Homework 3

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Section 1.4

2. (a) $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.3 + 0.6 - 0.18 = 0.72$

   (b) $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0}{P(B)} = 0$

5. $P(A) = .8$
   $P(B) = .5$
   $P(A \cup B) = .9$

   $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   $.9 = .8 + .5 - P(A \cap B)$
   $P(A \cap B) = .4 = .8 \cdot .5 = P(A) \cdot P(B)$
   Therefore, A and B are independent events.

6. A and $(B \cap C)$:

$$\begin{aligned} P(A \cap (B \cap C)) &= P(A \cap B \cap C) \\ &= P(A)P(B)P(C) \end{aligned}$$

   A and $(B \cup C)$:

$$\begin{aligned} P(A \cap (B \cup C)) &= P((A \cap B) \cup (A \cap C)) \\ &= P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) \\ &= P(A)P(B) + P(A)P(C) - P(A)P(B)P(C) \\ &= P(A)[P(B) + P(C) - P(B)P(C)] \\ &= P(A)[P(B) + P(C) - P(B \cap C)] \\ &= P(A)P(B \cup C) \end{aligned}$$

A' and $(B \cap C')$:

$$\begin{aligned} P(A' \cap (B \cap C')) &= 1 - P(A \cup (B' \cup C)) \\ &= 1 - [P(A) + P(B') + P(C) - P(A \cap B') - P(A \cap C) - P(B' \cap C) + P(A \cap B' \cap C)] \\ &= 1 - P(A) - P(B) - P(C) + P(A \cap B') + P(A \cap C) + P(B' \cap C) - P(A \cap B' \cap C) \\ &= [1 - P(A)][1 - P(B')][1 - P(C)] \\ &= P(A)P(B')P(C) \end{aligned}$$

A', B', and C' are mutually independent:
Condition (a):

$$\begin{aligned} P(A' \cap B' \cap C') &= 1 - P(A \cup B \cup C) \\ &= 1 - [P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)] \\ &= 1 - [P(A) + P(B) + P(C) - P(A)P(B) - P(A)P(C) - P(B)P(C) + P(A)P(B)P(C)] \\ &= 1 - P(A) - P(B) - P(C) + P(A)P(B) + P(A)P(C) + P(B)P(C) - P(A)P(B)P(C) \\ &= [1 - P(A)][1 - P(B)][1 - P(C)] \\ &= P(A')P(B')P(C') \end{aligned}$$

Condition (b):

$$\begin{aligned} P(A' \cap B') &= 1 - P(A \cup B) \\ &= 1 - [P(A) + P(B) - P(A \cap B)] \\ &= 1 - P(A) - P(B) + P(A)P(B) \\ &= [1 - P(A)][1 - P(B)] \\ &= P(A')P(B') \end{aligned}$$

$$\begin{aligned} P(A' \cap C') &= 1 - P(A \cup C) \\ &= 1 - [P(A) + P(C) - P(A \cap C)] \\ &= 1 - P(A) - P(C) + P(A)P(C) \\ &= [1 - P(A)][1 - P(C)] \\ &= P(A')P(C') \end{aligned}$$

$$\begin{aligned} P(B' \cap C') &= 1 - P(B \cup C) \\ &= 1 - [P(B) + P(C) - P(B \cap C)] \\ &= 1 - P(B) - P(C) + P(B \cap C) \\ &= [1 - P(B)][1 - P(C)] \\ &= P(B')P(C') \end{aligned}$$

7. (a) $P(A_1 \cap A_2' \cap A_3') = P(A_1) \cdot P(A_2') \cdot P(A_3') = .5 \cdot .3 \cdot .4 = .06$
$P(A_1' \cap A_2 \cap A_3') = P(A_1') \cdot P(A_2) \cdot P(A_3') = .5 \cdot .7 \cdot .4 = .14$
$P(A_1' \cap A_2' \cap A_3) = P(A_1') \cdot P(A_2') \cdot P(A_3) = .5 \cdot .3 \cdot .6 = .09$
$P(exactly\ one\ player\ successful) = .06 + .14 + .09 = 0.29$

(b) $P(A_1' \cap A_2 \cap A_3) = P(A_1') \cdot P(A_2) \cdot P(A_3) = .5 \cdot .7 \cdot .6 = .21$
$P(A_1 \cap A_2' \cap A_3) = P(A_1) \cdot P(A_2') \cdot P(A_3) = .5 \cdot .3 \cdot .6 = .09$
$P(A_1 \cap A_2 \cap A_3') = P(A_1) \cdot P(A_2) \cdot P(A_3') = .5 \cdot .7 \cdot .4 = .14$
$P(\textit{exactly one player misses}) = .21 + .09 + .14 = 0.44$

11. (a) If events A and B are mutually exclusive, then by definition, that means that their intersection, $A \cap B$, is always 0. Therefore $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = 0$. This may or may not be equal to $P(A)$. So, they can be independent if $P(A) = 0$ or $P(B) = 0$, but not always.

    (b) We have two cases:

    i. $P(B|A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{P(A)}{P(A)} = 1$. This means that if $P(B) = 1$, then they can be independent.

    ii. $P(A|B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{P(A)}{P(B)}$. This can equal $P(A)$ if $P(A) = 0$ or if $P(B) = 1$.

12. (a) $P(HHTHT) = .5 \cdot .5 \cdot .5 \cdot .5 \cdot .5 = (.5)^5 = 0.03125$

    (b) $P(THHHT) = (.5)^5 = 0.03125$

    (c) $P(HTHTH) = (.5)^5 = 0.03125$

    (d) $P(\textit{three heads occurring}) = \binom{5}{3} \cdot \left(\frac{1}{2}\right)^3 \cdot \left(\frac{1}{2}\right)^2 = 0.3125$

18. (a) 7

    (b) $\left(\frac{1}{2}\right)^7 = \frac{1}{128} = 0.0078125$

    (c) $32 + 16 + 8 + 4 + 2 + 1 = 63$

    (d) $\left(\frac{1}{2}\right)^{63} = 0.00000000000000000010842022$

    I think the statement is wrong and actually represents a higher probability than calculated above. I think it's closer to "1 in 9,223,372,036,854,775,808"

Section 1.5

2. (a) $P(G) = P((G \cap A) \cup (G \cap B)) = P(G|A)P(A) + P(G|B)P(B) = .85 \cdot .4 + .75 \cdot .6 = 0.79$

   (b) $P(A|G) = \dfrac{P(G|A)P(A)}{P(G)} = \dfrac{.85 \cdot .4}{.79} = .4304$

3. The percentage of patients with regular heartbeat and low blood pressure is 15.1%

	Low	Normal	High	
irregular	39	71.5	59.5	170
regular	151			830
	190	650	160	1000

4. Y = event that driver is in the 16-25 age group
A = event that driver has at least 1 accident

$$P(Y|A) = \frac{P(A|Y)P(Y)}{P(A)} = \frac{.05 \cdot .1}{.05 \cdot .1 + .02 \cdot .55 + .03 \cdot .2 + .04 \cdot .15} = 0.1786$$

8. Total probability of R:

$$\begin{aligned} P(R) &= P(R|B_1)P(B_1) + P(R|B_2)P(B_2) + P(R|B_3)P(B_3) + P(R|B_4)P(B_4) \\ &= .1 \cdot .4 + .05 \cdot .3 + .03 \cdot .2 + .02 \cdot .1 \\ &= 0.063 \end{aligned}$$

Posterior probabilities:

$$P(B_1|R) = \frac{P(R|B_1)P(B_1)}{P(R)} = \frac{.1 \cdot .4}{.063} = 0.6349$$

$$P(B_2|R) = \frac{P(R|B_2)P(B_2)}{P(R)} = \frac{.05 \cdot .3}{.063} = 0.2381$$

$$P(B_3|R) = \frac{P(R|B_3)P(B_3)}{P(R)} = \frac{.03 \cdot .2}{.063} = 0.09524$$

$$P(B_4|R) = \frac{P(R|B_4)P(B_4)}{P(R)} = \frac{.02 \cdot .1}{.063} = 0.03174$$