Centrifugal

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1 Mathematical Model

We begin with the fluid equations for mass conservation, momentum without the effect of gravity, and internal energy. The purpose of this initial portion is to understand the effects that the centrifugal force plays in a incompressible system. The continuity equation in its dimensionless form is given by

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0, \tag{1}$$

where we non-dimensionalize by a lengthscale L and viscous timescale τ_ν. In the continuity equation the term $\boldsymbol{u}$ is the velocity vector. The dimensionless form of the momentum equation is given by,

$$\frac{D\boldsymbol{u}}{Dt} = -\frac{\alpha T}{\mathrm{E}^2}\boldsymbol{s} - \frac{1}{\mathrm{E}}\hat{z} \times \boldsymbol{u} + \nabla^2 \boldsymbol{u} - \nabla P. \tag{2}$$

In the momentum equation we scale temperature in the centrifugal force with α and we apply constant temperature boundary conditions. The thermal energy equation with no internal heating ($Q = 0$) is given by,

$$\frac{DT}{Dt} = \frac{1}{\mathrm{Pr}}\nabla^2 T + Q. \tag{3}$$

The non-dimensional ratios we use to control our models are defined as,

$$\mathrm{E} = \frac{\nu}{L^2 \Omega} = \frac{\tau_\Omega}{\tau_\nu}, \tag{4}$$

$$\mathrm{Pr} = \frac{\nu}{\kappa}. \tag{5}$$

The Ekman number (E) is the ratio of kinematic viscosity over rotation rate and radius, and as such, E is a dimensionless measure of the rotation rate of our system. The Prandtl number (Pr) is the ratio of the kinematic viscosity and the thermal diffusivity, this number is set to one for all the cases.

2 Centrifugal Force Only

2.1 Aspect Ratio = 0.1

The $E = 10^{-5}$ case may be under resolved. May need to increase gridpoint resolution.

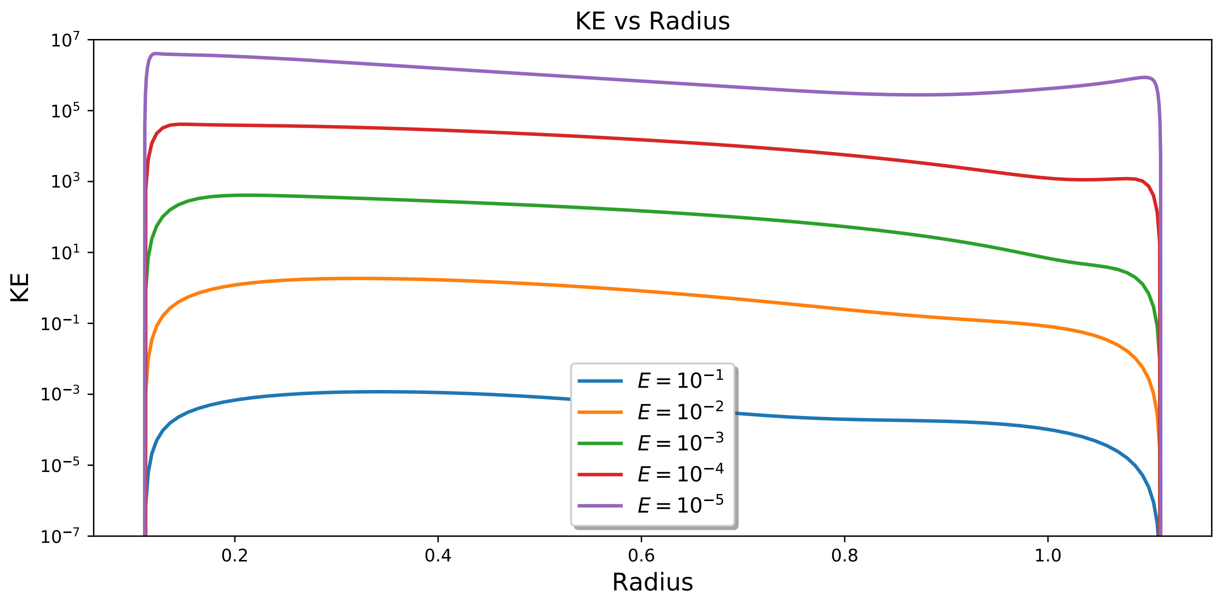

Figure 1: Kinetic energy shell average as a function of radius during equilibrated phase for a range of Ekman numbers.

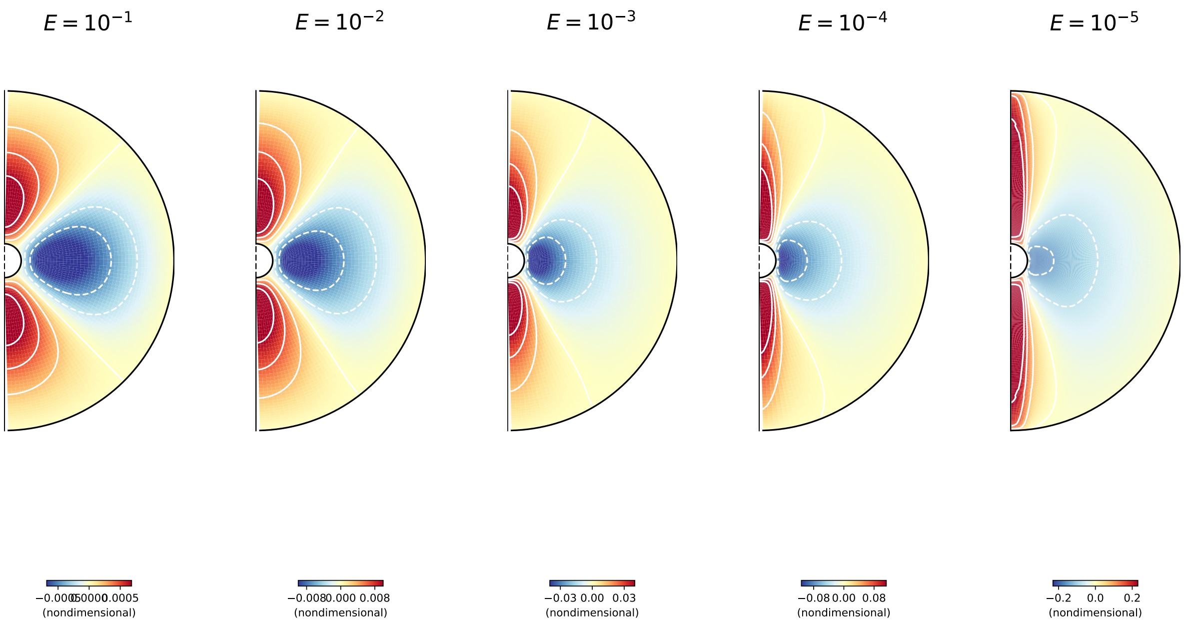

Figure 2: Temperature azimuthal average during equilibrated phase for a range of Ekman numbers.

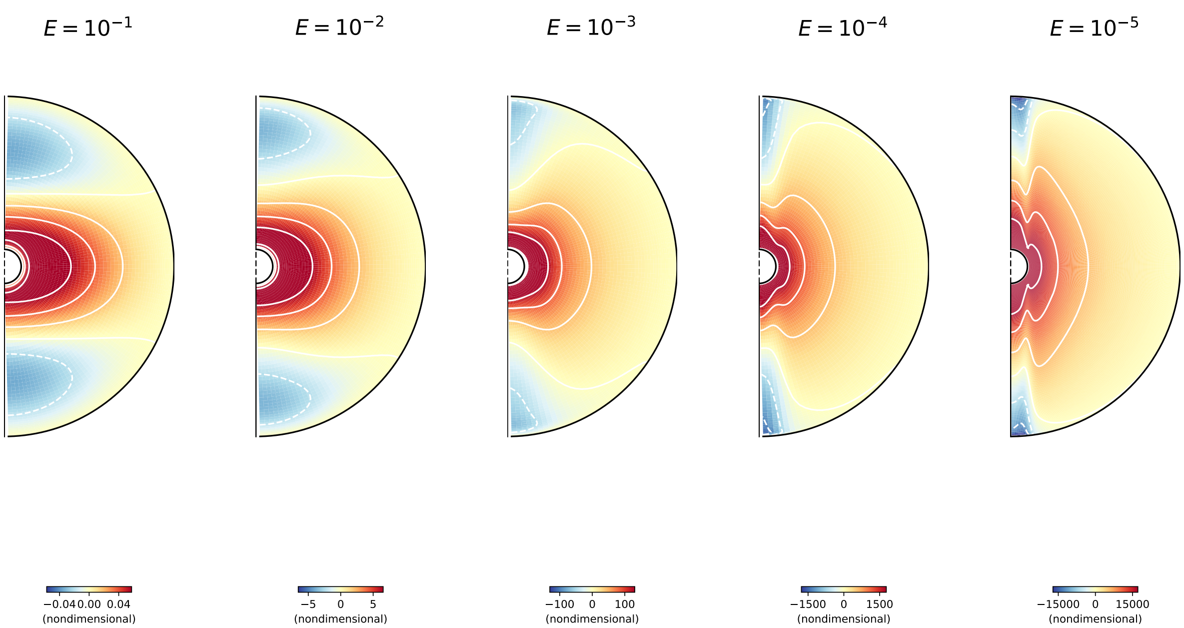

Figure 3: Angular velocity azimuthal average during equilibrated phase for a range of Ekman numbers.

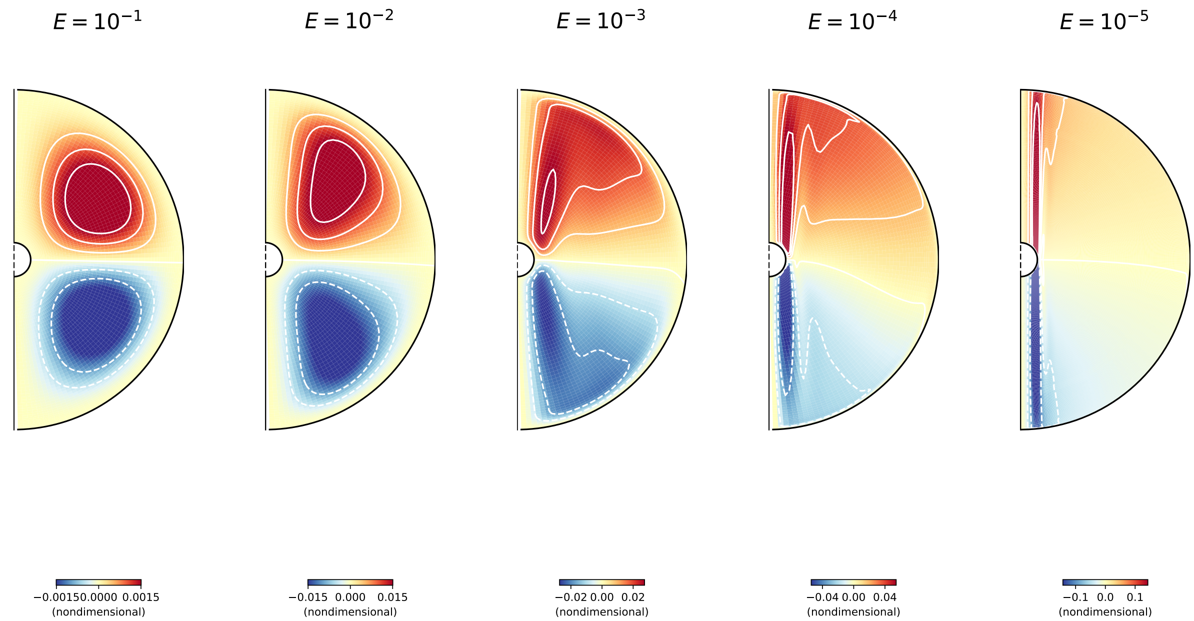

Figure 4: Mass flux azimuthal average during equilibrated phase for a range of Ekman numbers.

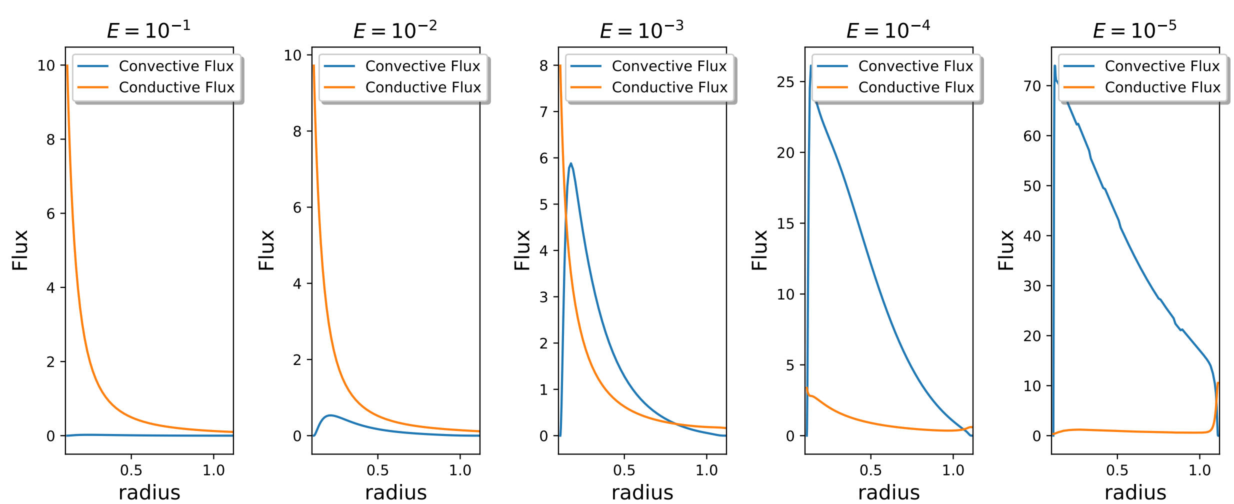

Figure 5: Azimuthal average of convective and conductive heat flux at the pole for a range of Ekman numbers.

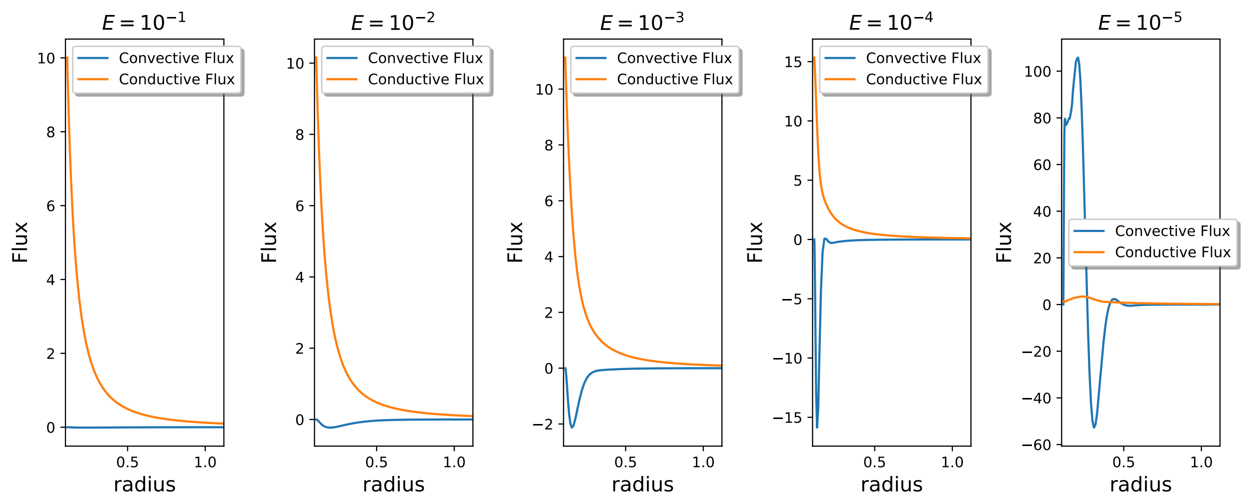

Figure 6: Azimuthal average of convective and conductive heat flux at the equator for a range of Ekman numbers.

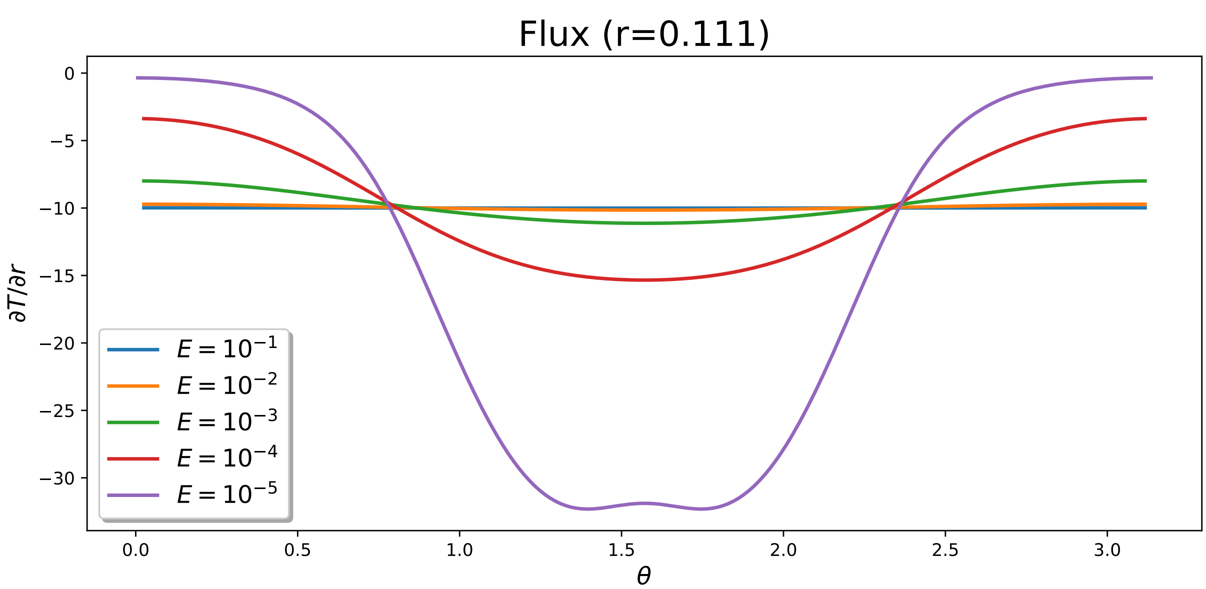

Figure 7: Conductive flux azimuthal average as a function of θ at the inner radius for a range of Ekman numbers.

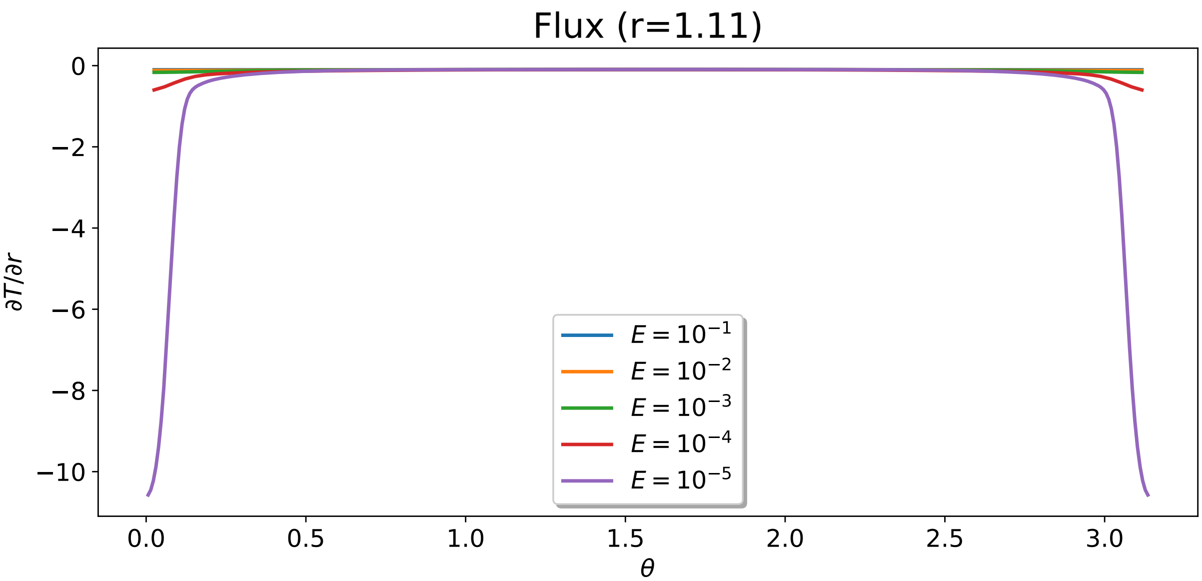

Figure 8: Conductive flux azimuthal average as a function of θ at the outer radius for a range of Ekman numbers.

Flux ratio values are not making any sense. The ratio is near 1 in some parts of the array, but for others it blows up especially at the outer and inner boundary. Need to ask Nick.

Aspect Ratio	E	Δt	N_r x N_θ x N_ϕ	KE	KE$_r$	KE$_\theta$	KE$_\phi$	Flux Ratio (r_i)	Flux Ratio (r_o)
0.1	10^{-1}	10^{-4}	64x96x192	2.72×10^{-4}	1.33×10^{-4}	9.82×10^{-5}	4.00×10^{-5}	6	4
0.1	10^{-2}	10^{-4}	64x96x192	3.71×10^{-1}	1.98×10^{-2}	1.08×10^{-2}	3.4×10^{-1}	6	4
0.1	10^{-3}	5×10^{-5}	64x96x192	67.23	2.12×10^{-1}	1.01×10^{-1}	66.92	6	4
0.1	10^{-4}	10^{-5}	64x96x192	9.77×10^{3}	3.511	2.016	9.76×10^{3}	6	4
0.1	10^{-5}	10^{-6}	128x384x768	1.21×10^{6}	66.14	58.68	1.21×10^{6}	6	4

Table 1: Details of numerical simulations performed for the incompressible model in this section. The following are specified, aspect ratio of the sphere, the Ekman number (E), the time-step size (Δt), the spatial resolution (N_r x N_θ x N_ϕ), kinetic energy and its components, and the flux ratio at the inner and outer boundary.

2.2 Aspect Ratio = 0.5

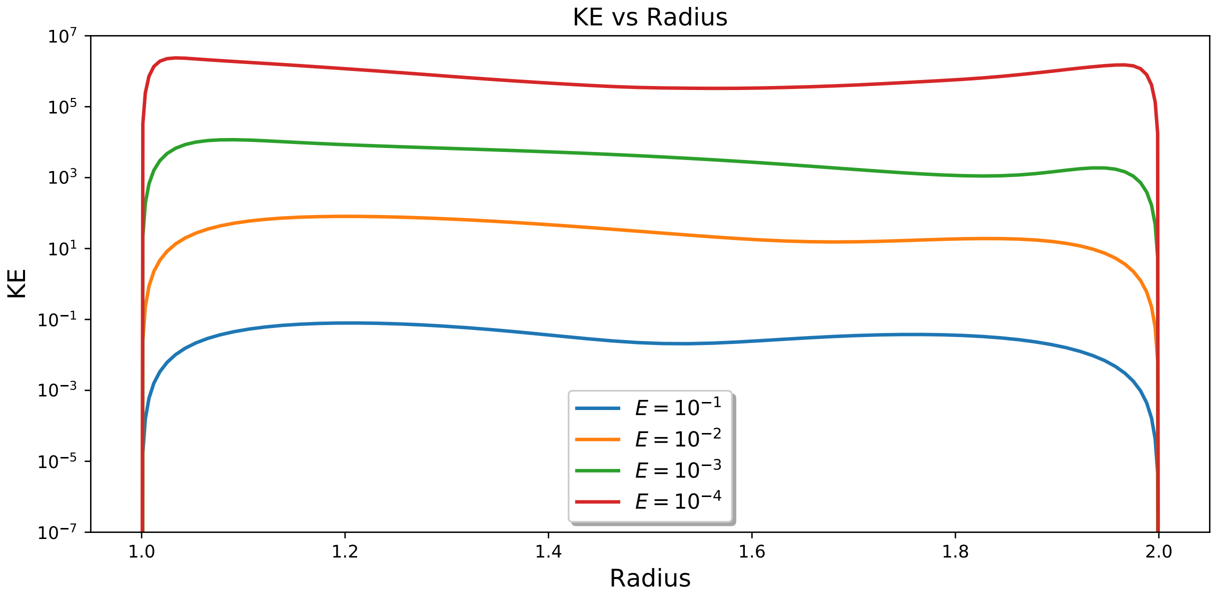

Figure 9: Kinetic energy shell average as a function of radius during equilibrated phase for a range of Ekman numbers with aspect ratio set to 0.5.

Aspect Ratio	E	Δt	N_r x N_θ x N_ϕ	KE	KE$_r$	KE$_\theta$	KE$_\phi$	Flux Ratio (r_i)	Flux Ratio (r_o)
0.5	10^{-1}	10^{-3}	64x96x192	3.22×10^{-2}	4.81×10^{-3}	2.07×10^{-2}	6.71×10^{-3}	6	4
0.5	10^{-2}	10^{-4}	64x96x192	28.3	1.98×10^{-2}	2.02	25.75	6	4
0.5	10^{-3}	5×10^{-5}	64x96x192	3.19×10^{3}	9.74	57.9	3.12×10^{3}	6	4
0.5	10^{-4}	10^{-5}	128x384x768	2.49×10^{big}	3.511	2.016	9.76×10^{3}	6	4

Table 2: Details of numerical simulations performed for the incompressible model in this section. The following are specified, aspect ratio of the sphere, the Ekman number (E), the time-step size (Δt), the spatial resolution (N_r x N_θ x N_ϕ), kinetic energy and its components, and the flux ratio at the inner and outer boundary.

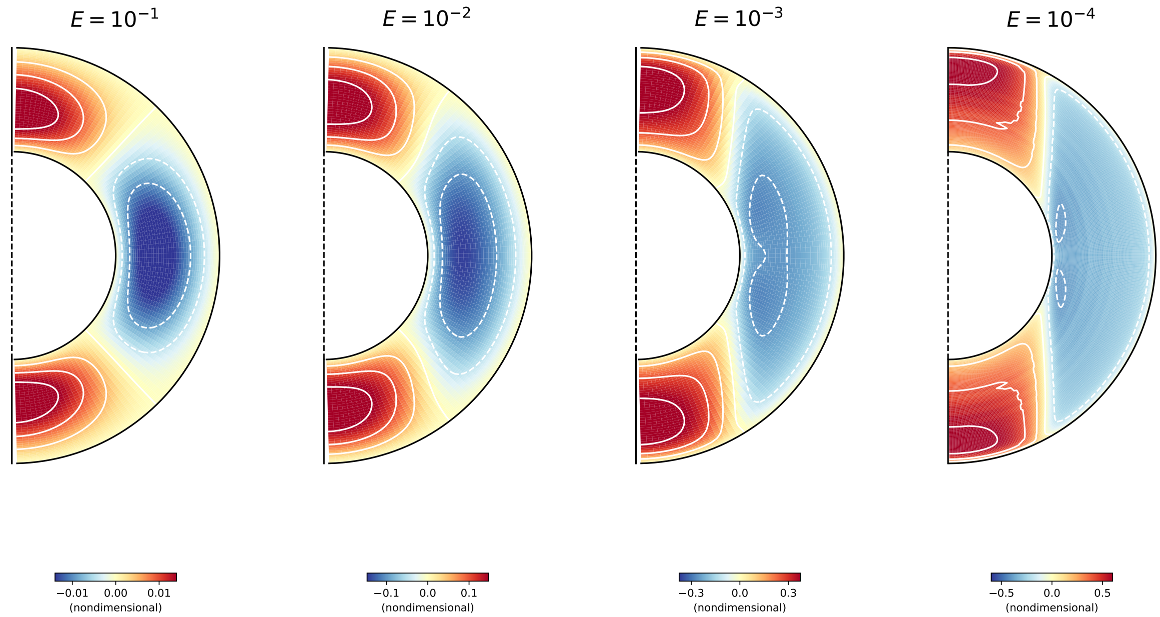

Figure 10: Temperature azimuthal average during equilibrated phase for a range of Ekman numbers.

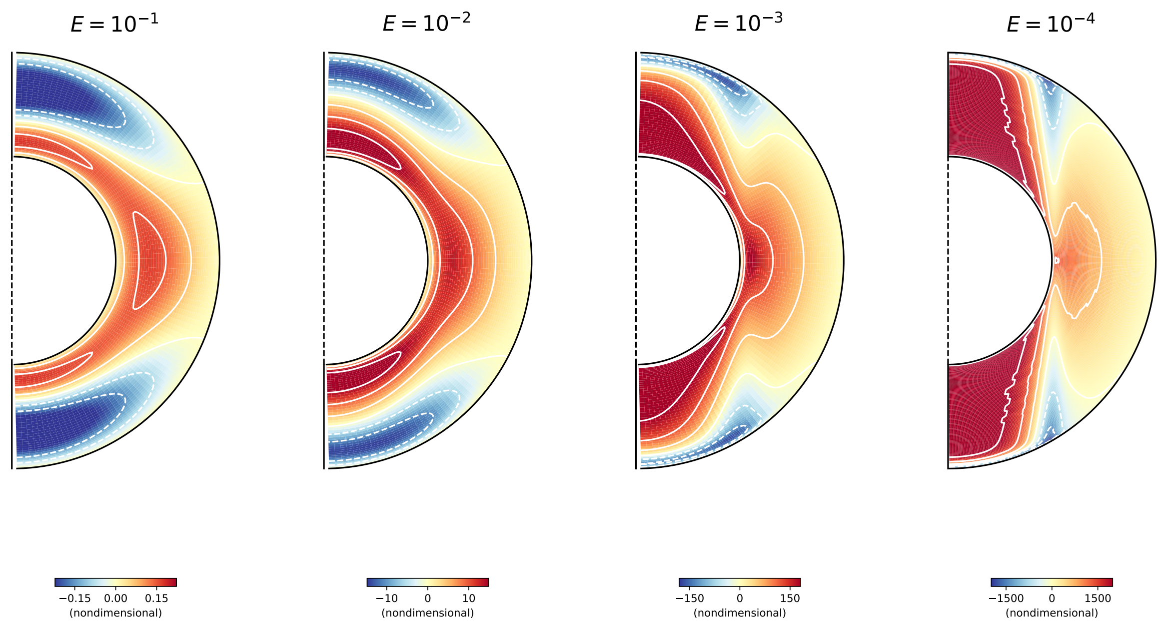

Figure 11: Angular velocity azimuthal average during equilibrated phase for a range of Ekman numbers.

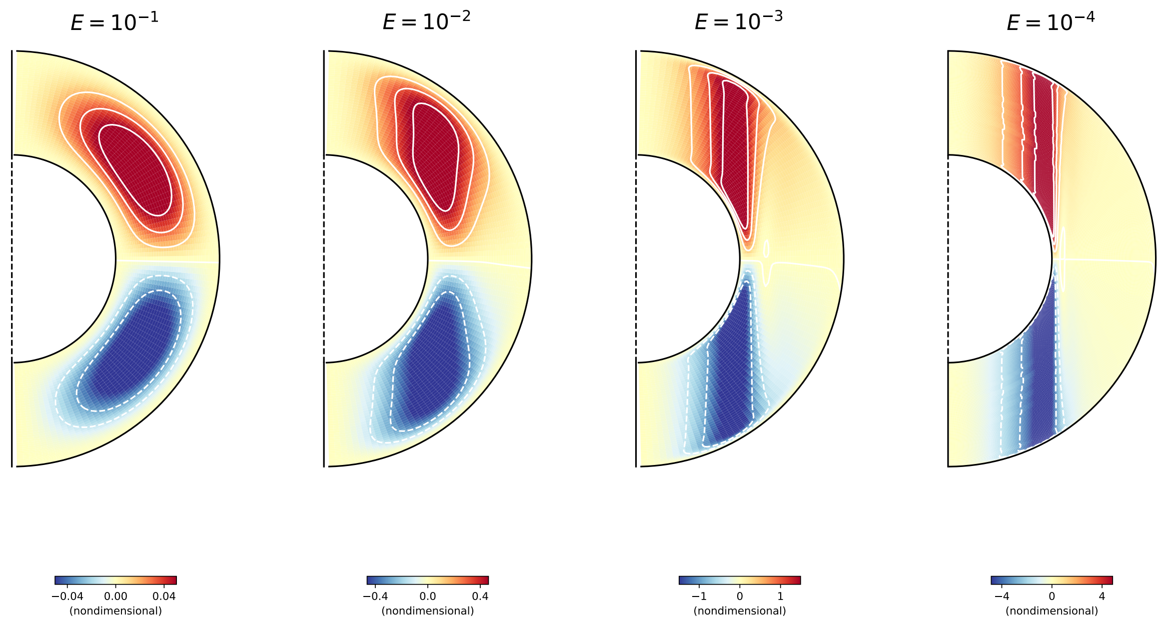

Figure 12: Mass flux azimuthal average during equilibrated phase for a range of Ekman numbers.

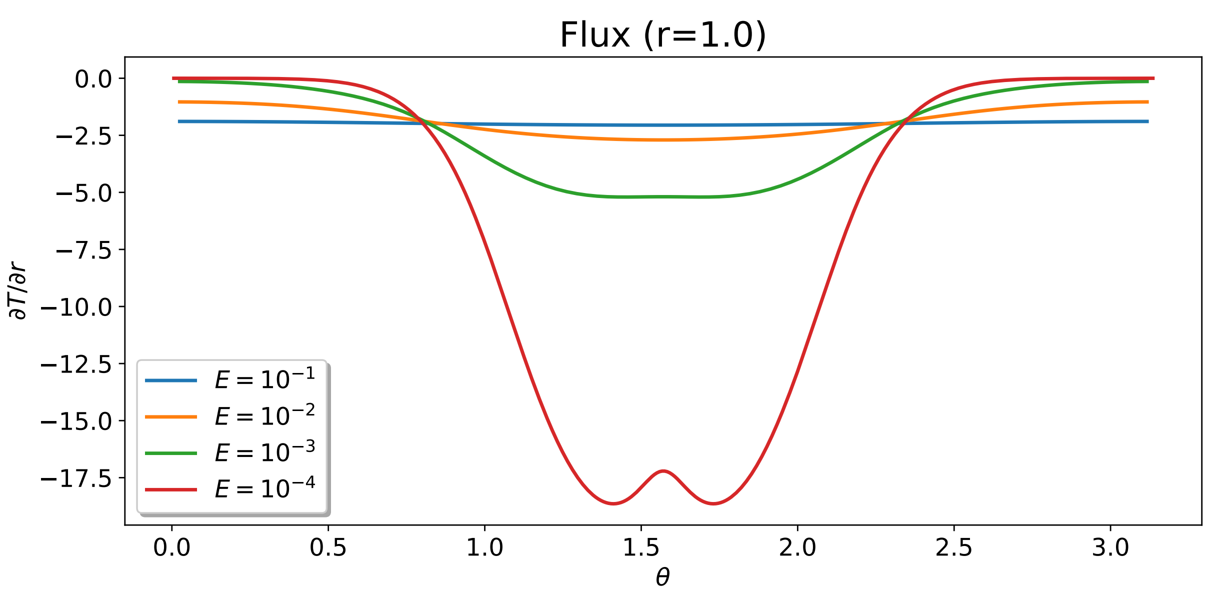

Figure 13: Conductive flux azimuthal average as a function of θ at the inner radius for a range of Ekman numbers.

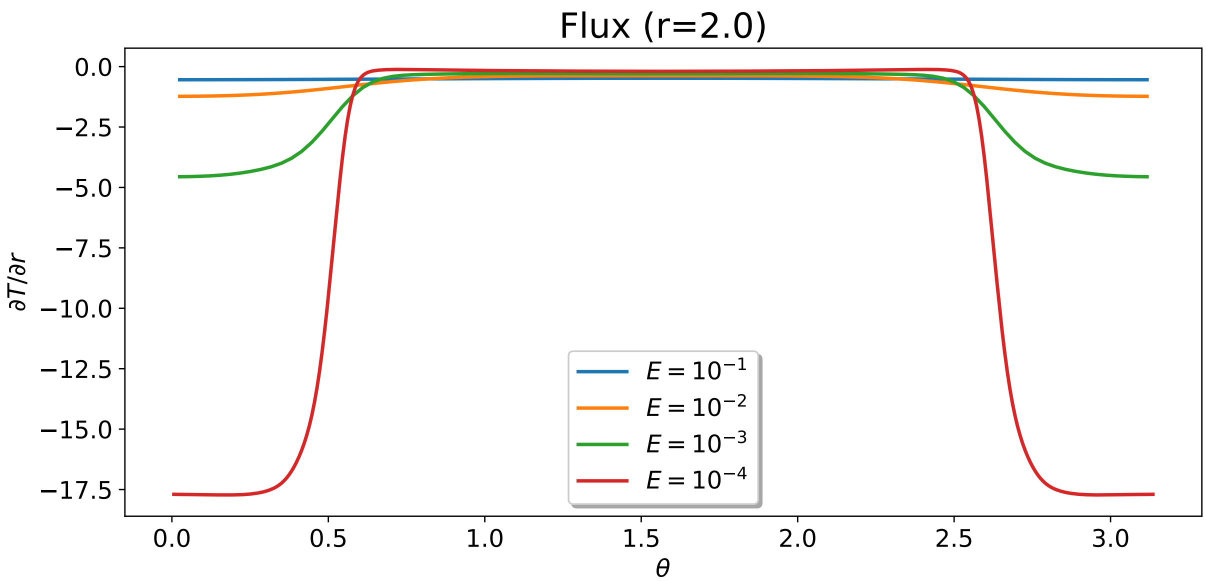

Figure 14: Conductive flux azimuthal average as a function of θ at the outer radius for a range of Ekman numbers.