Stat 414 Quiz #9

Spring 2016

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Start Time: 9:08 am/(pm) Stop time: 9:35 am/(pm)

You must show all of your work in order to receive full and/or partial credit. Tables/software are not allowed unless otherwise stated in the problem. 10 points

Suppose that $X_1, ..., X_n$ are independent random variables with comon MGF

$$M_X(t) = e^{2(2t+3t^2)} \quad \Rightarrow \quad \mu = 4 \quad \sigma^2 = 12$$

1. 3 points **Prove the distribution of $Y = \sum_{i=1}^{n} X_i$ using the moment generating function technique**

$$M_Y(t) = \prod_{i=1}^{n} M_X(t) = \prod_{i=1}^{n} e^{2(2t+3t^2)}$$

$$= \left(e^{2(2t+3t^2)}\right)^n$$

$$= e^{4nt + 6nt^2}$$

THIS IS THE mgf OF A NORMAL DISTRIBUTION WITH $\mu = 4n$ AND $\sigma^2 = 12n$

2. 2 points **Suppose n = 5. Find $E(X_1^2 X_2^2 X_3^2 X_4^2 X_5^2)$.**

$$E(X_i^2) = Var(X_i) + [E(X_i)]^2 = 12 + 16 = 28$$

~~$60 + 400 = 460$~~

$$E(X_1^2 X_2^2 X_3^2 X_4^2 X_5^2) = E(X_1^2)E(X_2^2)E(X_3^2)E(X_4^2)E(X_5^2)$$

~~$\approx 460^5 \approx 2.06 E15$~~

$$= 28^5 = 17210368$$

3. 2 points Suppose n = 4. Find $Var(X_1 + 3X_2 - 2X_3 - X_4)$.

$$Var(X_1 + 3X_2 - 2X_3 - X_4) = (1)^2(12) + (3)^2(12) + (-2)^2(12) + (-1)^2(12)$$

$$= 12 + 108 + 48 + 12$$

$$= 180$$

4. 3 points Suppose $n = 6$. Find

$$P\left(26.45 \leq \sum_{i=1}^{6}(X_i - 4)^2 \leq 151.08\right)$$

You may use tables/software for this problem.

$$W = \sum_{i=1}^{6} \frac{(X_i-4)^2}{12} \sim \chi^2(6)$$

$$P\left(\frac{26.45}{12} \leq \sum_{i=1}^{6} \frac{(X_i-4)^2}{12} \leq \frac{151.08}{12}\right)$$

$$= P(2.2042 \leq W \leq 12.59)$$

$$= P(W \leq 12.59) - P(W \leq 2.2042) = 0.95 - 0.1$$

$$= 0.85$$