Stat 414 Quiz #3

Spring 2016

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Start Time: 10:26 am/(pm) Stop time: 10:56 am/(pm)

You must show all of your work in order to receive full and/or partial credit. No work=No Credit. 2 pages, 10 points

A pharmaceutical company is trying to develop a new drug. It costs 2 million dollars for the first year for the research, 1 million per year for second year and third year, and 0.5 million per year for fourth and fifth year.
Let X be a random variable for the number of years to develop a new drug, and the probability mass function (pmf) of X is $f(x) = \frac{5-(x-3)^2}{c}$ for $x = 1, 2, 3, 4, 5$.

1. 1 points Find the value of c such that $f(x)$ is a valid pmf.

SET $\sum_{x=1}^{5} f(x) = 1$

$$1 = \frac{5-(1-3)^2}{c} + \frac{5-(2-3)^2}{c} + \frac{5-(3-3)^2}{c} + \frac{5-(4-3)^2}{c} + \frac{5-(5-3)^2}{c}$$

$$= \frac{1}{c} + \frac{4}{c} + \frac{5}{c} + \frac{4}{c} + \frac{1}{c} = \frac{15}{c}$$

$$\boxed{c = 15}$$

2. 3 points What is the mean and variance of X?

$$\mu_X = E(X) = \sum_{x=1}^{5} x \cdot \frac{5-(x-3)^2}{15} = 1\left(\frac{1}{15}\right) + 2\left(\frac{4}{15}\right) + 3\left(\frac{5}{15}\right) + 4\left(\frac{4}{15}\right) + 5\left(\frac{1}{15}\right)$$

$$= \frac{1+8+15+16+5}{15} = \frac{45}{15} = 3$$

$$VAR(X) = \cancel{E(X^2)} - \cancel{\mu_X^2}$$

$$E(X^2) - \mu_X^2 = \sum_{x=1}^{5} x^2 \cdot \frac{5-(x-3)^2}{15} = 1^2\left(\frac{1}{15}\right) + 2^2\left(\frac{4}{15}\right) + 3^2\left(\frac{5}{15}\right) + 4^2\left(\frac{4}{15}\right) + 5^2\left(\frac{1}{15}\right) - 9$$

$$= \frac{1+16+45+64+25}{15} - 9$$

$$= \frac{151}{15} - 9 = \frac{16}{15}$$

3. 2 points Let Y be a random variable for the total cost of development of new drug in millions. What is p.m.f of Y (in millions)?
Hint: it is easier to put p.m.f in table form.

Y	f(y)
2	1/15
3	4/15
4	5/15
4.5	4/15
5 ~~4.5~~	1/15

4. 3 points What is the mean and variance of Y?

$$\mu_Y = E(Y) = \sum_{y \in Y} y f(y) = 2\left(\frac{1}{15}\right) + 3\left(\frac{4}{15}\right) + 4\left(\frac{5}{15}\right) + 4.5\left(\frac{4}{15}\right) + 5\left(\frac{1}{15}\right)$$

$$= \frac{2 + 12 + 20 + 18 + 5}{15} = \frac{57}{15}$$

$$VAR(Y) = E(Y^2) - \mu_Y^2 = \sum_{y \in Y} y^2 f(y) - \mu_Y^2$$

$$= \left[2^2\left(\frac{1}{15}\right) + 3^2\left(\frac{4}{15}\right) + 4^2\left(\frac{5}{15}\right) + 4.5^2\left(\frac{4}{15}\right) + 5^2\left(\frac{1}{15}\right)\right] - \left(\frac{57}{15}\right)^2 = \frac{226}{15} - \frac{361}{25}$$

$$= \frac{47}{75}$$

5. 1 points What is $Var(3Y + 2)$?

$$Var(3Y+2) = E(9Y^2 + 6Y + 4) - \mu_Y^2$$

$$= 9E(Y^2) + 6E(Y) + 4 - \frac{361}{25}$$

$$= 9\left(\frac{226}{15}\right) + 6\left(\frac{57}{15}\right) + 4 - \frac{361}{25}$$

$$= \frac{3699}{25}$$