Proof of Admissibility

Consistency of Food and Proof of Admissibility Heuristic

Given a grid with food strategically placed at locations, we seek to output a heuristic value for the problem where Pacman must eat all the food. We therefore formulate a fully-connected graph, whose vertices are the locations of food and the location of Pacman. We weight edges by the Manhattan distance from one vertex to another. The heuristic is then calculated as the sum of path weights of the MST of the graph.

Definition 1.1. A heuristic from vertex u to v is *admissible* if $H(u, v) < T(u, v)$ where $T(u, v)$ is the true shortest path between vertices u and v and $H(u, v)$ is the computed heuristic value for u and v.

Definition 1.2. A heuristic $h(n)$ is consistent iff for every vertex v and every successor of v generated by any action a, the estimated cost of reaching the goal from v is no greater than the cost of getting to v plus the estimated cost of reaching the goal from v.

$$h(v) = c(v, a, v) + h(v) \tag{1}$$

Definition 1.3. Let $T = (V, E)$ be an MST with vertex set V and edge set E. Let $m(e)$ be the Manhattan distance between two vertices that share an edge e. The food search heuristic for MST T, given the current position of Pacman and the current positions of all uneaten food is given by:

$$H(T) = \sum_{i=0}^{|E|} m(e) \tag{2}$$

Definition 1.4. The Manhattan distance from $u = u_x, u_y$ to $v = v_x, v_y$ is given by $m(u, v) = |u_x - v_x| + |u_y - v_y|$

Lemma 1.1. *The Manhattan distance between two points is a consistent heuristic.*

Proof. Consider specifically the heuristic where $H(u, v) = |u_x - v_x| + |u_y - v_y|$ is such that for any node n with successor s, $h(a, n) \leq c(a, n, s) + h(a, s)$ where $c(a, n, s$ is the true path cost. We know that the Manhattan distance is the shortest path between two points. For Pacman, $h(a, n)$

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