Stat 414 Quiz #10

Spring 16

Student Name: Rolando Vicaría Date: 4/24/16

Start Time: 7:41 am/pm Stop time: 8:11 am/pm

You must show all of your work in order to receive full and/or partial credit. You may use the Normal tables (or software to calculate probabilities associated with the Normal) where appropriate. No other tables/software are allowed. 10 points

1. 4 points A soda dispensing machine has a variance in the amount of fill to be approximately $\sigma = 1$ ounce. Out of a sample of size $n = 30$ ounces of fill, find the probability that the sample mean will be within 0.3 ounces of the true population mean.

$$P(\mu - .3 \le \bar{X} \le \mu + .3) = P\left(\frac{-.3}{1/\sqrt{30}} \le Z \le \frac{.3}{1/\sqrt{30}}\right)$$

$$= P(-1.643 \le Z \le 1.643)$$

$$= \Phi(1.643) - \Phi(-1.643)$$

$$= 0.9495 - 0.0505 = 0.899$$

2. 3 points Airlines often oversell the number of seats on an airplane. A particular airline finds that 5% of the persons that make a reservation on a certain flight do not show up for the flight. Suppose that an airplane has 155 seats on a certain flight and the airline sells 160 tickets. What is the approximate probability that there will be a seat for everyone that is holding a reservation and is present to fly?

$$X \sim b(160, 0.95)$$

$$P(X \le 155) \approx P\left(\frac{X - 152}{\sqrt{160(0.95)(0.05)}} \le \frac{155.5 - 152}{\sqrt{160(0.95)(0.05)}}\right)$$

$$= P(Z \le 1.27) = 0.8980$$

3. 3 points Let $Z_1, Z_2, \ldots, Z_{10}$ be a random sample of size 10 from a standard normal population. Define

$$\bar{Z} = \frac{1}{10}\sum_{i=1}^{10} Z_i$$

Let Z_{11} be another independent observation from the same population.

(a) 1 point What is the distribution of $W = \sum_{i=1}^{10} Z_i^2$? Why?

$\chi^2(10)$ BECAUSE $Z_i^2 \sim \chi^2(1)$ AND VIA mgf TECHNIQUE WE SAW THAT SUM OF INDEPENDENT $\chi^2(r)$ R.V. IS $\chi^2(\sum_{i=1}^{n} r_i)$

(b) 1 point What is the distribution of $Y = \sum_{i=1}^{10}(Z_i - \bar{Z})^2$? Why?

$\chi^2(9)$ BECAUSE WE KNOW THAT $\frac{(n-1)S^2}{\sigma^2} \sim \chi^2(n-1)$ AND $(n-1)S^2 = \sum_{i=1}^{n}(X_i - \bar{X})^2$ AND $\sigma^2 = 1$ IN THIS CASE.

(c) 1 point What is the distribution of $U = \sum_{i=1}^{10}(Z_i - \bar{Z})^2 + Z_{11}^2$? Why?

$\chi^2(10)$ BY SAME RATIONALE OF PART (a), SUM OF INDEPENDENT χ^2 R.V.