Foreign Exchange Intervention Rule for Central Banks: A Risk-Based Framework

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Abstract

This paper presents a rule for foreign exchange interventions (FXI), designed to preserve financial stability in floating exchange rate arrangements. The FXI rule addresses a market failure: the absence of hedging solution for tail exchange rate risk (i.e. high volatility) in the market. Market impairment or overshoot of exchange rate between two equilibria could generate high volatility and threaten financial stability due to unhedged exposures to exchange rate risk in the economy. The rule uses the concept of Value at Risk (VaR) to define FXI triggers. While it provides to the market a hedge against tail risk, the rule allows the exchange rate to smoothly adjust to new equilibria. In addition, the rule is budget neutral over the medium term, encourages a prudent risk management in the market, and is more resilient to speculative attacks than other rules, such as fixed-volatility rules. The empirical methodology is backtested on Banco Mexico's FXIs data between 2008 and 2016.

Keywords: Foreign Exchange Interventions, Value at Risk, GARCH

JEL classification: E58, F31, G17

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1 Introduction

The 2019 IMF Annual Report on Exchange Arrangement and Exchange Restrictions[1] classified 66 exchange arrangements as either floating or "free" floating. In these arrangements, the supply and the demand in the foreign exchange (FX) market, absent official sector FX intervention (FXI), determines the exchange rate. The exchange rate risk could, then, be entirely managed by the private sector, often through the use of derivative instruments. The central bank transacts FX only to manage its FX reserves or in its role as fiscal agent for the government. However, a market failure constraining the private sector's ability to manage FX risks may significantly increase financial stability risks. Therefore, no central banks, in either floating or free-floating arrangements, rule out interventions on the FX market. Even central banks with a strong commitment to floating did intervene in the FX market in cases of exceptional stress, such as the COVID-19 pandemic.[2]

We conceptualize central banks' FXIs in floating exchange rate as a public good, specifically an insurance, provided by the public sector against exchange rate tail risks. The private sector might not be able to completely hedge FX risk either because markets participants have not internalized such risks, or because hedging markets are incomplete in a given jurisdiction, or because hedging costs cannot be optimally shared among participants. Under such circumstances, public intervention could be warranted to preserve financial stability. Our approach aligns with the literature supporting the idea that public intervention is desirable when a market failure prevents the market to supply optimally a desirable public good (Stiglitz (1993)).

Economies are more vulnerable to large variations in the exchange rate when unhedged exposures are larger. Exposures can be direct, such as a firm or a household borrows in FX while its income is denominated in local currency, or indirect, such as the exchange rate pass through on domestic prices, which ultimately affects the welfare of economic agents. FX market microstruc-

[1] Available at https://www.elibrary-areaer.imf.org/Pages/Home.aspx

[2] See, for example, https://www.fx-markets.com/foreign-exchange/4624581/chile-launches-biggest-fx-intervention-in-20-years, https://www.wsj.com/articles/norwegian-krone-soars-amid-signs-of-central-bank-intervention-11585068173

ture–in particular depth and liquidity–plays a role as liquid markets absorb shocks smoothly, reducing the likelihood of sudden volatility pikes. The macro-financial risk materializes when a large swing in exchange rate impacts vulnerable agents, leading to balance sheets impairments, which could have systemic implications. In other words, vulnerable economies–with pervasive unhedged exposures (e.g. dollarized small open economies) and shallow FX markets–would require more public insurance coverage against FX volatility compared to more robust economies with less significant exposures and resilient FX markets.

The rule proposed in this paper does not indicate how much of the risk should be covered by the central bank. Instead, this paper designs an operational framework that ensures that the central bank can easily provide an FX hedge to the market, in a consistent and transparent way. Public insurance provision entails pitfalls, such as the cost of the insurance and the proper alignment of incentives to avoid moral hazard reflected in excessive risk taking by private agents (McKinnon (2000)). Central banks habitually keep a fair degree of opacity about their intervention triggers, which makes it difficult to determine how much exchange rate risk they consider the market could manage on its own. Often, central banks do not formally assess exposures to unhedged exchange rate risk in the economy and how much risk their FXI are removing from the market. Although not formalized by central banks, FXI risk mitigation could be inferred ex-post from actual interventions if granular data on interventions are available. Other central banks, such as those in Colombia, Guatemala, and Mexico, use transparent intervention rules (Chamon et al. (2019)) that partially reveal how much of the risk they intend to cover themselves.

In this paper, an intervention is deemed rule-based when it reacts to predetermined parameters to deliver predictable responses. The most frequently used FXI rules are based on fixed-volatility triggers such as day-to-day exchange rate change, for example 2 percent depreciation from the previous day exchange rate close. The FXI rule can be disclosed or kept secret by the central bank, although, in the latter case, it may become transparent with experience. Patel & Cavallino (2019) surveyed 21 emerging market central banks and six out the 21 regularly use an intervention rule, while four do so occasionally.

Foreign exchange intervention rules are used to anchor agents' expectation and influence their behaviors such as to support the central bank objectives (Krugman (1991), Montoro & Ortiz (2013), and Fanelli & Straub (2020)). The shortcoming of many FXI rules is that they might not be flexible enough to accommodate a wide range of situations, and may have to be abandoned, challenging ex ante the authority's commitment to the rule (Kydland & Prescott (1977)). In this paper, we consider the rule as an input in the FXI decision without discussing what weight should be given to the rule compared with policy marker judgement. The objective of paper is more to help central bank in developing tools for FXI than to contribute to the rule versus discretion literature (Barro & Gordon (1983) and Taylor (2017)).

The recent conceptual framework developed by the IMF, the Integrated Policy Framework (IPF), provides new theoretical foundations to FXIs. The IPF integrates monetary policy, exchange rate policy, capital flows management, and financial stability into a unified approach (Basu et al. (2020)).

In the IPF, deviations from the uncovered interest rate parity (UIP) motivate FXIs in the spot market. Deviations from the uncovered interest rate parity (the UIP risk premium) signal market dysfunctions (International Monetary Fund (2020b)), which arise from imperfect arbitrage by financial intermediaries in the FX market. UIP-deviations based FXI would address FX market dysfunctions which originate from impaired arbitrage, and not from a structural adjustment of the real exchange rate, which should take place to allow the economy to absorb shocks. FXIs are expected to reduce the UIP risk premium, thereby improving economic welfare through a re-alignment of the real exchange with its equilibrium value. Therefore, the IPF presents UIP-based FXI as a useful complement to monetary policy, which could also have positive implications for financial stability.

In real time, however, it might be operationally challenging to distinguish between structural shocks and UIP deviations. The estimation of UIP deviations relies on the expected future spot rate, which might not be available or accurately measured when the FX shock hits the market. The UIP-based FXI, while optimal theoretically, cannot always be implemented in practice, particularly in markets with limited depth and liquidity, and scarce information. Still, these markets are precisely those where FXIs are the most needed. Therefore, a practical implementable approach is necessary.

This paper proposes an operational framework based on a VaR metric to determine the triggers of an FXI rule (the VaR rule). Like the UIP-based FXI, the VaR rule allows exchange rate to reach a new equilibrium, while reducing the risk of exchange rate overshot. The VaR-rule controls the FX risk on the market, thereby contributing to its stability. Since the central bank always leave a certain degree of risk on the market, the VaR rule encourages a prudent exchange rate risk management by the private sector and stimulates the demand for market-to-market FX hedging. Besides, VaR FXI has unique advantages, such as budget neutrality over the medium-term and robustness to market manipulation. Finally, the fixed frequency of FXIs under the VaR rule help to determine the maximum FXI amount consistent with both (i) the reserve adequacy constraint; and (ii) the minimum amount necessary to have a credible impact on the exchange rate.

The empirical methodology is backtested using Mexico data. The Mexican peso has floated for a long period in one of the most liquid FX markets among emerging economies. In addition, Banco Mexico's (BM) website provides detailed intervention data since 2008. Banco Mexico also implemented interventions both with a minimum bid rate; that is, a preannounced fixed-volatility rule, and without a minimum bid rate, providing a diversity of experiences in intervention strategies. Our paper focuses on one dimension of FXIs: preventing exchange rate tail risks, benchmarked against a risk-based metric. Other relevant motives may have been factored into BM's interventions without a minimum rate and are outside of the scope of this paper.

The rest of the paper is organized as follows: Section 2 formalizes the FXI rule. Section 3 presents the empirical framework based on a GARCH model. Section 4 provides the operational framework for central banks using the model for their FXIs. Section 5 back-tests the model on Mexico public data. Section 6 concludes.

2 VaR Interventions and Exchange Rate at Risk

The exchange rate at risk ($ERaR$) is defined as the percentile at a given threshold θ of the conditional distribution of the exchange rate returns r_t; that is, the Value-at-Risk (VaR). The VaR measures how much a financial variable (here, FX spot) might lose with a given probability and during a set period, for example, one day. Figure 1 illustrates the concept. Formally, assuming that the density $f(r_{t+1}|X_t)$ is properly defined over the real line, the exchange rate at risk is the minimum of the set of values verifying the following condition:

$$\mathbb{P}\left[r_t \leq ERaR|X_t\right] = \theta$$

Where $\mathbb{P}$ is the probability distribution function (PDF) of the density $f(r_{t+1}|X_t)$.

[Figure 1 about here.]

A conditional VaR depends on a set of variables, which can vary in real time. The distribution on which the VaR is estimated is based on the properties of time series and exogenous regressors, thereby the term "conditional." Following the literature (Sarno & Taylor, 2003), the dependent variable is the log-return r_t of the exchange rate e_t, at a daily frequency (see below). The conditional predictive density of exchange rate returns is $f(r_{t+1}|X_t)$ where X_t is a vector of explanatory variables.

$$r_{t+1} = \log\left(\frac{e_{t+1}}{e_t}\right)$$

The concept of VaR, as formalized in Jorion (2007), is frequently used in central and commercial banking for financial applications, managing risk exposure, and portfolio management. Alexander (2009) provides a comprehensive review of VaR for market risk analysis. Among many applications of the VaR model, one can cite, in the FX field, Al Janabi (2006), who proposes to use consistently a VaR framework for managing trading risk exposure of FX securities in the context of emerging and illiquid markets. Bredin & Hyde (2004) review the performance of several VaR methods using a portfolio based on the FX exposure of a small open economy.

Under the VaR approach, the policy decision determines how much risk will be transferred, which has macroeconomic underpinning. The quantile θ represents the central bank's commitment to absorb a predetermined and fixed share of the exchange rate risk through its FXIs. On the contrary, the risk transferred to the central bank varies with volatility with fixed-volatility triggers: the risk transfer is high with high volatility and low with low volatility, raising the issue of moral hazard as market participants know ex-ante that the central bank will hedge all volatility above a certain threshold. Regarding the optimal risk transfer; i.e. the level of θ, it is a function of the pervasiveness of exchange rate exposures in the economy due to currency mismatches in balance sheets and exchange rate pass-through to domestic prices.

Mathematically, the decision rule is formalized as an intervention region $\mathcal{R}_\theta\left(r_t\right)$, based on the economy tolerance thresholds for depreciation and appreciation, respectively (θ_l, θ_u). The decision rule is binary: if r_t falls within $\mathcal{R}_\theta$, then the central bank intervenes; otherwise, it does not. These regions evolve every day as a function of market conditions. In the rest of the paper, we assume that intervention regions are symmetric, that is, unhedged exposures to the exchange appreciation are as important as unhedged exposures to depreciation, to keep the likelihood of intervention against appreciation equal to the one of intervention against depreciation. Other assumptions can be made depending on the nature of the risk.

$$\mathcal{R}_\theta\left(r_t\right) = \{r_t \leq Q(r_t|X_{t-1}, \theta_l)\} \cup \{r_t > Q(r_t|X_{t-1}, \theta_u)\}$$

Where $Q(r_t|X_{t-1}, \theta)$ is the θ-quantile of the conditional log-returns distribution $f(r_{t+1}|X_t)$.

The VaR rule provides an objectifiable approach to FXIs. It gives the policy maker a clear anchor when deciding on intervention which is directly derived from the financial stability objective. Fixed-volatility triggers could be derived from an analysis of the volatility that the market could tolerate; however, they are one more step remote from the policy objective.

The VaR rule is financially sound for the central bank (see Appendix A). The central bank buys and sells FX with the same probability (with symmetric FXI, for instance 5% and 95%) at the extremes of the exchange rate distribution, and on the "good side" of the market (selling expensive and buying cheap), thereby using its resources efficiently. Profitability has been associated in the literature with efficient intervention to support FX markets (Friedman & Friedman (1953) and Sandri (2020)).

3 Empirical Framework

3.1 Specification

We use ARCH/GARCH (Autoregressive Conditional Heteroskedasticity/Generalized Autoregressive Conditional Heteroskedasticity) model for estimating the forward-looking exchange rate Value at Risk $Q(r_t|t-1,\theta)$. Engle (2001) conducted a comprehensive overview of the ARCH/GARCH models in financial econometrics in which he devoted an entire section to estimating VaR. Giot & Laurent (2004) model daily VaR using realized volatility and ARCH models and show that it has excellent forecasting performances. Chan et al. (2007) use nonlinear GARCH models to estimate VaR in the presence of a data generating process with heavy tails.

Other types of models could be used to estimate VaR, such as quantile regressions (Gaglianone et al. (2011)), copulas (Patton (2001)), and nonparametric kernel (Hoogerheide & van Dijk (2010)). However, the GARCH model is used in this paper because it is commonly used in the industry and by central banks. GARCH models relies on standard maximum likelihood estimators and are implemented in many statistical packages, making them easily operationalizable. Besides, as Jeon & Lee (2002) show, FX markets are quite efficient, and their features fit well the simple and robust approach of standard GARCH models. Finally, GARCH models are frequently used for the analysis of the FX markets. Hansen & Lunde (2005) argue that in the context of daily exchange rate returns, no alternative model can beat a GARCH(1,1) model, while Mcmillan & Speight (2012) show that an intraday GARCH(1,1) model generally provides superior forecasts compared with all other models.

More precisely, the VaR rule presented here uses an Exponential GARCH (EGARCH) model of volatility. The advantage of the EGARCH model is that it is a relatively parsimonious model which incorporates nonlinearities and asymmetries with only two equations. Exceeding a certain level of volatility could result in sudden shifts in expectations, closing large positions, and herding behavior among market participants. Besides, markets may not react equally to exchange rate appreciation and depreciation. The model presented is a proof of concept and could be refined by the central bank for implementation.

The EGARCH specification incorporates three components: drift, volatility, and the distribution of the error terms or innovations.

- **Drift**: $r_t = \boldsymbol{Intercept} + \boldsymbol{\rho} r_{t-1} + \boldsymbol{\beta} X_t + \varepsilon_t$ AR-X(p) for the average level of log-returns (r_t, the drift), and X_t a vector of exogenous regressors. The drift reflects the conditional exchange rate trend

- **Volatility**: $\log \sigma_t^2 = \boldsymbol{\omega} + \boldsymbol{\beta} g(r_{t-1})$with $g(r_t) = \boldsymbol{\alpha} r_t + \boldsymbol{\gamma}(|r_t| - \mathbb{E}[|r_t|])$: where σ_t is the volatility

- **Errors term**: $\varepsilon_t = \sigma_t \epsilon_t,\ \epsilon_t \sim TSK\ (\boldsymbol{\nu}, \boldsymbol{\lambda})$: the error term is parameterized for instance with a Tskew distribution (other distributions can be used, including Normal, T distribution and generalized error distributions).

The parameters in **bold** are estimated by the GARCH model.

The estimates use an EGARCH-X(1,1) with Tskew distributed innovations, estimated via Maximum Likelihood Estimation.[3] GARCH models are a standard approach to estimating and forecasting volatility (Engle, 2001). The number and order of lags are chosen based on AIC/BIC criteria (Akaike Information Criterion/Bayesian Information Criterion). An alternative model is the JP Morgan RiskMetrics model (Zumbach, 2007), with exogeneous regressors and innovations distributed with generalized error distribution, also known as generalized Gaussian distribution (GGD). Annex B presents different specifications and alternative models as a robustness exercise.

With the GARCH specification presented above, the VaR rule allows for a progressive adjustment of the exchange rate to its new equilibrium value. The estimated conditional volatility includes a drift that reflects the exchange rate trend and reduces the likelihood that the VaR rule triggers FXIs when the exchange rate is on an appreciating or depreciating trend. In addition, the conditional distribution changes with the volatility regime to keep the likelihood of interventions unchanged, thereby loosening the triggers as volatility increases. On the contrary, fixed-volatility rules do not adapt to trends and volatility regimes.

The exchange rate adjustment to a new equilibrium, even though necessary from a macroeconomic point of view, could be disruptive because of its impact on expectations and market functioning. Therefore, interventions in the tails of the volatility distribution during the adjustment process contribute to avoid that the exchange rate significantly overshoots its equilibrium level, harming financial stability.

The EGARCH includes several exogenous regressors (EGARCH-X) to improve the accuracy of the conditional volatility forecast. These exogenous variables can be used to factor in the specifics of a given market and customize the interventions metrics. Although our estimation is based on daily data, we incorporated some proxies for intraday volatility and liquidity too, hence capturing microstructure dimensions. Therefore, the VaR rule provides a single input for the intervention decision based on multiple variables (i.e., sources of information) that is more

[3] We built a Python wrapper around the comprehensive "ARCH" Python package, developed by Kevin Sheppard from Oxford University (Sheppard, 2020)

straightforward to interpret than large “traffic lights” dashboards, where sub-indicators might conflict. Our specification includes the following regressors:

- **The average exchange rate bid-ask spread over the day**, taken in absolute terms. The finance literature uses the bid-ask spread as a proxy of intraday market liquidity
- **The intraday spread (max–min over the day)**, which is another metric of intraday volatility
- **The daily interest rate differential between local and foreign currencies** captures the interest rate parity arbitrage. We used the daily interest rate differential instead of the cross-currency basis swap, as the former has a longer available time series
- **The one-month forward exchange rate**, expressed as the first difference from the day before. As the previous variable controls for interest rate parity, the forward rate could capture the impact of the cost of hedging on the spot market
- **The first-difference variation of the Chicago Board Options Exchange Volatility Index (VIX)**, which captures global risk sentiment[4]
- **The exchange rate of the US dollar vis-à-vis the euro**, to control for US dollar specific developments
- **Oil prices log returns**, as Mexico is an oil exporter and oil prices could impact term of trade (i.e. competitive shock)

The VaR rule is robust to speculative attacks because the conditional distribution is forecasted by the EGARCH-X model and updated in real-time. Forward-looking variables reduce the risk of opportunistic and strategic behaviors from market participants. Indeed, as market participants are changing behavior, the VaR rule is changing too. The adaptative feature of the VaR rule makes it more complicated and even impossible for market participants to take speculative positions because they would need to anticipate perfectly how their new behavior, as well as the behavior of all other market participants, will shift the conditional

[4] We tried with the VIX in level, but the p-value was higher than in the first difference

distribution of the exchange rate. On the contrary, with fixed thresholds, market participants know exactly when the central bank will intervene and can take speculative positions accordingly.

3.2 Estimation Results

As a proof of concept, the model is fitted against a real sample of Mexico FX data (against the US dollar) since 2001. The peso has been floating since 1994. Figure 2 presents the historical level, returns, and returns distribution of exchange rate. The bilateral spot exchange rate exhibits significant volatility in crisis times, such as the global financial crisis of 2008 and, more recently, the COVID-19 pandemic. The historical distribution shows that the returns are skewed to the right (depreciation). The model thus incorporates asymmetries to accurately capture the dynamics of this exchange rate series. Intervention data are available on the BM's website.[5]

[Figure 2 about here.]

The signs of the coefficients are consistent with macroeconomic and finance theory (see Sarno & Taylor (2003) for an exposition of exchange rate economics). Table 1 presents the output of the GARCH estimation process, where we integrate progressively the coefficients under different specifications, starting from the micro to the macro factors. We have tried to insert year dummies to control for structural breaks, but most of the coefficients were not significant.

- The bid-ask spread is positively correlated, suggesting that an increase in the bid-ask spread (market illiquidity) signals depreciation
- Forward points also contribute to predicting the movement in exchange rate log returns, with the same sign as expected

[5] https://www.banxico.org.mx/mercados/subastas-cambio-credito-banco.html

- A positive interest rate differential with the London Interbank Offered Rate (LIBOR) tends to appreciate the currency, due to carry-trade arbitrage
- The same results hold for the VIX, as an increase in global risk sentiment tends to depreciate emerging market currencies
- Changes in the EUR/USD exchange rate have the expected impact on local currency returns and contribute to the explanatory power of the model
- Similarly, with the oil price log returns, an increase in oil prices is associated with an appreciation of the local currency against the USD
- The model explains around 28 percent of the log-return variance

[Table 1 about here.]

Once fitted, the GARCH model provides an in-sample estimation of the conditional volatility, σ_t^2, over time, presented in Figure 3. Not surprisingly, the in-sample conditional volatility spikes during the global financial crisis and the COVID-19 crisis.

[Figure 3 about here.]

Figure 4 presents an out-of-sample plot of the conditional density, estimated through the fitted GARCH via expanding windows, over a 10-month timeframe (from January through end-October 2020). The plot of the conditional density shows that not only the conditional volatility widens significantly during the COVID-19 crisis (in March 2020), but that the skewness varies also substantially.

[Figure 4 about here.]

Like other types of forecasting density models, the GARCH model can also be used to produce the so-called fan charts, as shown in Figure 5. The fan chart presents the forecasted quantiles of the conditional distribution across time and provides an intuitive way to understand the uncertainty surrounding the mean forecasts. In this case, the uncertainty is precisely captured by the GARCH model, both through the estimation of the conditional density and also via higher moments of the distribution, including the skewness.

[Figure 5 about here.]

Finally, the quality of the GARCH forecasting density model is evaluated out-of-sample. The correctness of the density model specification is assessed via a probability integral transform (PIT) test (Diebold et al. (1998); Rossi & Sekhposyan (2019)). The PIT test is evaluated over four-month out-of-sample daily data, as presented in Figure 6. The PIT test outcome presented in Figure 6 indicates that the empirical distribution of the PIT is within the confidence band for all quantiles. This pattern suggests that the conditional density derived from the EGARCH-X models has a satisfactory out-of-sample accuracy and that it generates robust predictive distributions that capture well upside and downside risks.

Appendix B presents a series of alternative models (unconditional via Gaussian kernel, autoregressive, quantile regressions, and a series of GARCH and EGARCH models with different error terms specification) to benchmark the performance of our model, also using log score performance metrics against other models. The result of the tests is that the GARCH family of models dominates the alternative tested against an unconditional kernel density estimator and conditional density estimated from the quantile regression. Also, within the GARCH family (Gaussian, Tskew, GARCH versus EGARCH for different error terms specifications), there is no significant improvement in performance against our baseline model.

[Figure 6 about here.]

4 Operational Framework

4.1 Risk-Based Triggers

The VaR rule derives from a simple time series model and is, therefore, easy to operationalize and to communicate. The model requires a limited amount of data, available in the public domain, and their interpretation is unambiguous. VaR triggers are transparent and relatively easy to understand and to communicate to the market, while being part of a more global communication strategy from the central bank (see, for example, the Central Bank Transparency Code, International Monetary Fund (2020a)) FXIs are performed on the wholesale market, where participants are aware of the concept of VaR. The general public might be less familiar with VaR, but this is of little consequence from an operational standpoint as the general public does not directly participate in FXI.

In practical terms, the central bank estimates the model and assesses the intervention regions in real time. The central bank monitors the cumulated returns of the exchange rate—either depreciation or appreciation—compared with the previous day exchange rate. If more than one intervention per business day is allowed, the trigger for the second intervention would be based on the cumulated return of the exchange rate compared with the previous same-day trigger rate. For the rest of the paper, we will assume that the central bank does not intervene more than once a day. Once the cumulated returns reach the intervention regions, the central bank buys or sells FX. These regions evolve every day as a function of market conditions. On the specific day presented in Figure 7, foreign exchange intervention would have happened only if the exchange rate had depreciated by more than 3.3 percent or appreciated by more than 3.7 percent, for a 5 percent VaR (2.5 percent on each side). Thresholds would be computed every day, for each possible quantile, as shown on the fan chart (Figure 5).

[Figure 7 about here.]

Over time, the VaR-FXI rule can be represented by the region where the cumulative distribution function falls outside the central bank risk tolerance, for example, the 2.5th and 97.5th percentiles, as presented in Figure 8.

[Figure 8 about here.]

Had the BM followed the VaR FX intervention rule, it might have intervened in about 15 days over 10 months, from January through October 2020. Figure 9 presents the intervention days within the daily log-returns plot. It highlights with green dots (below ERaR 2.5th percentile) and red dots (above the ERaR 97.5th percentile) the occurrences falling in the intervention regions. The lower chart of Figure 9 presents the corresponding FX level in which the central bank would have intervened. Fifteen days of intervention correspond to about 7.5 percent of the period, even when using an intervention region of 5 percent. The frequency of interventions would increase when volatility is unusually high, which was the case during the COVID-19 crisis. However, this exercise is purely counterfactual, and it is likely that after the first FX intervention, the FX volatility would have decreased, hence reducing the need for future interventions.

[Figure 9 about here.]

4.2 FX Intervention Calibration under Risk-Based Interventions

One of the major advantages of the VaR rule is that it provides certainty about the frequency of FXIs over time. For example, with a VaR of 2.5 percent both for depreciation and appreciation, the central bank will, on average and over time, intervene 5 percent of the time. The central bank intervention frequency might temporarily deviate over a short period of time—depending on market conditions—but the VaR approach guarantees that the frequency is the same as the VaR over a sufficiently long period of time.

The fixed-frequency feature of the VaR rule can be used to calibrate the central banks' intervention budget. We assume that the central bank set a maximum amount for its intervention, which can be set for single intervention or for one business day (the later is often consider as the most practical). The budget could, then, be directly derived from the fixed frequency of intervention and the maximum FXI amount. The budget should remain within the constraint represented by the available FX reserves at the central bank for FXI. This is an important advantage compared to fixed-volatility rule where interventions frequency and, thus, the interventions budget, are not known ex-ante.

The VaR rule could contribute to the improve the efficiency of FXIs by maximizing the impact on the exchange rate for a given amount of FX sold/bought by the central bank. The maximum amount of daily intervention should be large enough to influence the exchange rate when the central bank intervenes. Accepting higher VaR (lower intervention frequency) is a solution if the intervention budget exceeds the FX reserves availability constraint, to keep the daily intervention at a credible amount. Another feature of the VaR rule is that it triggers interventions when price movement are unusually large, which should reinforce the intervention impact on the exchange rate.

Another substantial benefit of the VaR rule is that it improves the sustainability of the intervention policy. Should the central bank decide to operate with a symmetric rule (the same VaR for appreciation and depreciation), it will intervene the same number of times on both sides, hence offsetting selling FX with buying FX over time. As a result, a symmetric VaR rule would be budget neutral for the central bank over the medium term.

5 The Case of Mexico

The BM intervened via two types of auctions which differ by the reservation rate applied to each type of auctions. In the first type, the BM operated an auction every day with a preannounced reservation rate, that is, the minimum rate for eligible bids.[6] On many days, the auction with a minimum rate did not receive any demand, as the market was operating below the reservation rate. In the second type, the auction was organized when the BM found it opportune and did not have a reservation rate (an auction without a minimum rate). For some

[6] See https://www.banxico.org.mx/markets/auctions-with-minimum-price-e.html for more details

periods (for example, during 2015), daily auctions were preannounced without a minimum rate. The auctioned amount was also preannounced and later adjusted.

We are focusing in this paper on benchmarking the BM's FXI against a risk framework, while recognizing that the BM could have had other objectives in mind. The BM may have intervened for other reasons than preventing disorderly market conditions. The exact rationales for FXI is outside the scope of the paper. We are not assessing, here, the efficiency, the relevance, nor the rationale of the BM's interventions.

5.1 Interventions with a Minimum Price

"Auctions with a minimum price" were triggered 31 times from October 2008 through November 2016. The auction reservation price was set at the previous day's exchange rate close (MEX/USD) multiplied by 1.02 (2 percent depreciation) from October 9, 2008, to December 8, 2014, by 1.015 from December 9, 2014, to November 23, 2015, and by 1.01 from November 24, 2015, to January 5, 2017. Auctions with a minimum price were suspended after January 5, 2017. Figure 10 presents the auctions with a minimum price in green, with the corresponding FX daily log returns and level.

[Figure 10 about here.]

The ex-ante conditional cumulative distribution function (CDF) estimated via the GARCH model is used to benchmark the Banco Mexico FX interventions. Although the central bank did not use the model to intervene, one can always assess the corresponding FX rate conditional quantile when the central bank has intervened. The outcome is presented in Figure 11. The back-testing exercise shows that most FX interventions have occurred on the top of the conditional distribution—where the depreciation was the largest—and often above the 95th percentile. A few outliers occur at the median or below, but these cases are rare. Overall, the interventions with the minimum price approach were often but not always equivalent to a risk framework: the central bank intervened most of the time when the pressure on the exchange rate was the largest, with exceptions.

[Figure 11 about here.]

5.2 Interventions without a Minimum Price

The BM organized 319 "auctions without a minimum price" from September 2009 through November 2015. Interventions without a minimum price overlapped with "auctions with a minimum price," which could constrain the volatility distribution even in the absence of actual intervention under the rule—the mere presence of the rule influences market participants' behavior. Figure 12 presents the FX interventions without a minimum price on the Mexican pesos in red, with the corresponding FX daily log returns and level.

[Figure 12 about here.]

As in the case of interventions with a minimum price, the conditional CDF can be used to benchmark the FX interventions relative to the risk of the day, as presented in Figure 13. While interventions with a minimum price exhibited a pattern broadly consistent with large depreciation pressures on the exchange rate, the interventions without a minimum price have no clear risk patterns. Such interventions had occurred across the full conditional distribution, even when the pressures were not on the depreciation side but, on the contrary, the appreciation side. In this case, the fact that the central bank was selling USD could have, therefore, amplified the currency appreciation. This pattern likely suggests that the BM rationales for FX intervention under the no minimum price rule are likely broader than just the volatility of the exchange rate. For example, during certain periods, the motivation for interventions without a minimum price was to prevent excessive accumulation of foreign reserves, explaining that those interventions were not related to FX rate developments.

[Figure 13 about here.]

6 Conclusion

Foreign exchange interventions would benefit from a risk-based framework. Central banks use quantitative tools such as VaR to measure the risk on their foreign reserves portfolio or when they accept assets as collateral for their loans. In this paper, we argue that central banks could also use this concept when they intervene to support the FX market, as FXIs often aim at removing some exchange rate risk from the market.

The model of this paper has several policy implications. The VaR rule would help central banks concerned about financial stability risk arising from the exchange rate but committed to a floating exchange rate arrangement. Central banks could accompany the transition to exchange rate flexibility by formalizing the commitment to the float in a rule and by gradually increasing the VaR; i.e., reducing intervention frequency, to allow for more exchange rate volatility. Finally, the VaR rule could be used to deepen hedging markets by gradually transferring an increasing share of the exchange rate risk to the market.

A risk-based approach has convincing advantages relative to other types of rules. Compared with fixed-volatility rules, the VaR rule allows the central bank to control exactly the exchange rate risk to which the economy is exposed. It also facilitates preparing the intervention budget based on (i) the fixed FXI frequency; (ii) the maximum FXI amount; and (iii) the FX reserve adequacy constraint. Other advantages include being budget neutral over the medium term, forward looking, and combining several exogenous variables, reflecting both competitive shocks and market structure factors, into one trigger.

While it does not allow to identify the nature of the shocks, such as to guaranty the optimality of each FXI, the VaR rule is flexible enough to let the exchange rate timely adjust to new equilibria. It also allows the central bank to respond to market disfunctions reflected in high exchange rate volatility and contributes to prevent exchange rate overshot. Finally, the VaR rule is easier to implement and operationalize in markets at different stages of development than rules that would try determining the nature of shocks.

The paper argues that using VaR as an input would contribute to a better-informed FXI policy decision. It would help to anchor the decision to intervene on the risk hedging objective of the policymaker and the structural weaknesses of the economy, such as unhedged exposures to exchange rate risk. In summary, the VaR rule, while not currently widely used, could be considered as one approach to build a solid and consistent operational framework for FXIs.

Looking beyond interventions in the spot FX market, similar models and methods can be used by central banks to transfer risk from any markets to support financial stability. Considering the focus on UIP deviation in the IMF IPF, the VaR rule can be amended to target tail UIP deviations instead of tail spot volatility. The VaR would be a natural benchmark to determine how large deviations would signal market impairment. Benchmarking the deviations is important as forward rates deviate from the UIP most of the times and those deviations can be volatile, so there is a need to define a reasonable anchor, so that the frequency of central bank interventions is adequate. Moving on to the hedging market, the VaR rule can also be used to identify unusually large increases in the cost of hedging that could reflect hedging market impairment and spill over to the spot market. In that case, the relevant variable would be the spread between forward rates and the covered interest parity, which reflects the cost of hedging (Borio et al. (2016), Cerutti et al. (2019), Du et al. (2018), and Eguren Martin et al. (2018)).

How to determine the optimal degree of hedging provided by the central bank to the market and what should be left in the market is an open question. The paper discussed the structural factors underpinning direct and indirect exposures to exchange rate risk. However, it is, so far, left to the policymakers' judgment to translate those vulnerabilities into an optional level of hedging to be offered by the central bank; i.e., a target VaR. A method that estimates the extent of the vulnerabilities and combined them with their impacts on macro-financial stability is left for future research and would be most useful.

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A Comparative Static Financial Performances

This appendix presents a comparative static evaluation of the financial performance of the three different intervention strategies studied in this paper: the strategy without a minimum price, the strategy with a minimum price (rule-based), and the VaR rule strategy. We selected a one-year period of intervention, between October 2015 and October 2016. The frequency of intervention under the no minimum price and the minimum price strategies during this time was the same (18 interventions each), with similar volumes (USD 3.32 billion and USD 3.6 billion, respectively). Then, we constructed a series of interventions following the VaR rule at a similar frequency (18 interventions), corresponding to a 10 percent intervention threshold, only on the selling side to match the other strategies. We calibrated the intervention amount of the VaR rule to be at the average under the no minimum price strategy so that the total amount of intervention under the VaR rule exactly matched the intervention amount under the no minimum price strategy.

This financial benchmarking is purely a comparative exercise. It might be that some interventions without a minimum price complemented interventions with a minimum price and were executed at a less favorable time. The comparison is not taking such a selection effect into account and should not be interpreted as definite proof of the financial performance of one strategy against another. Under this caveat, the benchmarking shows that the rule-based strategies (either minimum price or VaR based) outperform the minimum price strategy, as presented in the table below.

Table 2 reports that on average, under the no minimum price rule, the central bank was selling USD when the local currency was slightly appreciating against the USD (by -6 bps; the negative value in this quotation means appreciation). Under the minimum price and the VaR rule schemes, on the contrary, the central bank was selling USD during a more favorable time. By design, the VaR rule is selecting days with particularly large depreciation, in the order of 160 bps. Likewise, the minimum price and VaR rule scheme were executed at much better rates in terms of level. While the VaR rule is designed to target the tails of the conditional distribution of the log returns, it does not necessarily guarantee that the intervention will happen at the most favorable exchange rate level. Our simulation shows that on average, it does nevertheless trigger interventions under better terms than for the two other schemes.

Looking at the performance against the strategy without a minimum price, computed against the gain realized by selling on average the same volume of USD at different points in time, the benchmarking exercise indicates that the minimum price intervention would outperform the strategy without a minimum price by 7 percent, while the VaR rule would outperform it by 8.3 percent.

[Table 2 about here.]

B Out-of-Sample Benchmarking

This appendix presents a series of alternative models to estimate the conditional density of foreign exchange log returns over time. Our baseline model relies on an EGARCH(1,1) with a Tskew parametrization of the error terms' distribution. The conditional benchmark models use the same sets of variables as our baseline model, presented in Section 3.1, and the same training/testing set.

We conduct benchmarking against:

- Different parametric variations of the GARCH model
  - EGARCH(1,1) with Gaussian errors
  - GARCH(1,1) with Tskew
  - GARCH(1,1) with Gaussian errors
- Density estimation via quantile regressions. The full-fledged density is obtained via quantile interpolation and resampling, following the rearrangement procedure of Chernozhukov et al. (2010).
- Unconditional distribution, estimated via Gaussian KDE on historical data

We assess the performance metrics of these models out-of-sample against our baseline model using (1) PIT tests and (2) a density log score, as explained in Diks et al. (2011); we use the uncensored version of the log score.

We summarize the results in Table 3. We define "pass the PIT test" as: if the estimated invert CDF distribution lies within the uniform distribution +/- 5 percent interval confidence bands, as defined in Rossi & Sekhposyan (2019), and "fail" otherwise. Regarding the density log score, we provide both the test statistic (the average difference of the log score across time) and the p-value of the test statistic, as explained in Diks et al. (2011). A positive test statistic indicates that the model outperforms the baseline model, while the p-value tested the null hypothesis that the log scores of the two models are the same.

[Table 3 about here.]

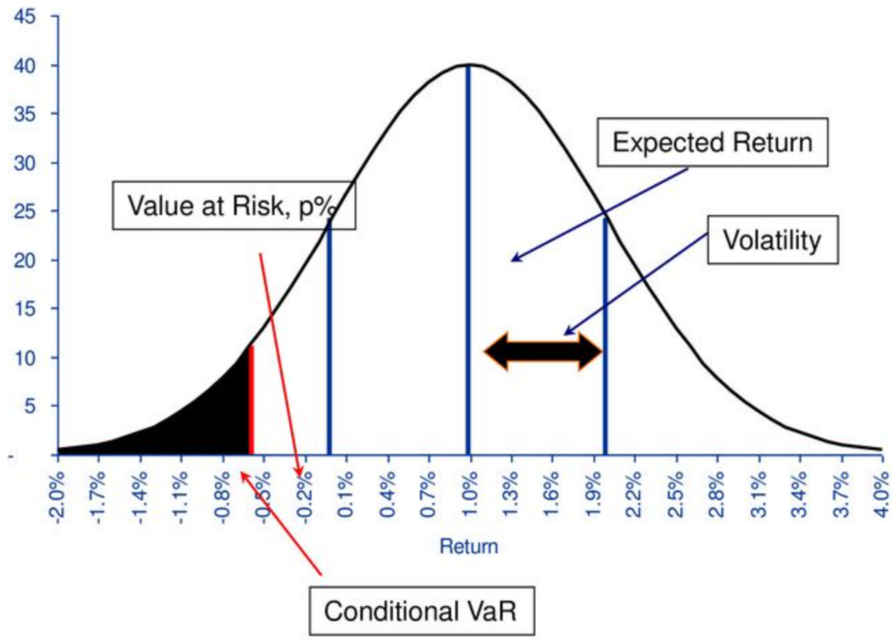
As Table 3 shows, the baseline model has a better log score than the unconditional distribution and the daily distribution fitted from the quantile regressions model with resampling, and these two models both fail the PIT test. However, the difference against variation of the GARCH model is less clear cut: while the other models outperform the baseline in terms of log score, the difference is not significant and they also fail the PIT test (albeit only by a very few numbers of percentiles). Therefore, while it is clear that the GARCH specification is appropriate to model FX returns—in line with the literature—variations around the type of GARCH and implied distributions are statistically equivalent.

[Figure 14 about here.]

[Figure 15 about here.]

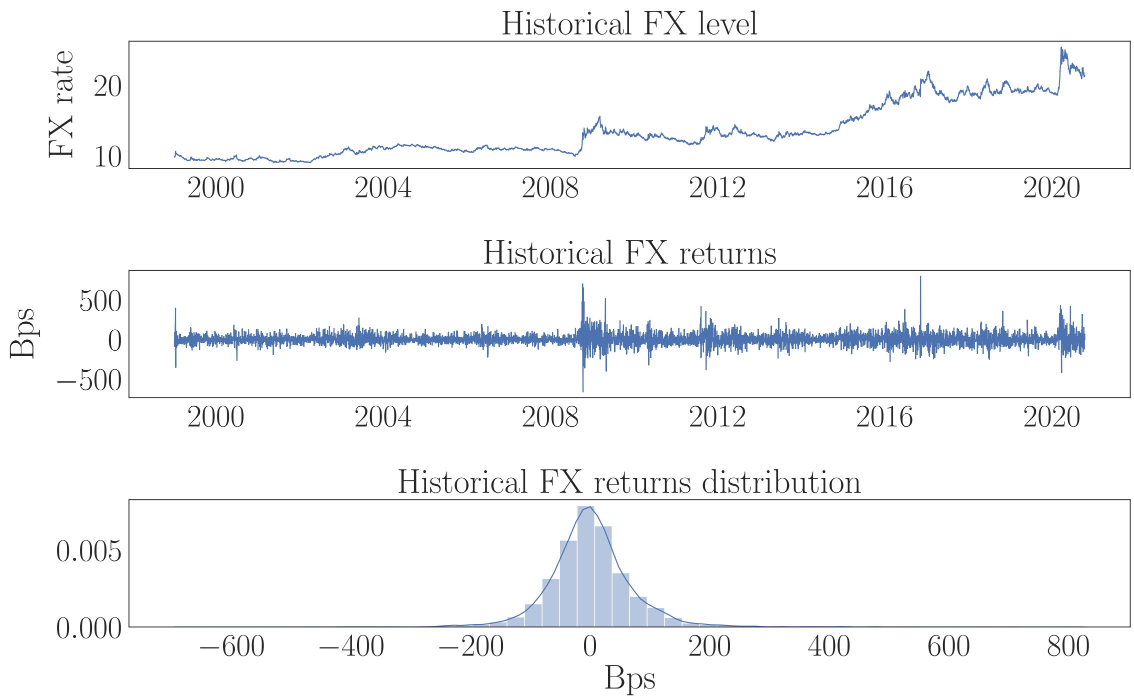
[Figure 16 about here.]

Figure 1: VaR Concept



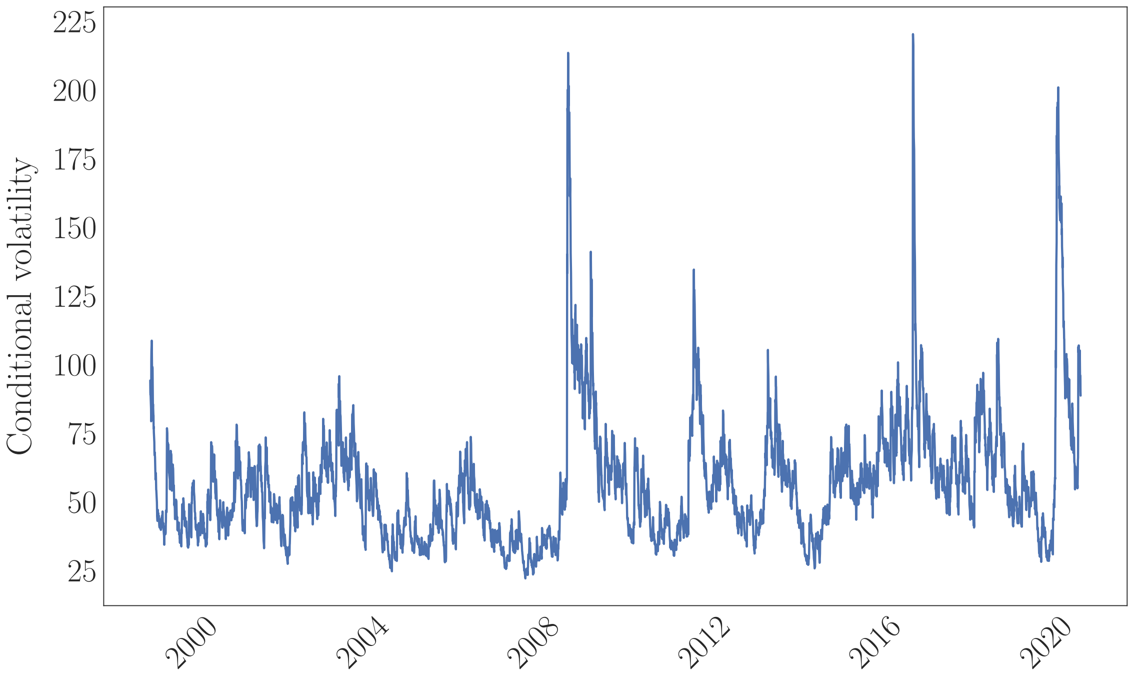
Source: semanticscholar.com

Figure 2: Mexican Peso against US Dollar



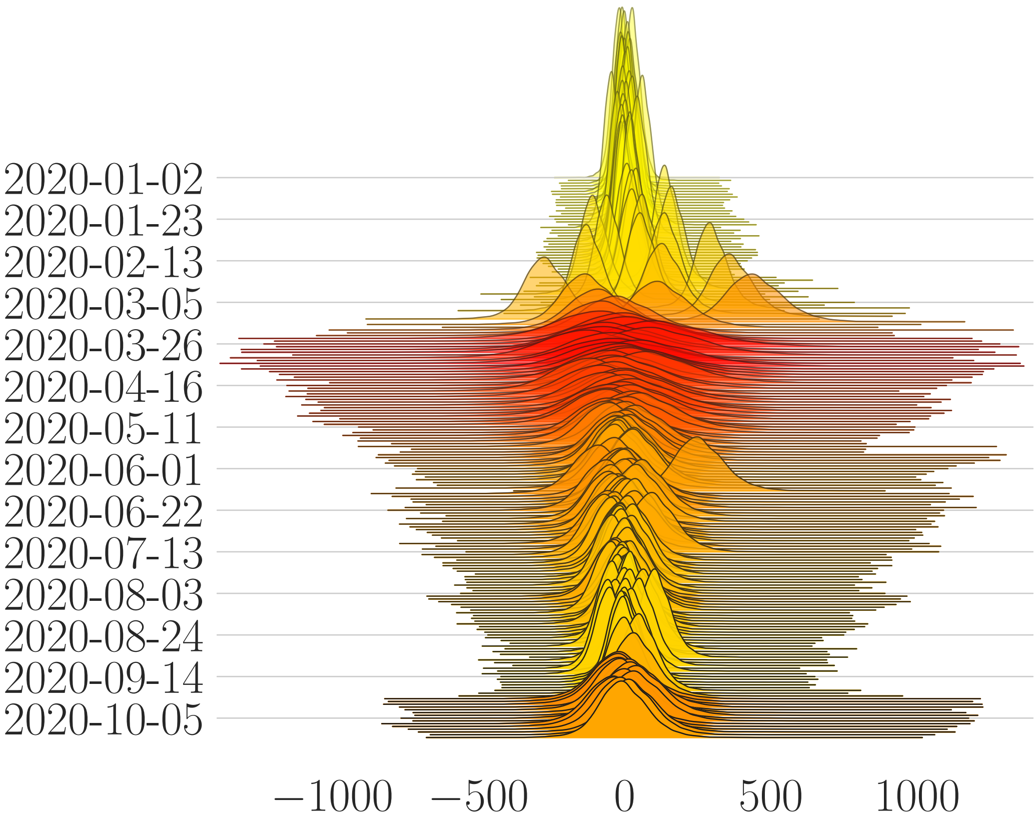
Sources: Bloomberg and authors calculations

Figure 3: Conditional FX Volatility over Time



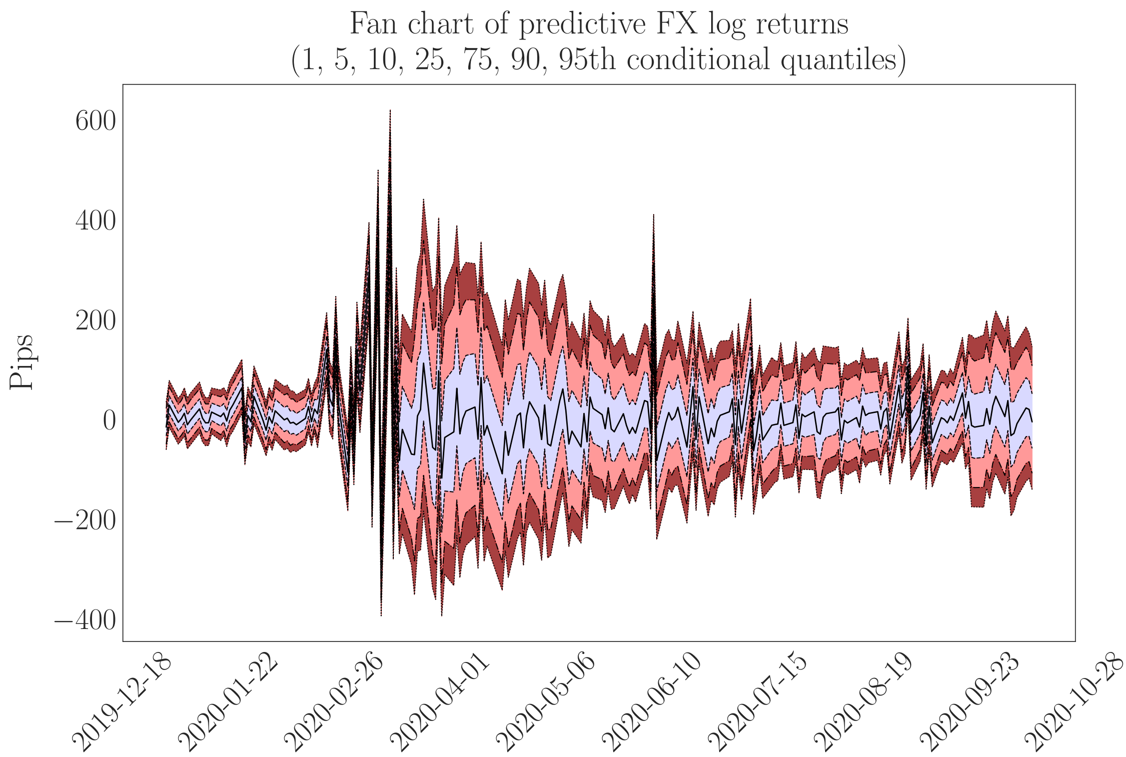
Sources: authors calculations

Figure 4: Out-of-Sample Conditional Density



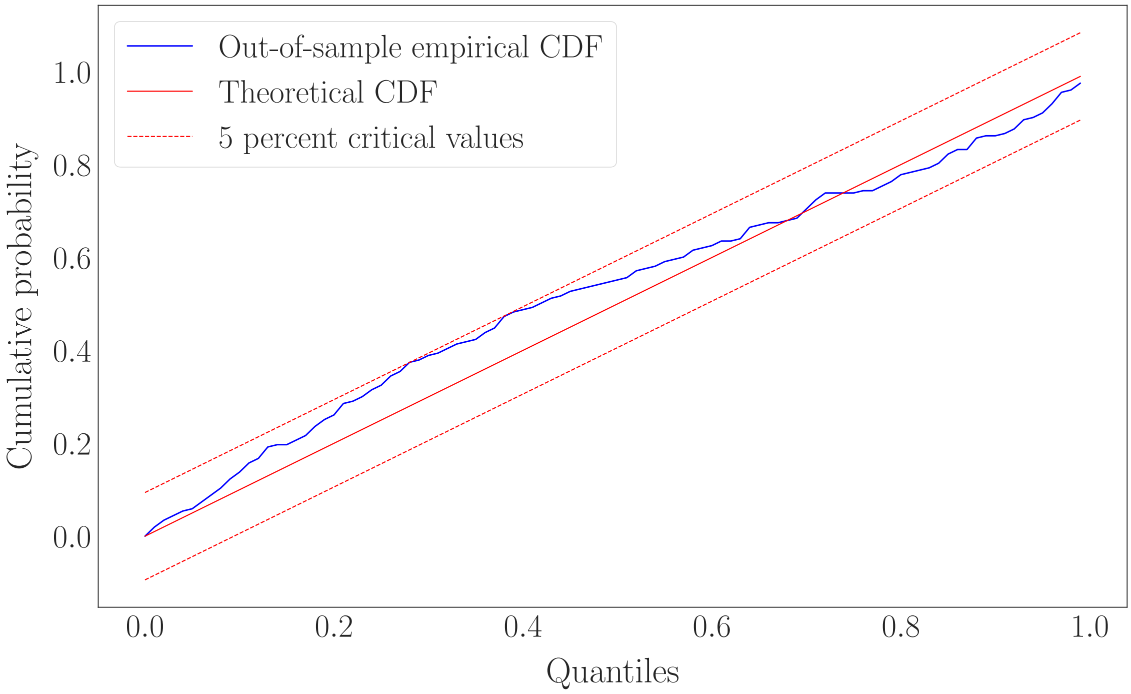
Sources: authors calculations

Figure 5: Out-of-Sample Fan Chart



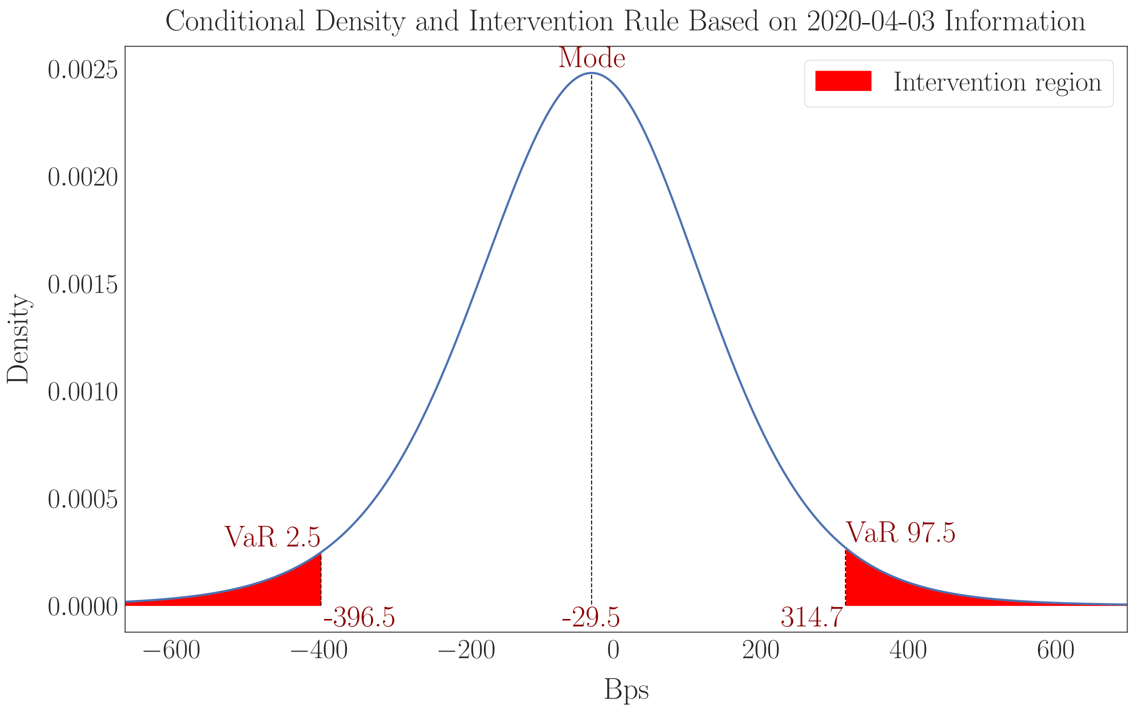
Sources: authors calculations

Figure 6: Probability Integral Transform Test



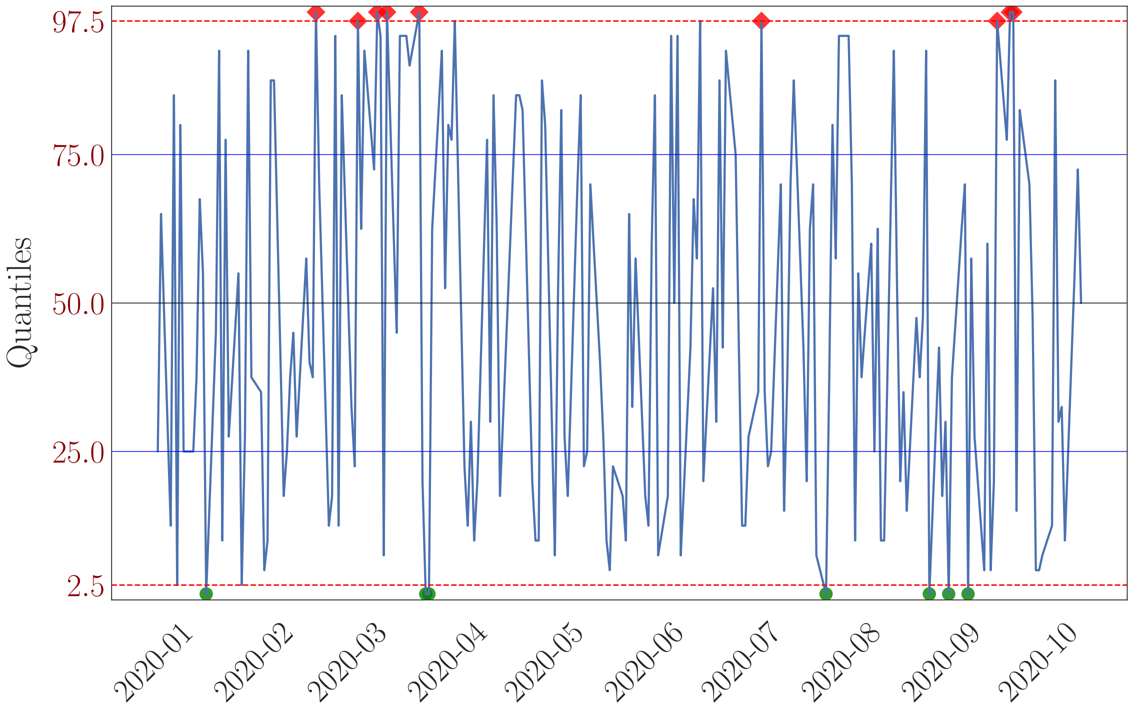
Sources: authors calculations

Figure 7: VaR FX Intervention Rule Based on a Given Information Set



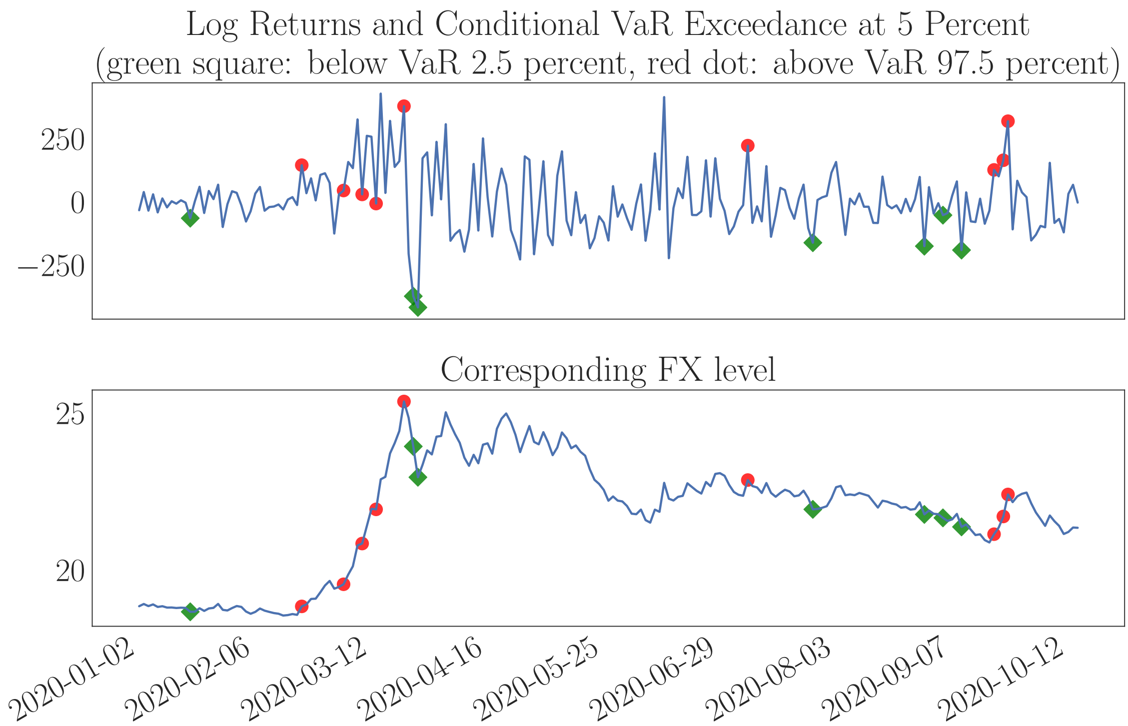
Sources: authors calculations

Figure 8: Conditional Cumulative Distribution Function and Intervention Thresholds



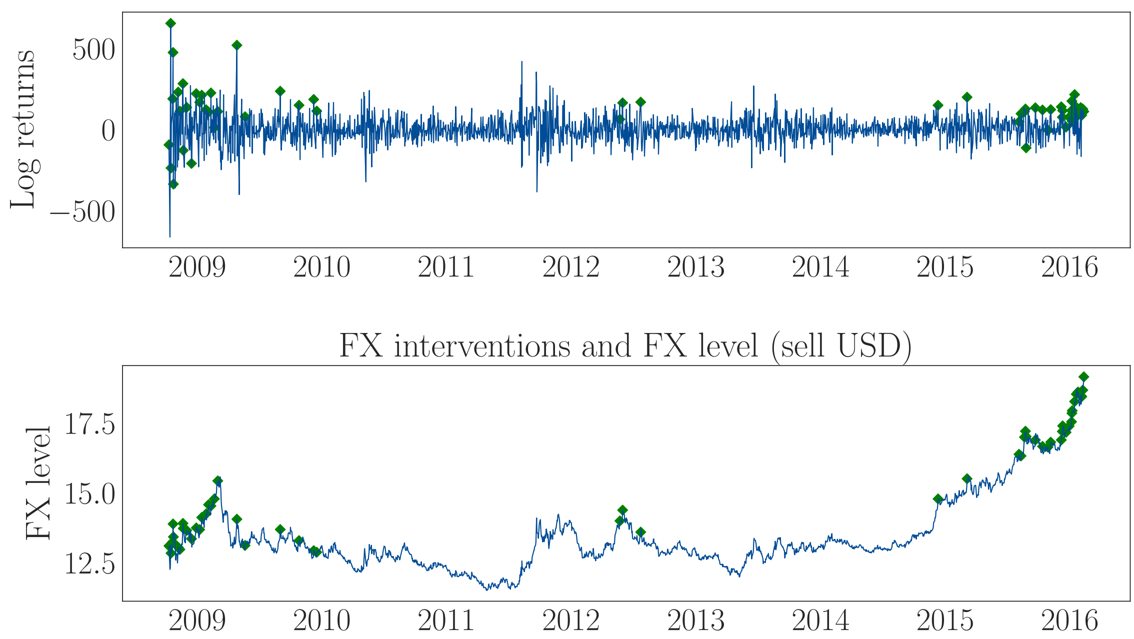
Sources: authors calculations

Figure 9: Conditional VaR Exceedance, Out-of-Sample



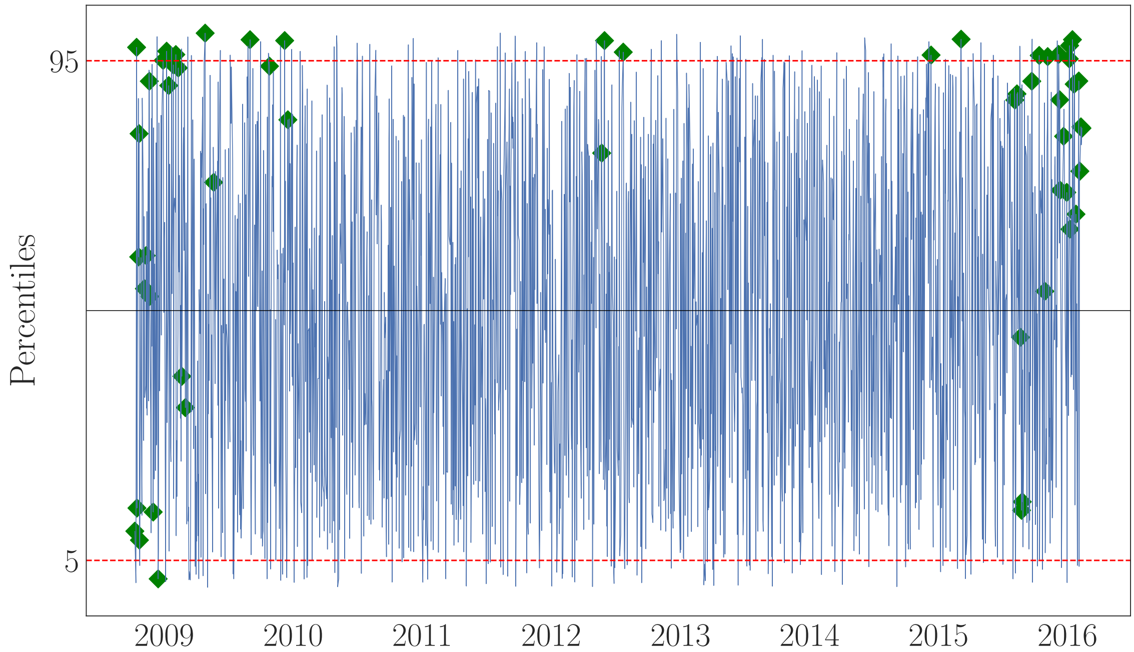
Sources: authors calculations

Figure 10: FX Interventions Log Returns with Minimum Price



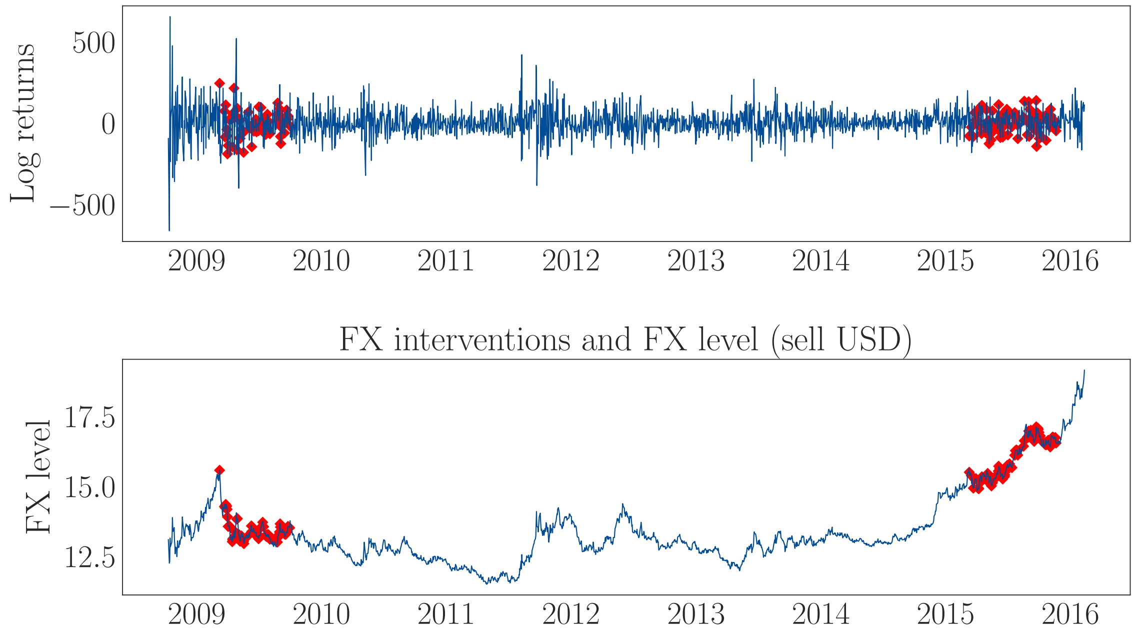
Sources: Banco Mexico and authors calculations

Figure 11: Conditional CDF of FX Intervention with Minimum Price



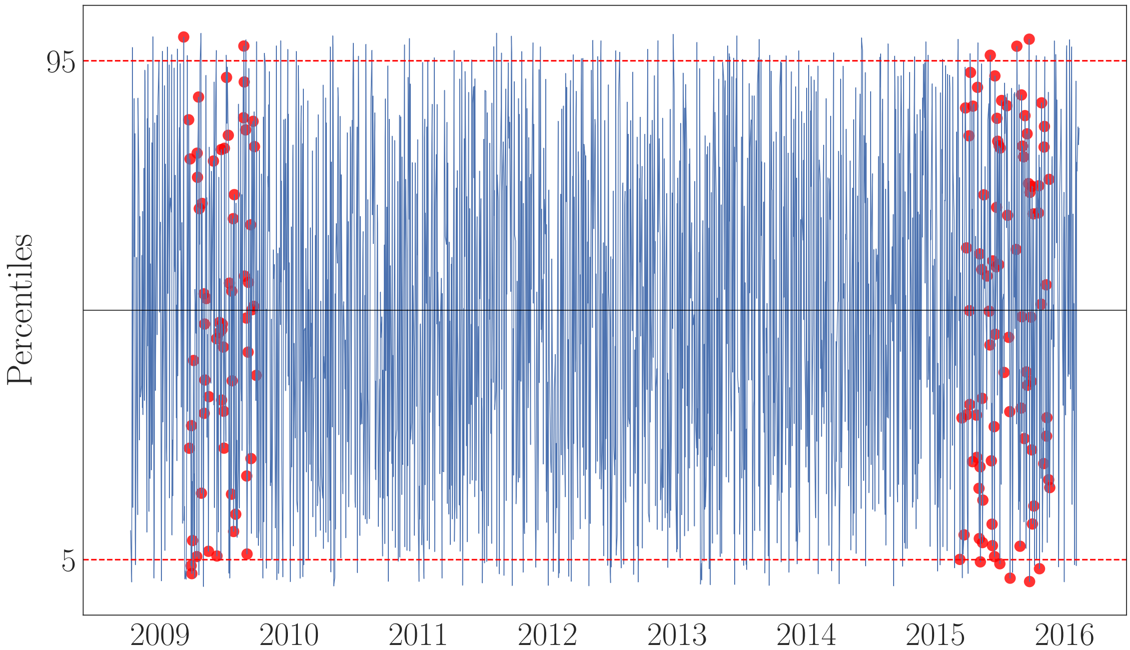
Sources: Banco Mexico and authors calculations

Figure 12: FX Interventions without a Minimum Price on the Mexican Peso/US Dollar



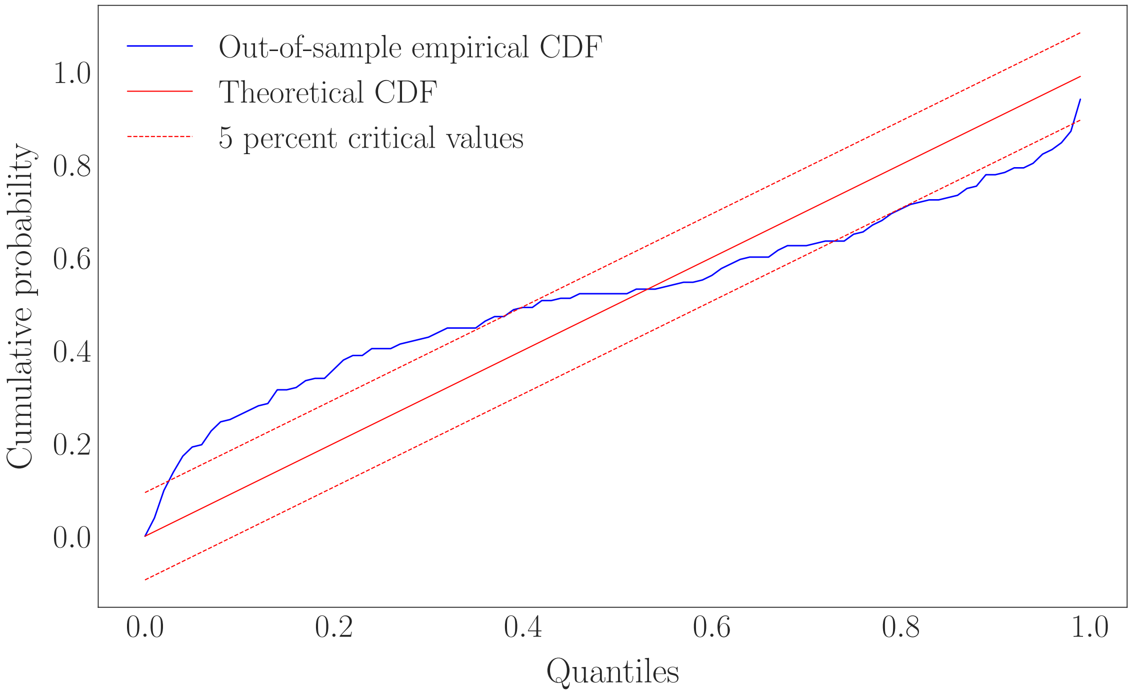
Sources: Banco Mexico and authors calculations

Figure 13: FX Interventions without a Minimum Price on the Mexican Peso/US Dollar



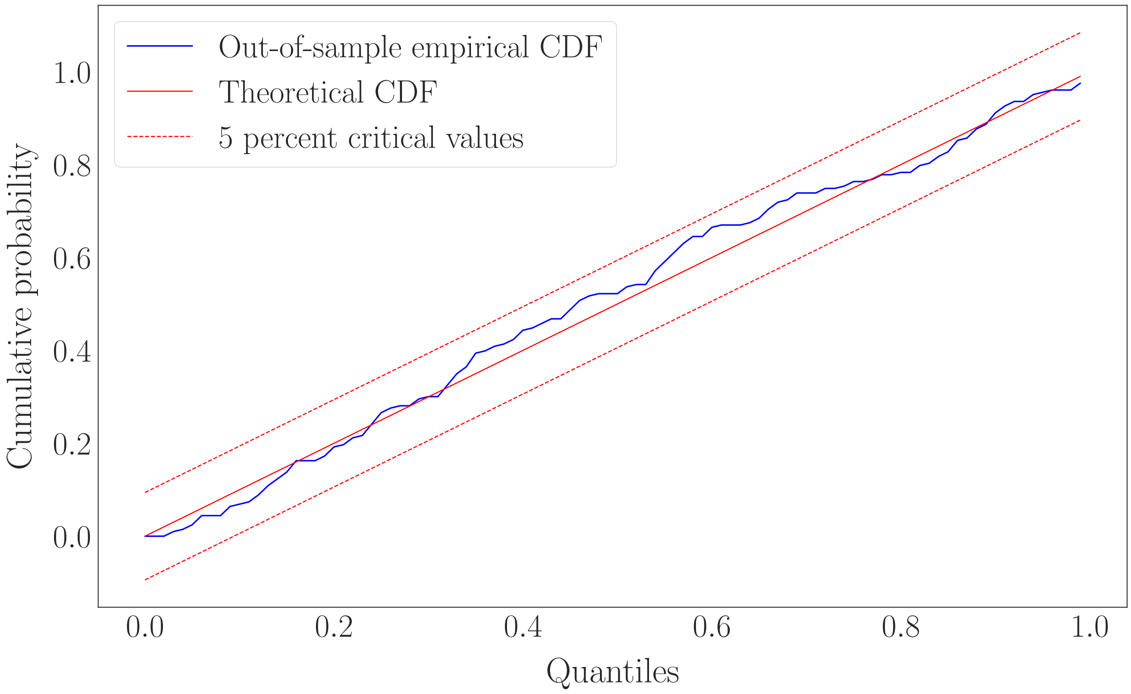
Sources: Banco Mexico and authors calculations

Figure 14: Unconditional Distribution PIT Test, Out-of-Sample



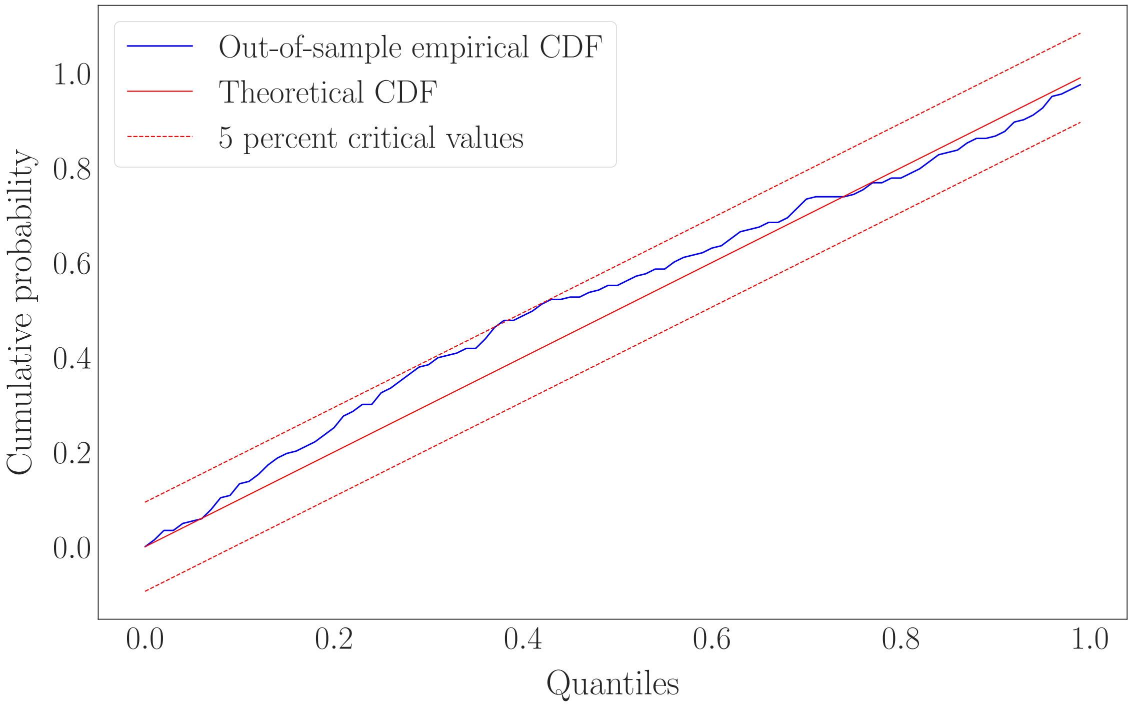
Sources: authors calculations

Figure 15: Quantile Regression Distribution PIT Test, Out-of-Sample



Sources: authors calculations

Figure 16: GARCH TSkew Benchmark PIT Test, Out-of-Sample



Sources: authors calculations

Table 1: Results of the GARCH Estimates

	Microstructure	CIP	Dollar move	Risk Appetite	Baseline
Intercept	-2.33***	-2.23	-1.84	-2.55	-1.63
Lag FX log returns	-0.07***	-0.08***	-0.08***	-0.08***	-0.08***
Bid ask abs	5.73***	24.55	-35.84	-2.48	3.43
Min max abs	35.55***	34.27	34.36***	34.44*	26.16*
Forward points first difference	23.29***	17.85***	26.44***	19.82***	19.44***
Interbank rate vs Libor		33.7***	39.31***	34.76***	33.87***
EURUSD log returns			-0.14***	-0.17***	-0.16***
VIX first diff				15.66***	15.37***
FX intervention dummy lag					2.23
Oil prices log returns					-0.02***
Omega	0.13***	0.13***	0.12***	0.11***	0.12***
Alpha	0.17***	0.17***	0.16***	0.16***	0.15***
Gamma	0.07***	0.06***	0.06***	0.05***	0.05***
Beta	0.98***	0.99***	0.99***	0.99***	0.99***
Nu	8.33***	8.67***	8.92***	8.71***	8.54***
Lambda	0.08***	0.08	0.09***	0.07*	0.08***
R2	5.8 %	6.7 %	10.4 %	27.3 %	27.6 %
R2 adjusted	5.8 %	6.6 %	10.4 %	27.2 %	27.5 %
Number of observations	5986	5986	5682	5682	5680
Significance *10%, **5%, ***1%					

Table 2: Financial Performances with and without Minimum Price

	No minimum price	Minimum price	VaR rule
Daily variation bps	-6.1	108.8	164.8
Average exchange rate	16.6	17.8	18
FX Performance against discretionary	0 %	6.9 %	8.3 %
Total volume bn USD	3.32	3.6	3.32
Number of interventions	18	18	18

Sources: authors calculations

Table 3: Results of the Out-of-Sample Benchmark Tests

	PIT	Logscore diff against Baseline	Diff pvalue
Baseline	Pass		
Unconditional	Fail	-6.36	0
Quantile Reg	Pass	-2.09	0.02
Gaussian EGARCH	Fail	1.235	0.892
TSkew GARCH	Fail	1.536	0.937
Gaussian GARCH	Fail	1.86	0.968

Sources: authors calculations