Lossy compression for lossless prediction

EECS Seminar: Advanced Topics in Machine Learning

Romain Graux

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Motivation

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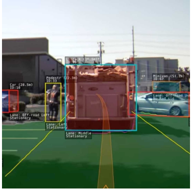

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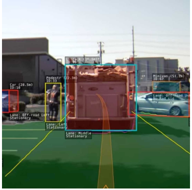

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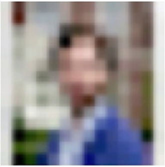

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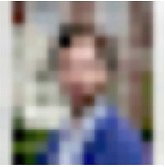

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Desired

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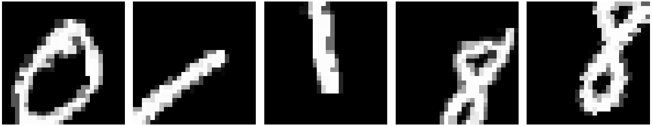
- Characterize minimum bit-rate to ensure high performance on desired tasks;
- Derive unsupervised objectives for training **task-centric** compressors;
- $> 1000x$ compression gains on Imagenet compared to JPEG (see Slide 12).

Intuition

Intuition: Augmented MNIST



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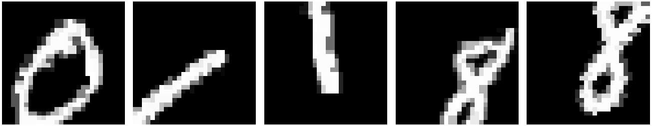

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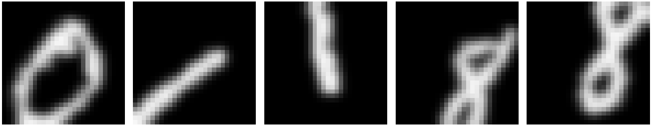

Standard neural compressor: 130 bit-rate

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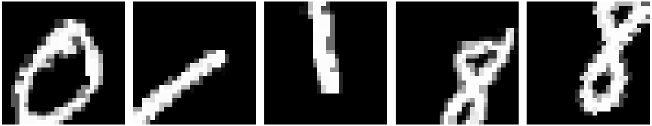

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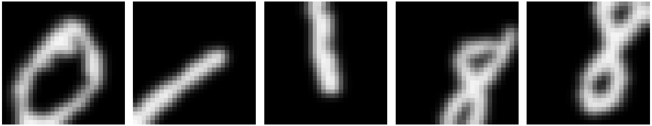

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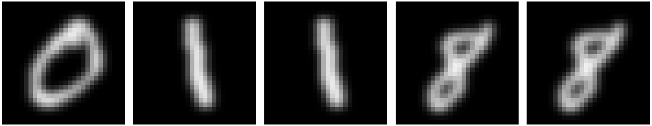

Their neural compressor: 48 bit-rate

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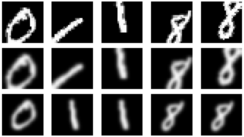

Prototypical digit ensures → high downstream performance
→ good compression rate

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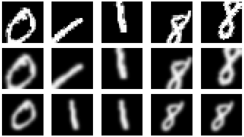

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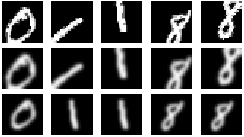

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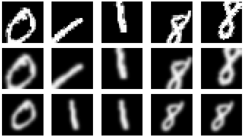

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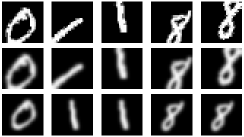

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$\Rightarrow$ The objective is **unsupervised**

Formalism

Goal:

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  - e.g. X: all images of machine learning researchers

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    - e.g. Y_1: how old is the person?
    - e.g. Y_2: does the person wear glasses?

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$$\sup_{Y \in \mathcal{T}} R[Y|Z] - R[Y|X] \underbrace{\leq \delta}_{\text{small}}$$

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Problem: Would assume access $\mathcal{T}$

Key assumption: Invariance structure

Tasks of interest to humans share structure.

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Proposition

Exists a "worst task" $M(X)$ that has all information to predict any invariant task $Y \in \mathcal{T}$

$$R[M(X)|Z] = \sup_{Y \in \mathcal{T}} R[Y|Z] - R[Y|X] \leq \delta$$

What we want:

$$x \sim x^+ \iff M(x) = M(x^+) \text{ for any } x, x^+ \in X$$

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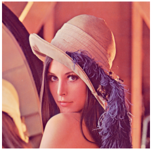

x_2: horizontal flip

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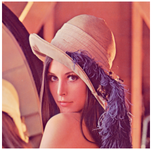

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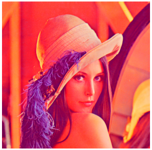

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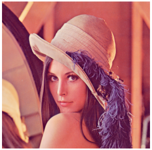

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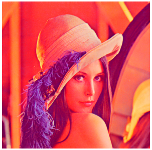

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$M(x)$: unaugmented

Theorem: Rate-Invariance

Using the rate-distortion therorem [Shannon, 1959] with $R[M(X)|Z]$ as distortion:

Theorem (Rate-Invariance)

The minimum achievable bit-rate for transmitting Z s.t. for any invariant $Y \in \mathcal{T}$ we have an excess Bayes risk $R[Y|Z] - R[Y|X]$ upper bounded by δ is

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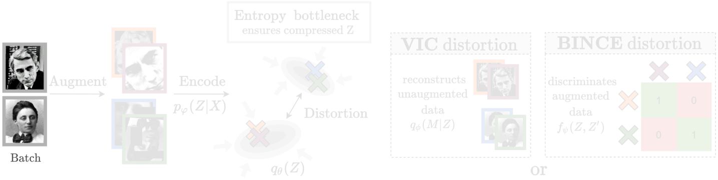
$\Rightarrow$ a δ decrease in *log-loss* save exactly δ bits

In practice

Training architecture



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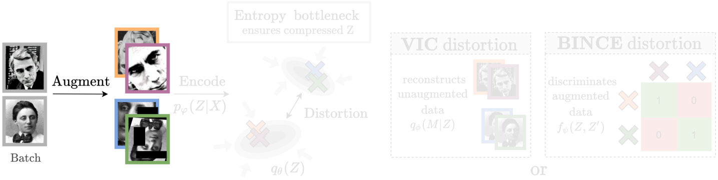

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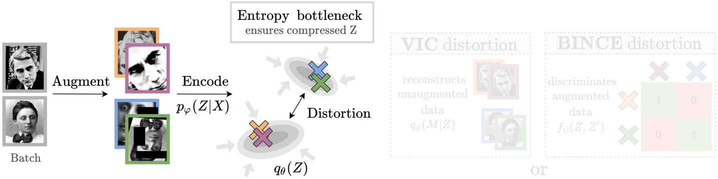

Entropy bottleneck: compressed Z

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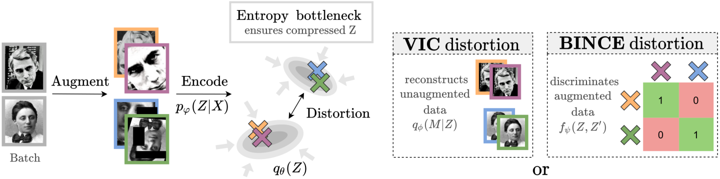

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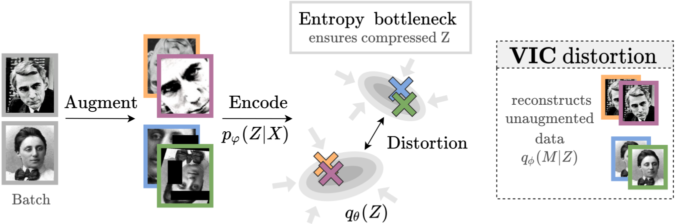
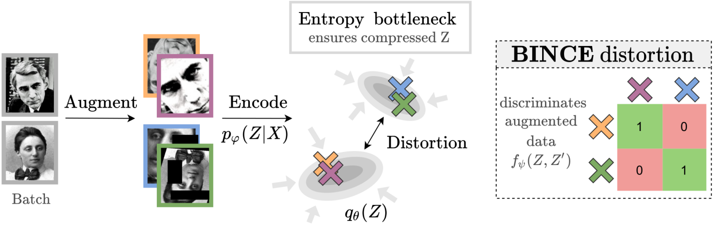
Variational Invariant Compressor (VIC)



Bottleneck InfoNCE (BINCE)



Performance