Stat 414 Exam #2

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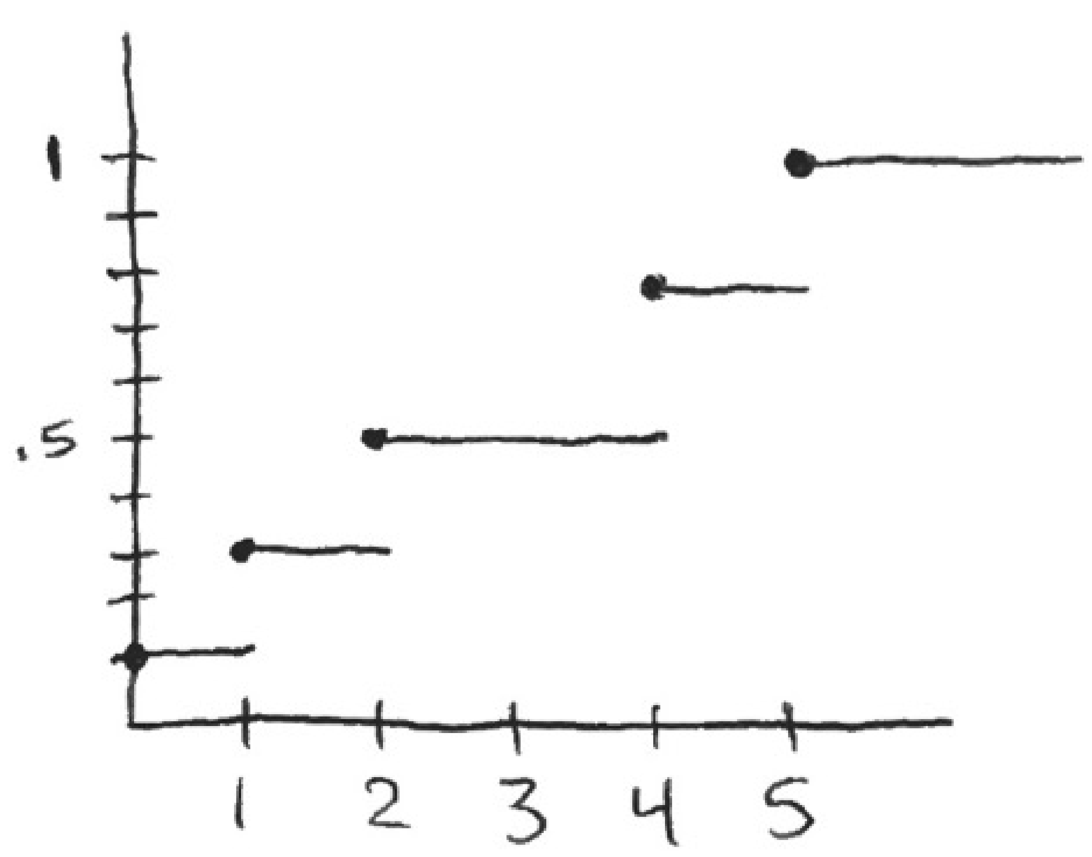
Start Time: 9:44 am/pm Stop time: 11:08 am/pm

You have 1 hour 30 min to complete and 10 minutes to scan/upload. You must show all of your work in order to receive full and/or partial credit. No work=No Credit. Tables/software are not permitted unless otherwise stated in the problem. 5 pages, 26 points

1. 5 points Let the discrete random variable X have the following cumulative distribution function (CDF), $F(x)$.

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{10} & 0 \le x < 1 \\ \frac{3}{10} & 1 \le x < 2 \\ \frac{5}{10} & 2 \le x < 4 \\ \frac{8}{10} & 4 \le x < 5 \\ 1 & 5 \le x \end{cases}$$

(a) 2 points Draw the graph of the CDF.



(b) 3 points Find the probability mass function of X. $f(x=0) = \frac{1}{10}$

$f(x=1) = f(x \le 1) - f(x \le 0) = \frac{3}{10} - \frac{1}{10} = \frac{2}{10} = \frac{1}{5}$

$f(x=2) = f(x \le 2) - f(x \le 1) = \frac{5}{10} - \frac{3}{10} = \frac{2}{10} = \frac{1}{5}$

$f(x=3) = f(x \le 3) - f(x \le 2) = \frac{5}{10} - \frac{5}{10} = 0$

$f(x=4) = f(x \le 4) - f(x \le 3) = \frac{8}{10} - \frac{5}{10} = \frac{3}{10}$

$f(x=5) = f(x \le 5) - f(x \le 4) = 1 - \frac{8}{10} = \frac{2}{10} = \frac{1}{5}$

x	0	1	2	3	4	5
f(x)	1/10	1/5	1/5	0	3/10	1/5

I OVERLOADED USE OF X BELOW

2. 3 points A couple decides to have children until they get a girl, but they agree to stop with a maximum of 3 children even if they haven't gotten a girl yet. You may assume the probabilities of each gender are the same and independent from one child to the next.

(a) 1 point Find the expected number of children.

Y	g(Y)
1	$f(x=1)$
2	$f(x=2)$
3	$f(x\geq 3)$

$$E(Y) = 1(f(x=1)) + 2(f(x=2)) + 3(1-f(x\leq 2))$$
$$= 1(.5) + 2(.5^1)(.5^1) + 3(1-(1-(.5)^2))$$
$$= .5 + .5 + .75 = \underline{1.75}$$

(b) 1 point Find the expected number of girls.

X	f(X)
0	$1-f(x\leq 3)$
1	$f(x\leq 3)$

$$E(x) = 0(1-f(x\leq 3)) + 1(f(x\leq 3))$$
$$= 0(1-(1-(.5)^3)) + 1(1-(.5)^3)$$
$$= 0 + 0.875 = \underline{0.875}$$

(c) 1 point Find the expected number of boys.

Z	f(Z)
0	$f(x=1)$
1	$f(x=2)$
2	$f(x=3)$
3	$f(x>3)$

$$E(Z) = 0f(x=1) + 1(f(x=2)) + 2(f(x=3)) + 3(f(x>3))$$
$$= 0(.5) + 1(.5^1)(.5^1) + 2(.5^2)(.5^1) + 3(1-f(x\leq 2))$$
$$= 0.25 + 0.25 + 3(1-(1-.5^2)) = \underline{1.25}$$

3. 3 points Suppose that a game is to be played with a fair die. In this game a player wins \$20 if a 2 turns up and \$40 if a 4 turns up. He loses \$30 if a 6 turns up. The player neither wins nor loses if any other face turns up. Find the moment generating function of the amount of money the player can win.

X	f(X)
1	0
2	20
3	0
4	40
5	0
6	-30

$$M(t) = E(e^{xt}) = \sum_{x\in S} e^{xt} f(x)$$
$$= e^{t}(0) + e^{2t}(20) + e^{3t}(0) + e^{4t}(40) + e^{5t}(0) + e^{6t}(-30)$$
$$= 20e^{t} + 40e^{4t} - 30e^{6t}$$

4. 4 points There is an airplane including four core engines. Each engine of the airplane will fail independently with probability $1-p$. Assume that this airplane will make a successful flight if at least 50 percent of its engines function well.

(a) 2 points Compute the probability that the airplane will make a successful flight in terms of p. X IS NUMBER OF FAILED ENGINES

$$P(\text{SUCCESS}) = P(X \le 2) = P(X=0) + P(X=1) + P(X=2)$$

$$= \binom{4}{0}(1-p)^0 p^4 + \binom{4}{1}(1-p)^1 p^3 + \binom{4}{2}(1-p)^2 p^2$$

$$= 1(1)p^4 + 4(1-p)p^3 + 6(1-p)^2 p^2$$

$$= p^4 + 4p^3 - 4p^4 + 6p^2 - 12p^3 + 6p^4$$

$$= 3p^4 - 8p^3 + 6p^2$$

(b) 2 points Suppose that the airplane has only two engines, and the other assumptions are the same. Compute the probability of successful flight of this airplane.

$$P(\text{SUCCESS}) = P(X \le 1) = P(X=0) + P(X=1)$$

$$= \binom{2}{0}(1-p)^0 p^2 + \binom{2}{1}(1-p)^1 p^1$$

$$= 1(1)p^2 + 2(1-p)p$$

$$= p^2 + 2p - 2p^2$$

$$= -p^2 + 2p$$

$$= p(2-p)$$

5. 3 points Each time a modem transmits one bit, the receiving modem analyzes the signal that arrives and decides whether the transmitted bit is 0 or 1. It makes an error with probability p, independent of whether any other bit is received correctly.

(a) 1 points If the transmission continues until the receiving modem makes three errors, what is the pmf of Z, the number of bits transmitted?

$$f(z) = \binom{z-1}{2} p^{3} (1-p)^{z-3}, \quad z = 3, 4, \ldots$$

(b) 2 points If $p = 0.25$, what is the probability of $Z = 12$ bits transmitted.

$$f(12) = \binom{11}{2} (.25^3)(.75^9) = 0.06453$$

6. 3 points The number of buses that arrive at a bus stop in s minutes is a poisson random variable X with expected value $s/5$.

(a) 1 points What is the probability that in a two-minute interval, three buses will arrive? $\lambda = \frac{2}{5}$

$$P(x=3; \lambda=\tfrac{2}{5}) = \frac{(2/5)^3 e^{-2/5}}{3!} = 0.00715$$

(b) 2 points How much time should you allow so that with probability 0.99 at least one bus arrives?

$\lambda = \frac{s}{5}$

$$P(X \geq 1) = 0.99$$

$$1 - P(X=0) = 0.99$$

$$P(X=0) = 0.01$$

$$\frac{(s/5)^0 e^{-s/5}}{0!} = 0.01$$

$$e^{-s/5} = 0.01$$

$$-s/5 = \ln 0.01$$

$$s = -5 \ln 0.01 \approx 23.026$$

7. 5 points A study indicates that an exploratory oil well drilled in a particular region should strike oil with probability 0.2.

(a) 1 point What is the probability that the first strike of oil comes on the third well drilled?

$P(X=3) = 0.8^2 \, 0.2^1 = 0.128$

(b) 1 point What is the probability that the third strike of oil comes on the fifth well drilled?

$P(X=5; r=3) = \binom{5-1}{3-1}(0.2^3)(0.8)^2 = 0.0307$

(c) 2 points Suppose that it costs \$4 per drill. What is the expected value and variance of the cost to find the three successful ones. Note: The dollars should be in the millions but as shown for simplicity.

$E(4X) = 4E(X) = 4\left(\frac{1}{p}\right) = 4\left(\frac{1}{.2}\right) = 20$

$VAR(4X) = 4^2 VAR(X) = 4^2\left(\frac{1-p}{p^2}\right) = 4^2\left(\frac{.8}{.2^2}\right) = 320$

(d) 1 point What assumptions are necessary to find the above parts?

EACH DRILL ATTEMPT IS AN INDEPENDENT BERNOULLI TRIAL WITH $p = 0.2$

THEREFORE, X FOLLOWS A GEOMETRIC DISTRIBUTION.