# Lossy compression for lossless prediction

## EECS Seminar: Advanced Topics in Machine Learning

Romain Graux

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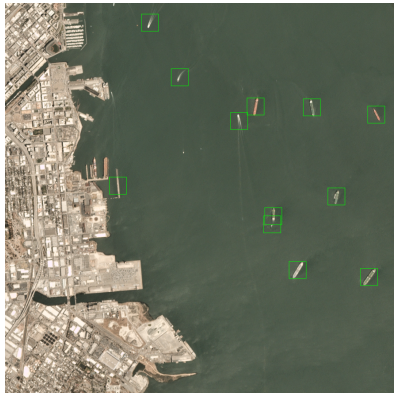

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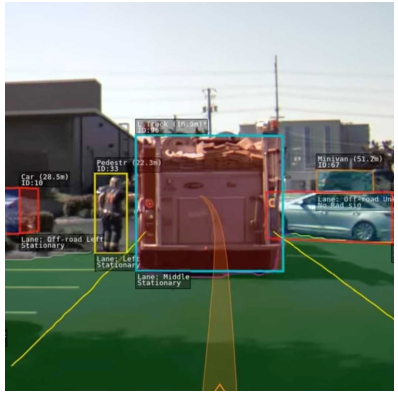

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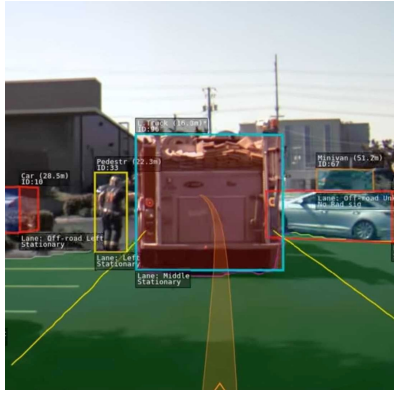

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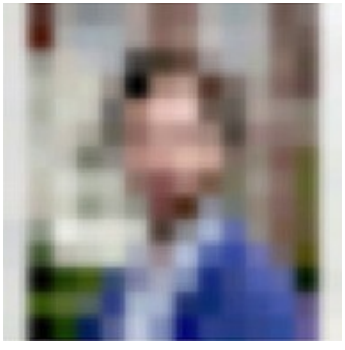

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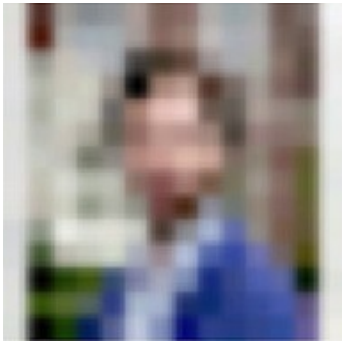

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Desired

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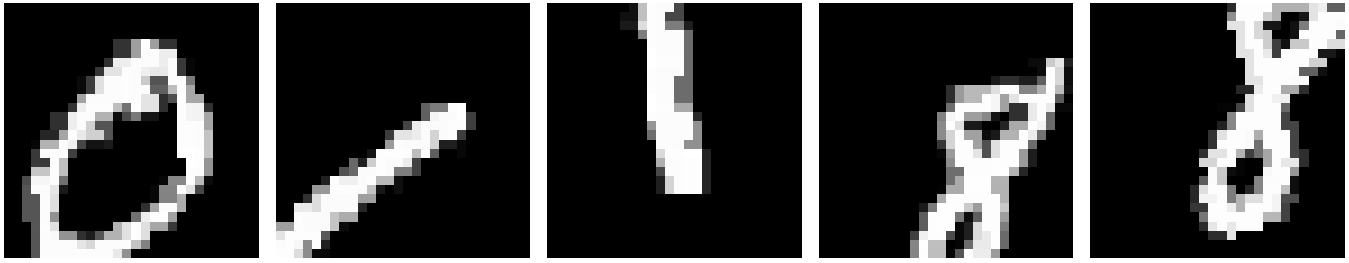
- Characterize minimum bit-rate to ensure high performance on desired tasks;
- Derive unsupervised objectives for training **task-centric** compressors;
- $> 1000x$ compression gains on Imagenet compared to JPEG (see Slide 9).

# Intuition: Augmented MNIST



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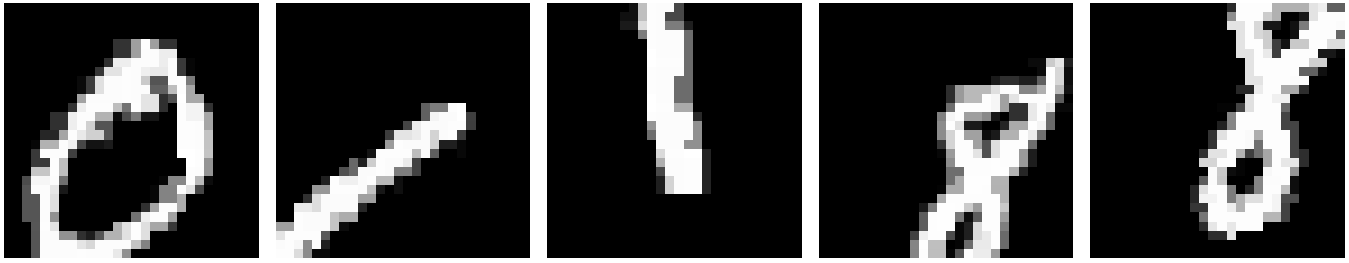

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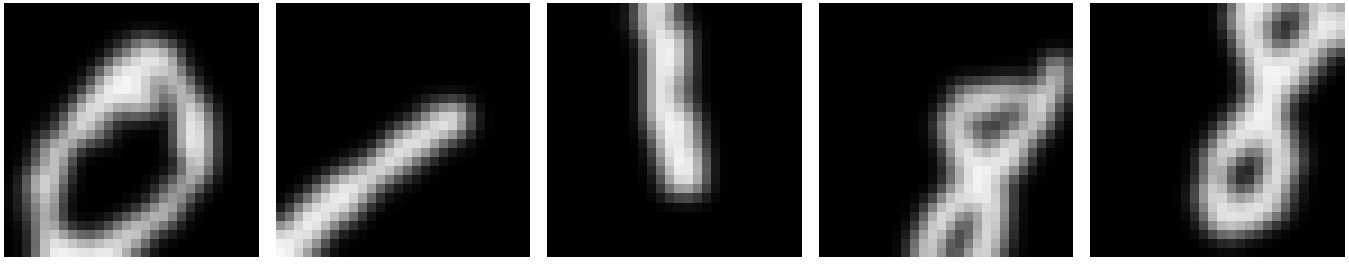

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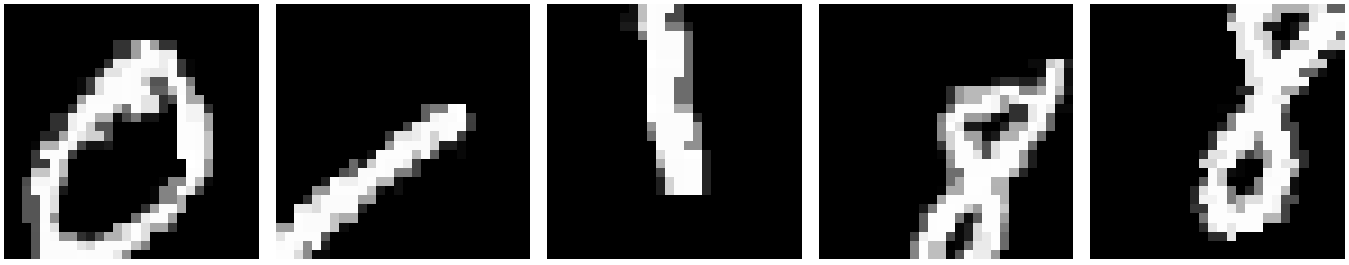

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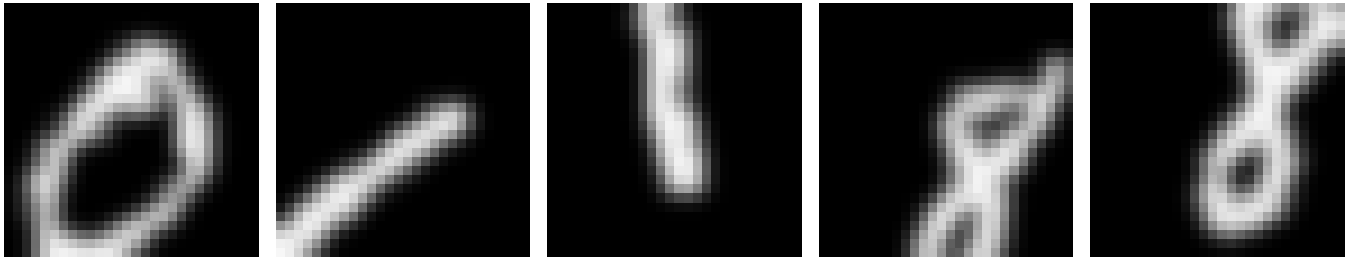

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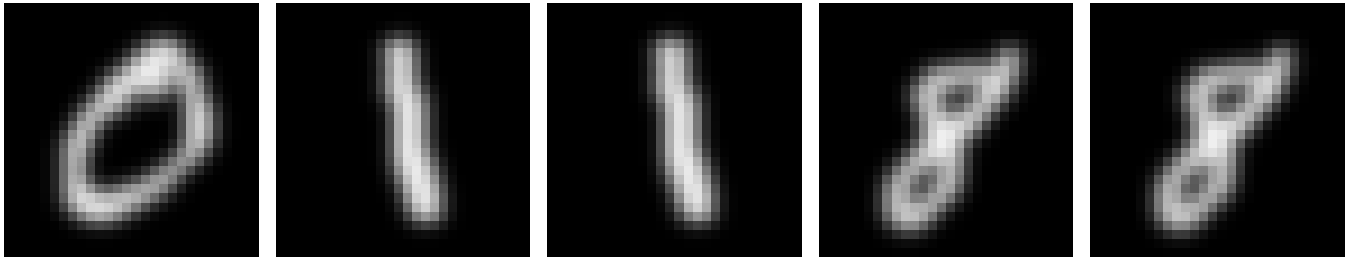

Their neural compressor: 48 bit-rate

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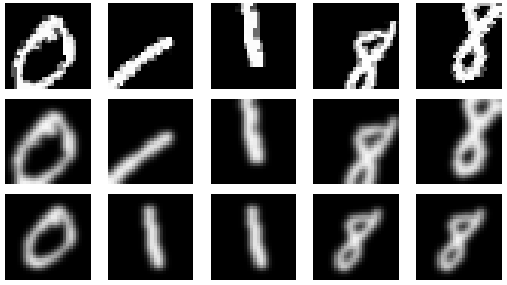

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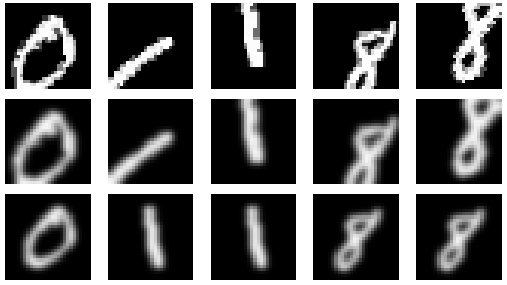

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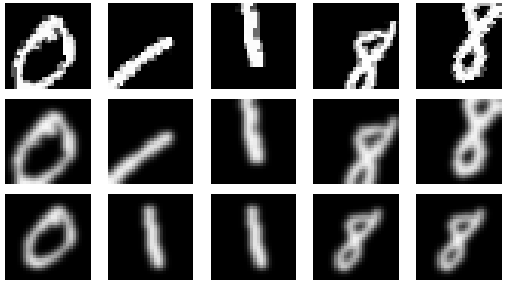

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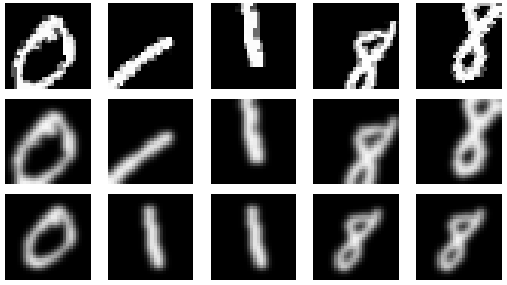

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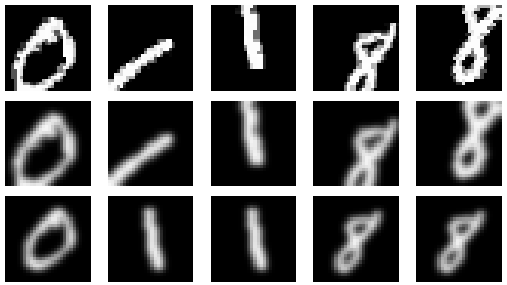

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⇒ The objective is **unsupervised**

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**Problem:** Would assume access $\mathcal{T}$

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Exists a "worst task" $M(X)$ that has all information to predict any invariant task $Y \in \mathcal{T}$

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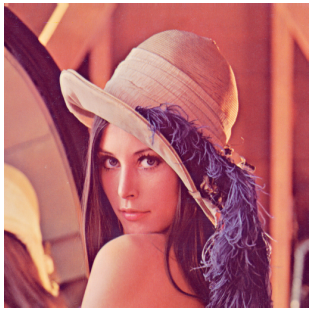

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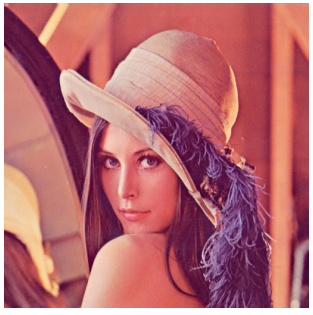

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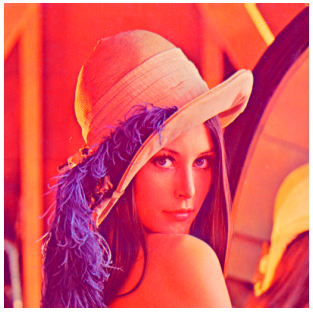

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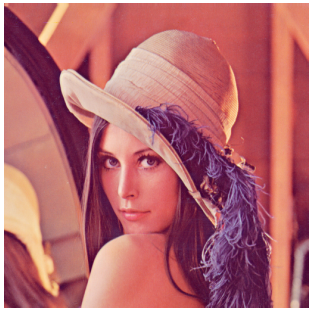

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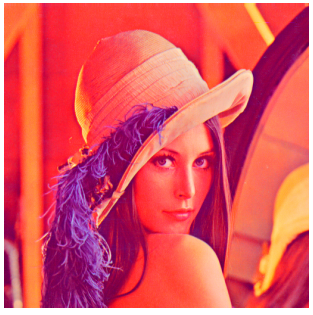

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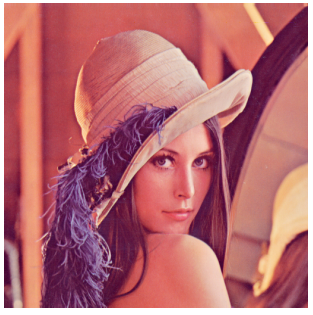

$M(x)$: unaugmented

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$\Rightarrow$ a $\delta$ decrease in *log-loss* save exactly $\delta$ bits

# Performance