FECHA:

HOJA:

Final 2017 B

$$k\frac{\partial T}{\partial z^*} = \frac{k}{H}\frac{\partial(T-T_\infty)}{\partial z}\frac{\Delta T}{\Delta T} = h(T-T_\infty)$$

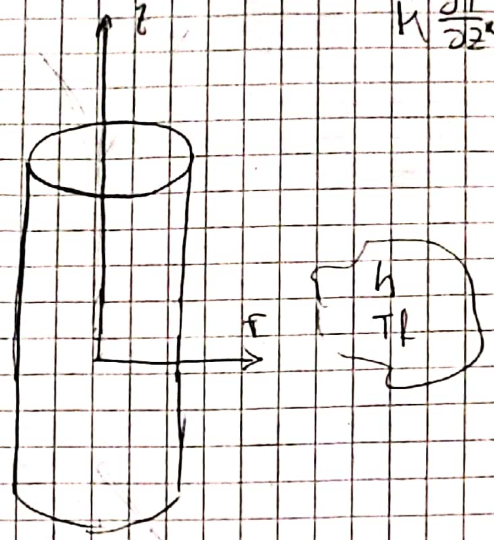
$$k\frac{\partial T^*}{\partial z} = hT^*$$

(1) V fijo.

$$V = \frac{\pi D^2}{4}H$$

$$\Rightarrow H = \frac{4V}{\pi D^2}$$

$\to$ fijo.



Esto si o si es un problema bidimensional

$$Bi_c^z = \frac{hH}{k} = \frac{4hV}{\pi k D^2} \qquad Bi^r = \frac{hR}{k}$$

(a)

$$Bi^z << 1 \qquad Bi^r << 1 \qquad \begin{cases} T^* = \frac{T - T_\infty}{\Delta T} = RZ \quad \Delta T \to T_0 - T_\infty \\ r^* = \frac{r}{R} \\ z^* = \frac{z}{H} \end{cases}$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha}\frac{\partial T}{\partial t} \rightsquigarrow \frac{1}{R^2}\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial T^*}{\partial r^*}\right) + \frac{1}{L^2}\frac{\partial T^*}{\partial z^{*2}} = \frac{1}{\alpha}\frac{\partial T^*}{\partial t}$$

Propongo $T^* = R(r^*,t)\,Z(z^*,t)$

$$\Rightarrow \begin{cases} \frac{\partial^2 Z}{\partial z^{*2}} = \frac{\partial Z}{\partial\left(\frac{\alpha t}{H^2}\right)} \\ \left.\frac{\partial Z}{\partial z^*}\right|_{\frac{1}{2},t} = -\frac{hH}{k}Z\left(\frac{1}{2}\right) \\ \left.\frac{\partial Z}{\partial z^*}\right|_{-\frac{1}{2},t} = \frac{hH}{k}Z\left(-\frac{1}{2}\right) \\ Z(z^*,0) = 1 \end{cases} \quad y \quad \begin{cases} \frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial R}{\partial r^*}\right) = \frac{\partial R}{\partial\left(\frac{\alpha t}{R^2}\right)} \\ \left.\frac{\partial R}{\partial r^*}\right|_1 = -\frac{hD}{k}R(1) \\ \left.\frac{\partial R}{\partial r^*}\right|_0 = 0. \\ R(r^*,0) = 1 \end{cases}$$

NOTA:

• En $\overline{Z}$:

$$\frac{\partial^2 Z}{\partial z^{*2}} = \frac{\partial Z}{\partial t_z^*} \quad , \quad \text{con } t_z^* = \frac{t\,\alpha}{H^2}$$

$$Z = X_{(z^*)}\, \mathcal{Z}_{(t^*)} \Rightarrow \quad \frac{X''}{X} = \frac{\dot{\mathcal{Z}}}{\mathcal{Z}} = -\lambda^2$$

Parte espacial

$$X = A\cos(\lambda z^*) + B\sin(\lambda z^*)$$

$$X'\left(\tfrac{1}{2}\right) = -\lambda A\sin\left(\lambda/2\right) + \lambda B\cos\left(\tfrac{\lambda}{2}\right) = -\frac{hH}{k}A\cos\left(\lambda/2\right) - \frac{hH}{k}B\sin\left(\lambda/2\right)$$

$$X'\left(-\tfrac{1}{2}\right) = \lambda A\sin\left(\tfrac{\lambda}{2}\right) + \lambda B\cos\left(\tfrac{\lambda}{2}\right) = \frac{hH}{k}A\cos\left(\tfrac{\lambda}{2}\right) - \frac{hH}{k}B\sin\left(\tfrac{\lambda}{2}\right)$$

Si sumo m.a.m

$$\cancel{2}\lambda\cos\left(\tfrac{\lambda}{2}\right)\cancel{B} = -\cancel{2}\frac{hH}{k}\cancel{B}\sin\left(\tfrac{\lambda}{2}\right)$$

$$\frac{tg\left(\lambda/2\right)}{\lambda} = -\frac{k}{hH} \rightsquigarrow tg = -\frac{k}{h}\lambda$$

Si Resto m.a.m

$$2\lambda A\sin\left(\tfrac{\lambda}{2}\right) = 2\frac{hH}{k}A\cos\left(\tfrac{\lambda}{2}\right)$$

$$\lambda\, tg\left(\lambda/2\right) = \frac{hH}{k} \rightsquigarrow tg = \frac{h}{k}\frac{1}{\lambda}$$

des bo mismo p

Yo se que esto tiene que ser simétrico

$\Rightarrow X = A\cos(\lambda x^*)$

$$\left.\begin{array}{l} X'\left(\tfrac{1}{2}\right) = -A\sin(\lambda/2)\lambda = -\dfrac{h}{K} A\cos(\lambda/2) \\ X'\left(-\tfrac{1}{2}\right) = \cancel{A\sin(\lambda/2)\lambda = \dfrac{h}{K} A\cos(\lambda/2)} \end{array}\right\} \rightarrow$$

$\Rightarrow \boxed{\lambda \, tg(\lambda/2) = \dfrac{hH}{K}}$ autonombres $\rightarrow Bi^z$

$$\Rightarrow Z = \sum_{n=1}^{\infty} A_n \cos\left(\lambda \frac{x}{H}\right) e^{-\frac{\alpha t}{H^2}\lambda^2}$$

$$A_n = \frac{\int_{-1/2}^{1/2} \cos(x^*\lambda)\, dx}{\int_{-1/2}^{1/2} \cos^2(x^*\lambda)\, dx} = \frac{4\sin(\lambda/2)}{\lambda + \sin(\lambda/2)}$$

$$Z = \sum_{n=1}^{\infty} \frac{4\sin(\lambda/2)}{\lambda + \sin(\lambda/2)} \cos\left(\lambda \frac{x}{H}\right) e^{-\frac{\alpha t}{H^2}\lambda^2}$$

Luego copio la solución p/ la parte radial

$$R = \sum_{n=1}^{\infty} \frac{2}{\xi} \frac{J_1(\xi)}{J_0^2(\xi) + J_1^2(\xi)} J_0\left(\xi \frac{2r}{D}\right) e^{-\xi^2 \frac{\alpha t}{R^2}}$$

con $\xi \cdot \dfrac{J_1(\xi)}{J_0(\xi)} = \dfrac{hR}{k} = Bi^r$

NOTA:

Con esto

$$T = T_\infty + (T_0 - T_\infty)\left[\sum_{n=1}^{\infty} \frac{4\sin(\lambda_n)}{\lambda_n + \sin(\lambda_n/2)} \cos\left(\lambda \frac{x}{H}\right) e^{-\frac{\lambda^2 \alpha t}{H^2}} \sum_{m=1}^{\infty} \frac{2}{\xi} \frac{J_1(\xi_m)}{J_0^2(\xi) + J_1^2(\xi)} J_0\left(\xi \frac{r}{R}\right) e^{-\frac{\xi^2 \alpha t}{R^2}}\right]$$

(a)

Los constantes del tiempo son

$$\frac{H^2}{\alpha \lambda_1^2} \quad y \quad \frac{R^2}{\alpha \xi_1^2} - \quad \rightsquigarrow \begin{cases} \dfrac{V^2}{\pi R^4 \alpha \lambda^2} \\ \dfrac{R^2}{\alpha \xi^2} \end{cases}$$

$$H = \frac{4V}{\pi D^2} \to \frac{16 V^2}{\pi D^4} \frac{1}{\alpha \lambda^2} = \frac{V^2}{\pi R^4 \alpha \lambda^2}$$

Si $Bi \ll 1 \rightsquigarrow \lambda_1 = 0{,}1\ i$

$\xi_1 = 0{,}14$

$$\Rightarrow \frac{H^2}{\cancel{\alpha} \lambda^2} = \frac{R^2}{\cancel{\alpha} \xi_1^2} \Rightarrow \frac{H}{R} = \sqrt{\frac{\lambda^2}{\xi^2}} = \frac{\lambda}{\xi}$$

$= 0{,}71$

Maximizar es que ambos sean iguales

(b) Si $Bi \gg 1 \to \infty$

$\lambda_1 = 1{,}57$

$\xi_1 = 2{,}41$

$$\Rightarrow \frac{H}{R} = 0{,}65$$

Si $Bi = 0 \Rightarrow$ Pared plana $\to$ todo el cambio de T se debe a la conducción $\to$ importa long de conducción

$Bi \to \infty \Rightarrow$ se debe a la convección $\to$ importa el área expuesta a la convección.

NOTA:

Escaneado con CamScanner