Exercice d'application

Reliability analysis and maintenance optimisation

We will work on a 12 years of feedback database (annual number of failures and annual unavailability time), for 1000 identical units of 5 different components.

1. Define the Pareto diagrams (80-20 to the nearest) for:

   → the number of failures

   → the mean failure time

   → the unavailability

   → Weighted Pareto chart (weights 2, 1 et 3)

   → Limited Weighted Pareto chart (weights 2, 1 et 3)

2. For each component, determine (with the Weibull paper AND with a linear regression) the coefficients of the Weibull distribution characterizing their lifetime and deduce the MTBF of each component.

Exercice d'application

Reliability analysis and maintenance optimisation

	Component 1		Component 2		Component 3		Component 4		Component 5	
REX (years)	N_1	T_1	N_2	T_2	N_3	T_3	N_4	T_4	N_5	T_5
1	80	2800	1	21	5	150	78	3744	0	0
2	74	2590	7	147	15	450	115	5520	0	0
3	67	2345	24	504	25	750	122	5856	0	0
4	62	2170	53	1113	33	990	118	5664	0	0
5	58	2030	91	1911	42	1260	106	5088	1	500
6	52	1820	130	2730	48	1440	93	4464	1	500
7	49	1715	160	3360	53	1590	79	3792	3	1500
8	45	1575	166	3486	58	1740	65	3120	6	3000
9	41	1435	147	3087	60	1800	53	2544	11	5500
10	37	1295	108	2268	61	1830	42	2016	19	9500
11	35	1225	66	1386	61	1830	32	1536	30	15000
12	32	1120	31	651	59	1770	26	1248	45	22500

Exercice d'application

Reliability analysis and maintenance optimisation

3. The S_1 system consists in a serial struture of all 5 considered components.
   Determine the equation of the reliability of S_1, denoted R_{S1}.
   Plot R_{S1} over 15 years for each weibull training method (weibull paper and linear regression).

4. Propose a DBN structure AND a Markov chain structure that could model the behavior of S_1. Compare the reliability over 15 years with both approaches (markovian and Weibull).

5. For each components, define (with a 1 month precision) the optimal systematic maintenance period (for some of these component, you will be able to compare with results with the one provided by the graphical abaque).

Component 1		Component 2		Component 3		Component 4		Component 5	
c_p	P	c_p	P	c_p	P	c_p	P	c_p	P
35	300	65	300	55	300	85	300	195	300

Exercice d'application

Reliability analysis and maintenance optimisation

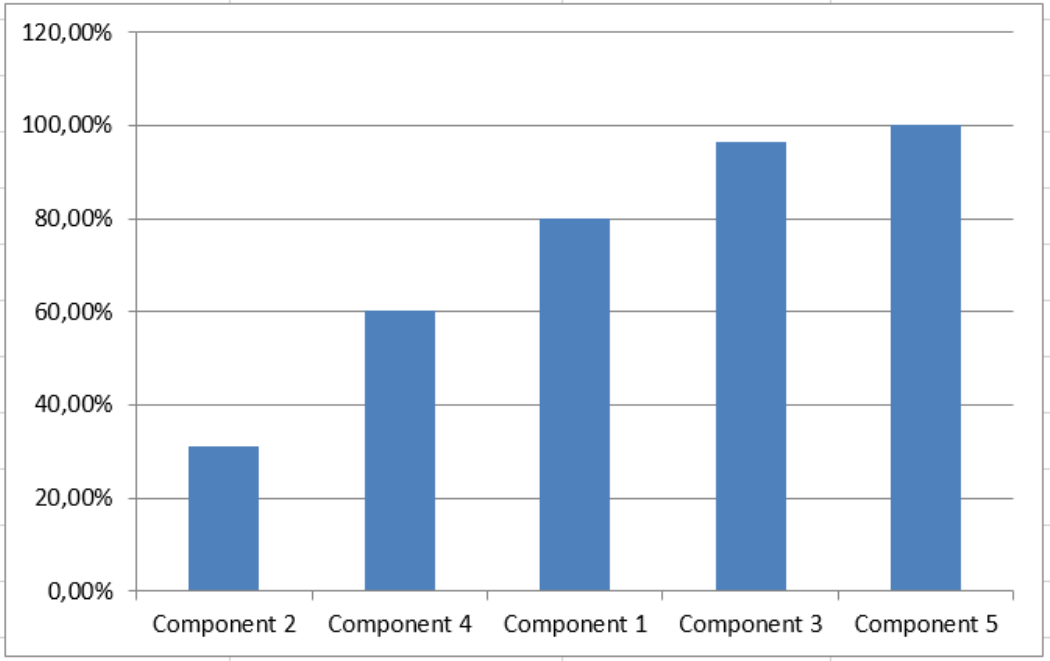
6. To improve the reliability of S_1, a redundancy of the three most impacting components (according to the limited weighted Pareto approach) is proposed to define the system S_2.

   Give the literal expression of the reliability of S_2.

   Knowing the costs of each redundant component (respectively 50, 70 and 110, in order of Pareto severity), define the optimal structure, guaranteeing a reliability of 0.5 after 5 years for the system S_2.

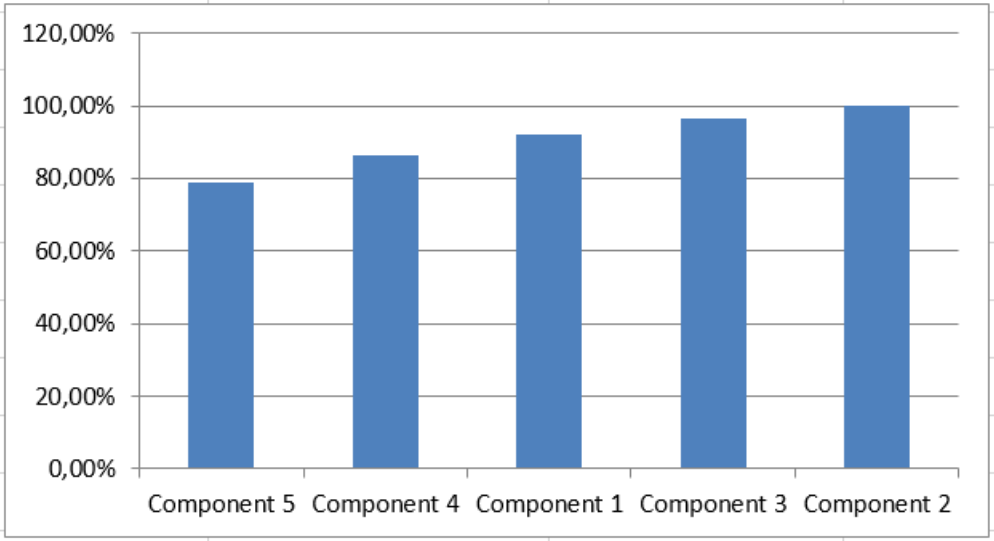
Pareto Diagram for failures

Equipement	Failures	Frequency	Cumulative frequency
Component 2	984	30,93%	30,93%
Component 4	929	29,20%	60,14%
Component 1	632	19,87%	80,01%
Component 3	520	16,35%	96,35%
Component 5	116	3,65%	100,00%
	3181		



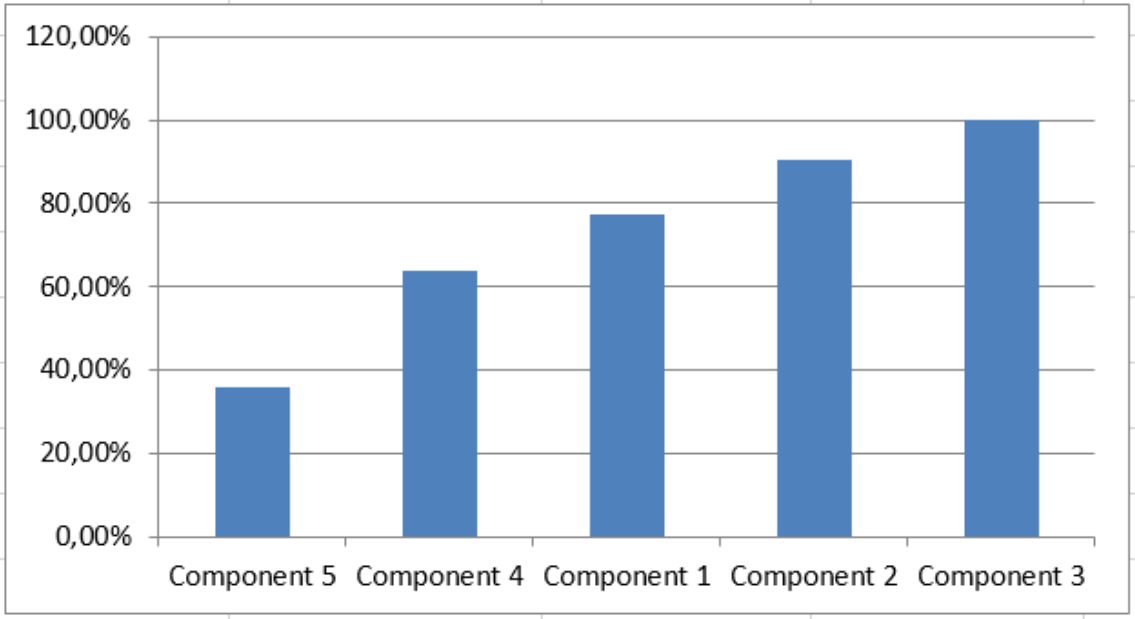
Pareto diagram for Mean Down Time

Equipement	Down time	Failures	Mean Down Time	Frequency	Cumulative frequency
Component 5	58000	116	500,00	78,86%	78,86%
Component 4	44592	929	48,00	7,57%	86,44%
Component 1	22120	632	35,00	5,52%	91,96%
Component 3	15600	520	30,00	4,73%	96,69%
Component 2	20664	984	21,00	3,31%	100,00%
			634,00		



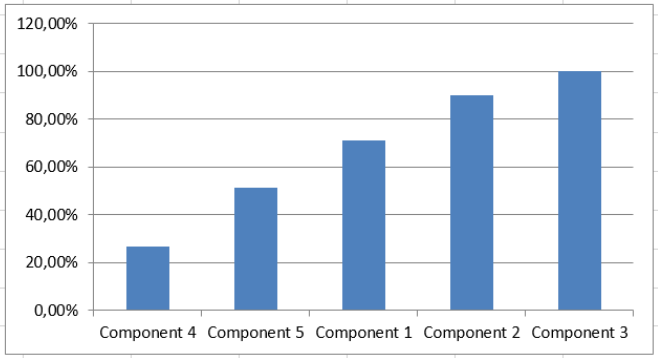
Pareto diagram for Unavailability

Equipement	Down time	Frequency	Cumulative frequency
Component 5	58000	36,03%	36,03%
Component 4	44592	27,70%	63,73%
Component 1	22120	13,74%	77,47%
Component 2	20664	12,84%	90,31%
Component 3	15600	9,69%	100,00%
	160976		



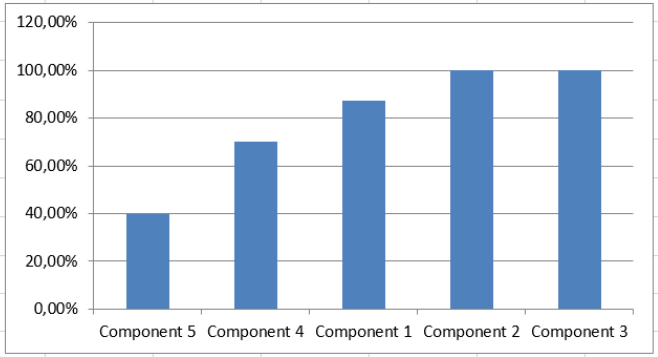
Weighted Pareto diagram

Score weights	2	1	3			
Criteria	**Failures**	**MDT**	**Unavailability**	**Score**	Frequency	Cum. Freq.
Component 4	4	4	4	**24**	26,67%	26,67%
Component 5	1	5	5	**22**	24,44%	51,11%
Component 1	3	3	3	**18**	20,00%	71,11%
Component 2	5	1	2	**17**	18,89%	90,00%
Component 3	2	2	1	**9**	10,00%	100,00%
		Score		**90**		



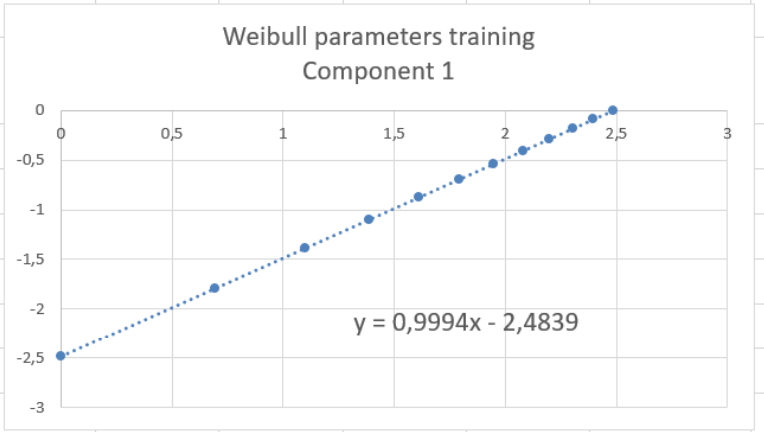
Limited weighted Pareto diagram

weighted points	20	10	30			
Criteria	**Failures**	**MDT**	**Unavailability**	**Score**	Frequency	Cum. Freq.
Component 5	0,00	**10,00**	**13,95**	**23,95**	39,92%	39,92%
Component 4	**7,30**	0,00	**10,73**	**18,03**	30,05%	69,97%
Component 1	**4,97**	0,00	**5,32**	**10,29**	17,15%	87,11%
Component 2	**7,73**	0,00	0,00	**7,73**	12,89%	100,00%
Component 3	0,00	0,00	0,00	**0,00**	0,00%	100,00%
		Score		**60**		
	20	10	30			



$N_{components}$ 1000

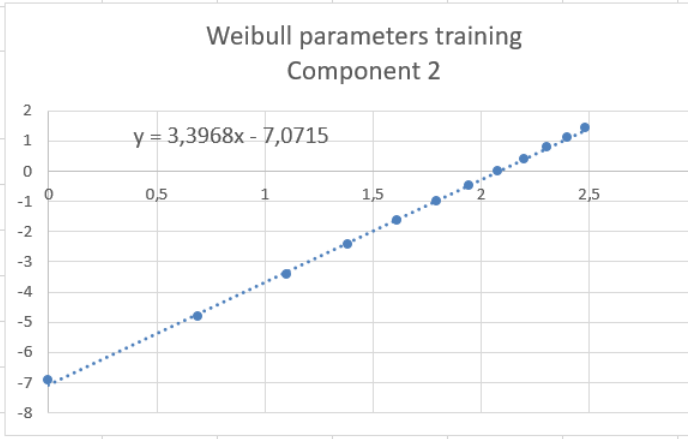
		Component 1					
t (years)	ln(t)	N_1	F_{1W}	Y_{1W}	F_{1REX}	Y_{1REX}	$\lvert Y_{1W}/Y_{1REX}\rvert$
1	0	80	0,080033	-2,4839	0,08	-2,48433	0,999827917
2	0,69314718	74	0,153602	-1,79117	0,154	-1,78835	1,001576277
3	1,09861229	67	0,221267	-1,38595	0,221	-1,38732	0,999011709
4	1,38629436	62	0,28351	-1,09844	0,283	-1,10058	0,99805694
5	1,60943791	58	0,340771	-0,87543	0,341	-0,87459	1,00095452
6	1,79175947	52	0,393449	-0,69322	0,393	-0,6947	0,997869835
7	1,94591015	49	0,441912	-0,53916	0,442	-0,53889	1,000498919
8	2,07944154	45	0,4865	-0,40571	0,487	-0,40425	1,003610229
9	2,19722458	41	0,527523	-0,28799	0,528	-0,28665	1,004696386
10	2,30258509	37	0,565266	-0,1827	0,565	-0,18343	0,995995136
11	2,39789527	35	0,599992	-0,08744	0,6	-0,08742	1,000250425
12	2,48490665	32	0,631942	-0,0005	0,632	-0,0003	1,477800334



β=	0,9994		T= 1,00021348
α=	12,00582		St. dev.= 0,00187379
A=	1	⇒	MTBF= 12,0058164

$N_{components}$ 1000

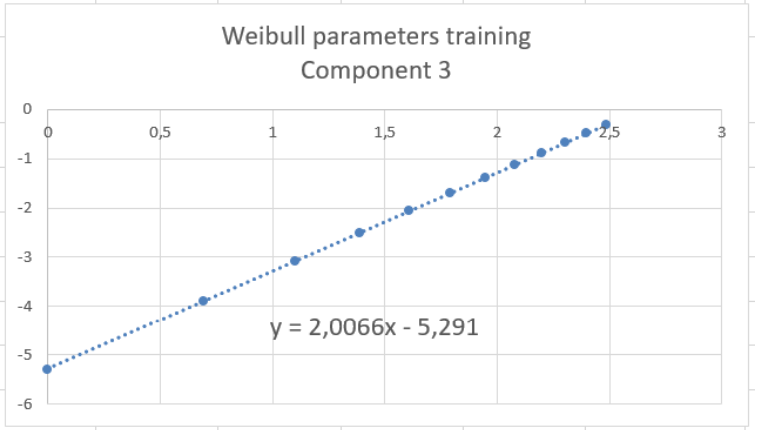
		Component 2					
t (years)	ln(t)	N_2	F_{2W}	Y_{2W}	F_{2REX}	Y_{2REX}	$\lvert Y_{2W}/Y_{2REX}\rvert$
1	0	1	0,000849	-7,0715	0,001	-6,90726	1,023778611
2	0,69314718	7	0,008902	-4,71702	0,008	-4,8243	0,977762022
3	1,09861229	24	0,034826	-3,33973	0,032	-3,4258	0,974876517
4	1,38629436	53	0,089882	-2,36254	0,085	-2,42102	0,975844091
5	1,60943791	91	0,182069	-1,60456	0,176	-1,64204	0,977175595
6	1,79175947	130	0,311573	-0,98525	0,306	-1,00708	0,978322941
7	1,94591015	160	0,467543	-0,46163	0,466	-0,46624	0,990126822
8	2,07944154	166	0,629158	-0,0081	0,632	-0,0003	24,57325221
9	2,19722458	147	0,772359	0,392032	0,779	0,41184	0,951905197
10	2,30258509	108	0,879588	0,749921	0,887	0,779493	0,962062055
11	2,39789527	66	0,946394	1,073671	0,953	1,117633	0,96066495
12	2,48490665	31	0,980402	1,369231	0,984	1,419528	0,964568002



β=	3,3968		T= 0,9761
α=	8,01899		St. Dev.= 0,01212
A=	0,8984	⇒	MTBF= 7,20426

$N_{components}$ 1000

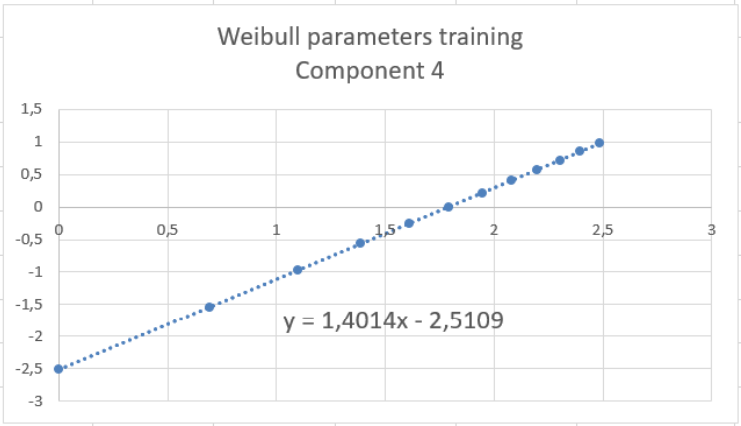
		Component 3					
t (years)	ln(t)	N_3	F_{3W}	Y_{3W}	F_{3REX}	Y_{3REX}	$\lvert Y_{3W}/Y_{3REX}\rvert$
1	0	5	0,005024	-5,291	0,005	-5,29581	0,999091331
2	0,69314718	15	0,020036	-3,90013	0,02	-3,90194	0,999536694
3	1,09861229	25	0,044634	-3,08652	0,045	-3,07816	1,002717672
4	1,38629436	33	0,078109	-2,50926	0,078	-2,51072	0,999420695
5	1,60943791	42	0,119498	-2,0615	0,12	-2,05703	1,002175098
6	1,79175947	48	0,167631	-1,69566	0,168	-1,69324	1,001427136
7	1,94591015	53	0,221191	-1,38634	0,221	-1,38732	0,999292695
8	2,07944154	58	0,278775	-1,11839	0,279	-1,11744	1,000852488
9	2,19722458	60	0,338955	-0,88205	0,339	-0,88189	1,00018521
10	2,30258509	61	0,400335	-0,67063	0,4	-0,67173	0,998371005
11	2,39789527	61	0,461606	-0,47938	0,461	-0,4812	0,996219419
12	2,48490665	59	0,521587	-0,30479	0,52	-0,30929	0,985444224



β=	2,0066		T= 0,99872781
α=	13,96841		St. Dev.= 0,00269146
A=	0,8862	⇒	MTBF= 12,3788076

$N_{components}$ 1000

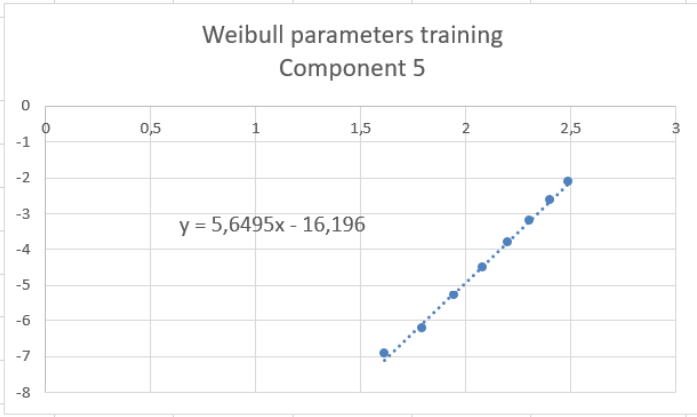
		Component 4					
t (years)	ln(t)	N_4	F_{4W}	Y_{4W}	F_{4REX}	Y_{4REX}	$\lvert Y_{4W}/Y_{4REX}\rvert$
1	0	78	0,077986	-2,5109	0,078	-2,51072	1,000073205
2	0,69314718	115	0,193042	-1,53952	0,193	-1,53976	0,99984356
3	1,09861229	122	0,315173	-0,9713	0,315	-0,97197	0,999314089
4	1,38629436	118	0,432534	-0,56815	0,433	-0,5667	1,002557328
5	1,60943791	106	0,539103	-0,25543	0,539	-0,25572	0,998872771
6	1,79175947	93	0,632147	7,2E-05	0,632	-0,0003	0,218850649
7	1,94591015	79	0,71097	0,216098	0,711	0,216182	0,999612515
8	2,07944154	65	0,776121	0,403229	0,776	0,402868	1,000897284
9	2,19722458	53	0,828855	0,568291	0,829	0,568769	0,999158681
10	2,30258509	42	0,870764	0,715943	0,871	0,716836	0,998754158
11	2,39789527	32	0,903528	0,84951	0,903	0,847174	1,002757939
12	2,48490665	26	0,928765	0,971448	0,929	0,9727	0,998713484



β=	1,4014		T= 1,00005046
α=	5,999693		St. Dev.= 0,00110617
A=	0,9114	⇒	MTBF= 5,46812015

$N_{components}$ 1000

		Component 5					
t (years)	ln(t)	N_5	F_{5W}	Y_{5W}	F_{5REX}	Y_{5REX}	$\lvert Y_{5W}/Y_{5REX}\rvert$
1	0	0	9,25E-08	-16,196			
2	0,69314718	0	4,64E-06	-12,2801			
3	1,09861229	0	4,59E-05	-9,98939			
4	1,38629436	0	0,000233	-8,36413			
5	1,60943791	1	0,000822	-7,10348	0,001	-6,90726	1,0284086
6	1,79175947	1	0,002301	-6,07345	0,002	-6,21361	0,97744428
7	1,94591015	3	0,005487	-5,20258	0,005	-5,29581	0,982395235
8	2,07944154	6	0,011631	-4,4482	0,011	-4,50433	0,987536534
9	2,19722458	11	0,022502	-3,78278	0,022	-3,80561	0,994000729
10	2,30258509	19	0,040433	-3,18755	0,041	-3,17332	1,004481541
11	2,39789527	30	0,068273	-2,64909	0,071	-2,60848	1,01556944
12	2,48490665	45	0,109179	-2,15752	0,116	-2,09315	1,030752964

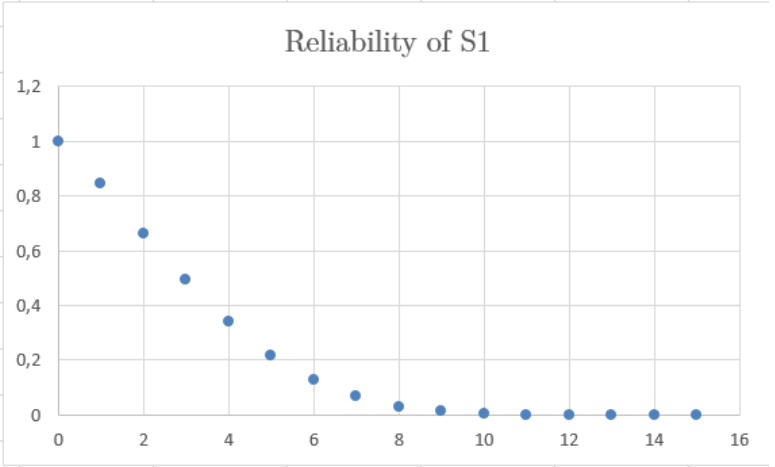


β=	5,6495		T= 1,00257367
α=	17,58071		St. Dev.= 0,01722947
A=	0,9241	⇒	MTBF= 16,2463356

	C_1	C_2	C_3	C_4	C_5
α	12,00582	8,018989	13,96841	5,999693	17,58071
β	0,9994	3,3968	2,0066	1,4014	5,6495

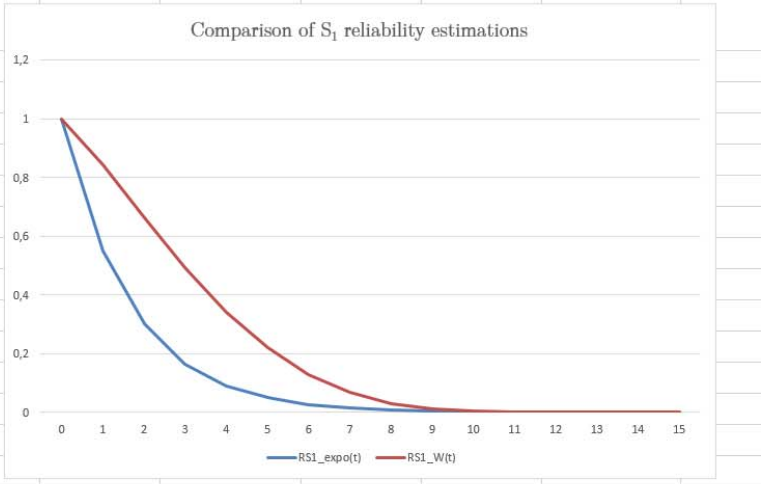
$$R_{S_1}(t) = \prod_{i=1}^{5} R_{C_i}(t) = \prod_{i=1}^{5} e^{-\left(\frac{t}{\alpha_i}\right)^{\beta_i}}$$

t	R_{c1}	R_{c2}	R_{c3}	R_{c4}	R_{c5}	R_{S1}
0	1	1	1	1	1	1
1	0,919967	0,999151	0,994976	0,922014	1	0,843245
2	0,846398	0,991098	0,979964	0,806958	0,999995	0,663362
3	0,778733	0,965174	0,955366	0,684827	0,999954	0,491729
4	0,71649	0,910118	0,921891	0,567466	0,999767	0,341056
5	0,659229	0,817931	0,880502	0,460897	0,999178	0,21864
6	0,606551	0,688427	0,832369	0,367853	0,997699	0,12756
7	0,558088	0,532457	0,778809	0,28903	0,994513	0,066523
8	0,5135	0,370842	0,721225	0,223879	0,988369	0,03039
9	0,472477	0,227641	0,661045	0,171145	0,977498	0,011894
10	0,434734	0,120412	0,599665	0,129236	0,959567	0,003893
11	0,400008	0,053606	0,538394	0,096472	0,931727	0,001038
12	0,368058	0,019598	0,478413	0,071235	0,890821	0,000219
13	0,338661	0,005736	0,420741	0,05206	0,833839	3,55E-05
14	0,311613	0,001309	0,366212	0,037674	0,758664	4,27E-06
15	0,286726	0,000227	0,315468	0,027008	0,665083	3,69E-07



	Component 1			Component 2			Component 3			Component 4			Component 5		
t (years)	N_1	T_1	UT_1 (years)	N_2	T_2	UT_2 (years)	N_3	T_3	UT_3 (years)	N_4	T_4	UT_4 (years)	N_5	T_5	UT_5 (years)
1	80	2800	78,33333333	1	21	0,9875	5	150	4,9107	78	3744	75,77142857	0	0	
2	74	2590	146,4583333	7	147	13,9125	15	450	29,7321	115	5520	226,7142857	0	0	
3	67	2345	199,6041667	24	504	71,7	25	750	74,5536	122	5856	362,5142857	0	0	
4	62	2170	246,7083333	53	1113	211,3375	33	990	131,4107	118	5664	468,6285714	0	0	
5	58	2030	288,7916667	91	1911	453,8625	42	1260	209,2500	106	5088	526,9714286	1	500	4,702380952
6	52	1820	310,9166667	130	2730	778,375	48	1440	287,1429	93	4464	555,3428571	1	500	5,702380952
7	49	1715	341,9791667	160	3360	1118	53	1590	370,0536	79	3792	550,7428571	3	1500	20,10714286
8	45	1575	359,0625	166	3486	1325,925	58	1740	462,9643	65	3120	518,1428571	6	3000	46,21428571
9	41	1435	368,1458333	147	3087	1321,1625	60	1800	538,9286	53	2544	475,4857143	11	5500	95,72619048
10	37	1295	369,2291667	108	2268	1078,65	61	1830	608,9107	42	2016	418,8	19	9500	184,3452381
11	35	1225	384,2708333	66	1386	725,175	61	1830	669,9107	32	1536	351,0857143	30	15000	321,0714286
12	32	1120	383,3333333	31	651	371,6125	59	1770	706,9464	26	1248	311,2571429	45	22500	526,6071429
	632		**3476,83333**	**984**		**7470,7**	**520**		**4094,7143**	**929**		**4841,45714**	**116**		**1204,47619**
	$MUT_{/632syst.}$=	5,50131857	years	$MUT_{/632syst.}$=	7,5921748	years	$MUT_{/632syst.}$=	7,87445055	years	$MUT_{/632syst.}$=	5,21147163	years	$MUT_{/632syst.}$=	10,3834154	years
	$MUT_{/1000syst.}$=	8,26083333	years	$MUT_{/1000syst.}$=	7,6787	years	$MUT_{/1000syst.}$=	10,3347143	years	$MUT_{/1000syst.}$=	5,76445714	years	$MUT_{/1000syst.}$=	12,6964762	years
	λ_{year}=	0,12105316		λ_{year}=	0,13023038		λ_{year}=	0,09676126		λ_{year}=	0,17347687		λ_{year}=	0,07876201	

t (years)	$R_{C1}(t)$	$R_{C2}(t)$	$R_{C3}(t)$	$R_{C4}(t)$	$R_{C5}(t)$	$R_{S1\_expo}(t)$	$R_{S1\_W}(t)$
0	1	1	1	1	1	1	1
1	0,885986857	0,877893161	0,9077727	0,840736595	0,924259862	0,54865597	0,843244636
2	0,784972711	0,770696402	0,824051275	0,706838022	0,854256292	0,30102337	0,663362197
3	0,695475505	0,6765891	0,74805125	0,594264591	0,789554802	0,16515827	0,491728626
4	0,616182157	0,593972944	0,679060503	0,499619089	0,729753813	0,09061507	0,341056068
5	0,545929292	0,521444785	0,616432586	0,420048808	0,674482158	0,0497165	0,21864041
6	0,48368617	0,45777281	0,559580673	0,353150404	0,623396786	0,02727725	0,127560439
7	0,428539597	0,401875619	0,507972058	0,296906468	0,576180627	0,01496583	0,066522846
8	0,37968045	0,352803858	0,461123167	0,249620133	0,532540627	0,00821109	0,030390055
9	0,336391889	0,309724094	0,418595022	0,209864781	0,492205926	0,00450506	0,011894348
10	0,298038792	0,271904664	0,379989133	0,176441001	0,454926181	0,00247173	0,0038928
11	0,264058453	0,238703245	0,344943761	0,148340406	0,420470009	0,00135613	0,001037695
12	0,233952319	0,209555946	0,31313053	0,124715208	0,388623553	0,00074405	0,000218986
13	0,20727868	0,183967732	0,284251346	0,104852639	0,359189151	0,00040823	3,54809E-05
14	0,183646186	0,161504013	0,258035612	0,088153451	0,331984115	0,00022398	4,27048E-06
15	0,162708107	0,141783269	0,234237684	0,074113832	0,306839592	0,00012289	3,68513E-07



S_2	C_1	C_4	C_5
	n	m	p
	2	3	1
	110	50	70

Cost= 440

R(5)= 0,5364

	R(t)						
t	C_1	C_2	C_3	C_4	C_5	S_1	S_2
0	1,0000	1,0000	1,0000	1,0000	1,0000	**1,0000**	1,0000
1	0,9200	0,9992	0,9950	0,9220	1,0000	**0,8432**	0,9873
2	0,8464	0,9911	0,9800	0,8070	1,0000	**0,6634**	0,9415
3	0,7787	0,9652	0,9554	0,6848	1,0000	**0,4917**	0,8495
4	0,7165	0,9101	0,9219	0,5675	0,9998	**0,3411**	0,7090
5	0,6592	0,8179	0,8805	0,4609	0,9992	**0,2186**	0,5364
6	0,6066	0,6884	0,8324	0,3679	0,9977	**0,1276**	0,3611
7	0,5581	0,5325	0,7788	0,2890	0,9945	**0,0665**	0,2126
8	0,5135	0,3708	0,7212	0,2239	0,9884	**0,0304**	0,1074
9	0,4725	0,2276	0,6610	0,1711	0,9775	**0,0119**	0,0457
10	0,4347	0,1204	0,5997	0,1292	0,9596	**0,0039**	0,0160
11	0,4000	0,0536	0,5384	0,0965	0,9317	**0,0010**	0,0045
12	0,3681	0,0196	0,4784	0,0712	0,8908	**0,0002**	0,0010

$$R(t) = \left(1 - \left(1 - R_{C_1}(t)\right)^n\right) * R_{C_2}(t) * R_{C_3}(t) * \left(1 - \left(1 - R_{C_4}(t)\right)^m\right) * \left(1 - \left(1 - R_{C_5}(t)\right)^p\right)$$