Stat 414 Quiz #5

Spring 2016

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Start Time: 10:32 am/pm Stop time: 10:58 am/pm

You must show all of your work in order to receive full and/or partial credit. 10 points

The density of X is given by

$$f(x) = \frac{1}{4} + cx, \qquad 0 \le x \le 1.$$

1. 2 points **Find the value of c that makes the pdf above valid.**

$$1 = \int_0^1 \frac{1}{4} + cx\, dx = \frac{1}{4}\int_0^1 1\,dx + c\int_0^1 x\,dx$$

$$= \frac{1}{4}[x]_0^1 + c\left[\frac{x^2}{2}\right]_0^1$$

$$= \frac{1}{4} + \frac{1}{2}c = \frac{2c+1}{4} \Rightarrow c = \frac{4-1}{2} = \boxed{\frac{3}{2}}$$

2. 1 points **Find the mean of X.**

$$E(X) = \int_0^1 x\left(\frac{1}{4} + \frac{3}{2}x\right)dx = \frac{1}{4}\int_0^1 x\,dx + \frac{3}{2}\int_0^1 x^2\,dx = \frac{1}{4}\left[\frac{x^2}{2}\right]_0^1 + \frac{3}{2}\left[\frac{x^3}{3}\right]_0^1$$

$$= \frac{1}{8} + \frac{1}{2} = \frac{1}{8} + \frac{4}{8} = \boxed{\frac{5}{8}}$$

3. 2 points **Find the variance of X.**

$$E(X^2) = \int_0^1 x^2\left(\frac{1}{4} + \frac{3}{2}x\right)dx = \frac{1}{4}\int_0^1 x^2\,dx + \frac{3}{2}\int_0^1 x^3\,dx = \frac{1}{4}\left[\frac{x^3}{3}\right]_0^1 + \frac{3}{2}\left[\frac{x^4}{4}\right]_0^1$$

$$= \frac{1}{4}\left(\frac{1}{3}\right) + \frac{3}{2}\left(\frac{1}{4}\right) = \frac{1}{12} + \frac{3}{8} = \frac{11}{24}$$

$$Var(X) = E(X^2) - [E(X)]^2 = \frac{11}{24} - \left(\frac{5}{8}\right)^2 = \frac{11}{24} - \frac{25}{64} = \boxed{\frac{13}{192}}$$

4. 2 points **Find the median of X.**

$$F(x) = \int_0^x \frac{1}{4} + \frac{3}{2}t\,dt = \frac{1}{4}\int_0^x 1\,dt + \frac{3}{2}\int_0^x t\,dt = \frac{1}{4}[t]_0^x + \frac{3}{2}\left[\frac{t^2}{2}\right]_0^x$$

$$= \frac{1}{4}x + \frac{3}{2}\left(\frac{x^2}{2}\right) = \frac{3x^2+x}{4}$$

$$F(\pi_{0.5}) = \frac{3(\pi_{0.5})^2 + (\pi_{0.5})}{4} = 0.5$$

$$3\pi_{.5} + \pi_{.5} = 2 \Rightarrow 3\pi_{.5} + \pi_{.5} - 2 = 0$$

$$\pi_{.5} = \frac{-1 \pm \sqrt{1^2 - 4(3)(-2)}}{2(3)}$$

$$= \frac{2}{3} \text{ or } -1$$

5. 3 points **Find the 25th percentile of X.**

$$F(\pi_{.25}) = \frac{3(\pi_{.25})^2 + (\pi_{.25})}{4} = .25$$

$$3\pi_{.25} + \pi_{.25} - 1 = 0$$

$$\pi_{.25} = \frac{-1 \pm \sqrt{1^2 - 4(3)(-1)}}{2(3)}$$

$$\pi_{.25} = .4343 \text{ or } -.7676$$