

We use an alphabet composed of a blank space and the 26 letters of the English alphabet.

$$k = 2, l = 3$$

The plaintext is SZAB, which split into groups is: SZ / AB. Numerically: $SZ \mapsto 19 \cdot 27 + 26 = 539$, $AB \mapsto 1 \cdot 27 + 2 = 29$.

We choose $p = 37, q = 31$, so $n = pq = 1147, \phi(n) = 1080$. We pick $e = 17, \gcd(e, 1080) = 1$.

The public key is: $K_E = (1147, 17)$

We compute the cyphertext:

$$c = m^e \mod n = \begin{cases} 539^{17} \mod 1147 = 104 \\ 29^{17} \mod 1147 = 1050 \end{cases}$$

We calculated the powers using the repeated squaring modular exponentiation algorithm:

$$539^{17} \mod 1147 = 539^{2^0+2^4} = 539 \cdot (((539^2)^2)^2)^2 = 539 \cdot ((330^2)^2)^2 = 539 \cdot (1082^2)^2 = 539 \cdot 784^2 = 539 \cdot 1011 \mod 1147 = 104$$

$$29^{17} \mod 1147 = 29^{2^0+2^4} = 29 \cdot (((29^2)^2)^2)^2 = 29 \cdot ((841^2)^2)^2 = 29 \cdot (729^2)^2 = 29 \cdot 380^2 = 29 \cdot 1025 \mod 1147 = 1050$$

The literal equivalents are:

$$\begin{cases} 104 = 0 \cdot 27^2 + 3 \cdot 27 + 23 \mapsto _CW \\ 1050 = 1 \cdot 27^2 + 11 \cdot 27 + 24 \mapsto AKX \end{cases}$$

The cyphertext is $_CWAKX$

The private key is $K_D = d = e^{-1} \mod \phi(n) = 953$

Using the private key we decrypt the cyphertext:

$$m = c^d \mod n = \begin{cases} 104^{953} \mod 1147 = 539 \\ 1050^{953} \mod 1147 = 29 \end{cases}$$

$$104^{953} \mod 1147 = 539^{1110111001_2} = 104^{2^0+2^3+2^4+2^5+2^7+2^8+2^9} = 104 \cdot 608 \cdot 330 \cdot 1082 \cdot 1011 \cdot 144 \cdot 90 = 539$$

$$1050^{953} \mod 1147 = 1050^{2^0+2^3+2^4+2^5+2^7+2^8+2^9} = 1050 \cdot 1025 \cdot 1120 \cdot 729 \cdot 1025 \cdot 1120 \cdot 729 = 29$$

$$539 = 19 \cdot 27 + 26 \mapsto SZ$$

$$29 = 1 \cdot 27 + 2 \mapsto AB$$

The result is SZAB, which is what we encrypted in the first place.