Dynamic Model and Parameters

This report describes the equations of motion and dynamic models used by the Coriolis SIH for quadrotor X as well as the QGroundControl parameters associated with it.

This model assumes a North-East-Down (NED) inertial reference frame $\mathcal{F}_I$ as well as a Front-Right-Down body frame $\mathcal{F}_B$ attached to the center of mass (CM) of the vehicle.

Equations of motion

The equations of motion of a rigid body are given by

$\dot{p}_I = v_I$	Inertial position
$\dot{v}_I = \frac{1}{m}(W_I + F_{a,I} + C_{IB}(q)\, T_B)$	Inertial velocity, conservation of linear momentum
$\dot{q} = \frac{1}{2} q \otimes \omega_B$	Quaternion (attitude)
$\dot{\omega}_B = I^{-1}(M_{t,B} + M_{a,B} - \omega_B \times I\omega_B)$	Body rates, conservation of angular momentum

q is the quaternion used for the representation of the attitude. The Direct Cosine Matrix (DCM) C_{IB} can be obtained from q. It allows to perform transformation from the body to inertial frame.

The inertia matrix is assumed constant, so its inversion is computed once. The inertia is a 3x3 matrix composed of six (repeated) entries

$$I = \begin{bmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{bmatrix}$$

Weight force

The weight force in the inertial frame is a constant given by

$W_I = (0; 0; mg)$

where m is the mass in kg and $g = 9.81\ [m/s^2]$ is the gravity acceleration.

Aerodynamic forces

The aerodynamic forces are modeled as a first order drag in order to stop the vehicle in absence of horizontal thrust.

$F_{a,I} = -K_{DV}\, v_I$

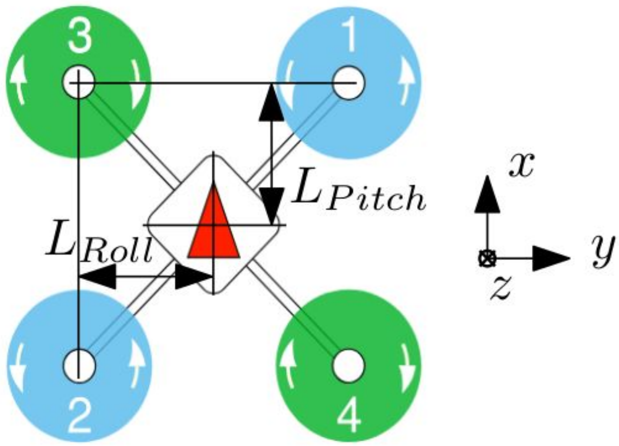
Thruster force

The quadrotor is equipped with 4 thrusters, that produce forces T1 to T4 pulling the vehicle upward (opposite to z axis)

$T_B = [0; 0; -(T_1 + T_2 + T_3 + T_4)]$

where the thrust force T_i of each motor i is related to its normalized signal u_i as $T_i = T_{max}\, u_i$

Thruster moments



The quad thrusters produce moments in the body frame as

$M_{t,Bx} = L_{Roll}(-T_1 + T_2 + T_3 - T_4)$

$M_{t,By} = L_{Pitch}(T_1 - T_2 + T_3 - T_4)$

$M_{t,Bz} = Q_1 + Q_2 - Q_3 - Q_4$

Where L_{Roll} is the arm length generating the rolling moment, i.e. distance from the left motors to the CM, and L_{Pitch} is the arm length generating the pitching moment, i.e. the distance from the front motors to the CM.

The torque Q_i of each motor i is related to its normalized signal u_i as $Q_i = Q_{max}\, u_i$

Aerodynamic Moments

The aerodynamic moments are modeled as a first order drag moments to stop the vehicle rotation in absence of thruster moments.

$M_{a,B} = -K_{D\omega}\, \omega_B$

Integration step

Forward Euler is used for the integration step. Rewriting the equations of motion in discrete time, with a sampling time Δt gives

$p_I^{k+1} = p_I^{\,k} + v_I^{\,k}\, \Delta t$

$v_I^{k+1} = v_I^{\,k} + \frac{1}{m}(W_I^{\,k} + F_{a,I}^{\,k} + C_{IB}(q^k)\, T_B^{\,k})\, \Delta t$

$q^{k+1} = \left(q^k + \frac{1}{2} q^k \otimes \omega_B^{\,k}\, \Delta t\right) / \left\| q^k + \frac{1}{2} q^k \otimes \omega_B^{\,k}\, \Delta t \right\|$

$\omega_B^{\,k+1} = \omega_B^{\,k} + I^{-1}(M_{t,B}^{\,k} + M_{a,B}^{\,k} - \omega_B^{\,k} \times I\omega_B^{\,k})\, \Delta t$

Note that the quaternion is normalized at every time step to ensure $\|q\| = 1$

Sensors Reconstruction

In this section $\eta_i \in N(0, \sigma_i)$ denotes a white gaussian noise with a standard deviation σ_i.
The IMU[1] reconstruction and sensor noise standard deviations are taken from [1].

The IMU signals are reconstructed from the states as follows

$u_{acc} = C_{BI}(\dot{v}_I - g) + \eta_{acc}$

$u_{gyro} = \omega_B + \eta_{gyro}$

$u_{mag} = C_{BI}\mu_I + \eta_{mag}$

where g is the gravity acceleration vector, μ_I is the magnetic field assumed constant (i.e. taken at the initial location).

The AMSL[2] altitude is taken as

$h = h_0 - p_{I,z}$

where h_0 is the AMSL ground altitude at the initial location.

The barometric altitude, pressure, and temperature are reconstructed as follows

$h_{baro} = h + \eta_{h,baro}$

$P = P_0 (1 + ah/T_0)^{-g/(aR)}$

$\theta = \theta_0 + ah$

where $P_0 = 1013.25\ [mbar]$ is the MSL pressure, $a = -6.5 \cdot 10^{-3}\ [K/m]$ is the temperature gradient according to the standard atmosphere model, T_0 is the ground temperature in Kelvin, θ_0 its corresponding temperature in degrees Celsius, and $R = 287.1\ [J/(kg\ K)]$ is the ideal gas constant.

The GPS latitude, longitude, altitude and velocity are reconstructed as follows

$lat = lat_0 + p_{I,x}/R_E + \eta_{lat}$

$lon = lon_0 + p_{I,y}/R_E/\cos lat_0 + \eta_{lon}$

$h_{gps} = h + \eta_{h,gps}$

$v_{gps} = v_I + \eta_{gps}$

where lat_0 and lon_0 are the initial geodetic locations in radians, and $R_E = 6371000\ [m]$ is the radius of Earth

Limitations:

1. The barometric model for the pressure and temperature are valid in the Troposphere only (up to $11000\ [m]$ AMSL), which is realistically not a limitation for a drone.
2. This set of models for the sensors reconstruction are defined under the assumption of a local flat Earth (i.e. the Earth is big enough that it can be approximated locally as a flat plane). Therefore those models are valid only for flight missions no more than few km around the initial position.

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[1] Inertial Measurement Unit
[2] Above Mean Sea Level

3. The initial position cannot be at the geographic North or South pole. The GPS model for the latitude and longitude assumes small angles and therefore the division by $\cos lat_0$ would become a singularity at the geographic poles.

QGroundControl Parameters

All the SIH parameters start by “SIH_”, so they can be sorted out easily when searching them

Name	**Type**	**Description**	**Default**	**Units**
SIH_MASS	Float	vehicle mass	1.0	kg
SIH_IXX	Float	vehicle inertia term about x axis	0.025	$kg \cdot m^2$
SIH_IYY	Float	vehicle inertia term about y axis	0.025	$kg \cdot m^2$
SIH_IZZ	Float	vehicle inertia term about z axis	0.035	$kg \cdot m^2$
SIH_IXY	Float	vehicle inertia cross term x and y	0.0	$kg \cdot m^2$
SIH_IXZ	Float	vehicle inertia cross term x and z	0.0	$kg \cdot m^2$
SIH_IYZ	Float	vehicle inertia cross term y and z	0.0	$kg \cdot m^2$
SIH_T_MAX	Float	maximum thrust of one propeller	5.0	N
SIH_Q_MAX	Float	maximum torque of one propeller	0.1	Nm
SIH_L_ROLL	Float	arm length generating the rolling moment	0.2	m
SIH_L_PITCH	Float	arm length generating the pitching moment	0.2	m
SIH_KDV	Float	First order drag coefficient	1.0	$N/(m/s)$
SIH_KDW	Float	First order angular damper coefficient	0.025	$Nm/(rad/s)$

The following parameters are related to the initial location and if modified should be coherent among each others. i.e. the magnetic field should correspond to the magnetic field present at the given latitude and longitude.

Name	**Type**	**Description**	**Default**	**Units**
SIH_LAT0	Int32	Initial geodetic latitude with 7 digits	454671160	10^{-7} *deg*
SIH_LON0	Int32	Initial geodetic longitude with 7 digits	-737578370	10^{-7} *deg*
SIH_H0	Float	Ground altitude at this location	32.34	*m*
SIH_MU_X	Float	North magnetic field at this location	0.2903	*G*
SIH_MU_Y	Float	East magnetic field at this location	-0.0832	*G*
SIH_MU_Z	Float	Down magnetic field at this location	0.9500	*G*

References

[1] Bulka, Eitan, and Meyer Nahon. "Autonomous fixed-wing aerobatics: from theory to flight." *2018 IEEE International Conference on Robotics and Automation (ICRA)*. IEEE, 2018.