Raise the bar!

We define :

$$\pi_{impl} \equiv GD\,\frac{impl = fun\ n\ a\ ->\ match\ \ n\ with\ 0\ ->\ a\ |\ \_\ ->\ impl\ (n-1)\ (a*n*n)}{impl \to fun\ n\ a\ ->\ match\ \ n\ with\ 0\ ->\ a\ |\ \_\ ->\ impl\ (n-1)\ (a*n*n)}$$

$$\pi_{bar} \equiv GD\,\frac{bar = fun\ n\ ->\ impl\ n\ 1}{bar \to fun\ n\ ->\ impl\ n\ 1}$$

Base case : n = 0

$$\pi_0 \equiv APP\,\frac{\pi_{impl}\qquad\qquad 0\to 0\ \ 0\to a\ \ a\to 1\ \ 1=1}{\pi_{bar}\ 0\to\ \pi_{impl}\ 0\ 1\qquad match\ \ n\ with\ 0\ ->\ a\ |\ \_\ ->\ impl\ (n-1)\ (a*n*n)\to 1}{bar\ 0\to\ 0!*0!+1\to 1}$$

Now for n$\geq$0

$$\pi_1 \equiv APP\,\frac{APP\,\frac{BYI.H(\pi_0)}{\pi_3}}{OP\,\frac{\frac{a\to 1\quad \pi_2}{impl\ n\ (a*(n+1)!*(n+1)!)\to a*(n+1)!*(n+1)!\ \pi_2}\quad \frac{\pi_2\to(n+1)!*(n+1)!\quad \pi_3}{}}{\pi_{impl}\qquad PM\,\frac{impl\ n\ (a*(n+1)!*(n+1))\to(n+1)!*(n+1)!}{impl\ (n+1-1)\ (a*(n+1)!*(n+1)!)\to\ (n+1)!*(n+1)!}}{\pi_{bar}\ n\to\ \pi_{impl}\ n\ 1\qquad match\ \ n\ with\ 0\ ->\ a\ |\ \_\ ->\ impl\ (n+1-1)\ (a*(n+1)!*(n+1)!)\to\ (n+1)!*(n+1)!}{bar\ n\to\ n!*n!}$$