Student Name: _Rolando Vicaría_   Date:_4/3/16_

Start Time: _11:03_   am/(pm)   Stop time: _11:23_   am/(pm)

You must show all of your work in order to receive full and/or partial credit. Tables/software are not allowed unless otherwise stated in the problem. 10 points

1. Suppose X is a continuous random variable with the following joint probability density

$$f(x,y) = 6(1-y), \qquad 0 \le x \le y \le 1$$

(a) 2 points What is the marginal pdf of Y?

$$f_Y(y) = \int_0^y 6(1-y)\,dx = 6x - 6xy \Big|_0^y = 6y - 6y^2$$

$$= 6y(1-y) \quad 0 \le y \le 1$$

(b) 2 points What is $h(x|y)$, the conditional pdf of X, given that $Y = y$?

$$h(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{6(1-y)}{6y(1-y)} = \frac{1}{y} \qquad x \le y \le 1, \quad 0 \le x \le 1$$

(c) 3 points Find $\mu_{x|y}$.

$$\mu_{x|y} = E(x|y) = \int_0^y x \cdot \frac{1}{y}\,dx = \frac{x^2}{2y}\Big|_0^y = \frac{y}{2} \quad 0 \le y \le 1$$

(d) 3 points Find $\sigma^2_{x|y}$.

$$E(X^2|y) = \int_0^y x^2 \frac{1}{y} \, dx = \frac{x^3}{3y}\bigg|_0^y = \frac{y^2}{3} \qquad 0 \leq y \leq 1$$

$$\sigma^2_{x|y} = E(X^2|y) - E(x|y)^2$$

$$= \frac{y^2}{3} - \left(\frac{y}{2}\right)^2 = \frac{y^2}{3} - \frac{y^2}{4} = \frac{4y^2}{12} - \frac{3y^2}{12} = \frac{y^2}{12}$$

$$0 \leq y \leq 1$$