Stat 414 Quiz #4

Spring 2016

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Start Time: 10:58 am/pm Stop time: 11:17 am/pm

You must show all of your work in order to receive full and/or partial credit. No tables/software are allowed for this quiz. 10 points, 2 pages

1. 1 points Suppose that a particular trait of a person (such as eye color, etc) is classified on the basis of one pair of genes and suppose that d represents a dominant gene and r represents a recessive gene. Thus a person with dd genes is pure dominant, one with rr is pure recessive, and one with rd is hybrid. The pure dominant and hybrid are alike in appearance. Offspring receive 1 gene from each parent. If, with respect to a particular trait, two hybrid parents have a total of four children. What is the probability that two or three of the four children have the appearance of the dominant trait? You may assume that each child is equally likely to inherit either of the two genes.

WE CAN MODEL THIS AS A BINOMIAL RANDOM VARIABLE WITH $n = 4$ and $p = 3/4$

$$P(2 \le X \le 3) = P(X=2) + P(X=3)$$

$$= \binom{4}{2}(3/4)^2(1/4)^2 + \binom{4}{3}(3/4)^3(1/4)^1$$

$$= 6\left(\frac{9}{16}\right)\left(\frac{1}{16}\right) + 1\left(\frac{27}{64}\right)\left(\frac{1}{4}\right)$$

$$= \frac{81}{256} \approx 0.3164$$

2. 6 points Let X be a discrete random variable with moment generating function $M(t)$. Prove that for $n \geq 1$

$$M^{(n)}(t) = E\left[X^n e^{tX}\right]$$

where $M^{(n)}(t)$ is the n^{th} derivative of $M(t)$. **Hint:** You might find proof by induction helpful.

BY DEF. $M(t) = E(e^{tX}) = \sum_{x \in S} e^{tx} \cdot f(x)$

WHEN $n=1$

$$\cancel{M} \; M^{(1)}(t) = \frac{d}{dt}\left[\sum_{x \in S} e^{tx} \cdot f(x)\right] = \sum_{x \in S} x\, e^{tx} \cdot f(x)$$

$$= E\left[X^1 e^{tX}\right]$$

ASSUMING $M^{(n)}(t) = E(X^n e^{tX})$,

$$M^{(n+1)}(t) = \frac{d}{dt}\left[M^{(n)}(t)\right] = \frac{d}{dt}\left[E(X^n e^{tX})\right]$$

$$= \frac{d}{dt}\left[\sum_{x \in S} x^n e^{tx}\right]$$

$$= \sum_{x \in S} x^n \cdot x \cdot e^{tx}$$

$$= \sum_{x \in S} x^{n+1} e^{tx}$$

$$= E(X^{n+1} e^{tX})$$