Homework 1

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Previous edition:

2. (a) Population: All families that have three children.
   Sample: One family selected at random
   Sample space (in terms of of their children as 3-tuples from youngest to oldest): {ggg, ggb, gbg, bgg, gbb, bgb, bbg, bbb}
   Type of data: categorical

   (b) Population: All rats in the cage.
   Sample: One rat selected at random.
   Sample space: {m, f}
   Type of data: categorical

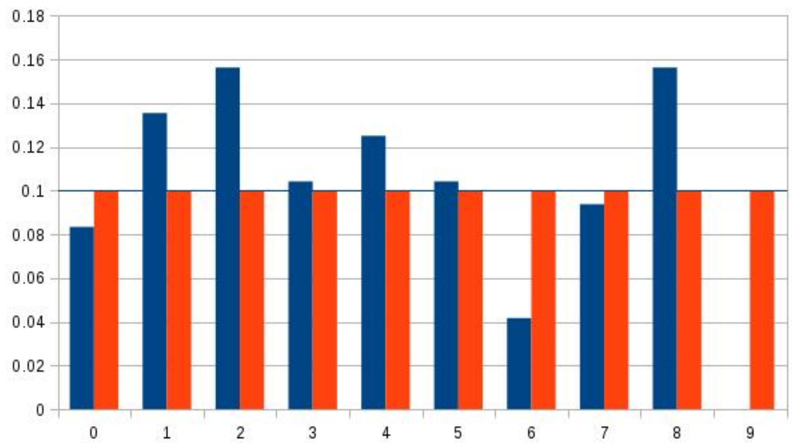
   (c) Population: All 3 digit integers.
   Sample: One 3 digit integer selected at random.
   Sample space: { 100, 101, ..., 998, 999 }
   Type of data: quantitative discrete

3. (a) With true random digits, I would assign a probability of 1/10 to each digit. The p.m.f. of X would be: $f(x) = \dfrac{1}{10}$

   (d)

Digit	Frequency	Relative Frequency
0	8	.0833
1	13	.1354
2	15	.1563
3	10	.1042
4	12	.125
5	10	.1042
6	4	.0417
7	9	.0938
8	15	.1563
9	0	0

   The figure below shows the relative frequencies of the 10 digits in blue and the probability histogram in orange. We can see that the mean value of the blue bars is actually .1, the expected probability of each digit.



13. (a) Airline with better on-time performance per city:
    Los Angeles: Alaska Airlines (.889 vs .856)
    Phoenix: Alaska Airlines (.948 vs .921)
    San Diego: Alaska Airlines (.914 vs .855)
    San Francisco: Alaska Airlines (.831 vs .713)
    Seattle: Alaska Airlines (.858 vs .767)

    (b) Airline with better overall on-time performance: America West (.867 vs .891)

    (c) This is an example of Simpson's paradox where Alaska Airlines shows better on-time performance in each individual city, but America West has better on-time performance overall. The city there they both had the best performance, Phoenix, America West had many more flights.

Current edition:

1. Given:
   $P(PT \cap C) = .28$
   $P(PT' \cap C') = P((PT \cup C)') = 1 - P(PT \cup C) = .08$
   $P(PT \cup C) = .92$
   $P(C) = P(PT) - .16$

   Using theorem 1.1-5:
   $P(PT \cup C) = P(PT) + P(C) - P(PT \cap C)$
   $.92 = P(PT) + P(PT) - .16 - .28$
   $P(PT) = (.92 + .16 + .28)/2 = .68$

2. Given:
   $P(M) = .85$
   $P(S) = .23$
   $P(M \cap S) = .17$

   We're looking for the probability $P(M' \cap S') = 1 - P(M \cup S)$

To find $P(M \cup S)$ we apply theorem 1.1-5:
$P(M \cup S) = P(M) + P(S) - P(M \cap S) = .85 + .23 - .17 = .91$

Therefore, $P(M' \cap S') = 1 - P(M \cup S) = 1 - .91 = .09$

3. (a) $P(A) = 12/52$ {JH, JD, JC, JS, QH, QD, QC, QS, KH, KD, KC, KS}
   (b) $P(A \cap B) = 2/52$ {JH, JD}
   (c) $P(A \cup B) = 16/52$ {JH, JD, JC, JS, QH, QD, QC, QS, KH, KD, KC, KS, 9H, 9D, 10H, 10D}
   (d) $P(C \cup D) = 1$
   (e) $P(C \cap D) = 0$

4. (a) {TTTT, TTTH, TTHT, THTT, HTTT, TTHH, THTH, HTTH, HTHT, HHTT, THHT, HHHT, HHTH, HTHH, THHH, HHHH}
   (b)
      i. $P(A) = 5/16$
      ii. $P(A \cap B) = 0$
      iii. $P(B) = 10/16 = 5/8$
      iv. $P(A \cap C) = 4/16 = 1/4$
      v. $P(D) = 4/16 = 1/4$
      vi. $P(A \cup C) = 9/16$
      vii. $P(B \cap D) = 4/16 = 1/4$

6. (a) $P(A \cup B) = .4 + .5 - .3 = .6$
   (b) $P(A \cap B') = .4 - .3 = .1$
   (c) $P(A' \cup B') = 1 - P(A \cap B) = 1 - .3 = .7$

7. Given:
   $P(A \cup B) = .76$
   $P(A \cup B') = .87$

   We can rewrite the second fact as $P(B) - P(A \cap B) = .13$

   From theorem 1.1-5: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
   $.76 = P(A) + .13$
   $P(A) = .76 - .13 = .63$

8. Given:
   $P(L' \cap R') = 1 - P(L \cup R) = .21 \Rightarrow P(L \cup R) = .79$
   $P(L) = .41$
   $P(R) = .53$

   $P(L \cap R) = P(L) + P(R) - P(L \cup R) = .41 + .53 - .79 = .15$

9. (a) $P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3) = \frac{1}{3} + \frac{1}{3} + \frac{1}{3} - \frac{1}{9} - \frac{1}{9} - \frac{1}{9} + \frac{1}{27} = \frac{19}{27}$

(b) $P(A_1 \cup A_2 \cup A_3)$ can also be expressed as 1 minues the intersection of the complements of A_1, A_2, and A_3, i.e. $1 - P(A_1' \cap A_2' \cap A_3')$.
These are independent events because the occurence of one does not change the probability of the others occuring. Therefore, we can express the intersection as a product of the individual probabilities: $1 - (P(A_1') \times P(A_2') \times P(A_3'))$
Rewriting the complements: $1 - ((1 - P(A_1)) \times (1 - P(A_2)) \times (1 - P(A_3))$.
Since we know that $P(A_i) = 1/3$ for i = 1,2,3, the previous expression can be reduced to $1 - (1 - 1/3)^3$.

11. (a) Based on the last line of the problem that indicates we only care about the number, the sample space consists of the numbers of all the possible slots: {1,2,3,...,36,0,00}. It may be useful to also add along side each number, the color of the slot. But aside from the green ones, the problem doesn't describe how those colors are assigned.

(b) $P(A) = 2/38 = 1/19$

(c) $P(B) = 4/38 = 2/19$

(d) $P(D) = 18/38 = 9/19$

15. (a) By theorem 1.1-6, we know that the probability of the union of all these events is equal to the sum of the probabilities of the events minus the probabilities of the intersections of all possible subsets of the events plus the probability of the intersection of all the events. Since these events are mutually exclusive, all the intersection terms will be 0. Thus, the theorem will reduce to the sum of the individual probabilities of each event. Also, since the events are exhaustive, their sum must equal 1. In other words: $\sum_1^m P(A_i) = 1$. If all $P(A_i)$ are equal, then then $P(A_i) = 1/m$ for i = 1,2,...,m.

(b) Part (a) shows that the $P(A_i) = 1/m$ for all i = 1,2,...,m. By the same argument above, the theorem 1.1-6 will reduce to a sum of the individual probabilities of events A_i for i = 1,2,...,h. Therefore, the $P(A)$ where $A = A_1 \cup A_2 \cup \cdots \cup A_h$ is equal to: $P(A) = \sum_1^h P(A_i) = \sum_1^h \frac{1}{m} = \frac{h}{m}$