How Well Do WGANs Estimate the Wasserstein Metric?

Machine Learning 2021 Course

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Introduction

What are the methods for evaluating the Wassertstein-1 distance and how good are they?

Ways for computing Wasserstein-1 distance:

1. Gradient Penalty (GP)
2. Weight Clipping (WC)
3. c-transform
4. (c, ϵ)-transform
5. Lipschitz Penalty (LP)

What is Wasserstein distance?

Wasserstein Distance is a measure of the distance between two probability distributions (Earth Mover's distance).

Experiments

- Datasets: MNIST, CIFAR-10.
- Models: MLP (two hidden layers (width=128), ReLU activation), CNN (DCGAN).
- Optimizers: Adam, RMSProp.

Weight Clipping

$$\max_{\omega} \left\{ \frac{1}{N} \sum_{i=1}^{N} \varphi_\omega(x_i) - \frac{1}{N} \sum_{i=1}^{N} \varphi_\omega(y_i) \right\}. \tag{1}$$

- learning rate: 5×10^{-5}
- optimizer: RMSprop
- MLP, $\epsilon = 0.05$ and $\epsilon = 0.08$ - MNIST and CIFAR10 respectively. CNN, $\epsilon = 0.03$ and $\epsilon = 0.2$ - MNIST and CIFAR10 respectively.

Gradient Penalty

$$\max_{\omega}\left\{\frac{1}{N}\sum_{i=1}^{N}\varphi_{\omega}\left(x_i\right)-\frac{1}{N}\sum_{i=1}^{N}\varphi_{\omega}\left(y_i\right)-\right.\\ \left.\frac{\lambda}{M}\sum_{i=1}^{M}\left(1-\|\nabla_{z=z_i}\varphi_{\omega}(z)\|\right)^2\right\}. \tag{2}$$

- beta values: $(0, 0.9)$
- MLP, learning rate: 5×10^{-3}. CNN, learning rate: 8×10^{-3} and 10^{-2} - MNIST and CIFAR-10 respectively.
- optimizer: Adam
- $\lambda = 10$

Lipschitz Penalty

$$\max_{\omega} \left\{ \frac{1}{N} \sum_{i=1}^{N} \varphi_\omega(x_i) - \frac{1}{N} \sum_{i=1}^{N} \varphi_\omega(y_i) - \frac{\lambda}{M} \sum_{i=1}^{M} \max\{0, \|\nabla_{z=z_i} \varphi_\omega(z)\| - 1\}^2 \right\}. \tag{3}$$

- beta values: $(0, 0.9)$
- MLP, learning rate: 5×10^{-3}. CNN, learning rate: 8×10^{-4} and 10^{-3} - MNIST and CIFAR-10 respectively.
- optimizer: Adam
- $\lambda = 10$

c-transform

$$\varphi_\omega^c(y_i) \approx \widehat{\varphi_\omega^c}(y_i) = \min_j \{c(x_j, y_i) - \varphi_\omega(x_j)\}, \tag{4}$$

$$\max_\omega \left\{ \frac{1}{N} \sum_{i=1}^{N} \varphi_\omega(x_i) + \frac{1}{N} \sum_{i=1}^{N} \widehat{\varphi_\omega^c}(y_i) \right\}. \tag{5}$$

- learning rate: 10^{-3}
- optimizer: RMSprop
- MLP, $\epsilon = 0.05$ and $\epsilon = 0.08$ - MNIST and CIFAR10 respectively. CNN, $\epsilon = 0.03$ and $\epsilon = 0.2$ - MNIST and CIFAR10 respectively.

(c,ϵ)-transform

$$
\begin{aligned}
&\varphi_\omega^c\left(y_i\right) \approx \widehat{\varphi_\omega^{(c,\epsilon)}}\left(y_i\right) = \\
&-\epsilon \log\left(\frac{1}{N}\sum_{j=1}^{N}\exp\left(-\frac{1}{\epsilon}\left(c\left(x_j, y_i\right) - \varphi_\omega\left(x_j\right)\right)\right)\right),
\end{aligned}
\tag{6}
$$

$$
\max_\omega \left\{\frac{1}{N}\sum_{i=1}^{N}\varphi_\omega\left(x_i\right) + \frac{1}{N}\sum_{j=1}^{N}\widehat{\varphi_\omega^{(c,\epsilon)}}\left(y_j\right)\right\}. \tag{7}
$$

- learning rate: 10^{-4}
- optimizer: RMSprop
- MLP, CNN, $\epsilon = 12$ and $\epsilon = 1$ - MNIST and CIFAR10 respectively.

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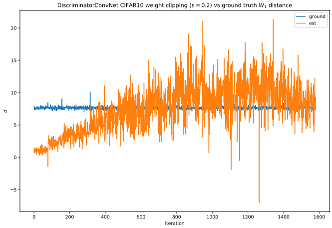
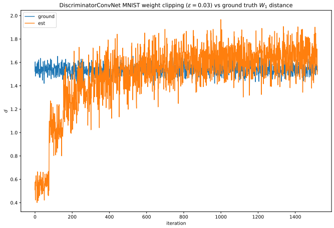
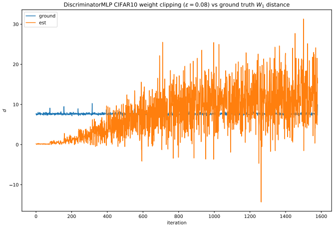
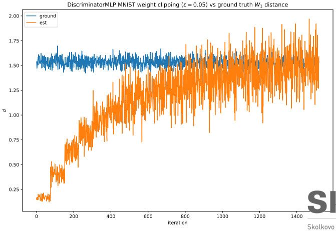
Weight clipping

CNN, MLP / CIFAR-10, MNIST



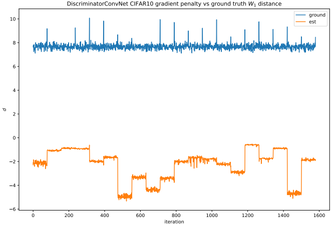


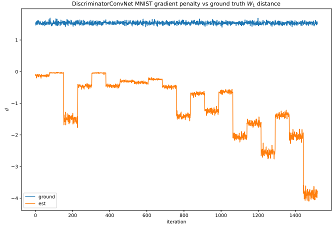


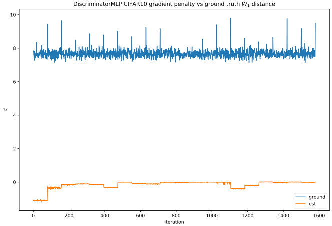


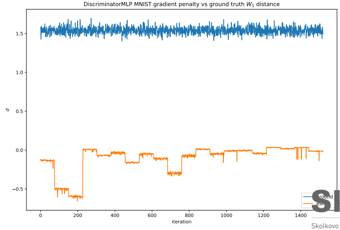
Gradient penalty

CNN, MLP / CIFAR-10, MNIST









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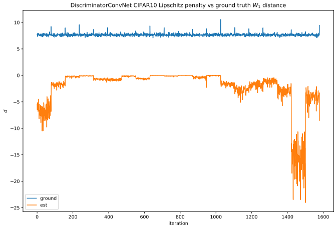
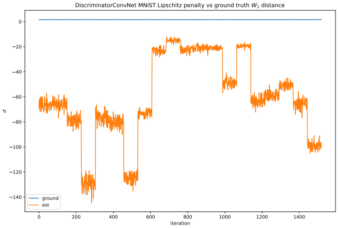
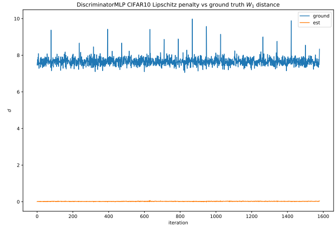
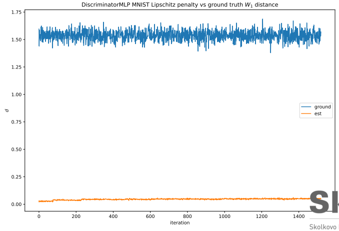
Lipschitz penalty

CNN, MLP / CIFAR-10, MNIST



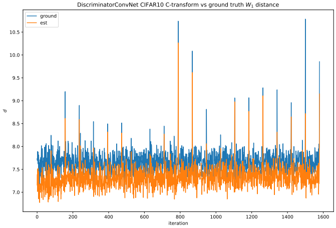


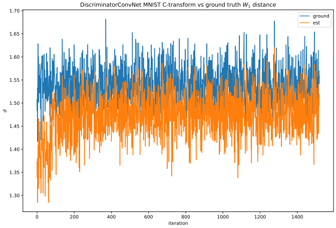


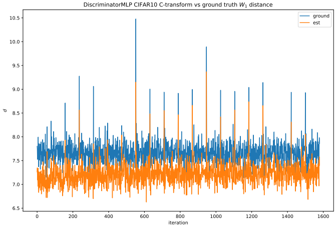


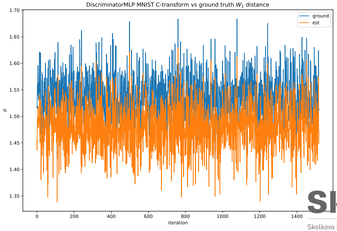
c-transform

CNN, MLP / CIFAR-10, MNIST



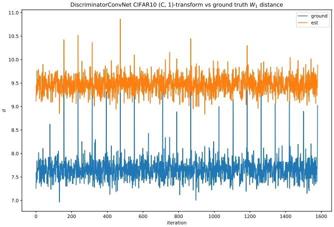


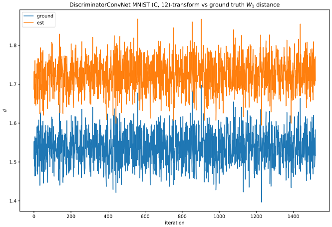


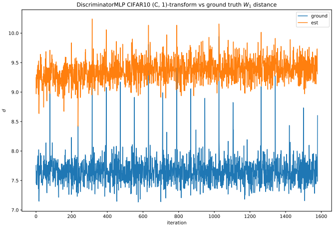


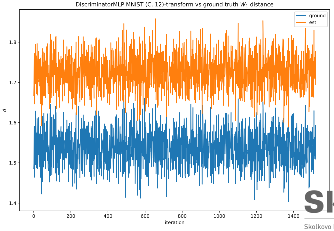
(c, ϵ)-transform

CNN, MLP / CIFAR-10, MNIST









Results

Approximation

MLP	MNIST	CIFAR-10
WC	0.407 ± 0.018	4.854 ± 0.157
GP	1.641 ± 0.009	7.819 ± 0.017
LP	1.491 ± 0.002	7.642 ± 0.011
c-transform	0.059 ± 0.001	0.448 ± 0.005
(c, ϵ)-transform	0.189 ± 0.001	1.696 ± 0.008

ConvNet	MNIST	CIFAR-10
WC	0.184 ± 0.011	3.056 ± 0.109
GP	2.612 ± 0.049	9.926 ± 0.063
LP	61.65 ± 1.659	9.761 ± 0.18
c-transform	0.065 ± 0.001	0.344 ± 0.006
(c, ϵ)-transform	0.186 ± 0.001	1.807 ± 0.007

Results

Stability

MNIST	WC	GP	LP	c-T	(c,ϵ)-T
$N=64, M=64$	0.08	0.19	0.01	1.45	1.69
$N=64, M=512$	0.06	-0.66	0.02	1.3	1.64
$N=512, M=64$	0.0	-0.38	0.01	1.44	1.65
$N=512, M=512$	0.0	-0.18	0.02	1.3	1.59
Ground truth	1.36	1.36	1.36	1.36	1.36
CIFAR-10	**WC**	**GP**	**LP**	**c-T**	**(c,ϵ)-T**
$N=64, M=64$	0.05	-0.74	0.02	7.21	9.22
$N=64, M=512$	0.04	-0.14	-0.04	6.49	9.19
$N=512, M=64$	0.0	-0.6	0.01	6.98	9.17
$N=512, M=512$	0.01	-2.9	0.01	6.3	9.13
Ground truth	6.89	6.89	6.89	6.89	6.89

Conclusion

	WC	GP	LP	c-transform	(c, ϵ)-transform
Approximation	$\checkmark$	$\times$	$\times$	$\checkmark$	$\checkmark$
Stability	$\times$	$\times$	$\times$	$\checkmark$	$\checkmark$

Further improvements: work in (c, ϵ)-transform with smaller values of ϵ without nan's.