Review Exercises II

Roly Vicaría
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1. $4(14)(6) + 4(14)(5) + 4(6)(5) + 14(6)(5) = 1156$
2. Florida can generate more distinct permutations (5040 vs 3780).
3. Distinguishable permutations = 420. Of those, $2 \cdot \frac{5!}{2!3!} = 20$ start with 2 or 3. Therefore, 400 start with either 4 or 5.
4. $\binom{9}{2}\binom{4}{1} = 144$
5. $\binom{10}{5} = 252$
6. ACCLLUUS - $8! = 40320$ permutations
   AABEGLR - $7! = 5040$ permutations
   8 permutations = CALCULUS, 2 permutations = ALGEBRA
   $\frac{8}{40320} = \frac{1}{5040} < \frac{2}{5040}$
   Monkey has a better chance of rearranging AABEGLR to spell ALGEBRA.
7. $P(\text{"full house among 5 fair dice"}) = \dfrac{\binom{6}{1}\binom{5}{3}\binom{5}{1}\binom{5}{2}}{6^5} = 0.3858$
8. A = bridge hand contains 4 aces
   B = bridge hand contains 4 kings
   $$P(A \cup B) = P(A) + P(B) - P(A \cap B) = \frac{\binom{48}{9}}{\binom{52}{13}} + \frac{\binom{48}{9}}{\binom{52}{13}} - \frac{\binom{44}{5}}{\binom{52}{13}} = 0.00528$$
9. $\binom{12}{1}\binom{11}{1}\binom{10}{1}$ is the same as $12 \cdot 11 \cdot 10$, which is computing the permutations of the remaining denominations. So it counts 3, 2, 1 as distinct from 1, 2, 3. Since order doesn't matter, $\binom{12}{3}$ is the correct choice.

10. $P(\text{unanimous decision}) = \dfrac{\binom{23}{12}}{\binom{25}{12}} = 0.26$

11. $P(\text{at least one 6 among 6 dice}) = 1 - P(\text{no 6's}) = 1 - \dfrac{5^6}{6^6} = 0.6651$

    $P(\text{at least two 6's among 12 dice}) = 1 - P(\text{one 6}) - P(\text{no 6's}) = 1 - \dfrac{12 \cdot 5^{11}}{6^{12}} - \dfrac{5^{12}}{6^{12}} = 0.6187$

    $P(\text{at least three 6's among 18 dice}) = 1 - P(\text{two 6's}) - P(\text{one 6}) - P(\text{no 6's})$
    $= 1 - \dfrac{\binom{18}{2}5^{16}}{6^{18}} - \dfrac{18 \cdot 5^{17}}{6^{18}} - \dfrac{5^{18}}{6^{18}} = 0.5973$

12. $P(.25 < y < .5) = \displaystyle\int_{.25}^{.5} 6y(1-y)dy = 3y^2\Big|_{.25}^{.5} - 2y^3\Big|_{.25}^{.5} = 0.3438$

    W = number of observations of Y lying between .25 and .5

    $P(W = 3) = \binom{5}{3}(0.3438)^3(1-0.3438)^2 = 0.1750$

13. $F_Y(y) = \begin{cases} 0, & y < 0 \\ \dfrac{3y^2}{2} - y^3, & 0 \le y \le 1 \\ 1/2, & 1 < y < 2 \\ \dfrac{y}{2} - \dfrac{1}{2}, & 2 \le y \le 3 \\ 1, & y > 3 \end{cases}$

14. $X \sim b(n, p_X)$, $Y \sim b(m, p_Y)$, $W = 4X + 6Y$

	X	Y	W
μ	np_X	mp_Y	$4np_X + 6mp_Y$
σ^2	$np_X(1-p_X)$	$mp_Y(1-p_Y)$	$16np_X(1-p_X) + 36mp_Y(1-p_Y)$

15. $E(e^{3X}) = \displaystyle\sum_{x=0}^{10} e^{3x}\binom{10}{x}(1/3)^x(2/3)^{10-x} = 467592000$

16. $X \sim b(10, 0.01)$

    $P(X \ge 1) = 1 - P(X = 0) = 1 - \binom{10}{0}(0.01)^0(0.99)^{10} = 0.09562$

17. $X \sim Poisson(6.4)$

    $P(X < 3) = \dfrac{6.4^0 e^{-6.4}}{0!} + \dfrac{6.4^1 e^{-6.4}}{1!} + \dfrac{6.4^2 e^{-6.4}}{2!} = 0.04632$

18. $X \sim N(100, 16^2)$

    $z_{0.02} = 2.054$