

# Deep Q Learning: From Paper to Code

Markov Decision Processes

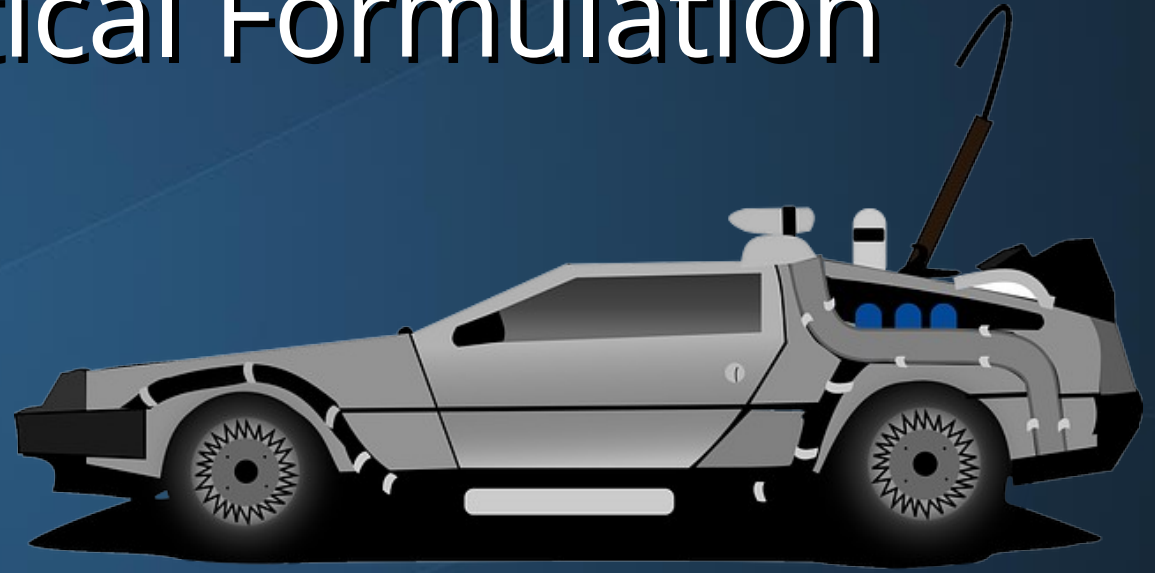
# Last Time ...

- Interactions of agent and environment
- Agent learns and acts
- Environment is what is acted upon
- Rewards tell the agent what is good

# Mathematical Formulation



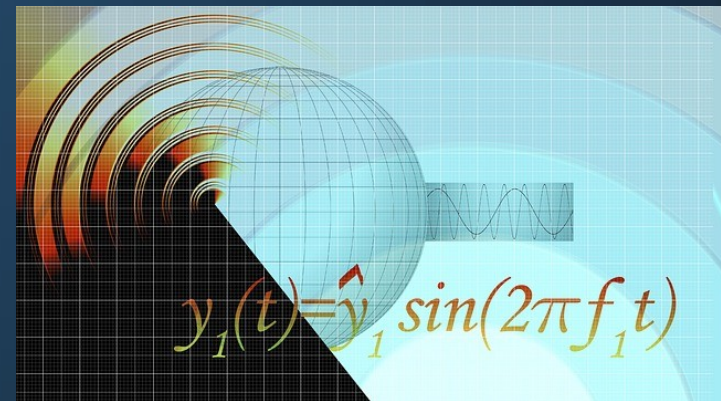
$(S_1, A_1, R_1, S_2, A_2, R_2, \dots)$



Actions affect all future rewards

State depends only on  
previous state and action

**Markov Decision Process**



Mathematical abstraction

# Probabilistic Transitions

Actions cause state transitions

$$p(s', r | a, s) \neq 1$$

$$\sum_{s', r} p(s', r | a, s) = 1$$

Probabilities define our dynamics

$$r(s, a) = E[R_t | S_{t-1} = s, A_{t-1} = a] = \sum_{r \in R} r \sum_{s' \in S} p(s', r | s, a)$$

Expected reward  $\rightarrow$  outcome \* probability

# Maximizing Rewards & Episodic Tasks



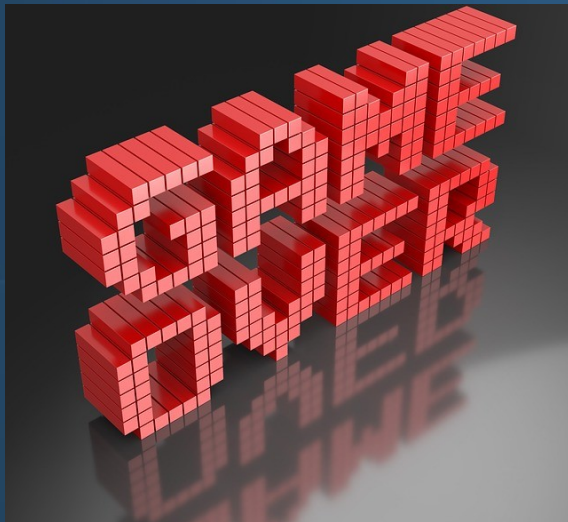
Series of rewards → expected return

Sum of rewards that follow current time

$$G_t = R_{t+1} + R_{t+2} + R_{t+3}, \dots, R_T$$

Episode: discrete period of game play

# Episodic Game Play



Terminal state is unique



$$G_T = 0$$

Ensures sum over rewards finite

# Reward Discounting

Not all tasks are episodic!



$$\sum_{t=0}^{\infty} R_t \rightarrow \infty$$

Fix by discounting

Discount factor  $\rightarrow \gamma$

# Reward Discounting

$$0 \leq \gamma \leq 1$$

$1 \rightarrow \gamma$  Far sighted

$0 \rightarrow \gamma$  Myopic

$$0.95 \leq \gamma \leq 0.99$$

$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} = \sum_{k=0}^{\infty} \gamma^k R_{t+k+1}$$



# Reward Discounting

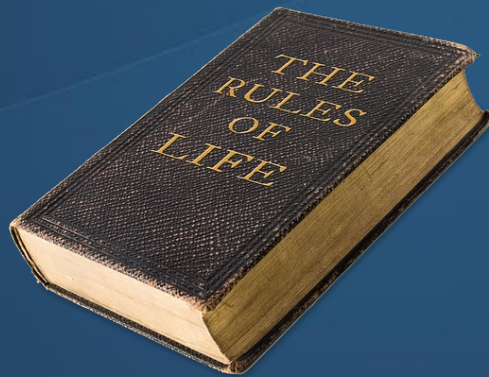
$$G_t = R_{t+1} + \gamma R_{t+2} + \gamma^2 R_{t+3} + \gamma^3 R_{t+4} + \dots$$

$$G_t = R_{t+1} + \gamma(R_{t+2} + \gamma R_{t+3} + \gamma^2 R_{t+4} + \dots)$$

$$G_t = R_{t+1} + \gamma G_{t+1}$$

But wait... how can we know future rewards?

# The Policy



Mapping of states to actions



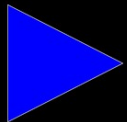
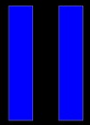
Can be probabilistic

$\Pi$

# Next Exercise

SFFF  
FHFH  
FFFH  
HFFG

- Frozen Lake environment
- Reasonable deterministic policy
- 1000 games
- Plot win % over trailing 10 games



# Summary

- MDP determined by previous states and actions
- Governed by probability distribution
- Agent maximizes rewards over time
- Policy tells us how agent will act in some state

# Up Next

