Homework 6

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STAT414 Spring 2016

February 21, 2016

Section 2.5

1. (a) $P(X \geq 13) = P(X > 12) = q^{12} = (0.9)^{12} = 0.2824$

   (b) $P(X = 30) = \binom{29}{2}(0.1)^3(0.9)^{27} = 0.02361$

2. $P(X = 10) = \binom{9}{4}(0.5)^5(0.5)^5 = 126\left(\frac{1}{2^10}\right) = \frac{63}{512}$

5. (a) $R'(0) = \frac{M'(0)}{M(0)} = \frac{E(X)}{1} = \mu$

   (b) $R''(0) = \frac{M(0)M''(0) - [M'(0)]^2}{[M(0)]^2} = \frac{(1)(\sigma^2 + [M'(0)]^2) - [M'(0)]^2}{1^2} = \frac{\sigma^2}{1} = \sigma^2$

6. (d) Using the result from 2.5-5, the mean, μ, of the negative binomial distribution can be calculated as follows:

$$\mu = R'(0) = \frac{M'(0)}{M(0)} = \frac{rp^{-1}}{1} = \frac{r}{p}$$

   And variance, σ^2, can be calculated as follows:

$$\begin{aligned}
\sigma^2 = R''(0) &= \frac{M(0)M''(0) - [M'(0)]^2}{[M(0)]^2} \\
&= \frac{(1)(rp^{-2}(r + 1 - p)) - (rp^{-1})^2}{(1)^2} \\
&= rp^{-2}(r + 1 - p) - (rp^{-1})^2 \\
&= r^2p^{-2} + rp^{-2} - rp^{-1} - r^2p^{-2} \\
&= rp^{-2} - rp^{-1} \\
&= rp^{-2}(1 - p) \\
&= \frac{r(1 - p)}{p^2}
\end{aligned}$$

7. Start by writing out $E(X^r)$ at $r=1$ and $r=2$:

$$E(X^1) = \sum_{x\in S} x \cdot f(x) = 5^1$$
$$E(X^2) = \sum_{x\in S} x^2 \cdot f(x) = 5^2$$

   This tells us that the probability distribution has a mean of 5 with a variance of 0. So we can infer that the pmf is $f(x) = 1$ for $x = 5$.

   The mgf is then $M(t) = E(e^{Xt}) = \sum_{x=5} e^{xt} f(x) = e^{5t}$.

9. $1 + \frac{4}{3} + \frac{4}{2} + \frac{4}{1} = \frac{25}{3} = 8.333$

Section 2.6

1. (a) $P(2 \le X \le 5; \lambda = 4) = P(X \le 5; \lambda = 4) - P(X \le 1; \lambda = 4) = 0.785 - 0.092 = 0.693$
   (b) $P(X \ge 3; \lambda = 4) = 1 - P(X \le 2; \lambda = 4) = 1 - 0.238 = 0.762$
   (c) $P(X \le 3; \lambda = 4) = 0.433$
2. $P(X = 2; \lambda = 2) = \frac{2^2 e^{-2}}{2!} = \frac{4}{2e^2} = 0.2707$
3. $P(X > 10; \lambda = 11) = 1 - P(X \le 10; \lambda = 11) = 1 - 0.460 = 0.540$
4.

$$\begin{aligned} 3P(X=1) &= P(X=2) \\ 3\left(\frac{\lambda^1 e^{-1}}{1!}\right) &= \frac{\lambda^2 e^{-2}}{2!} \\ 3\left(\frac{\lambda}{e}\right) &= \frac{\lambda^2}{2e} \\ 3\left(\frac{2e}{e}\right) &= \frac{\lambda^2}{\lambda} \\ 6 &= \lambda \end{aligned}$$

   Therefore, $P(X = 4; \lambda = 6) = \frac{6^4 e^{-6}}{4!} = 0.1339$

5. $P(X \le 1; \lambda = 1.5) = 0.558$
6. $P(X = 0; \lambda = 0.5) = 0.607$
8. (a) $P(X \le 1; \lambda = 5) = 0.040$
   (b) $P(4 \le X \le 6; \lambda = 5) = P(X \le 6; \lambda = 5) - P(X \le 3; \lambda = 5) = 0.762 - 0.265 = 0.497$

9. (a) We are given that random variable X, the number of requests per day, follows a Poisson distribution with $\lambda = 3$. Therefore, we can define random variable Y, the number of newspapers sold per day, as follows:

Y	$g(y)$
0	$f(X=0) = 0.0498$
1	$f(X=1) = 0.1494$
2	$f(X=2) = 0.2240$
3	$f(X=3) = 0.2240$
4	$f(X \geq 4) = 1 - f(X \leq 3) = 0.353$

Therefore, the expected value of Y, the number sold, is

$$E(Y) = 0(0.0498) + 1(0.1494) + 2(0.2240) + 3(0.2240) + 4(0.353) = 2.6814$$

(b) We are looking for n, such that $P(X > n) < 0.05$. We can rewrite that as

$$\begin{aligned} P(X > n) &< 0.05 \\ 1 - P(X \leq n) &< 0.05 \\ -P(X \leq n) &< -0.95 \\ P(X \leq n) &> 0.95 \end{aligned}$$

By looking at the table in Appendix B, we can see that for $\lambda = 3$, the first x value where $P(X \leq x) > 0.95$ is when $x = 6$.

10. If X is a Poisson random variable with $\mu = 9$, that means that $\mu = \sigma^2 = \lambda = 9$, $\sigma = 3$.

$$\begin{aligned} P(\mu - 2\sigma < X < \mu + 2\sigma; \lambda = 9) &= P(9 - 2(3) < X < 9 + 2(3); \lambda = 9) \\ &= P(3 < X < 15; \lambda = 9) \\ &= P(X \leq 14; \lambda = 9) - P(X \leq 3; \lambda = 9) \\ &= 0.959 - 0.021 \\ &= 0.938 \end{aligned}$$