*, volume 1. Cambridge University Press, 2010. doi: 10.1017/CBO9780511780875.
- [2] Alexander Engel, Graeme Smith, and Scott E. Parker. Quantum algorithm for the vlasov equation. *Physical Review A*, 100(6), Dec 2019. ISSN 2469-9934. doi: 10.1103/physreva.100.062315. URL <http://dx.doi.org/10.1103/PhysRevA.100.062315>.
- [3] C. Holland and P.H. Diamond. On the dynamics of large-scale structures in electron temperature gradient turbulence. *Physics Letters A*, 344(5):369 – 382, 2005. ISSN 0375-9601. doi: <https://doi.org/10.1016/j.physleta.2005.06.090>. URL <http://www.sciencedirect.com/science/article/pii/S0375960105010352>.
- [4] W. Horton. Drift waves and transport. *Rev. Mod. Phys.*, 71:735–778, Apr 1999. doi: 10.1103/RevModPhys.71.735. URL <https://link.aps.org/doi/10.1103/RevModPhys.71.735>.
- [5] N.T. Howard, C. Holland, A.E. White, M. Greenwald, and J. Candy. Multi-scale gyrokinetic simulation of tokamak plasmas: enhanced heat loss due to cross-scale coupling of plasma turbulence. *Nuclear Fusion*, 56(1):014004, dec 2015. doi: 10.1088/0029-5515/56/1/014004. URL <https://doi.org/10.1088/0029-5515/56/1/014004>.
- [6] Pijush K. Kundu, Ira M. Cohen, and David R. Dowling. *Fluid Mechanics*. Elsevier, 2016.
- [7] Guang Hao Low and Isaac L. Chuang. Optimal hamiltonian simulation by quantum signal processing. *Phys. Rev. Lett.*, 118:010501, Jan 2017. doi: 10.1103/PhysRevLett.118.010501. URL <https://link.aps.org/doi/10.1103/PhysRevLett.118.010501>.
- [8] Dastgeer Shaikh and P. K. Shukla. Fluid turbulence in quantum plasmas. *Phys. Rev. Lett.*, 99:125002, Sep 2007. doi: 10.1103/PhysRevLett.99.125002. URL <https://link.aps.org/doi/10.1103/PhysRevLett.99.125002>.
- [9] Catherine F. Westbury and R. N. Henriksen. Turbulence in Cooling Flows. , 388:64, Mar 1992. doi: 10.1086/171130.