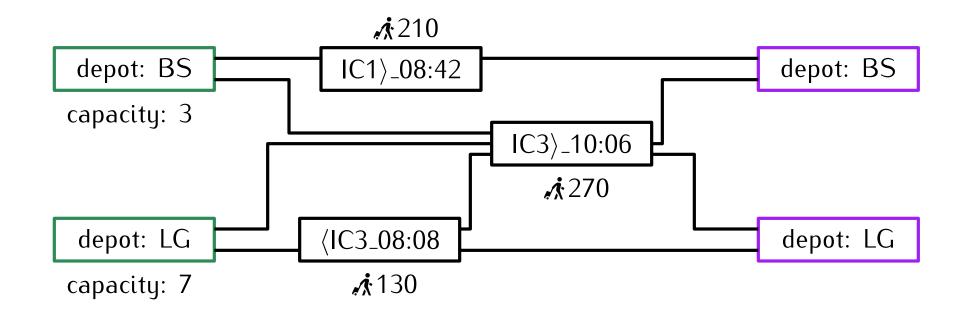
# Collaboration SBB - ETH

# **RSSched**

Rolling Stock Scheduling with Maintenance Regulations



by Leon Sering

# Rolling Stock Scheduling

# What does **rolling stock** mean?

rolling stock: "Vehicles that drive on rails."

for the project:

vehicle: multiple wagen that can drive by itself but are never

uncoupled (smallest unit)

formation: one or more vehicles that are coupled and form a train

type: vehicle can be of different types, only vehicles of the

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schedule: collection of tours, one for each vehicle

tour: sequence of activities covering one day

• **start:** spawning at a depot in the morning

• **service trip:** brings customers from a to b

• **dead-head trip:** driving from a to b without customers

• maintenance: maintenance check after given distance

• end: de-spawning at a depot in the evening



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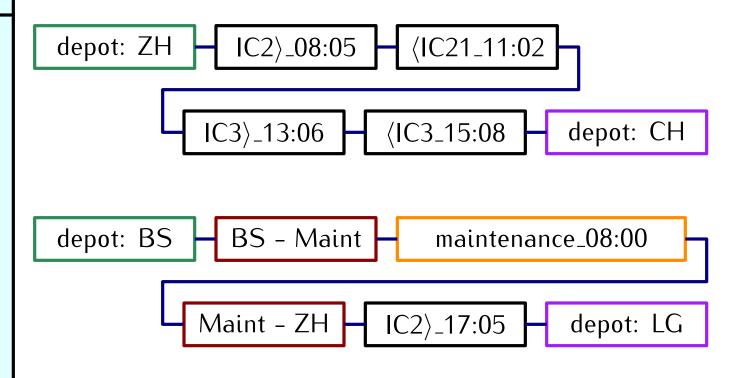
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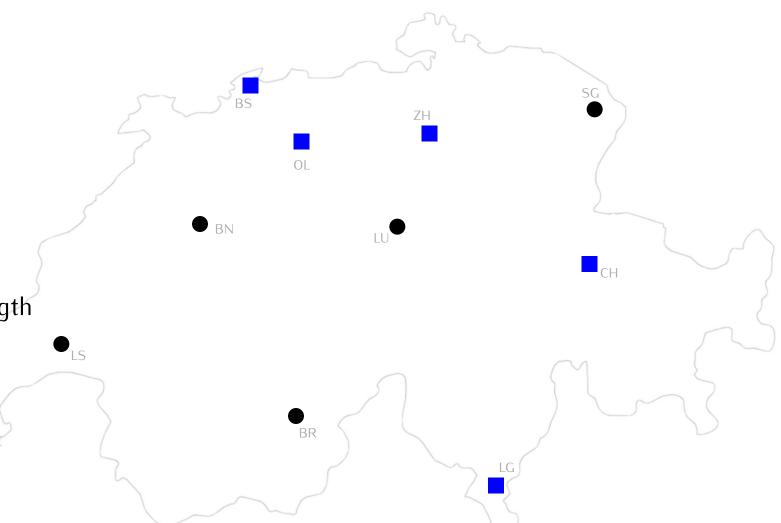
# Phase 1 Basic Scheduling

### input:

- **locations** (finite set *L*)
- depots: location, capacity
- routes origin, destination, distance, duration
- service trips route, departure time, passenger demand
- dead head trips:
  - distance matrix  $\mathbb{R}^{L \times L}_{\geq 0}$
  - travel-time matrix  $\mathbb{R}^{L \times L}_{>0}$
- vehicle type infos passenger capacity, max formation length

### input:

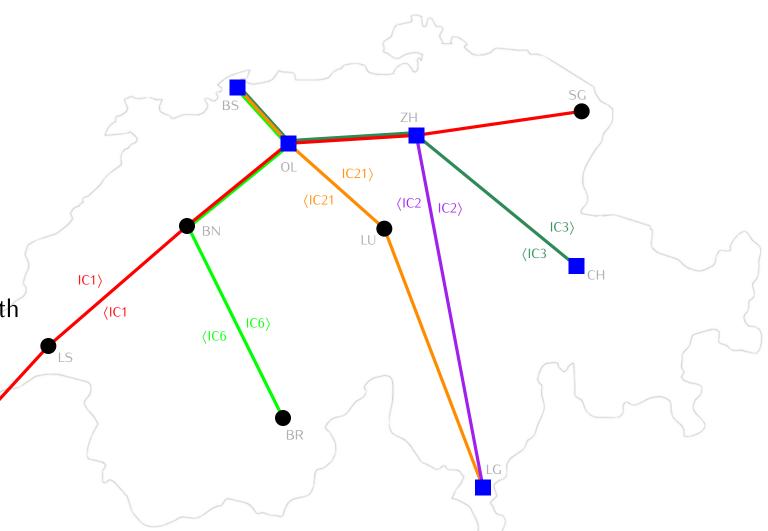
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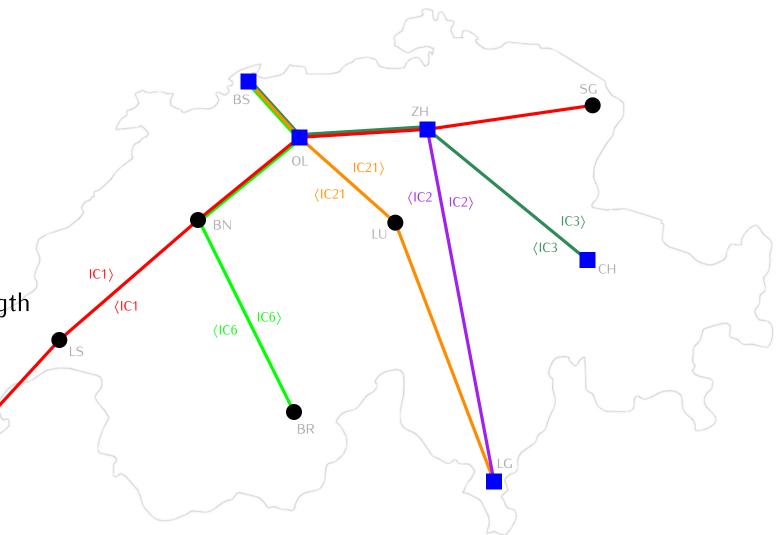
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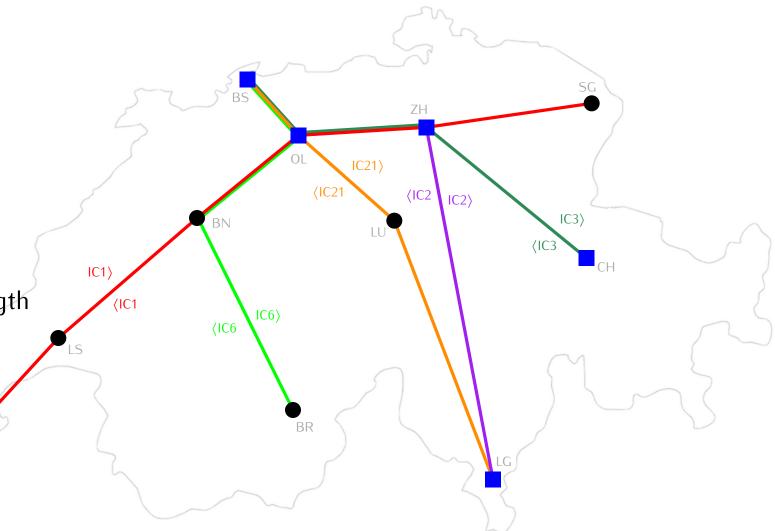


IC1>_05:42 IC1>_06:42	⟨IC1_05:07 ⟨IC1_06:07	\langle IC21_07:02 \langle IC21_08:02
IC1)_0 <b>7</b> :42	⟨IC1_07:07	(IC21_09:02
: :	: :	: :
IC1>_20:42	⟨IC1_21:07	\langle IC21_22:02

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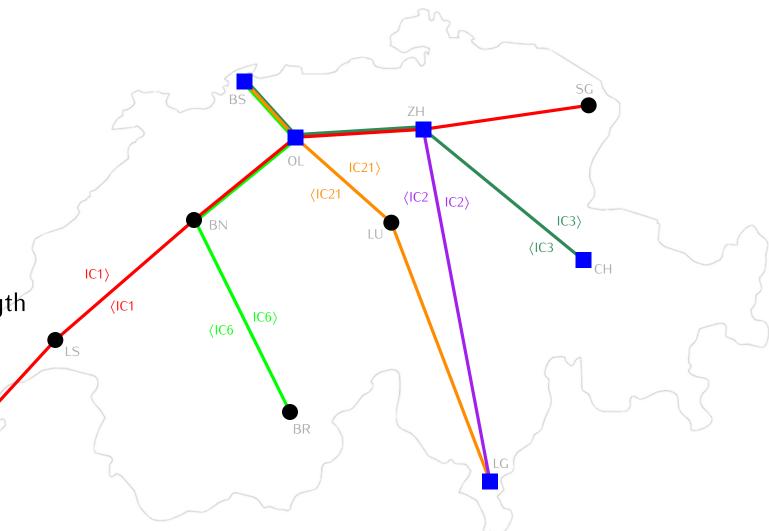


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# output (schedule):

- a list of tours starting and ending at a depot
- cyclic (every day the same schedule)
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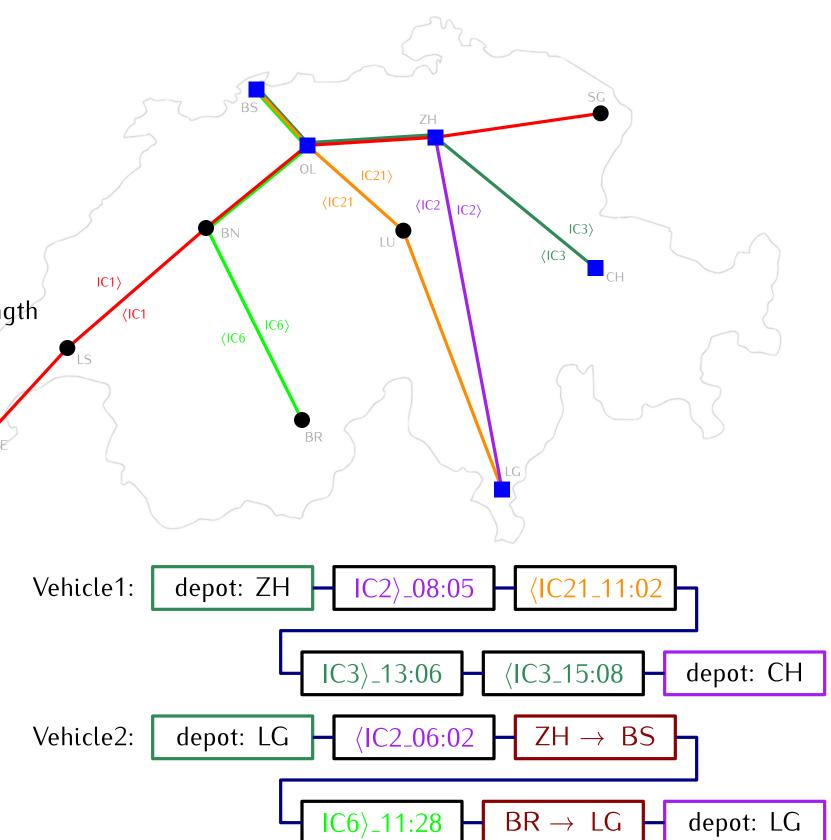
C1 >_05:42	⟨IC1_05:07 ♣ 20 ⟨IC1_06:07 ♣ 130 ⟨IC1_07:07 ♣ 310	 ⟨IC21_07:02
;	⋮	 :
:	⟨IC1_21:07 ⁂110	⟨IC21_22:02 <b>☆</b> 140

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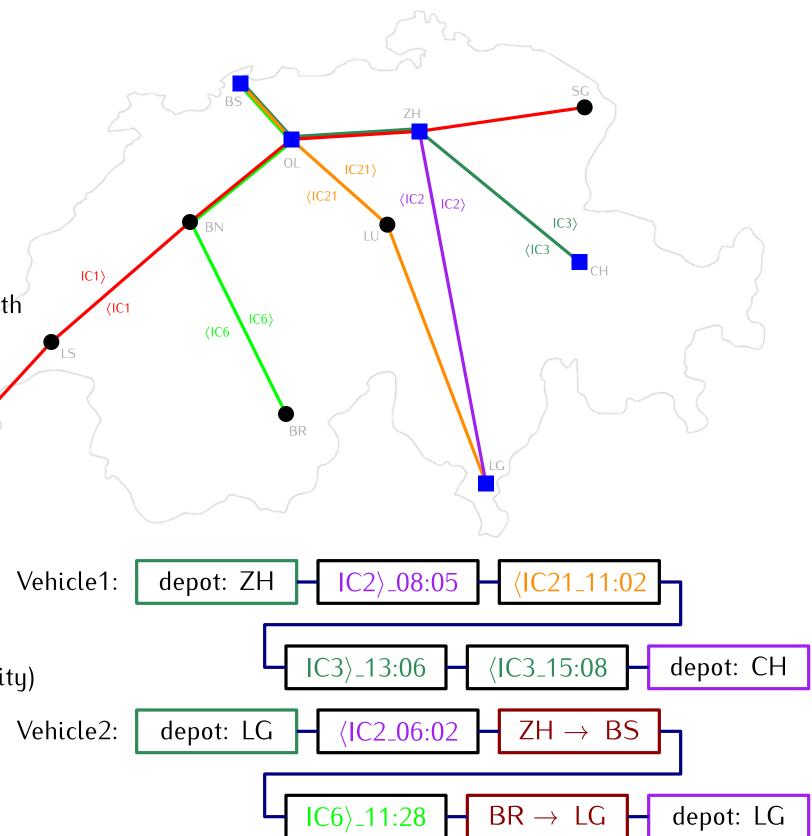
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## feasibility:

- 1. for each pair of consecutive activities  $a_1$ ,  $a_2$  in same tour:
  - $a_1$ .endLocation =  $a_2$ .startLocation
  - $a_1$ .endTime  $\leq a_2$ .startTime -gapTime
- 2. for each depot:
  - • # spawning vehicles ≤ depots capacity
  - # spawning vehicles = # de-spawning vehicles (cyclicity)



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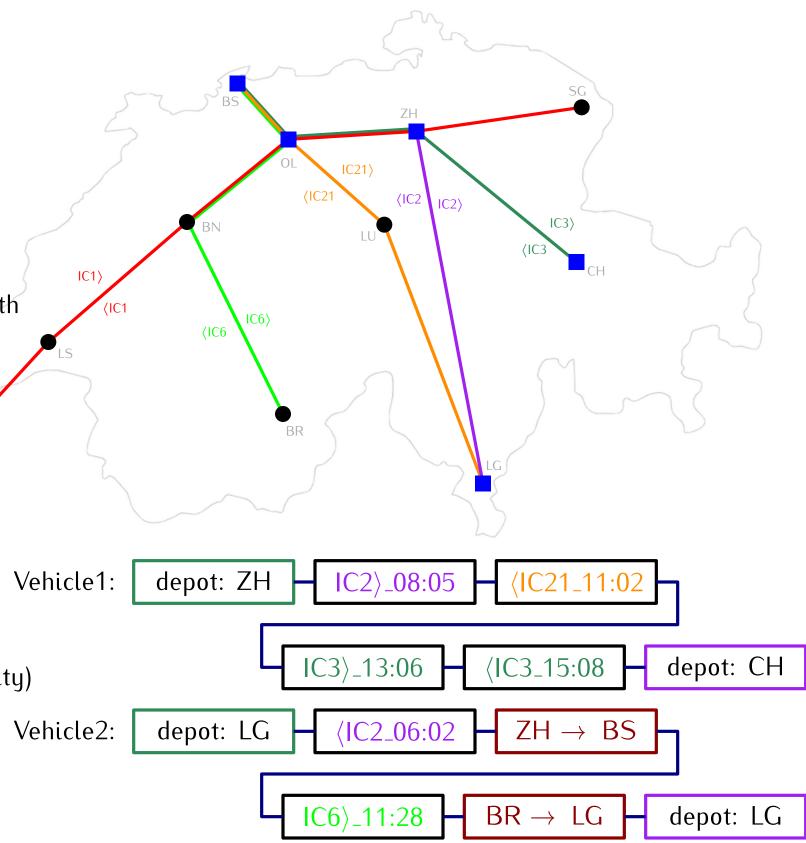
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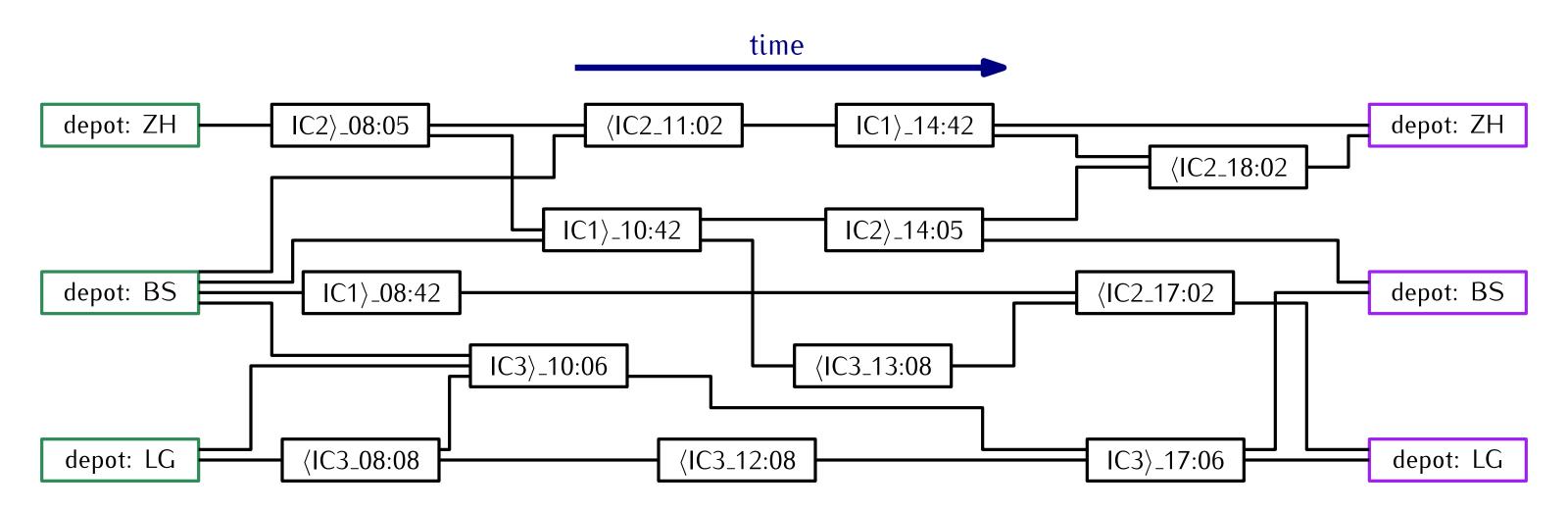
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hierachical objective: (to be minimized from top to bottom)

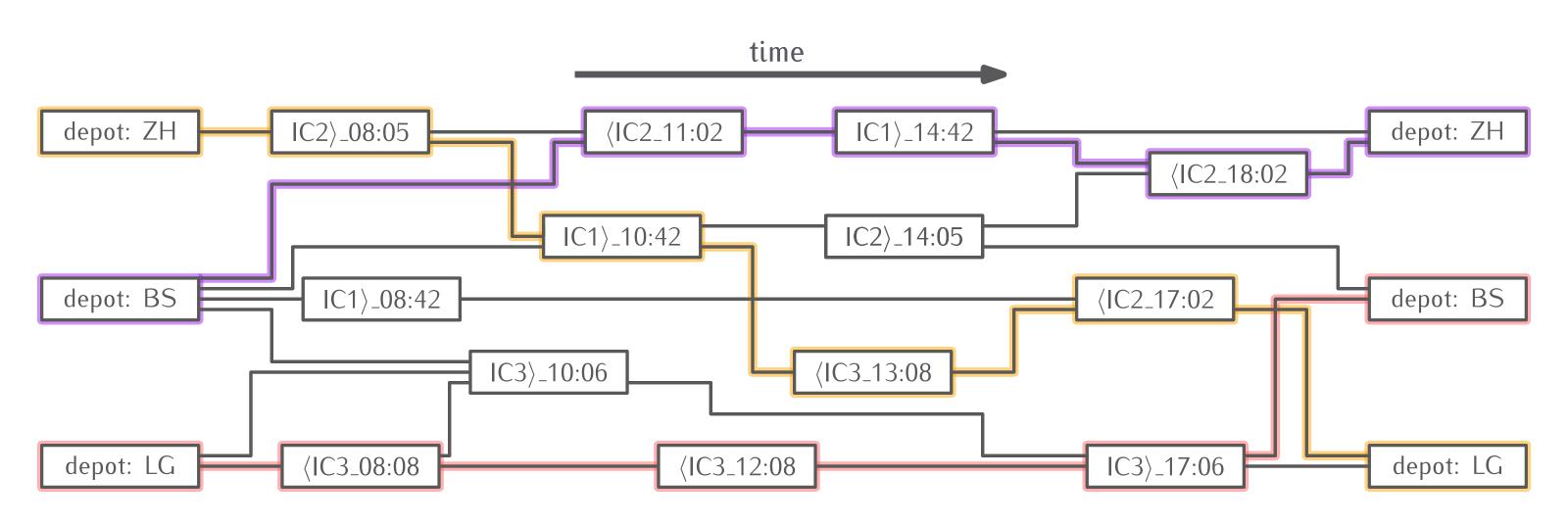
- 1. # unserved passengers
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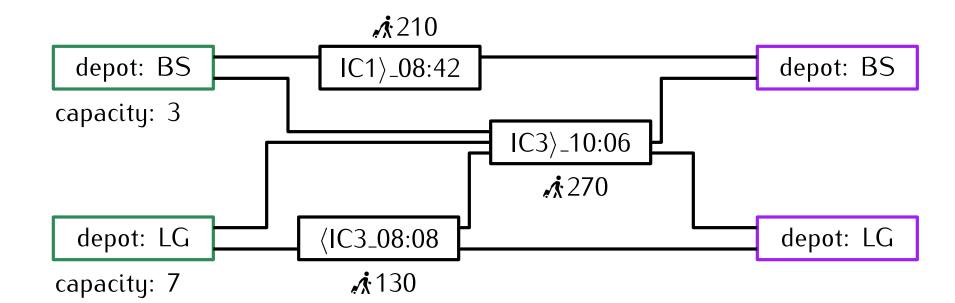
# Phase 1 - Modeling



# Phase 1 - Modeling



### Model



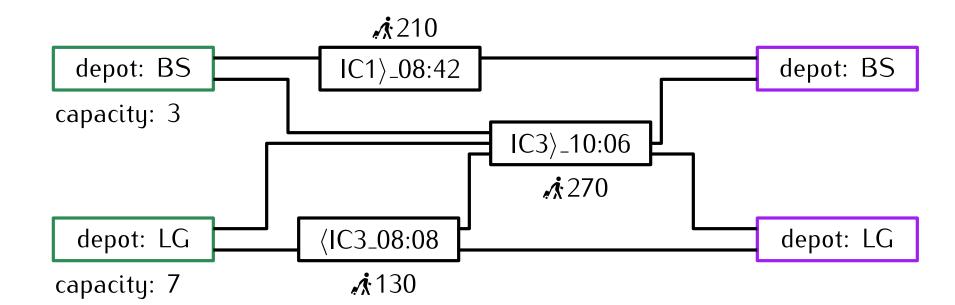
#### train info:

- passenger capacity per train: 200
- maximal vehicle in formation: 5

# hierachical objective:

- 1. # unserved passengers
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#### Model



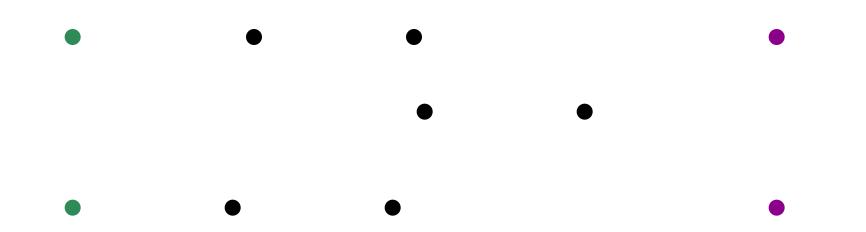
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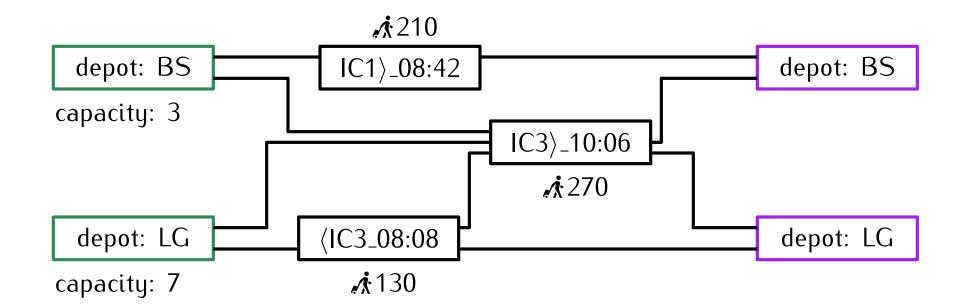
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#### Min-Cost-Circulation



#### Model



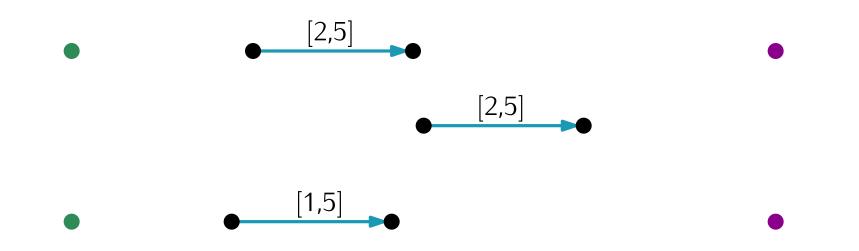
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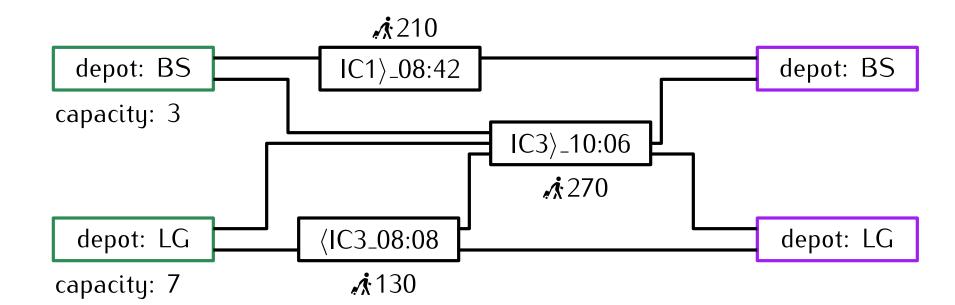
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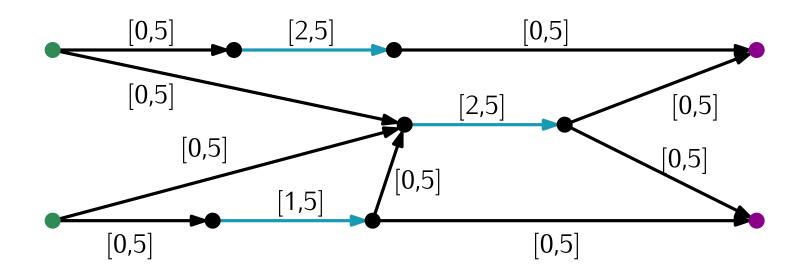
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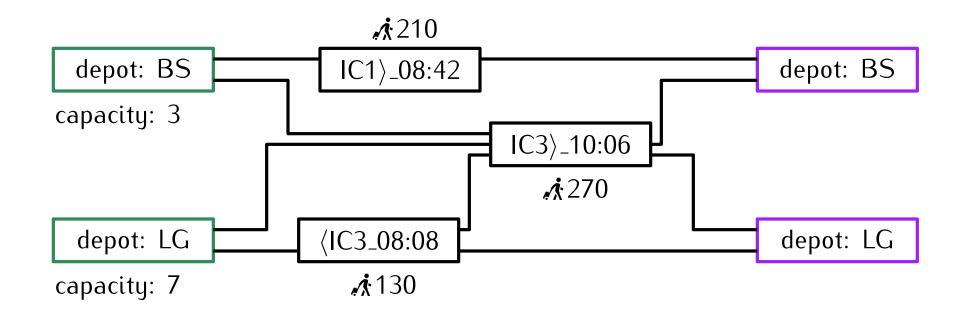
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#### Model



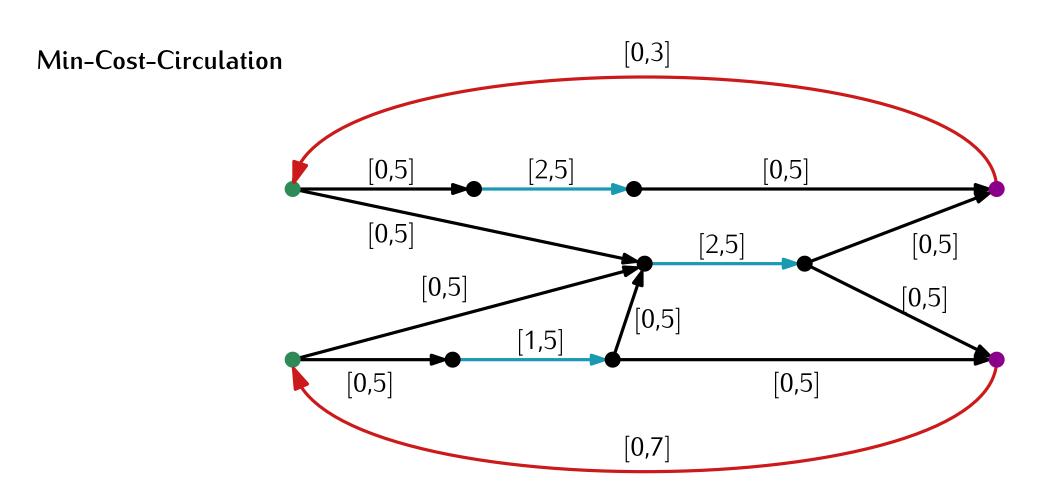
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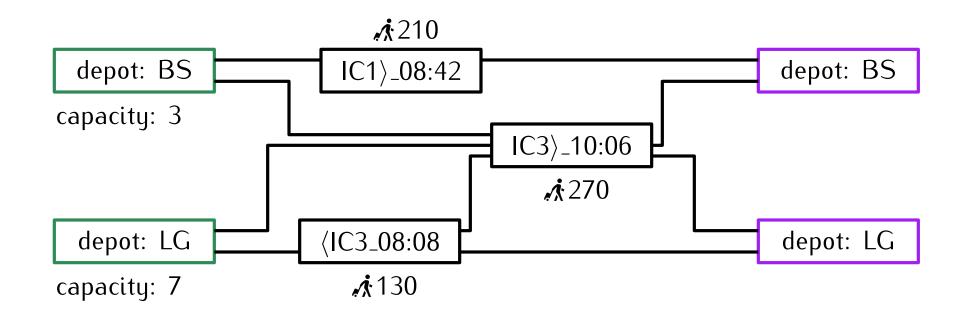
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#### Model

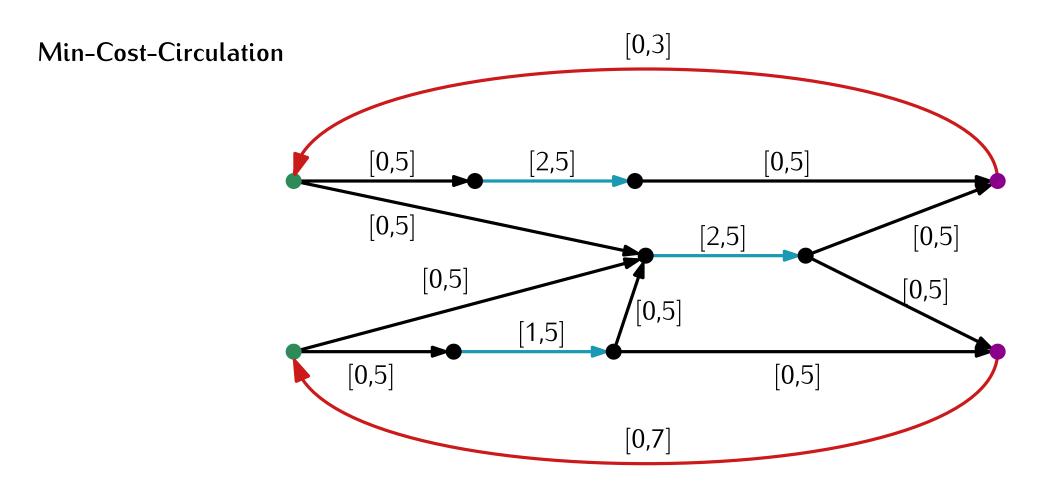


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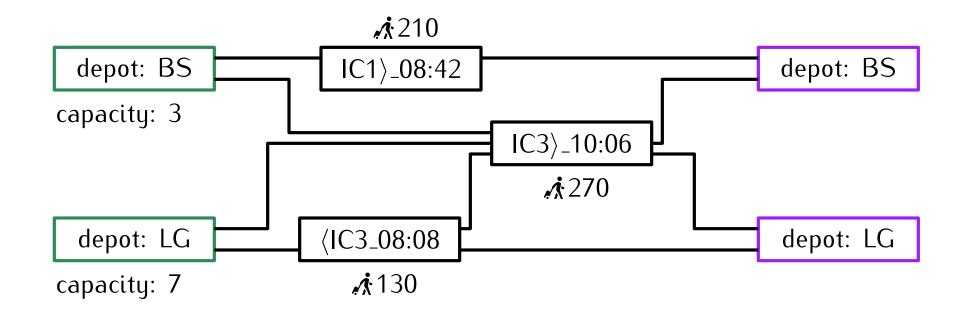
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#### costs:

- 1. forward arcs: distance traveled
- 2. depot arcs: sum of all distances

#### Model

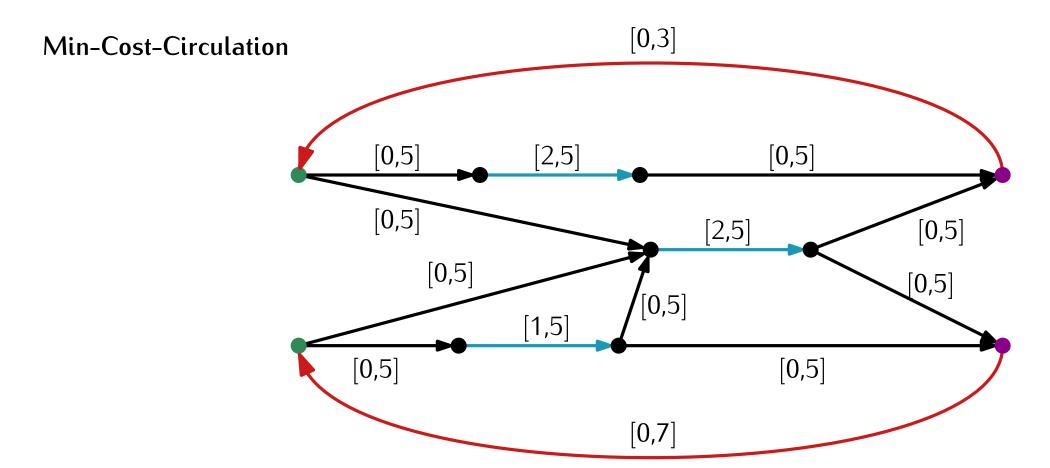


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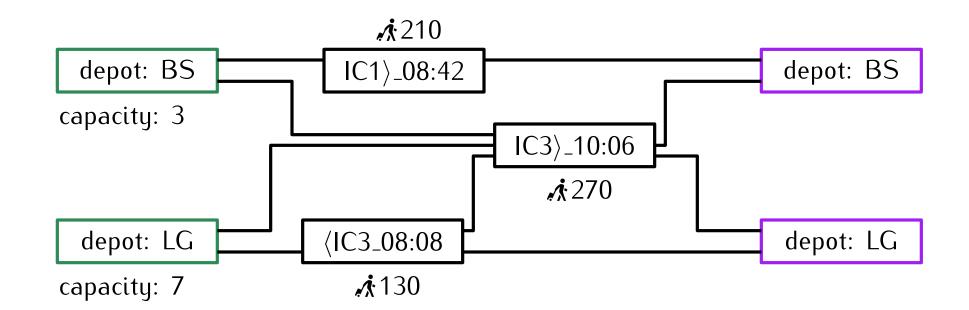
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- 1. create flow network
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#### Model



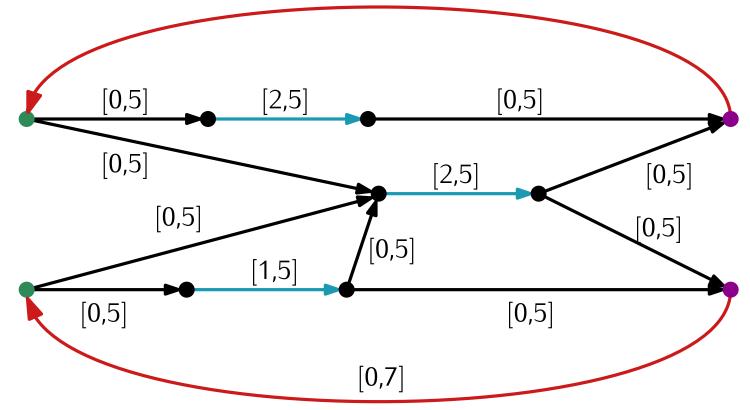
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# Min-Cost-Circulation



[0,3]

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### running time

- 1200 service trips
- 120 vehicle

- $\Rightarrow$  0.5 seconds
- Lenovo X1 Yoga

# Phase 2 Model Extensions

# seated / standing passengers

• assumption: passengers travelling < 15 minutes can stand (no seat needed)



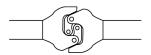
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# strengthen / weaken train formation during a trip

- split service trips into segments
- within a segment: no coupling allowed
- between segments: vehicles can be (de-)coupled



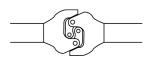
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 vehicles can be towed unused on a service trips (saves staff cost)



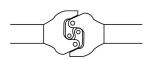
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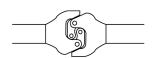
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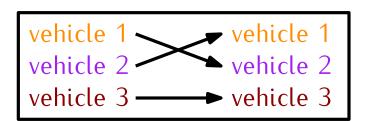






#### maintenance

- maintenance stations with multiple maintenance slots
- maintenance slots can be used by each type
- $\bullet$  vehicles must go to maintenance every 15 000 km (  $\sim$  every 20 days)
- schedule is a single day repeated each day
- ullet map vehicles arriving at a depot in the evening to vehicles departure early in the morning (on the next day)



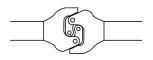
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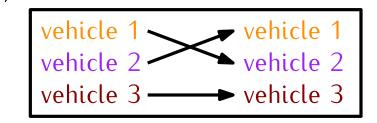
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### new hierachical objective:

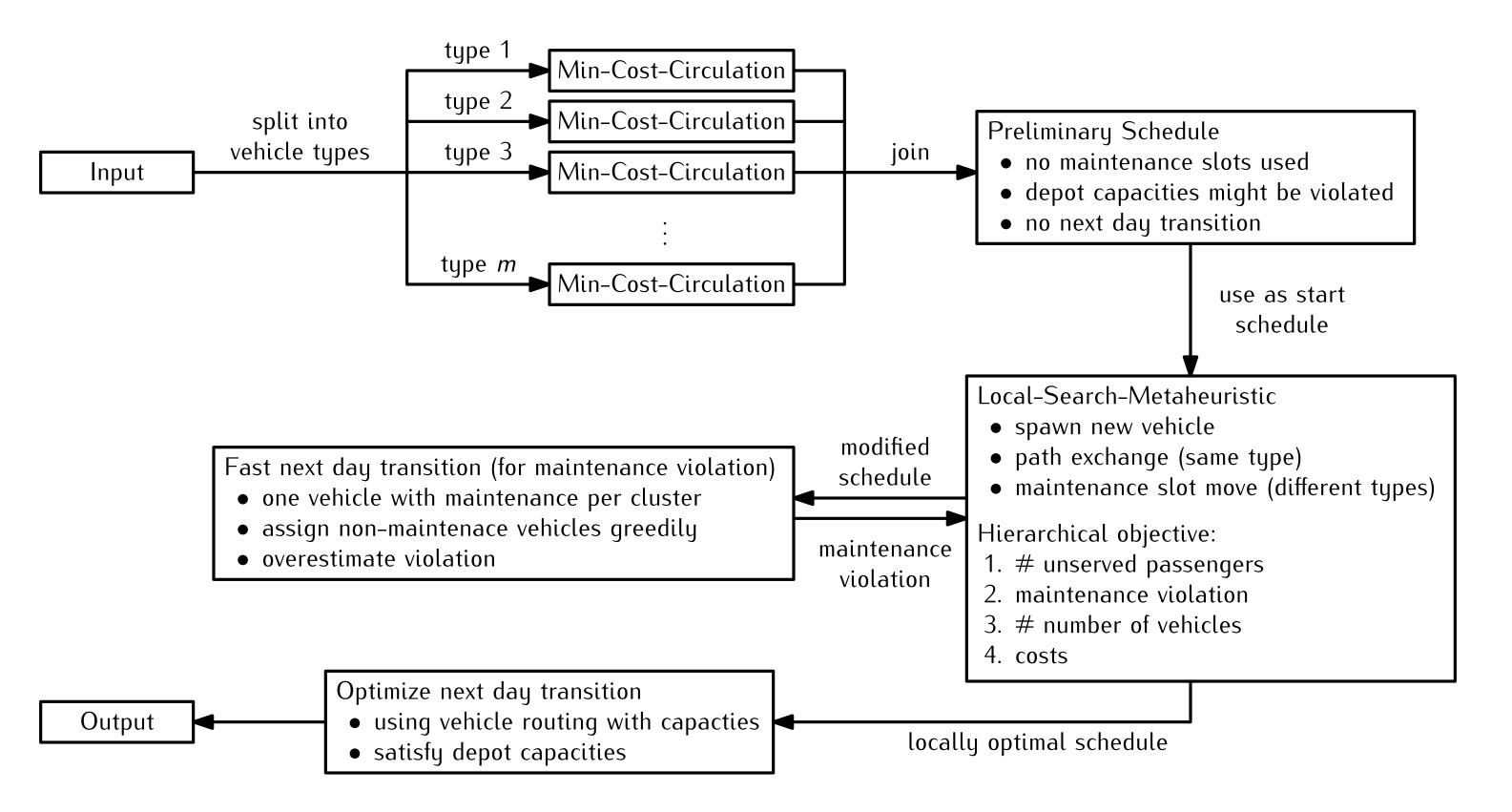
- 1. # unserved passengers
- 2. maintenance violation
- 3. # number of vehicles
- 4. costs

#### costs are a linear combination of

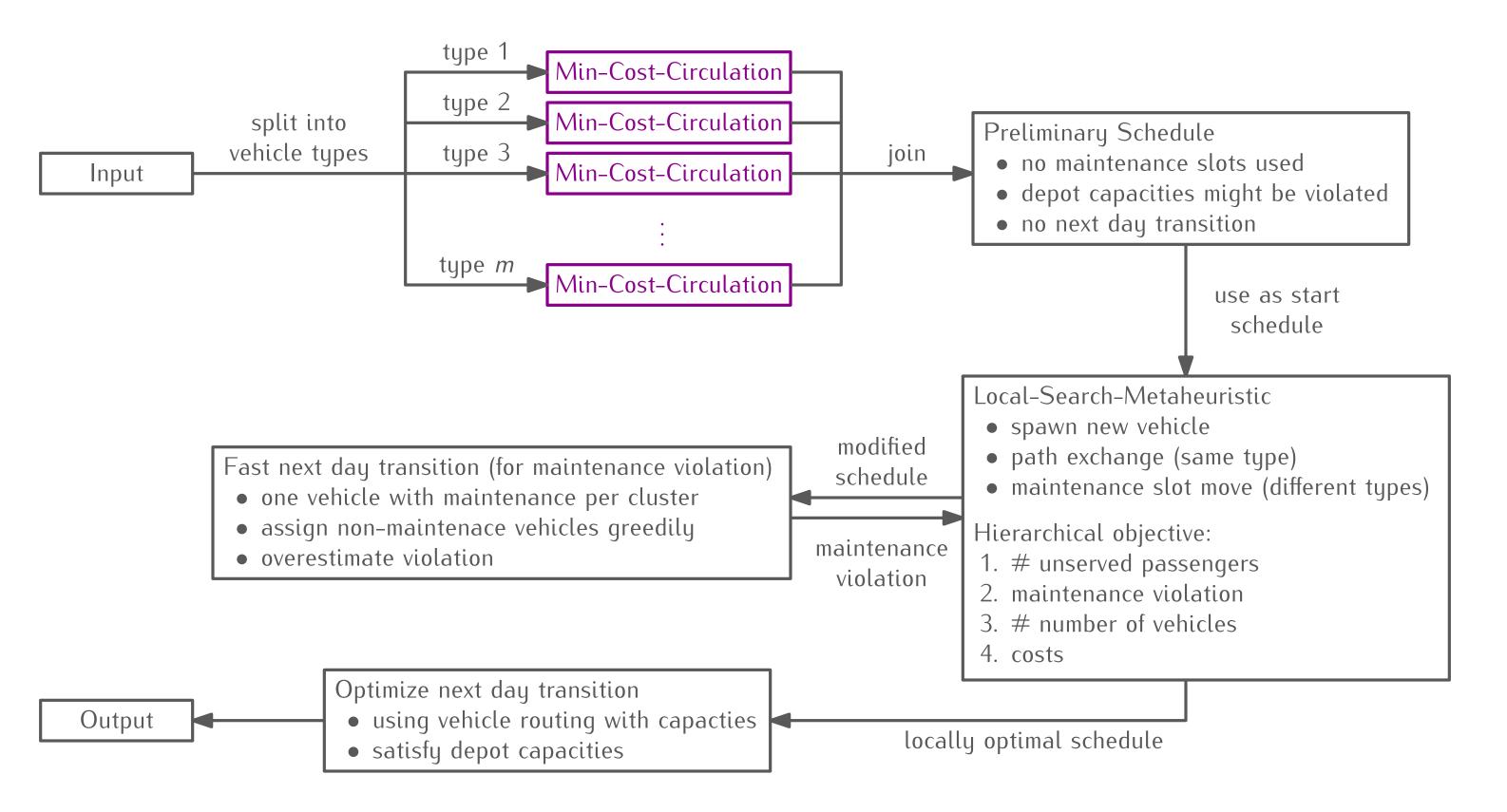
- total service trip duration
- total dead head trip duration
- mainteance duration
- idle duration
- staff cost (each train formation pays this only once)



# Phase 2 - Algorithm Overview

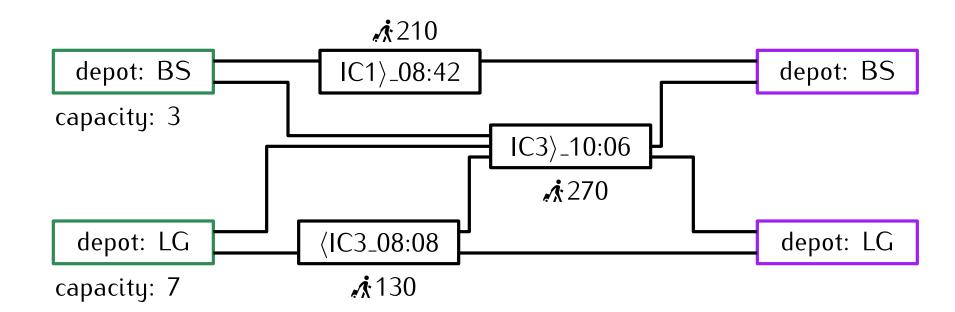


# Phase 2 - Algorithm Overview

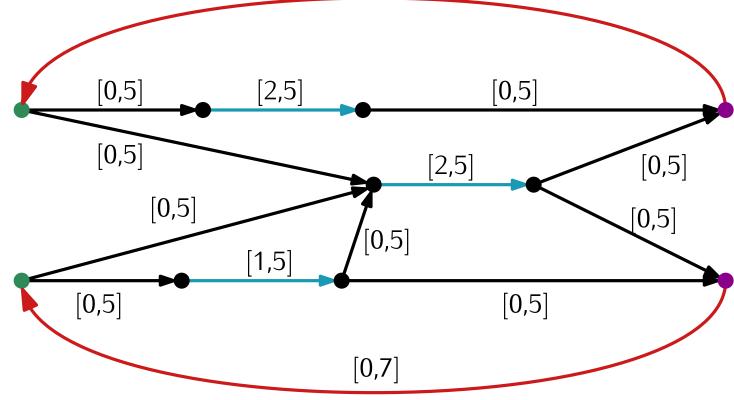


# Min-Cost-Circulation

#### Model







[0,3]

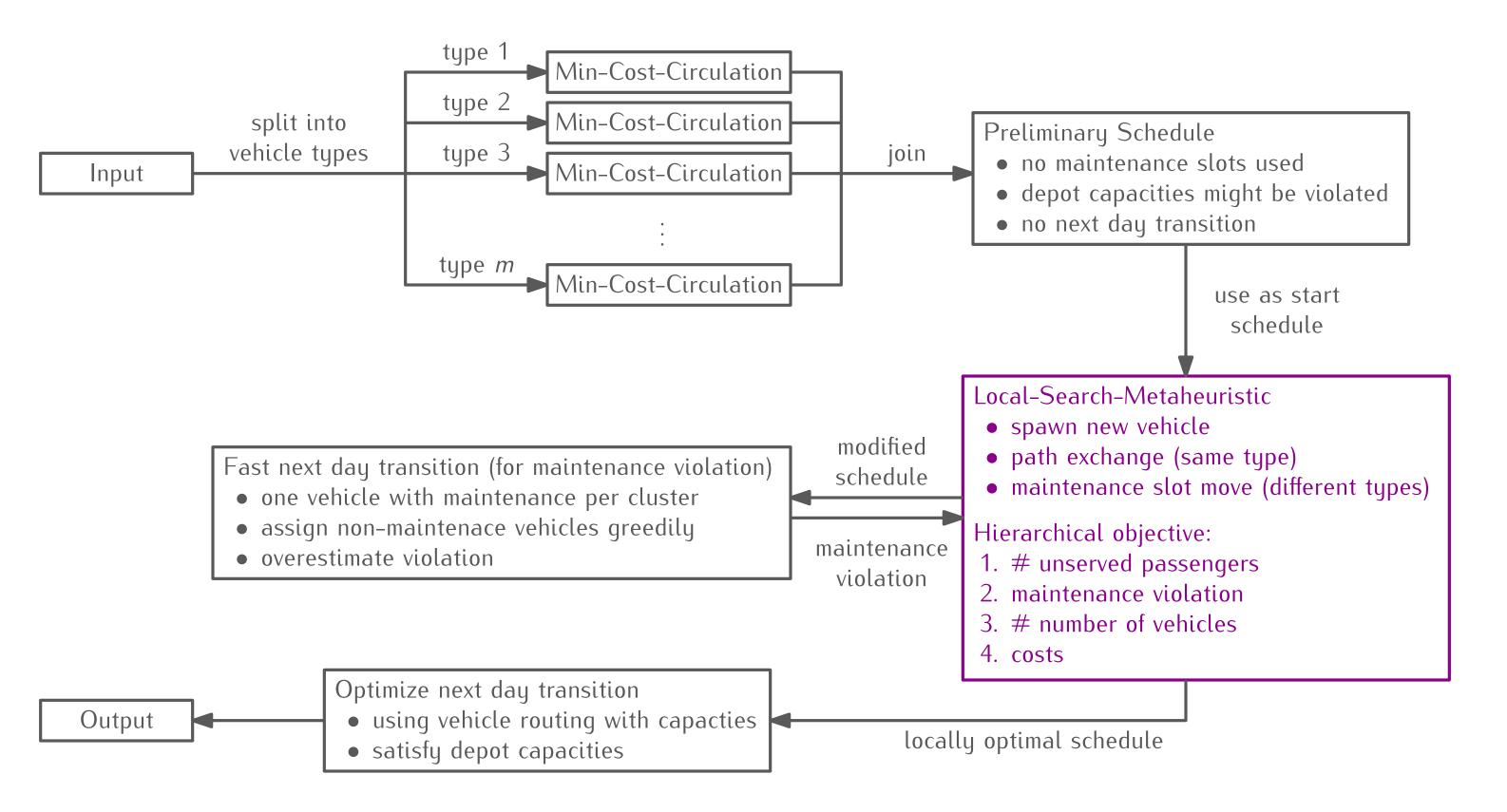
# costs:

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#### costs:

- 1. forward arcs: new costs
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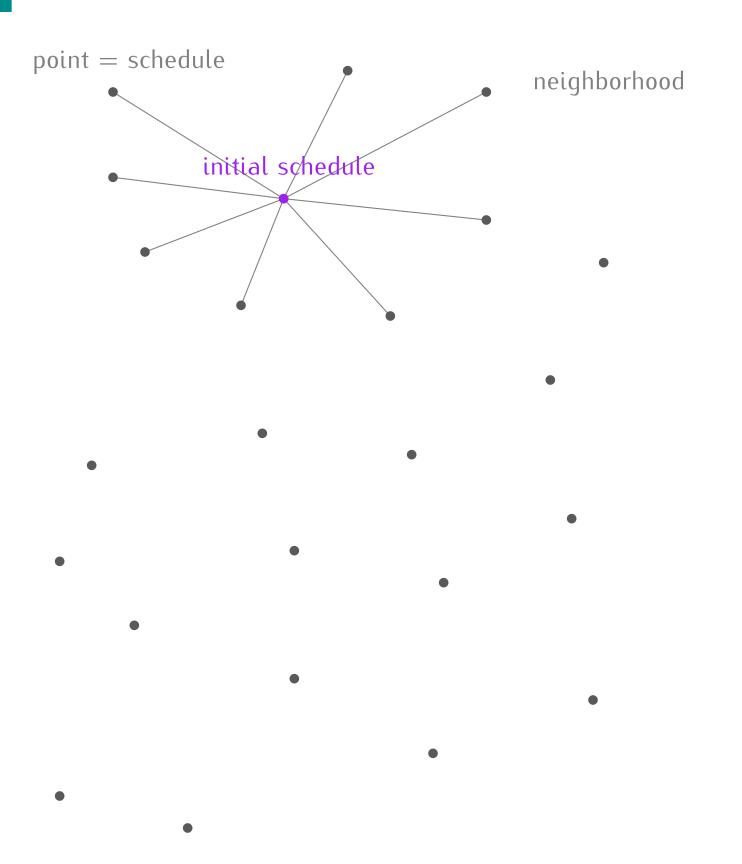
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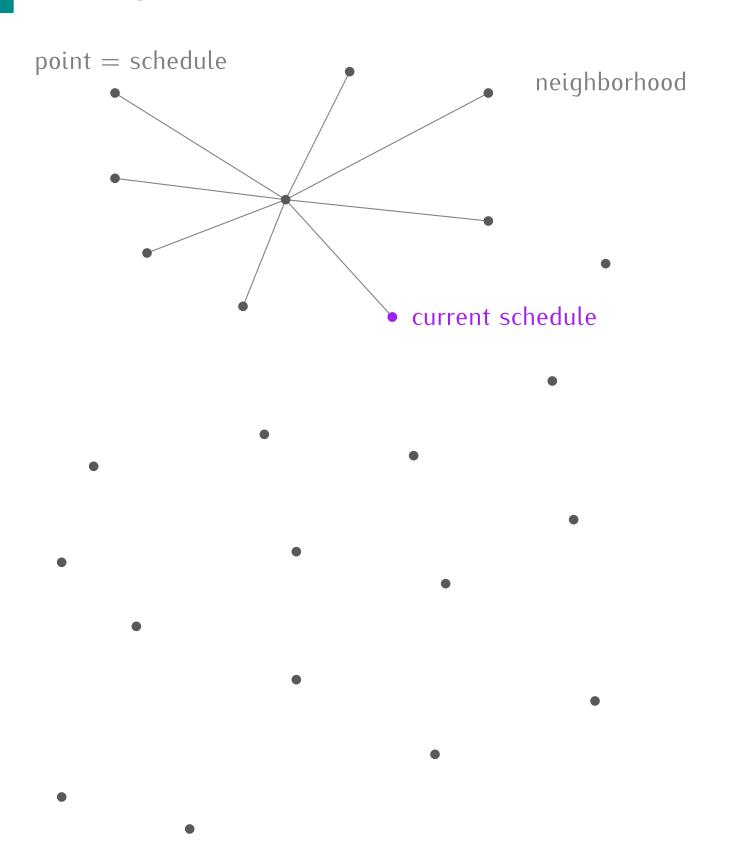


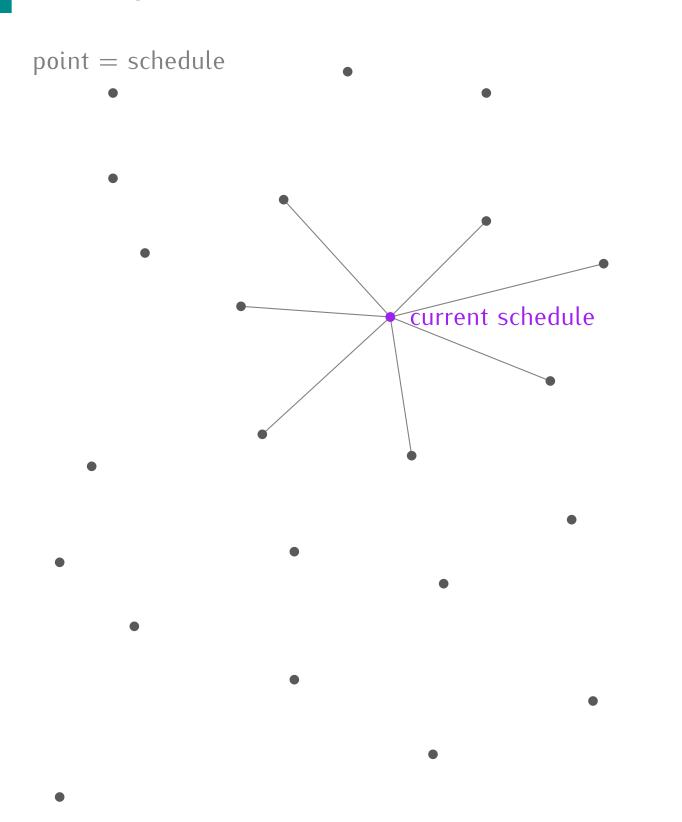
# Local Search

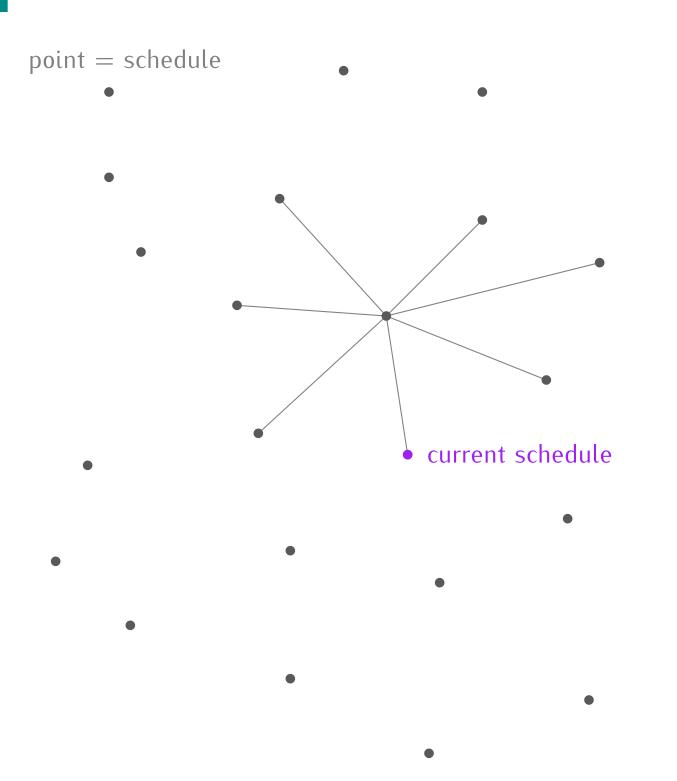
```
point = schedule
```

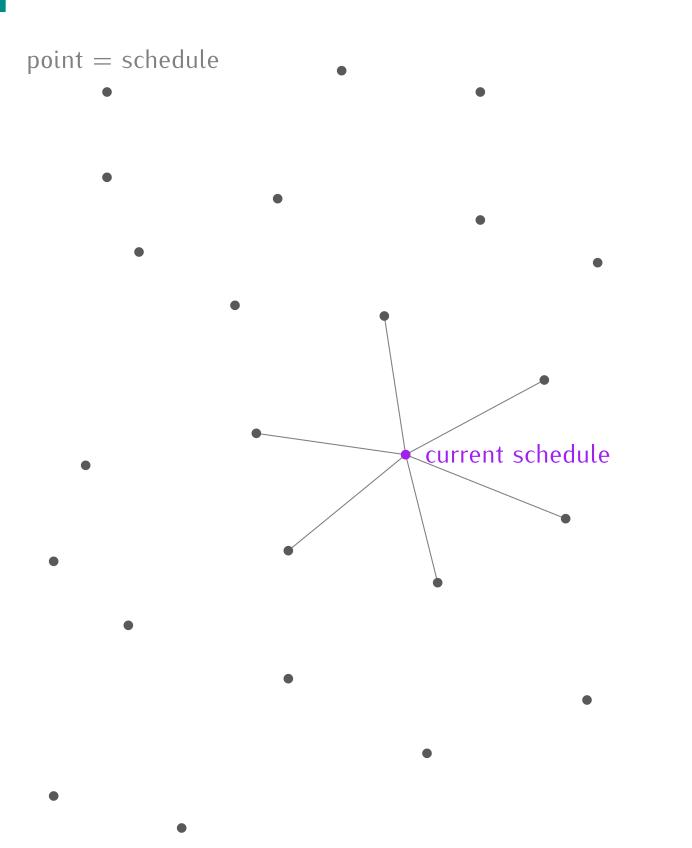
```
point = schedule
              initial schedule
```

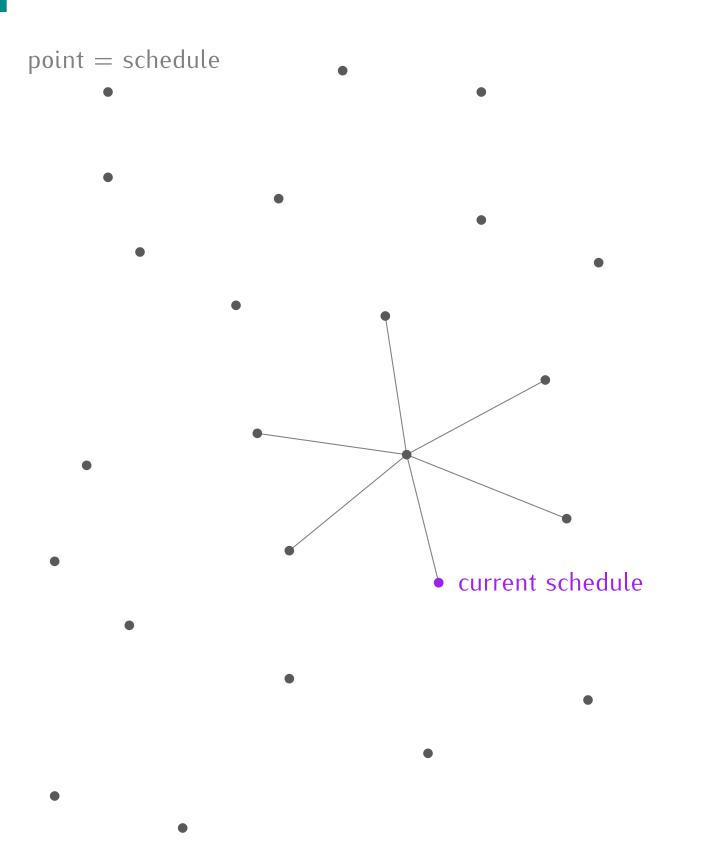


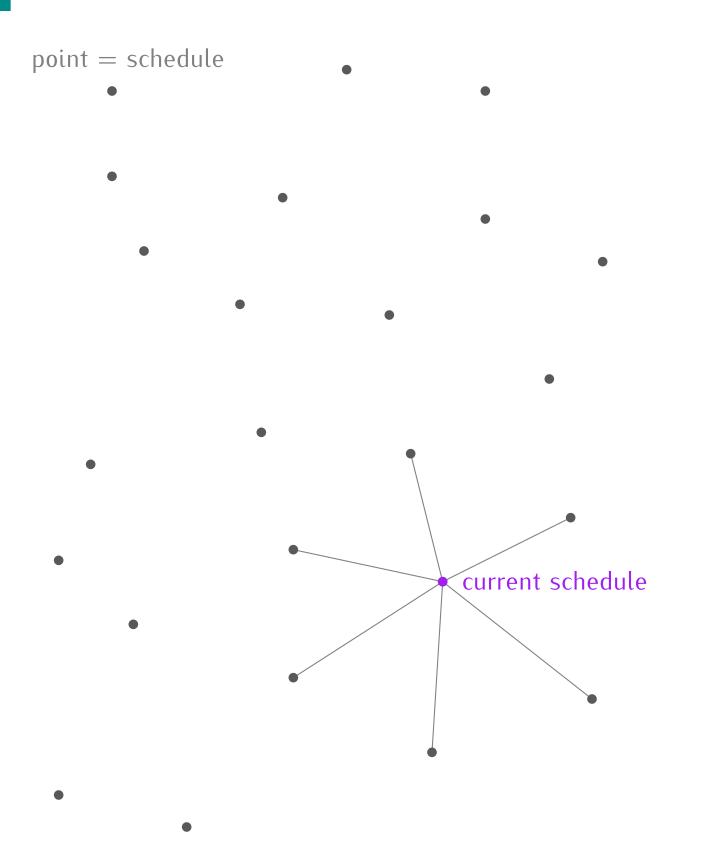


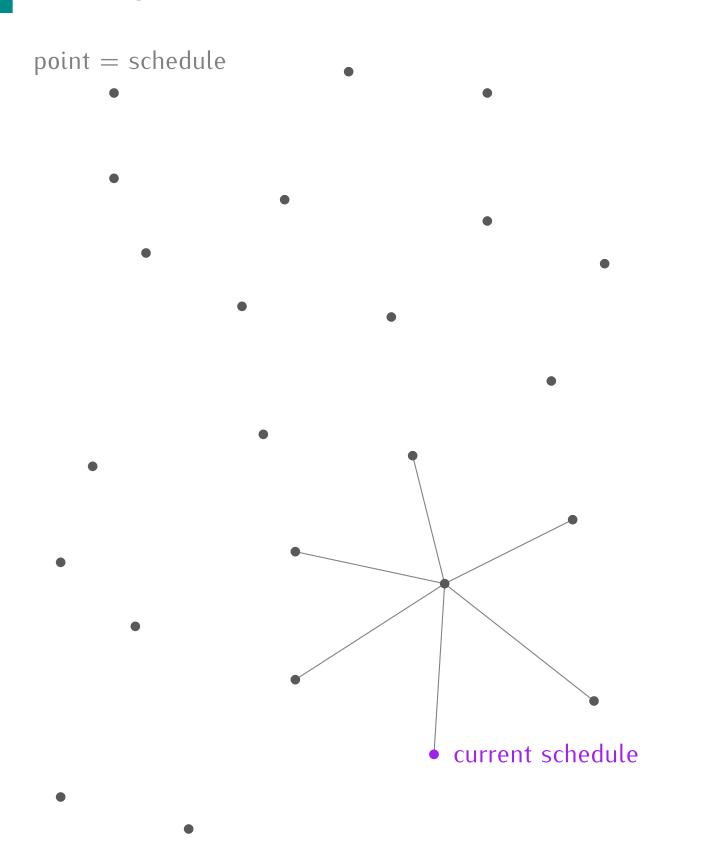


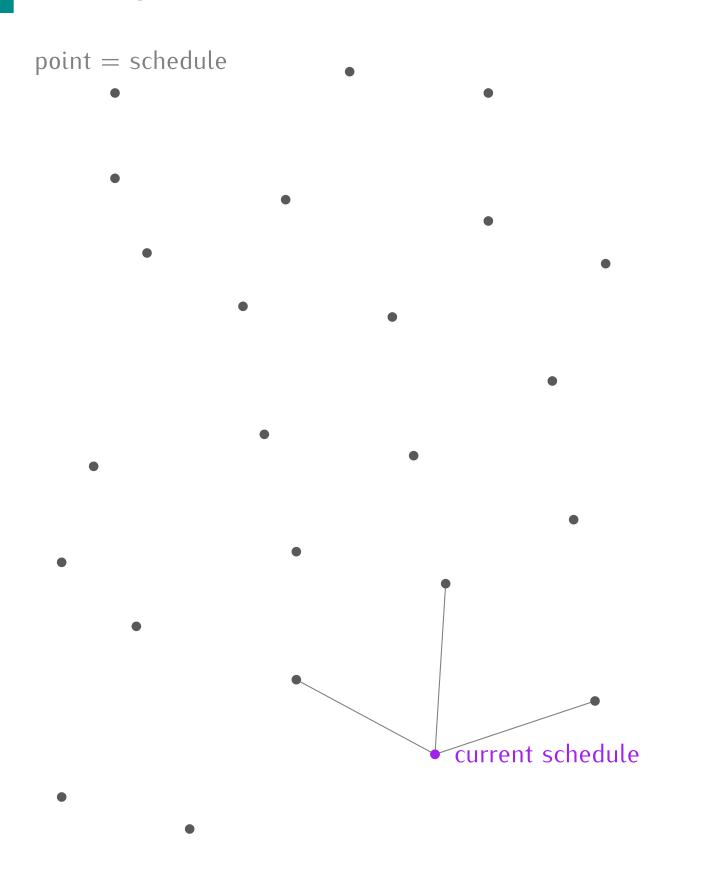


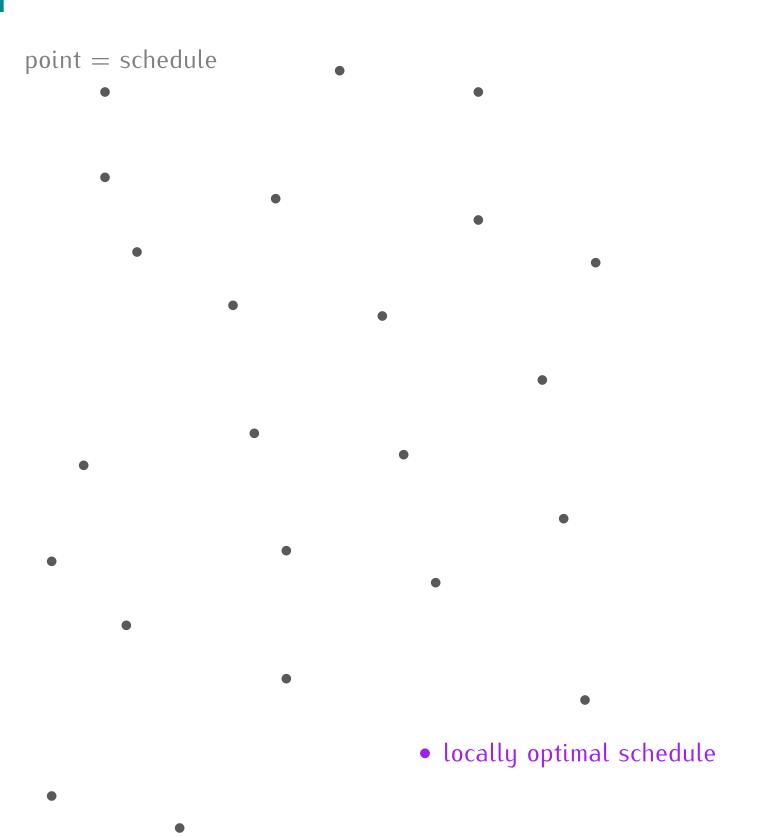




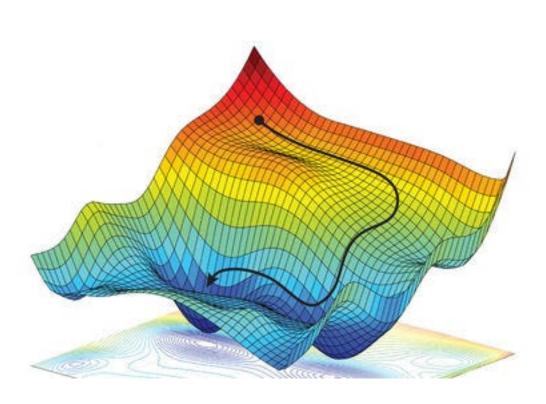


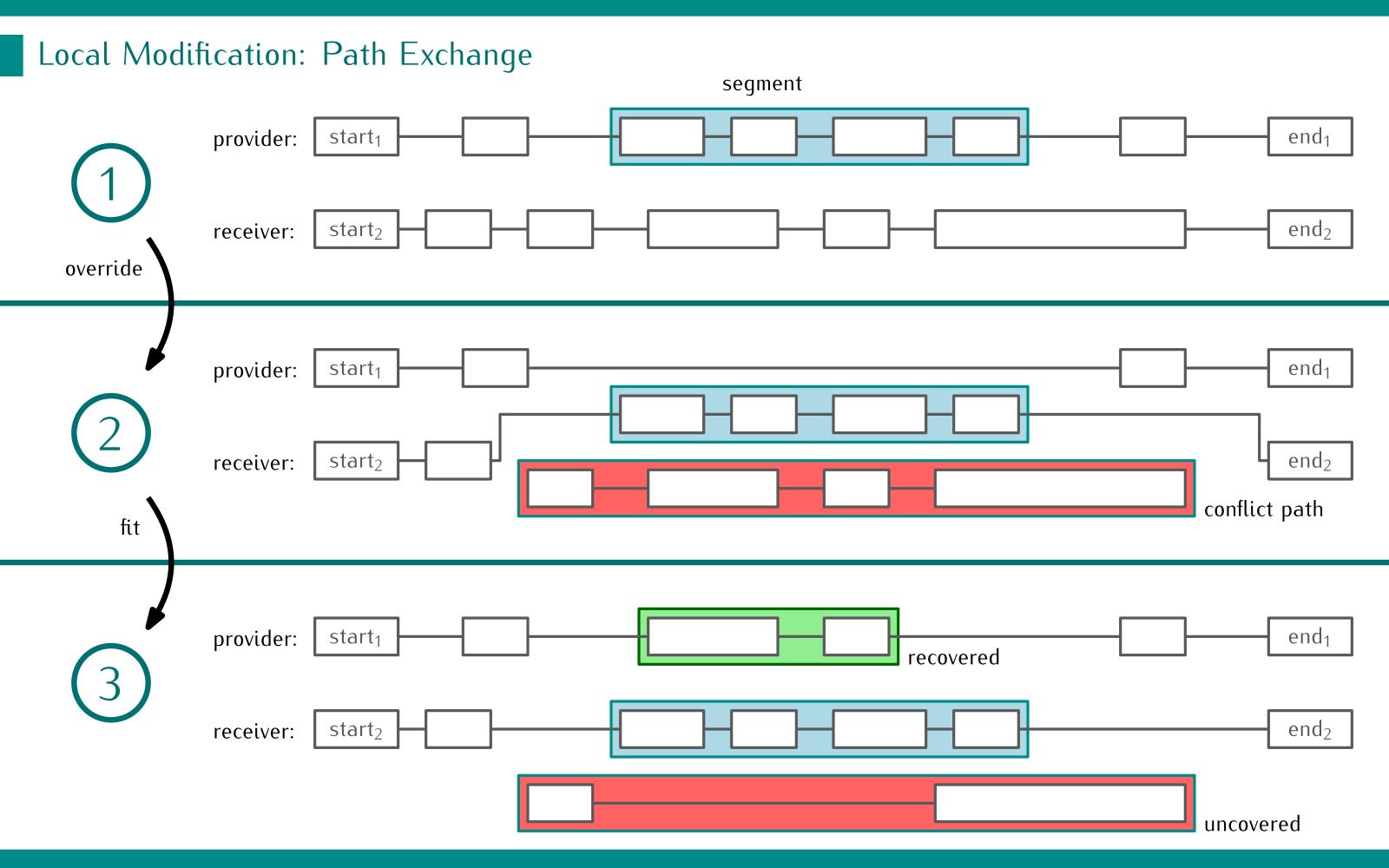




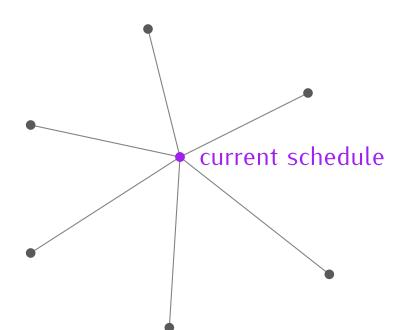






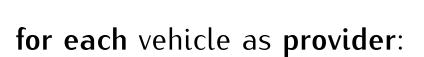


given the current schedule, consider the following modificaitons:



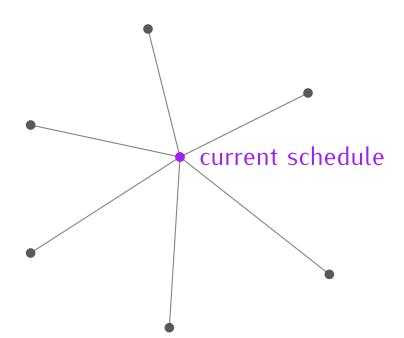
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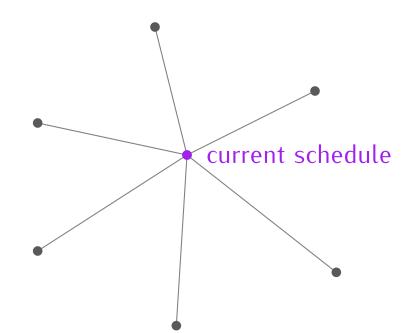


dead-head trips slower than service trips

for each segment of provider's tour that can be removed



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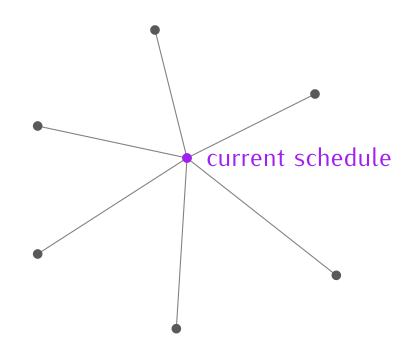
dead-head trips slower than service trips

for each segment of provider's tour that can be removed

for each vehicle of the same type (that is not the provider) as receiver:

(if segment is mainteance slot also different types are allwed)

given the current schedule, consider the following modificaitons:



for each vehicle as provider:

dead-head trips slower than service trips

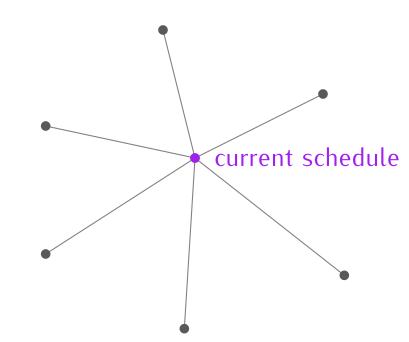
for each segment of provider's tour that can be removed

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PathExchange(segment, provider, receiver)

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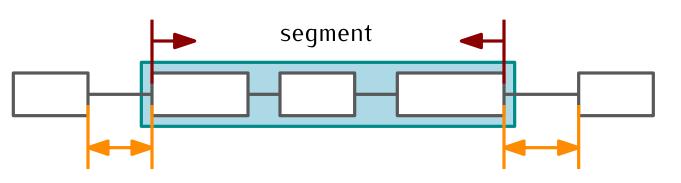


for each vehicle as provider:

dead-head trips slower than service trips

for each segment of provider's tour that can be removed

- restrict length: start of first node to end of last node
- restrict overhead time before and after

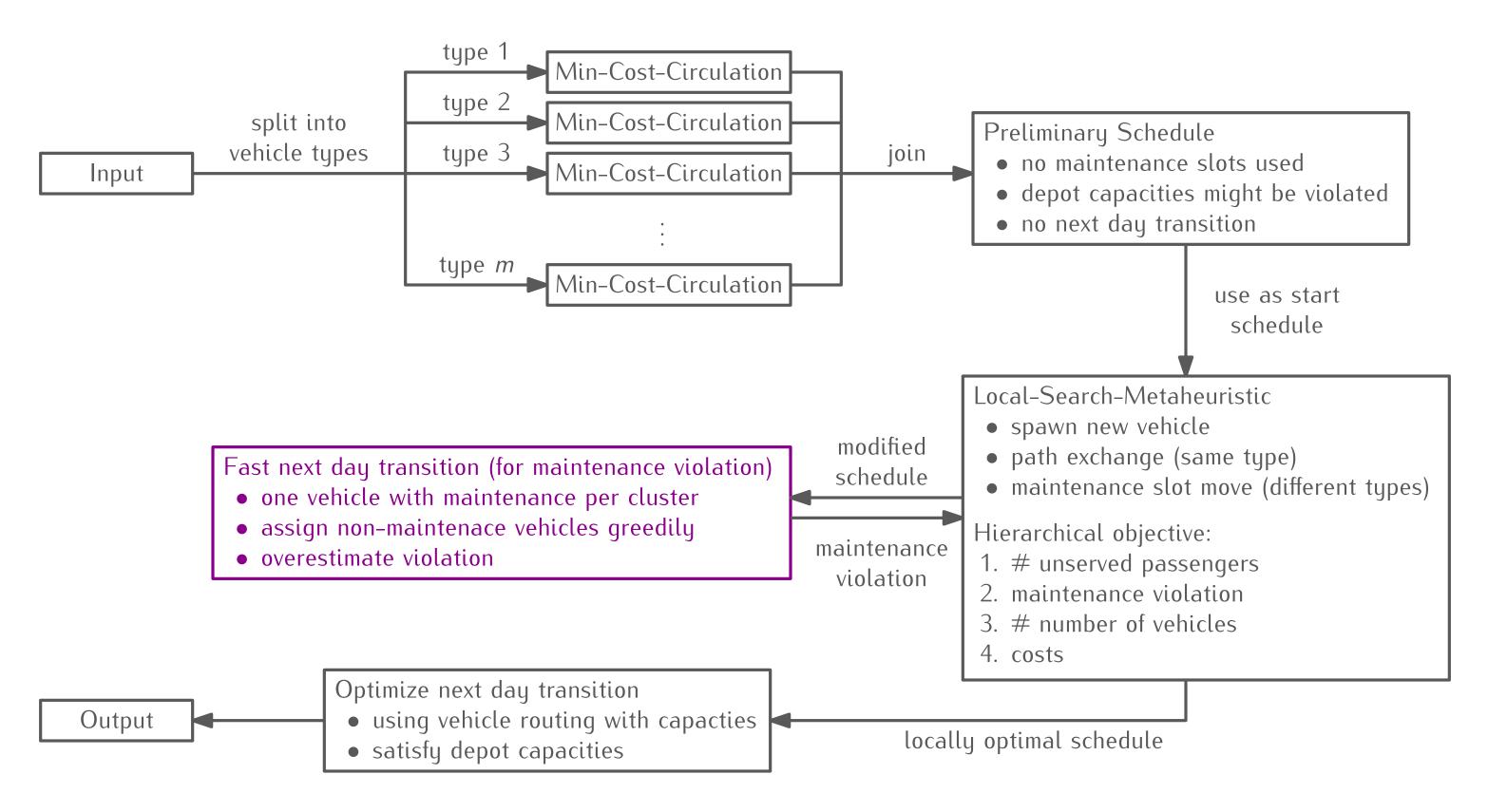


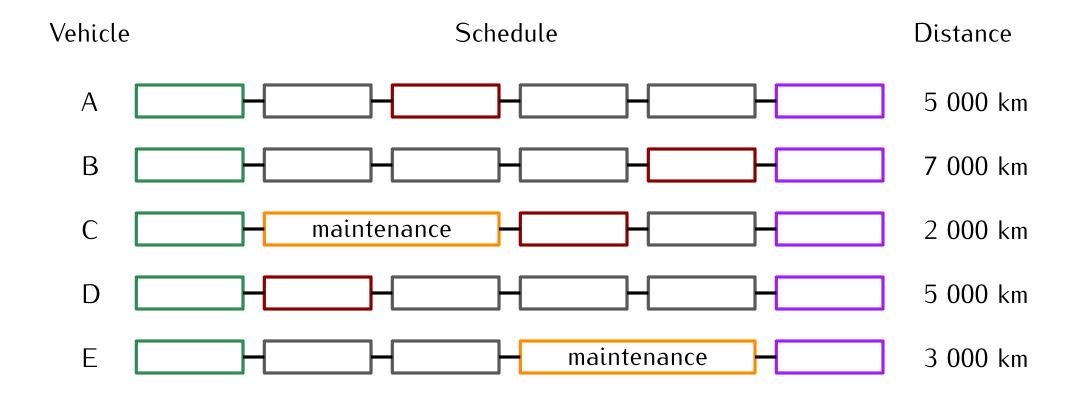
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### Phase 2 - Algorithm Overview





maintenance all 15 000 km

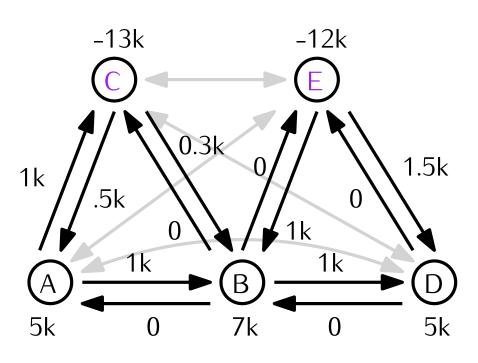
Vehicle	Schedule	Distance	Maintenance Counter	
А		5 000 km	= 5 000 km	
В		7 000 km	= 7 000 km	
С	maintenance — — — — — — — — — — — — — — — — — — —	2 000 km - 15 000 km	= - 13 000 km	
D		5 000 km	= 5 000 km	
Ε	maintenance	3 000 km - 15 000 km	= - 12 000 km	

maintenance all 15 000 km

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#### Model

- consider types individually
- fully connected directed graph
  - one node per vehicle labeled with maintenance counter
  - arcs  $(v_1, v_2)$  are labelled with dead-head-distance between end depot of  $v_1$  to start depot of  $v_2$



maintenance all 15 000 km

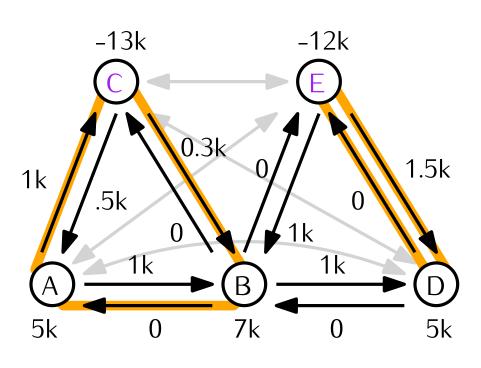
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- a set of disjoint cycles
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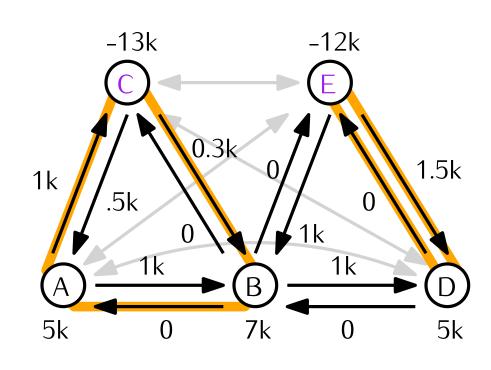
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#### **Next Day Transition**

- a set of disjoint cycles
- all nodes are covered

#### **Objective:**

• minimize maintenance violation:  $\max_{C \in \mathcal{C}} (\max \{ d(C), 0 \})$  where  $d(C) = \sup$  over all node and arc labels of cycle C



- 1. put all maintenance vehicles in a cycle (and sort them)
- 2. for each non-maintenance vehicle (in decreasing order):
  - (a) put vehicle into the first fitting cycle (or the last if none are fitting)
  - (b) resort cycles

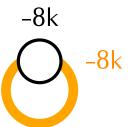
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maintenance vehicles: (decreasing)		-8k	-11k	-14k	
non-maintenance vehicles:	<b>O</b> 7k	<b>O</b>	O	O	O
(decreasing)		5k	4k	4k	3k

#### Algorithm

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maintenance vehicles:
(decreasing)



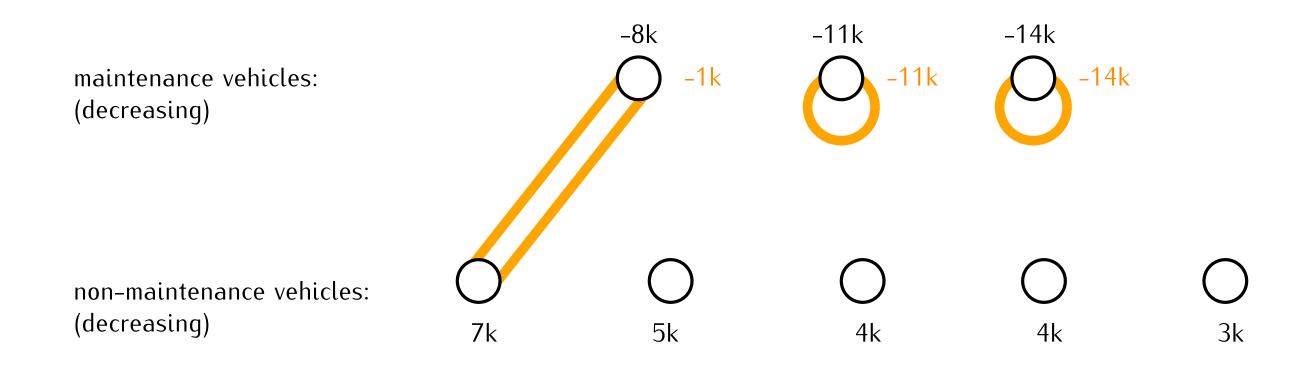
non-maintenance vehicles:
(decreasing)

$$O_{4k}$$

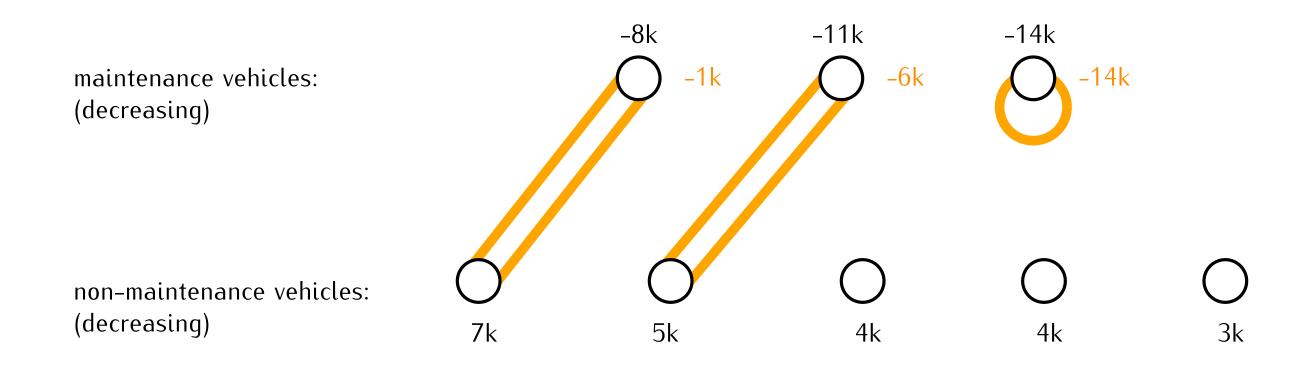
$$\bigcirc$$

$$\frac{1}{3k}$$

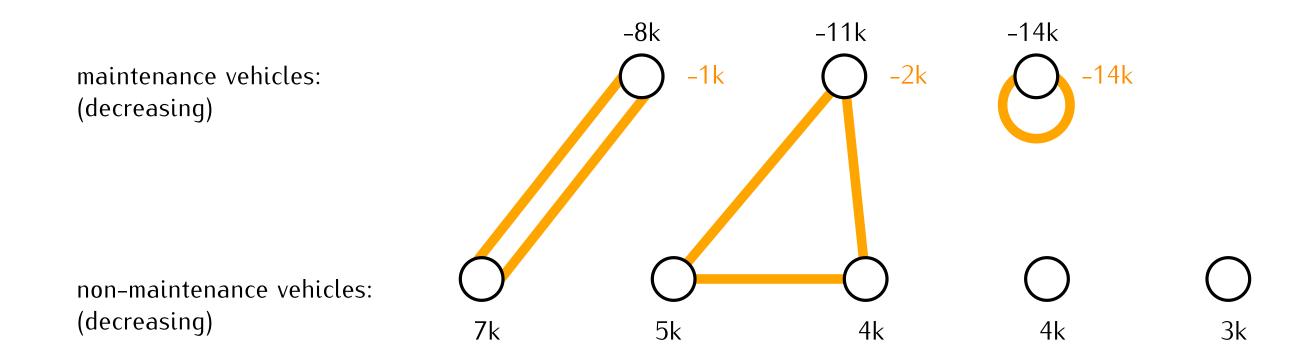
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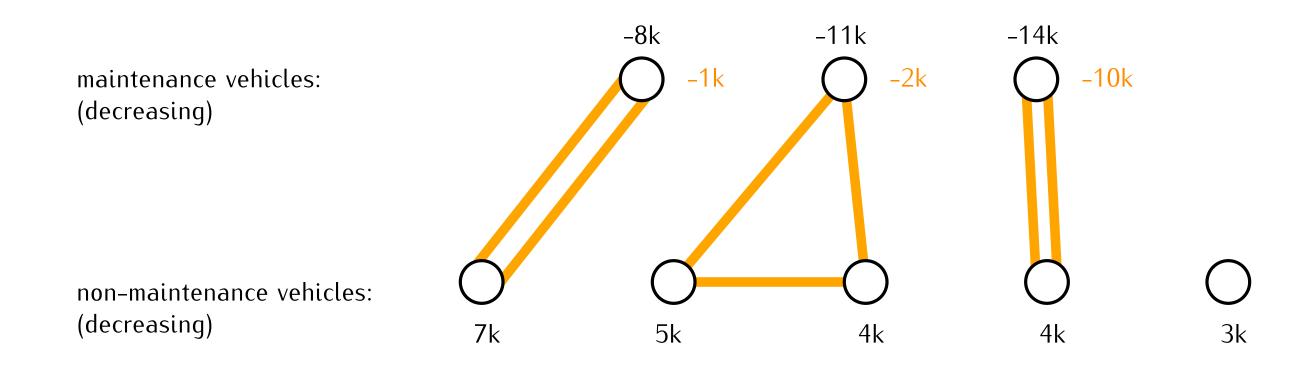
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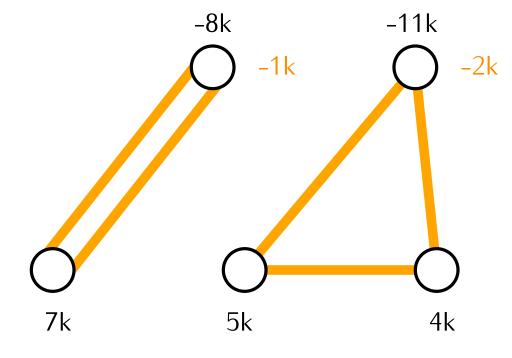


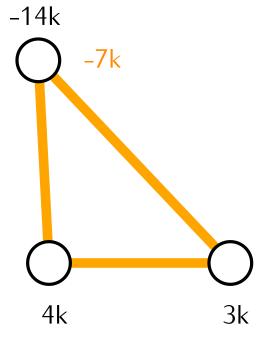
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maintenance vehicles:
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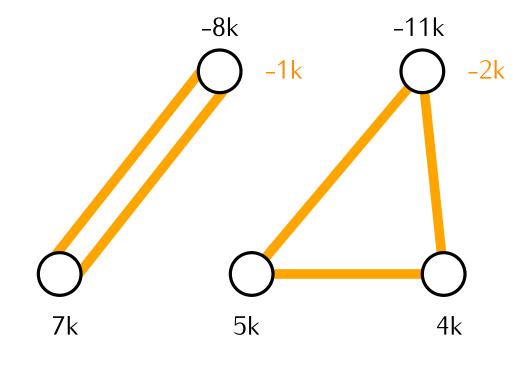


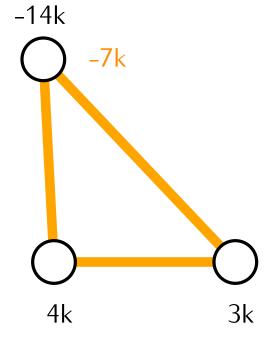
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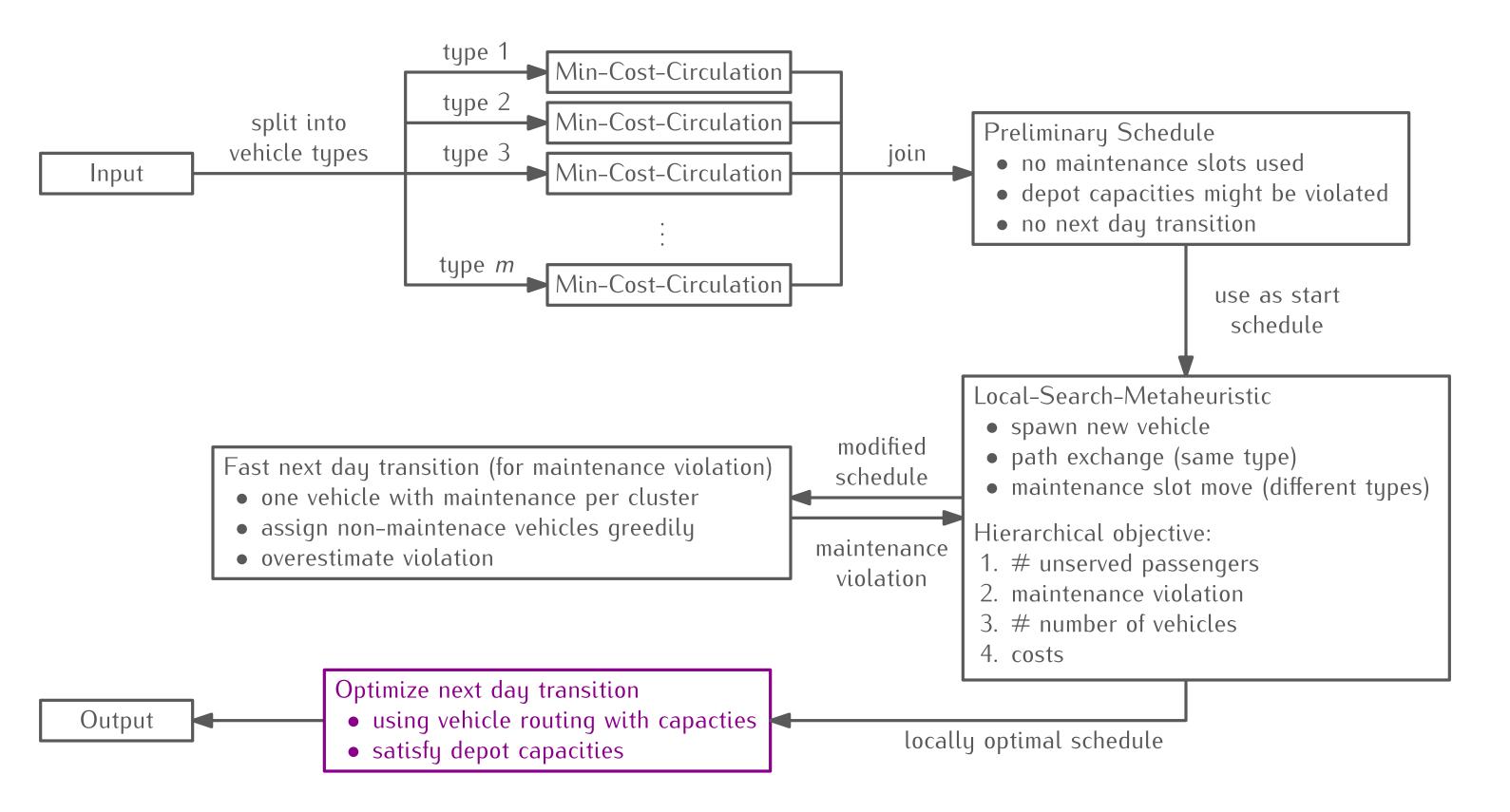
non-maintenance vehicles:
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(dead-head trips between the depots are ignored for the slides)

### Phase 2 – Algorithm Overview



#### Local Search (again):

- still: only one maintenance vehicle per cycle
- exchange vehicles between two cycles
- reoptimize cycles with 3-opt TSP solver

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#### Neighborhood:

for each cycle1:

for each other cycle2:

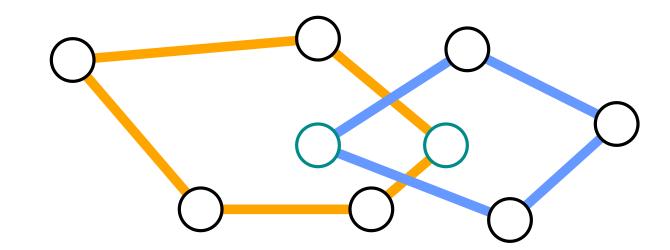
for each vehicle1 of cycle1

for each vehicle2 of cycle2

VehicleExchange(cycle1, vehicle1, cycle2, vehicle2)

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for each other cycle2:

for each vehicle1 of cycle1

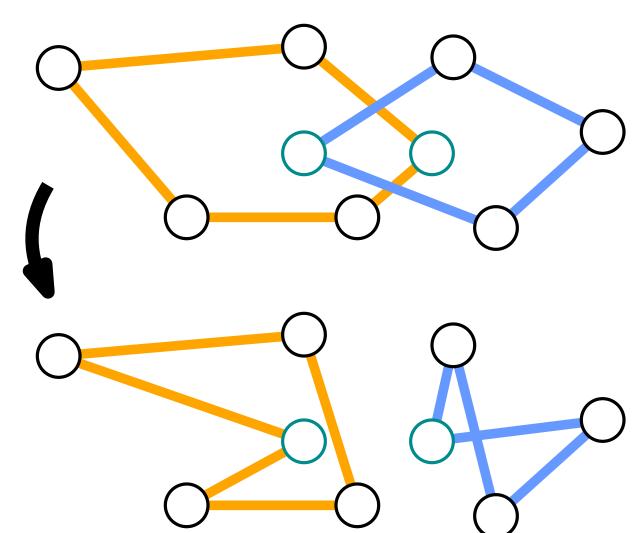
for each vehicle2 of cycle2

VehicleExchange(cycle1, vehicle1, cycle2, vehicle2)

#### **Local Search** (again):

- still: only one maintenance vehicle per cycle
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Vehicle Exchange



#### Neighborhood:

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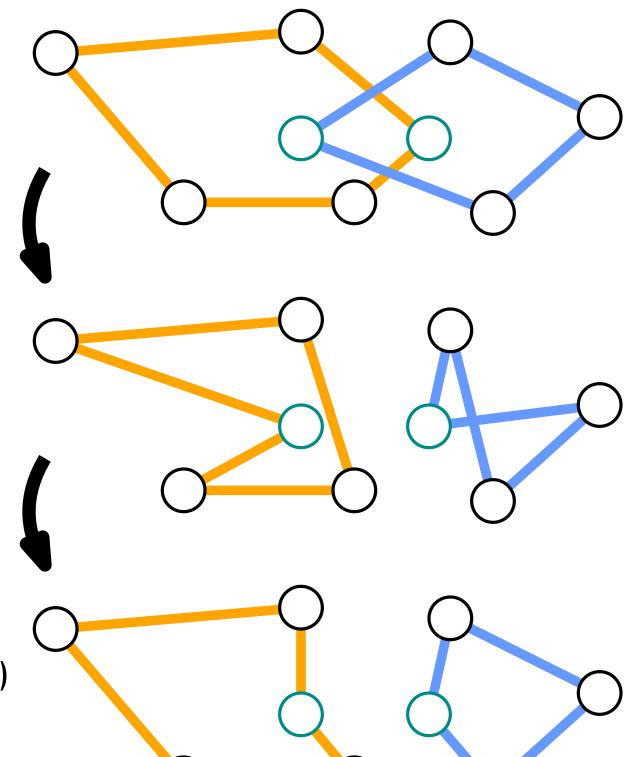
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Vehicle Exchange

TSP reoptimization



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