Review Exercises III

Roly Vicaría
STAT414 Spring 2016

May 1, 2016

1. San Diego Reader carried for 309.375 days. That's $\frac{309.375 - 266}{16} = 2.71$ standard deviations above the mean which is still within the realm of possibility since 99.7 percent of values lie between ± 3 standard deviations about the mean.

2. $\frac{660 - 565}{75} = 1.2667$

   That means that about 10.2% or 434 scored better.

3. $\mu = \frac{103.5 + 144.5}{2} = 124$

   $z_{0.1} = 1.282$

   $\sigma = \frac{144.5 - 124}{1.282} = 15.99$

4. $P(X \geq 4) = 1 - P(X \leq 3) = 1 - [.3 + (.7)(.3) + (.7)^2(.3)] = 0.343$

5. $P(x = 7) = \binom{6}{3}(0.3)^4(0.7)^3 = 0.05557$

6.

$$\begin{aligned} P(X \leq 1) &= \int_0^1 \frac{xe^{-2x}}{\Gamma(2)(1/2)^2} dx \\ &= 4\int_0^1 xe^{-2x} dx \end{aligned}$$

Doing integration by parts with $u = x$ and $dv = e^{-2x}dx$, we get,

$$\begin{aligned}
4\int_0^1 xe^{-2x}dx &= 4\left[-\frac{1}{2}xe^{-2x}\Big|_0^1 - \int_0^1 -\frac{1}{2}e^{-2x}dx\right] \\
&= 4\left[-\frac{e^{-2}}{2} - \frac{1}{4}e^{-2x}\Big|_0^1\right] \\
&= 4\left[-\frac{e^{-2}}{2} - \frac{e^{-2}}{4} + \frac{1}{4}\right] \\
&= -3e^{-2} + 1 \\
&= 0.594
\end{aligned}$$

7. $P(X > 20) = e^{-20/20} = 0.3679$

8. $E(X) = \int_0^3 \frac{1}{9}y^3 dy = \frac{y^4}{36}\Big|_0^3 = \frac{81}{36} = 2.25$

9. (a) $f_Y(y) = \dfrac{3y^2}{39}$

   $P(X = 1|Y = 2) = \dfrac{f(1,2)}{f_Y(2)} = \dfrac{4/39}{12/39} = \dfrac{1}{3}$

   (b) $h(x|y) = \dfrac{f(x,y)}{f_Y(y)} = \dfrac{xy^2/39}{3y^2/39} = \dfrac{x}{3}$

   $E(X|Y = 2) = \sum_x xh(x|2) = 1\left(\frac{1}{3}\right) + 2\left(\frac{2}{3}\right) = \frac{5}{3}$

10. (a) $f_X(x) = \int_0^\infty 2e^{-(x+y)}dy = -2e^{-(x+y)}\Big|_0^\infty = 2e^{-x}$

    $P(X < 1) = \int_0^1 2e^{-x}dx = -2e^{-x}\Big|_0^1 = -2e^{-1} + 2$

    $P(X < 1, Y < 1) = \int_0^1\int_0^y 2e^{-(x+y)}dxdy = \int_0^1 \left[-2e^{-(x+y)}\Big|_0^y\right] = \int_0^1 -2e^{-2y} + 2e^{-y}dy$

    $= e^{-2y}\Big|_0^1 - 2e^{-y}\Big|_0^1 = e^{-2} - 1 - (2e^{-1} - 2) = \dfrac{1-2e}{e^2} + 1$

    $P(Y < 1|X < 1) = \dfrac{P(X < 1, Y < 1)}{P(X < 1)} = \dfrac{e^{-2} - 2e^{-1} + 1}{-2e^{-1} + 2} = 0.3161$

    (b) $h(y|x) = \dfrac{2e^{-(x+y)}}{2e^{-x}} = e^{-y}$

11. $P(\text{even}) = 4/10$, $P(\text{odd}) = 6/10$

    $M(t) = E(e^{tx}) = e^{-3t}\left(\frac{6}{10}\right) + e^{5t}\left(\frac{4}{10}\right)$