1 Deep reinforcement Q-learning

Paper 1: Playing Atari with Deep Reinforcement Learning:
`https://www.cs.toronto.edu/~vmnih/docs/dqn.pdf`
Paper 2: Human-level control through deep reinforcement learning:
`https://web.stanford.edu/class/psych209/Readings/MnihEtAlHassibis15NatureControlDeepRL.pdf`

1.1 Optimal action-value function

$$\begin{aligned} Q^*(s,a) &= max_\pi \mathbf{E}[R_t | s_t = s, a_t = a, \pi] \\ &= max_\pi \mathbf{E}_{s' \sim \varepsilon}[r + \gamma . max_{a'} Q^*(s', a') | s, a] \end{aligned}$$

with:

- Environment: ε
- State (observation): s
- Action: $a \in$ set of discrete actions $\mathbf{A} = \{1, 2, ..., k\}$
- Policy mapping observation (s) to action (a): $\pi = P(a|s)$
- Reward: r, Discount factor: $\gamma \in]0, 1[$, Horizon: T
- Future discounted return at time t:

$$R_t = \sum_{t'=t}^{T} \gamma^{(t'-t)} . r_{t'} = r_t + \gamma . r_{t+1} + \gamma^2 . r_{t+2} + ...$$

1.2 Tabular Q-learning

Update the Q-table with the Bellman equation at each iteration i:

$$Q_{i+1}(s,a) = \mathbf{E}[r + \gamma . max_{a'} Q_i(s', a') | s, a]$$

$$Q_i \longrightarrow Q^* \text{ when } i \longrightarrow \infty$$

Problems:

- No generalisation: Q is estimated separately at each iteration.
- No scaling: The MDP and Q-table grow very fast very large.
- No learning in large state space: every state is a new state.

1.3 DQN

1.3.1 Problem with function approximators

Non-linear Q-function approximators (such as NN) make RL unstable:

- Consecutive actions in the observation sequence are highly correlated.
- Small updates to Q may significantly change the policy.

Solution, 2 key ideas:

- Replay memory.
- Target network.

1.3.2 Replay memory

Role: Removing correlations in the observation sequences and smoothing over changes in the data distribution.

Dataset of agent experiences over N last timesteps, with:

- Experience at timestep t: $e_t = (s_t, a_t, r_t, s_{t+1})$
- Replay memory dataset of past experiences: $D = \{e_{t-1}, e_{t-2}, ..., e_{t-N}\}$

The Q-network is updated over a sample of mini-batch experiences linearly drawn at random from the pool of stored experiences in the replay memory.

1.3.3 Target network

Role: Adjusting the action-values toward target values that are only periodically updated, thereby reducing correlations with the targets. Generating the targets using an older set of weights adds a delay between an update to Q and the time this update affects the targets y_i, reducing divergence.

Target network $\widehat{Q}$, with:

- Weights: θ_i^-
- Target network update frequency: C

The Q-network, with weights θ_i is cloned every C steps to obtain the target network $\widehat{Q}$, with weights θ_i^-. This separate network is held fixed for the next C updates and is used for generating the targets y_i in the Q-learning updates.

1.3.4 Learning strategy

Off-policy: states and rewards are obtained with a π behavior policy different from the online policy that is being learned, with π the ϵ-greedy policy:

$$\epsilon\text{-greedy strategy: } a = \begin{cases} argmax_a Q(s,a,\theta) & \text{with probability } 1-\epsilon \\ \text{rand a} \in \mathbf{A} & \text{with probability } \epsilon \end{cases}$$

Model-free: does not construct an estimate of the environment ε.

1.3.5 State space

A state is a sequence of observations and actions, with:

- Observation: x
- State: $s_t = (x_1, a_1, x_2, a_2, ..., x_{t-1}, a_{t-1}, x_t) = (s_{t-1}, a_{t-1}, x_t)$
- State observation preprocessing function: $\phi_s = \phi(s) \sim s$

Assuming a finite horizon, the MDP is large but finite.

1.3.6 Q-Network

Neural network function approximator with weights θ:

$$Q(s,a,\theta) \approx Q^*(s,a)$$

Loss function at iteration i:

$$\begin{aligned} L_i(\theta_i) &= \mathbf{E}_{(s,a,r,s')\sim U(D)}[(y_i - Q(s,a,\theta_i))^2] \\ &= \mathbf{E}_{(s,a,r,s')\sim U(D)}[(r + \gamma.max_{a'}Q(s',a',\theta_i^-) - Q(s,a,\theta_i))^2] \end{aligned}$$

with:

- Q updates on samples (mini-batch) of experiences (s,a,r,s'): $U(D)$
- Target: $y_i = r + \gamma.max_{a'}Q(s',a',\theta_i^-)$
  - Weights of the target network: θ_i^-
  - Error clipping between -1 and 1
  - The target y_i changes over time, contrary to supervised learning
- Minimize with mini-batch gradient descent.

Gradient:

$$\nabla_{\theta_i} L_i(\theta_i) = \mathbf{E}_{(s,a,r,s')\sim U(D)}[(r+\gamma.max_{a'}Q(s',a',\theta_i^-) - Q(s,a,\theta_i)).\nabla_{\theta_i}Q(s,a,\theta_i)]$$

1.4 DQN algorithm

DQN algorithm

Initialize replay memory D to capacity N
Initialize action-value function Q with random weights θ
Initialize target action-value function $\widehat{Q}$ with weights $\theta^- = \theta$
for episode = 1,M **do**
    Initialize sequence $s_1 = x_1$ and preprocessed sequence $\phi_1 = \phi(s_1)$
    for t = 1,T **do**
        With probability ϵ select a random action a_t
        otherwise select action $a_t = argmax_a Q(\phi(s_t), a, \theta)$
        Execute a_t in emulator ε and observe reward r_t and image x_{t+1}
        Set $s_{t+1} = (s_t, a_t, x_{t+1})$ and preprocess $\phi_{t+1} = \phi(s_{t+1})$
        Store transition $(\phi_t, a_t, r_t, \phi_{t+1})$ in D
        Sample random minibatch of transitions $(\phi_j, a_j, r_j, \phi_{j+1})$ from D
        Set $y_i = \begin{cases} r_j & \text{if episode terminates at step } j+1 \\ r_j + \gamma . max_{a'} \widehat{Q}(\phi_{j+1}, a', \theta^-) & otherwise \end{cases}$
        Perform a gradient descent step on $(y_i - Q(\phi_j, a_j, \theta))^2$ w.r.t. θ
        Every C steps reset $\widehat{Q} = Q$
    end for
end for

1.5 Atari Q-network

1.5.1 Input

Fixed length state representation preprocessed by function ϕ, with:

- Size: $32 \times (84 \times 84 \times 4)$:
  - Mini-batch size: 32
  - Image cropped and reduced x: 84×84 pixels
  - Agent history length: 4 frames

1.5.2 Neural network

CNN:

- CONV1: hwc = $8 \times 8 \times 32$, s=4

- Relu
- CONV2: hwc = $4 \times 4 \times 64$, s=2
- Relu
- CONV3: hwc = $3 \times 3 \times 64$, s=1
- Relu
- FC1: 256 logits
- Relu
- FC2: $|\mathbf{A}| \in$ [4,18] logits

with:

- Optimizer: RMSprop or Adam
- Loss function: huber (not quadratic)

1.5.3 Output

Actions:

- Predicted Q-values of the individual action for the input state.
- Compute Q-values for all actions in a state with a single forward pass.

1.5.4 Hyperparameters

- Training: 10000000 timesteps
- Replay memory dataset size: $N = 1000000$ timesteps
- Target network update frequency: $C = 10000$ timesteps
- Reward: $r \in \{-1, 0, 1\}$
- Frame Skipping: k = 4 frames:
  - The agent sees 1/4 frames and the action is repeated over 4 frames.
- α, γ, ϵ, RMSprop, other: see paper 2.

1.6 Double DQN

Paper 3: Deep Reinforcement Learning with Double Q-learning:
`https://arxiv.org/pdf/1509.06461.pdf`

1.6.1 Overestimation in DQN

DQN sometimes learns unrealistically high value functions because it includes a maximization step over estimated action values, which tends to prefer over-estimated to underestimated values. The max operator uses the same values to select and to evaluate an action for the targets, i.e. $max_{a'}Q(s', a', \theta_i^-)$, resulting in overoptimistic Q-value estimates and suboptimal policies.

1.6.2 Decoupling action selection and action evaluation

Double DQN solution to overestimation:

- Decoupling action selection and action evaluation for the target values.

Double DQN reduces overestimation by decomposing the max operation when approximating the target values into action selection and evaluation:

- The action is selected by the online network.
- The action Q-value is evaluated by the target network.

1.6.3 Double DQN target

DQN target:

$$y_i = r + \gamma . max_{a'}Q(s', a', \theta_i^-)$$

Double DQN target:

$$y_i = r + \gamma . Q(s', argmax_{a'}Q(s', a', \theta_i), \theta_i^-)$$

1.6.4 Hyperparameters

- Target network update frequency: $C = 30000$ timesteps
- Mostly unchanged from DQN: see paper 3.

1.7 Dueling DDQN

Paper 4: Dueling Network Architectures for Deep Reinforcement Learning:
`https://arxiv.org/pdf/1511.06581.pdf`

1.7.1 Decoupling value function and advantage function

For many states, some actions do not affect the environment in any relevant way. But for single stream Q-networks, it is important to estimate the state values for every state to choose the optimal action. The dueling architecture can learn which state are (or are not) valuable, without having to learn each action for each state, by separating state values and action advantages.

The dueling neural network architecture decouples the value and advantage in Deep Q-Networks with two separate function estimators:

- State-value function approximator: $V(s)$
- State-dependent action advantage approximator: $A(s, a)$

For many states, it is unnecessary to estimate the value of each action choice. The dueling network learns a general state value that is shared across many similar actions, leading to faster convergence and more efficient Q-functions.

1.7.2 Advantage function

In the dueling network architecture:

- Q measures the value of choosing a particular action when in this state.
- V measures how good it is to be in a particular state.
- A measures the relative importance of each action, deducting V.

with:

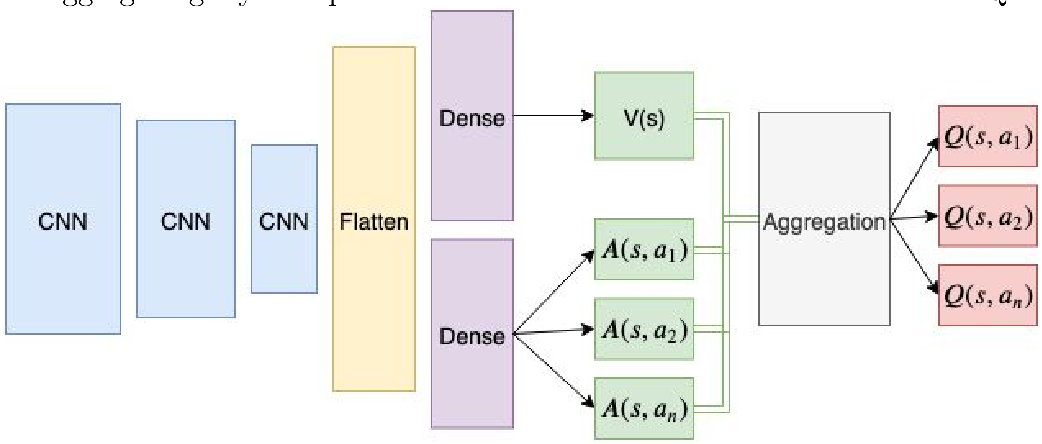
- $Q^\pi(s, a) = V^\pi(s) + A^\pi(s, a)$
- $V^\pi = \mathbf{E}_{a\sim\pi(s)}[Q^\pi(s, a)]$
- $\mathbf{E}_{a\sim\pi(s)}[A^\pi(s, a)] = 0$

Thus, the advantage function, relating the V and Q functions is defined as:

$$A^\pi(s, a) = Q^\pi(s, a) - V^\pi(s)$$

1.7.3 Dueling network architecture

1.7.3.1 Architecture The dueling deep Q-network architecture consists of two streams for the value and advantage functions, while sharing a common convolutional feature learning module. The two streams are combined via an aggregating layer to produce an estimate of the state-value function Q.



1.7.3.2 Value stream $V(s)$
Estimate the scalar state-value $V(s, \theta, \beta)$:

- $\mathrm{FC1}_V$: 512 logits
- Relu
- $\mathrm{FC2}_V$: 1 logits

with:

- Weights of the convolutional learning module: θ
- Weights of the value stream: β

1.7.3.3 Advantage stream $A(s, a)$
Estimate the $|\mathbf{A}|$-dimensional action-advantage vector $A(s, a, \theta, \alpha)$:

- $\mathrm{FC1}_A$: 512 logits
- Relu
- $\mathrm{FC2}_A$: $|\mathbf{A}| \in [4{,}18]$ logits

with:

- Weights of the convolutional learning module: θ
- Weights of the advantage stream: α
- Number of valid actions: $|\mathbf{A}|$

1.7.3.4 Aggregating layer $Q(s, a)$

The value and advantage streams are combined to produce a single state-action value Q-function $Q(s, a, \theta, \alpha, \beta)$, with:

$$Q(s, a, \theta, \alpha, \beta) = V(s, \theta, \beta) + (A(s, a, \theta, \alpha) - \frac{1}{|\mathbf{A}|}. \sum_{a'} A(s, a', \theta, \alpha))$$

1.7.4 Hyperparameters

- Since the two streams propagate gradients, the combined gradient is rescaled by $1/\sqrt{2}$ when backpropagated to the CNN learning module.
- Gradient error clipping between -10 and 10.
- Mostly unchanged from DDQN: see paper 4.