Stat 414 Quiz #6

Spring 2016

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Start Time: 10:17 am/~~pm~~ Stop time: 10:47 am/~~pm~~

You must show all of your work in order to receive full and/or partial credit. No tables or software are allowed on this quiz. 10 points

1. 6 points Suppose the number of phone calls at State College's 911 dispatch follows a Poisson distribution with mean of 0.2 calls per hour. Let X be the waiting time for the next phone call. Make sure you explain any property or assumption you use to answer these question. If you do not, you will lose points.

   (a) 2 points What is $P(X < 2)$?

   $\lambda = \frac{1}{5}$ $\theta = 5$

   SINCE $P(X=2)=0$, WE CAN USE CDF, $P(X \leq 2)$

   $F(2) = 1 - e^{-2/5} \approx 0.3297$

   X FOLLOWS EXPONENTIAL DISTRIBUTION WITH $\theta = 5$.

   (b) 3 points What is $P(X > 4|X > 2)$?

   $$P(X>4 \mid X>2) = \frac{P(X>4 \cap X>2)}{P(X>2)} = \frac{P(X>4)}{P(X>2)}$$

   $$= \frac{e^{-4/5}}{e^{-2/5}} = e^{-2/5} = 0.6703$$

   (c) 1 point Let Y be the waiting time until the 5th phone call. What is **mgf** of Y?

   $Y \sim \text{GAMMA}(5, 5)$

   $$M(t) = \frac{1}{(1-5t)^5}$$

   (FROM HOMEWORK)

2. 4 points Let X be a Gamma random variable with parameters $\theta = 1$ and $\alpha = 2$. Find $P(|X+4| \leq 6)$.

$-6 \leq X+4 \leq 6$

$-10 \leq x \leq 2$

$P(X \leq 2)$

SINCE GAMMA IS DEFINED FOR $X > 0$

$$P(X \leq 2) = \int_0^2 \frac{x^1 e^{-x/1}}{\Gamma(2)(1)^2}\, dx = \int_0^2 x e^{-x}\, dx$$

$u = x \qquad dv = e^{-x}\, dx$

$du = dx \qquad v = -e^{-x}$

$$= -xe^{-x}\Big|_0^2 + \int_0^2 e^{-x}\, dx$$

$$= -2e^{-2} + \left[-e^{-x}\right]_0^2$$

$$= -2e^{-2} + \left[-e^{-2} + 1\right]$$

$$= 0.5940$$