Stat 414 Exam #1

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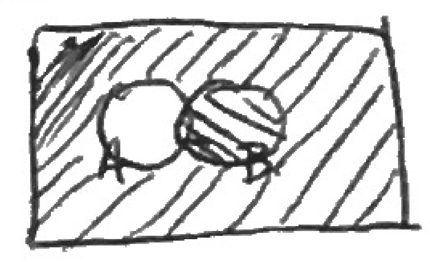
Start Time: 7:08 am/pm Stop time: 8:19 am/pm

You have 1 hour 30 min to complete and 10 minutes to scan/upload. You must show all of your work in order to receive full and/or partial credit. No work=No Credit. 5 pages, 32 points

1. 6 points A baking company has two retail stores. It is known that 30% of the potential customers buy bread from store 1 alone, 50% but from store 2 alone, and 10% buy from both stores. Let A be the event that a randomly chosen potential customer buys from store 1 and B be the event they buy from store 2.



   (a) 2 points Find $P(A' \cup B)$

$$P(A' \cup B) = 1-[P(A) - P(A \cap B)]$$

$$= 1-[.3 - .1] = \boxed{.8}$$

   (b) 2 points Find $P(A|B')$

$$P(A|B') = \frac{P(A \cap B')}{P(B)} = \frac{P(A) - P(A \cap B)}{P(B)}$$

$$= \frac{.3 - .1}{.5} = \boxed{.4}$$

   (c) 1 point Are the events mutually exclusive (disjoint)? Justify.

NO, BECAUSE THEIR INTERSECTION IS NOT 0.

   (d) 1 point Are events A and B independent? Justify.

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{.1}{.5} = .2 \neq .3 = P(A)$$

NO, NOT INDEPENDENT.

2. 4 points Let A and B are two events in a sample space. If $P(A) = 0.9$ and $P(B) = 0.8$. Show that $P(A \cap B) \geq 0.7$. ~~ASSUMING NOT MUTUALLY EXCLUSIVE.~~

~~IF A AND B ARE INDEPENDENT: $P(A \cap B) = P(A)P(B) = 0.72$~~

$P(A \cup B)$ must BE BETWEEN 0 AND 1.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= .9 + .8 - P(A \cap B)$$

$$= 1.7 - P(A \cap B)$$

THEREFORE, $P(A \cap B)$ MUST BE ≥ 0.7

3. 6 points A company has 10 employees and they are to be divided into three jobs, A, B, and C. Three employees are going to job A, four to job B, and three to job C.

(a) 2 points In how many ways can the job assignments be made?

$$\binom{10}{3}\binom{7}{4}\binom{3}{3} = \frac{10!}{3!\cancel{7!}} \cdot \frac{\cancel{7!}}{4!\cancel{3!}} \cdot \frac{\cancel{3!}}{3!0!}$$

$$= \frac{10!}{3!4!3!} = \underline{4200}$$

(b) 4 points Suppose three of the employees are women. What is the probability of all women being assigned to the same job?

Job C: $\binom{7}{3}\binom{4}{4} = \frac{7!}{3!\cancel{4!}} \cdot \frac{\cancel{4!}}{4!0!} = \frac{7!}{3!4!} = 35$

Job B: $\binom{7}{3}\binom{4}{1}\binom{3}{3} = \frac{7!}{3!1!3!} = 140$

Job A: $\binom{7}{4}\binom{3}{3} = 35$

$$P(3 \text{ WOMEN IN SAME JOB}) = \frac{35 + 140 + 35}{4200} = \underline{0.05}$$

4. 8 points You have a fair six-sided die that you roll once. Let R_i denote the event that the roll is i. Let G_j denote the event that the roll is greater than j. Let E denote the event that the roll of the die is even-numbered.

(a) 2 points What is $P(R_3|G_1)$?

$$P(R_3|G_1) = \frac{P(R_3 \cap G_1)}{P(G_1)} = \frac{1/6}{5/6} = \frac{1}{5}$$

(b) 2 points What is the conditional probability that 6 is rolled given that the roll is greater than 3?

$$P(R_6|G_3) = \frac{P(R_6 \cap G_3)}{P(G_3)} = \frac{1/6}{3/6} = \frac{1}{3}$$

(c) 2 points What is $P(G_3|E)$?

$$P(G_3|E) = \frac{P(G_3 \cap E)}{P(E)} = \frac{2/6}{3/6} = \frac{2}{3}$$

(d) 2 points Given that the roll is greater than 3, what is the conditional probability that the roll is even?

$$P(E|G_3) = \frac{P(E \cap G_3)}{P(G_3)} = \frac{2/6}{3/6} = \frac{2}{3}$$

$$\frac{P(G_3|E)P(E)}{P(G_3)} = \frac{(2/3)\cancel{(3/6)}}{\cancel{(3/6)}} = \frac{2}{3}$$

5. 2 points Five job applicants are ranked according to their abilities with applicant number 1 being the best, number 2 being the second best, and so on. The hiring committee does not know the ranking system and suppose they hire three applicants at random. What is the probability they hire at least one of the two best applicants?

$\binom{5}{3}$ WAYS TO HIRE 3 APPLICANTS $= 10$

P(HIRING AT LEAST 1 OF TOP 2) = 1 − P(NOT HIRING EITHER)

ONLY ONE WAY OF HIRING 3 APPLICANTS WITHOUT HIRING EITHER OF TOP 2.

P(HIRING AT LEAST 1) $= 1 - \frac{1}{10} = \boxed{\frac{9}{10}}$

6. 1 points An unbiased die is tossed once. If the face is odd, an unbiased coin is tossed repeatedly; if the face is even, a biased coin with probability of heads $p = 1/3$ is tossed repeatedly. (Successive tosses of the coin are independent in each case.) If the first n throws result in heads, what is the probability that the unbiased coin is being used?

$$P(\text{bias} \mid n \text{ heads}) = \frac{P(n \text{ heads} \mid \text{bias})\, P(\text{bias})}{P(n \text{ heads} \mid \text{bias})\, P(\text{bias}) + P(n \text{ heads} \mid \text{not bias})\, P(\text{not bias})}$$

$$= \frac{(1/3)^n (1/2)}{(1/3)^n (1/2) + (1/2)^n (1/2)}$$

$$= \frac{(1/3)^n}{(1/3)^n + (1/2)^n}$$

$$= \boxed{\frac{1}{1 + (3/2)^n}}$$

7. 2 points Only two training methods, A and B, are used for teaching a particular skill. The failure rate for method A is 20% and 10% for method B. Method B, however is expensive and time consuming and it only used 30%. A worker is trained by one of the methods, but fails to learn it. What is the probability that the worker was trained by method A?

$P(A) = .7 \qquad P(B) = .3$

$P(F|A) = .2 \qquad P(F|B) = .1$

$$P(A|F) = \cancel{P(A \cap F)\, P(F)} \; \frac{P(F|A)\,P(A)}{P(F|A)P(A) + P(F|B)P(B)}$$

$$= \frac{.2(.7)}{.2(.7) + .1(.3)} = \underline{0.824}$$