Stat 414 Quiz #7

Spring 2016

Student Name: Rolando Vicaría Date: 3/27/16

Start Time: 10:56 am/pm Stop time: 11:21 am/pm

You must show all of your work in order to receive full and/or partial credit. 10 points

A stone store has three types of stones (limestone, marble, and granite). It sells a packet of two random stones. Let X denote the number of limestones in a packet, and let Y denote the number of marbles in a packet. The joint probability mass function of X and Y is:

$$f(x, y) = \frac{2xy + 1}{c}$$

for $x = 0, 1, 2,\ y = 0, 1, 2,$ and $x + y \leq 2$.

1. 3 points Find c that makes the joint PMF valid.

$(x,y) \in \{(0,0), (0,1), (1,0), (1,1), (2,0), (0,2)\}$

$$1 \equiv \sum_{(x,y)\in S} f(x,y) = \frac{2(0)(0)+1}{c} + \frac{2(0)(1)+1}{c} + \frac{2(1)(0)+1}{c} + \frac{2(1)(1)+1}{c} + \frac{2(2)(0)+1}{c} + \frac{2(0)(2)+1}{c}$$

$$= \frac{1}{c} + \frac{1}{c} + \frac{1}{c} + \frac{3}{c} + \frac{1}{c} + \frac{1}{c}$$

$$= \frac{8}{c}$$

$$\boxed{c = 8}$$

		0	1	2
X	0	1/8	1/8	1/8
	1	1/8	3/8	0
	2	1/8	0	0

2. 7 points **Find the correlation coefficient between X and Y.**

$$\cancel{Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X \sigma_Y}}$$

$$\mu_X = \sum_{(x,y)\in S} x\, f(x,y)$$

$$= 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 1\left(\tfrac{1}{8}\right) + 1\left(\tfrac{3}{8}\right) + 2\left(\tfrac{1}{8}\right)$$

$$= \frac{6}{8} = \frac{3}{4}$$

$$\mu_Y = \sum_{(x,y)\in S} y\, f(x,y)$$

$$= 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 1\left(\tfrac{1}{8}\right) + 1\left(\tfrac{3}{8}\right) + 2\left(\tfrac{1}{8}\right)$$

$$= \frac{6}{8} = \frac{3}{4}$$

$$\sigma_X^2 = E(X^2) - \mu_X^2 = \sum_{(x,y)\in S} x^2 f(x,y) = 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 1\left(\tfrac{1}{8}\right) + 1\left(\tfrac{3}{8}\right) + 4\left(\tfrac{1}{8}\right) - \frac{9}{16}$$

$$= \frac{7}{16}$$

$$\sigma_Y^2 = E(Y^2) - \mu_Y^2 = \sum_{(x,y)\in S} y^2 f(x,y) = 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 0\left(\tfrac{1}{8}\right) + 1\left(\tfrac{1}{8}\right) + 1\left(\tfrac{3}{8}\right) + 4\left(\tfrac{1}{8}\right) - \frac{9}{16}$$

$$= \frac{7}{16}$$

$$E(XY) = \sum_{(x,y)\in S} xy\, f(x,y) = (0)(0)\tfrac{1}{8} + (1)(0)\tfrac{1}{8} + (0)(1)\tfrac{1}{8} + (1)(1)\tfrac{3}{8} + (2)(0)\tfrac{1}{8} + (0)(2)\tfrac{1}{8}$$

$$= \frac{3}{8}$$

$$Cov(X,Y) = E(X,Y) - \mu_X\mu_Y = \frac{3}{8} - \left(\frac{3}{4}\right)\left(\frac{3}{4}\right) = \frac{3}{8} - \frac{9}{16} = -\frac{3}{16}$$

$$Corr(X,Y) = \frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{-\frac{3}{16}}{\sqrt{\frac{7}{16}\cdot\frac{7}{16}}} = -\frac{3}{\cancel{16}}\cdot\frac{\cancel{16}}{7} = \boxed{-\frac{3}{7}}$$