

Foreign Exchange Intervention Rule for Central Banks: A Risk-Based Framework

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Abstract

This paper presents a new rule for central banks' foreign exchange (FX) interventions, using the concept of Value at Risk (VaR). The VaR rule is used to design an intervention policy that consistently transfers a given share of exchange rate risk from the market to the central bank balance sheet, depending on the economy exposure to exchange rate risk. A VaR-based intervention rule is desirable for countries under floating exchange rate arrangements, where central banks intervene in the FX market to preserve financial stability. This approach is consistent with the price and financial stability mandates of many central banks, including inflation targeters. The VaR rule has other appealing features for central banks, including being forward looking and budget neutral over the medium term. The VaR rule is back-tested on Banco Mexico's publicly available FX interventions data between 2008 and 2016, both with and without a preannounced fixed volatility threshold.

Keywords: Foreign Exchange Interventions, Value at Risk, Exchange Rate at Risk, GARCH

JEL classification: E58 (Central Banks and Their Policies), F31 (Foreign Exchange), G17 (Financial Forecasting and Simulations)

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1 Introduction

The 2018 IMF Annual Report on Exchange Arrangement and Exchange Restrictions classified 65 exchange arrangements as either floating or free floating, of which 38 implemented inflation targeting. In those arrangements, the supply and demand in the foreign exchange (FX) market determine the exchange rate with no predictable path. As a result, the exchange rate risk is not managed by the authorities and remains fully with the private sector, despite the financial stability risk that it entails. As hinted by the breakdown between floating and “free” floating, not all central banks are comfortable with ruling out participation in the FX market. Even those with a strong commitment to floating, for example, Chile, Mexico, and Norway, did intervene in the FX market in cases of exceptional stress, such as the COVID-19 pandemic.¹

Typical FX intervention objectives in a floating exchange rate arrangement are variants of preserving market functioning, for example, smoothing excessive exchange rate volatility and addressing disorderly market conditions, or correcting exchange rate misalignment. The literature adds objectives such as an adequate stock of reserves, external competitiveness, and price stability (Patel & Cavallino (2019), Chamon et al. (2019)). These all circle back to notions of market stability (having enough foreign reserves to intervene later) or exchange rate equilibrium (external and price stability), both of which are underpinned by the notion of macrofinancial risk to the economy.

The main contribution of this paper is to look at intervention through the prism of risk. Each economy presents to a different extent unhedged exposures to exchange rate risk. Unhedged exposures include any direct or indirect exposition to exchange rate risk by any economic agents. They fundamentally depend on structural features of the economies, chiefly their size and degree of openness. The degree of resilience to foreign exchange risk—that is, large swings in the exchange rate—that the economy can absorb varies substantially across countries. As interventions in the FX spot market transfer exchange rate risk off the market, FX interventions can be desirable even for a floating exchange rate regime, as maintaining orderly conditions on the FX market is part of a broad financial stability mandate.

Central banks usually keep a fair degree of opacity about their intervention triggers, which makes it difficult to determine how much exchange rate risk they consider the market could manage on its own and when to intervene. However, it could be inferred from actual intervention if enough data on interventions are published. Other central banks, such as those in Colombia, Guatemala, and Mexico, use transparent intervention rules (Chamon et al., 2019) that reveal,

¹See, for example,

<https://www.fx-markets.com/foreign-exchange/4624581/chile-launches-biggest-fx-intervention-in-20-years>,
<https://www.wsj.com/articles/norwegian-krone-soars-amid-signs-of-central-bank-intervention-11585068173>

at least partially, their risk tolerance.

This paper proposes an empirical methodology based on Value at Risk to determine the trigger of an FX intervention rule anchored to the risk tolerance of the central bank (the VaR rule) that absorbs a constant amount of exchange rate risk and leaves the rest in the market. This risk-based strategy is beneficial both from a financial stability angle and because of its support of the development of the FX risk hedging market. This methodology can also be used to reverse engineer central banks' risk tolerance based on their intervention in the FX spot market.

There are important conceptual caveats to mention. Our paper does not explore the efficiency of FX interventions, as we use a simple reduced-form framework without causal identification. The paper focuses on the timing (the trigger) for intervention and not on the optimal intervention amount. Further work could explore these issues.

This paper relates to the rule-versus-discretion literature. The paper argues that the VaR rule offers many of the benefits attached to the rule-based approach while minimizing common pitfalls of rules. In this sense, it contributes to constant efforts of central banks to improve the design of policy rules (Taylor, 2017) in the domain of exchange rate policy. The most remarkable advantage of the VaR-based intervention is that it keeps the risk transfers constant, while it would increase or decrease, possibility without limit, with exchange rate volatility under fixed-volatility triggers.

The empirical methodology is applied to the case of Mexico. The peso has floated for a long period in one of the most liquid FX markets among emerging economies. In addition, Banco Mexico's (BM) website provides detailed intervention data since 2008. Banco Mexico also implemented interventions both with a minimum bid rate (that is, a preannounced fixed volatility rule) and without a minimum bid rate, providing a diversity of experiences in intervention strategy. Our paper focuses on only one dimension of FX interventions: preventing excessive volatility on the FX market benchmarked against a risk-based metric. Other relevant reasons and motives may well have been factored into BM intervention without a minimum price.

The rest of the paper is organized as follows: Section 2 presents a brief literature review. Section 3 explains the concept of the exchange rate at risk and the formalization of the FX intervention rule. Section 4 presents the empirical framework based on a GARCH model. Section 5 provides the operational framework for central banks using the model for their FX interventions. Section 6 back-tests the model on Mexico public data. Section 7 concludes.

2 Literature Review

The literature on policy rules mainly focuses on the monetary policy decision. The debate is essentially whether the monetary policy decision should be guided by a pre-established reaction function (the rule) or by policymakers' expert judgment (discretion). The reasoning is that rules reduce the cost of disinflation policy if the monetary authorities have not established an inflation aversion reputation by curbing an inflation expectation of rational economic agents (Kydland & Prescott, 1977). Once a reputation has been established, the rule may not be superior to discretion based on sound judgment (Barro & Gordon, 1983).

While less explored in the literature, the same arguments could apply to FX intervention rules. They can likely signal the commitment to a floating exchange rate (within boundaries) and convince rational economic agents (which may challenge the central bank commitment to float) that the exchange rate will experience at least a certain degree of volatility. In addition, FX rules serve to anchor market expectations (Montoro & Ortiz, 2013) and to provide some sense of safety to the market, thereby contributing to its stability.

More generally, central banks' commitment can be used to steer agents' behavior. For example, Krugman (1991) shows that a commitment to intervene as the exchange rate leaves a target zone causes change in the behavior of economic agents, even when there is no explicit intervention. Also, Fanelli & Straub (2020) show that commitment to future interventions is necessary to have an impact on exchange rates today. Finally, Basu et al. (2018) show that commitment has additional benefits over discretion when there are capital outflows and FX reserves may run out.

An important aspect to consider for FX interventions is the source of shock that the central bank would like to mitigate. In micro-founded optimal policy frameworks, the rationale for FX interventions depends on the shock generating the exchange rate movement. For example, in Basu et al. (2020), exchange rate movements owing to permanent real shocks—for example, productivity and commodity prices, and fundamental changes in world interest rates—should generally be accommodated unless they trigger financial constraints. Financial stability depends both on cyclical (exchange rate volatility) and structural factors such that domestic FX hedging, not just the exchange rate volatility, could motivate the central banks to smooth movements associated with higher uncovered interest rate parity (UIP) premia. Therefore, according to this literature, only those exchange rate movements associated with identifiable global financial shocks and growing bid-ask spreads should be included in the rule, while movements arising from commodity price shocks should not be included. However, in practice, the diagnosis of the shock requires judgment (Basu et al. (2020), Cavallino (2019)), and identifying in real time the source of the shock is often not possible. Therefore, the FX intervention rule we present here

has a somewhat narrow focus. The rationale for the VaR rule is to preserve orderly market conditions by preventing excess volatility and tail risks to materialize, irrespective of their source. One important aspect is that any source of shocks could potentially degenerate into financial stability risks, as long as it creates risks to unhedged exposure. This is the reason we do not identify the source of the shock in this paper, which also has benefits in terms of implementation.

In this paper, an intervention is deemed rule based when it reacts to predetermined parameters to deliver predictable responses. The most used rules are based on fixed-volatility triggers such as day-to-day exchange rate change, for example, 2 percent depreciation from the previous day exchange rate close. The rule can be disclosed or kept secret by the central bank, although, in the latter case, it may become transparent with experience. [Patel & Cavallino \(2019\)](#) surveyed 21 emerging market central banks and six out the 21 regularly use an intervention rule, while four do so occasionally.

Some studied the efficiency of FX rules and usually find them less efficient than discretion. However, in some cases, the understanding of rules includes tactics such as "leaning against the wind," that is, delaying the adjustment of the exchange rate, which would not be considered a rule in this paper ([Chutasripanich & Yetman, 2015](#)). In other cases, it involves central banks with an already established preference for floating, for which the literature indicates that the rule may not be superior to discretion ([Fatum & King, 2005](#)). While country-specific empirical work on rule-based interventions (fixed-volatility triggers) was performed for Canada ([Fatum & King, 2005](#)) and Columbia ([Kuersteiner et al., 2018](#)), none was completed for Mexico, to our knowledge.

The concept of VaR, as formalized in [Jorion \(2007\)](#), is frequently used for financial applications, for managing risk exposure, for portfolio allocation, and so on. [Alexander \(2009\)](#) provides a comprehensive review of VaR for market risk analysis. Among many applications of the VaR model, one can cite in the FX field [Al Janabi \(2006\)](#), who proposes to consistently use a VaR framework for managing trading risk exposure of FX securities, in the context of emerging and illiquid markets. [Bredin & Hyde \(2004\)](#) review the performance of a number of VaR methods using a portfolio based on the FX exposure of a small open economy.

Using ARCH/GARCH (Autoregressive Conditional Heteroskedasticity/Generalized Autoregressive Conditional Heteroskedasticity) models for estimating VaR is also standard practice in the literature. [Engle \(2001\)](#) conducts a comprehensive overview of the ARCH/GARCH models in financial econometrics and devotes an entire section to estimating VaR. [Giot & Laurent \(2004\)](#) model daily VaR using realized volatility and ARCH models, and show that it has excellent forecasting performances. [Chan et al. \(2007\)](#) use nonlinear GARCH models to estimate VaR in the presence of a data generating process with heavy tails. Other types of

models could be used to estimate VaR, such as quantile regressions (Gaglianone et al., 2011), copulas (Patton, 2001), and nonparametric kernel (Hoogerheide & van Dijk, 2010). However, the GARCH model is used for this paper because it is a standard model used by market participants and central banks around the world, with widespread implementation on many statistical packages. Besides, as Jeon & Lee (2002) show, FX markets are quite efficient and their features fit well the simple and robust approach of standard GARCH models.

Finally, GARCH models are frequently used for the analysis of the FX markets. Hansen & Lunde (2005) argue that in the context of daily exchange rate returns, nothing can beat a GARCH(1,1) model, while Mcmillan & Speight (2012) show that an intraday GARCH(1,1) model generally provides superior forecasts compared with all other models.

3 VaR Interventions and Exchange Rate at Risk

The Exchange Rate at Risk (*ERaR*) is the percentile at a given threshold θ of the conditional distribution of the exchange rate returns r_t (that is, the VaR). It measures how much a set of investments (here, FX) might lose with a given probability and during a set period, for example, one day. VaR models are commonly used in central and commercial banking for risk analysis and management. Formally, the *ERaR* is the minimum of the set of values verifying the following condition, assuming that the density $f(r_{t+1}|X_t)$ is properly defined over the exchange rate returns support:

$$\mathbb{P}[r_t \leq ERaR|X_t] = \theta$$

Where \mathbb{P} is the probability distribution function (PDF) of the density $f(r_{t+1}|X_t)$.

A conditional VaR depends on a set of variables, which can vary in real time. The distribution on which the VaR is estimated is based on the properties of time series and exogenous regressors, thereby the term "conditional." Following the literature (Sarno & Taylor, 2003), the dependent variable is the log-return r_t of the exchange rate e_t , at a daily frequency (see below). The conditional predictive density of exchange rate returns is $f(r_{t+1}|X_t)$ where X_t is a vector of explanatory variables.

$$r_{t+1} = \log\left(\frac{e_{t+1}}{e_t}\right)$$

The VaR approach fixes how much risk remains in the market and how much is absorbed by the central bank. On the contrary, with fixed-volatility triggers, the risk transferred to the central bank varies with volatility; it is not fixed, as with a VaR. The risk removed from the market can be expressed in USD terms. The GARCH model used to estimate the model delivers

a full-fledged conditional density of the exchange rate logreturns. It is therefore straightforward to estimate the expected shortfall of exchange rate depreciation or appreciation that the central bank is preventing for occurring. If the central bank knows the aggregate level of FX exposure in the system, the risk in USD terms removed from the market is the product of the expected exchange rate shortfall with the system-wide FX exposures.

Under the VaR approach, the policy decision determines how much risk will be transferred, which has macroeconomic underpinning. The quantile θ represents the central bank's commitment to absorb a predetermined portion of the exchange rate risk. Our model gives full discretion to the policymaker to decide on the risk tolerance. Conceptually, the risk tolerance should be a function of the pervasiveness of exchange rate exposures in the economy due to (1) currency mismatches in balance sheets; and (2) exchange rate pass-through to domestic price. The VaR can be expressed as a percentage of the distribution or in nominal terms, if unhedged exposures can be quantified, by applying the percentage to the estimated nominal amount of the unhedged exposure. Then, the VaR can be presented as a percentage of GDP to emphasize macroeconomic implications.

Mathematically, the decision rule is formalized as an intervention region $\mathcal{R}_\theta(r_t)$, based on the economy tolerance thresholds for depreciation and appreciation, respectively (θ_l, θ_u) . The decision rule is binary: if r_t falls within \mathcal{R}_θ , then the central bank intervenes; otherwise, it does not. These regions evolve every day as a function of market conditions. For the rest of the paper, we assume that intervention regions are symmetric, that is, unhedged exposures to the exchange appreciation are as important as unhedged exposures to depreciation, to keep the likelihood of intervention against appreciation equal to the one of intervention against depreciation. Other assumptions can be made depending on the nature of the risk.

$$\mathcal{R}_\theta(r_t) = \{r_t \leq Q(r_t|X_{t-1}, \theta_l)\} \cup \{r_t > Q(r_t|X_{t-1}, \theta_u)\}$$

Where $Q(r_t|X_{t-1}, \theta)$ is the θ -quantile of the conditional log-returns distribution $f(r_{t+1}|X_t)$.

The VaR-based FX interventions formalized above are conceptually appealing, as they are rooted in the principles of modern financial analysis. The points below explain why VaR-based FX intervention complies with these principles:

- VaR-based FX interventions provide an objectifiable approach to FX interventions by shifting the policy focus on the exchange rate risk instead of the exchange rate level. Central banks should let the market operate as long as current FX developments are within their risk tolerance and intervene otherwise. Another benefit of the VaR rule is that it allows the policymaker to anchor its risk tolerance as a function of market and macroeconomic conditions (see more discussion below).

- VaR-based FX interventions are forward-looking, that is, conditional VaRs are forecasted. Central bankers should intervene to prevent imminent risks. Also, forward-looking variable thresholds reduce the risk of opportunistic and strategic behaviors from market participants, who might otherwise take speculative positions when the central bank intervenes in fixed thresholds. Indeed, as market participants are changing behavior, the VaR rule is changing too. This complicates the possibility for market participants to take speculative positions, as they would need to anticipate perfectly how their new behavior, as well as the rest of the market, will shift the conditional FX distribution. On the contrary, with fixed thresholds (for example, plus or minus 2 percent), market participants know exactly when the central bank will intervene.
- VaR-based FX interventions are adaptive to market conditions. The VaR-based triggers loosen or tighten with volatility to keep the likelihood of FX interventions unchanged, thereby accommodating both exchange rate trends and changes in volatility regime. VaR-based FX interventions are, thus, ideal to accompany the transition to more exchange rate flexibility in a smooth and controlled fashion. For example, policymakers could increase the risk tolerance progressively as the FX market depth increases.
- VaR-based FX interventions capture the FX market's nonlinear and asymmetric reactions to shocks. Exceeding a certain level of volatility could result in sudden shifts in expectations, closing large positions, and herding behavior among market participants. Besides, markets may not react equally to exchange rate appreciation and depreciation.
- VaR-based FX interventions incorporate information from several indicators. The VaR includes several exogenous variables (for example, bid-ask spread, FX forward points, interest rate differential, and so on) that bring additional information to the estimation of the conditional volatility. These exogenous variables can be used to factor in the specifics of a given market and customize the measure. Our current modeling approach is parsimonious, but it could be augmented to incorporate more complex models, capturing, for example, structural breaks or regime shifts.
- VaR-based FX interventions support the development of the hedging market, as this makes sure that interventions leave some risk within the market. The central bank is not hedging the market against all risks concerning a certain fixed threshold; it is hedging only against tail risks, which are typically difficult to hedge in the market. Via the VaR rule, the central bank provides a public good (the hedge), while limiting moral hazards stemming from market overconfidence in central bank interventions. VaR-based FX interventions also moderate hedging costs by absorbing some of the risk to preserve market functioning.
- Symmetric VaR-based FX interventions are unbiased and budget neutral. If symmetric, this assumes an equal probability of buying and selling FX, making interventions budget

neutral over time. Besides, VaR-based FX interventions avoid bias in the exchange rate policy in favor of a weak or strong currency. However, asymmetric VaR rule FX interventions could be used, for example, to accumulate reserves by choosing a higher buying frequency than selling.

- VaR-based FX interventions derive from a simple time series model and are, therefore, easy to operationalize. The model requires a limited amount of data, available in the public domain, and their interpretation is unambiguous. VaR triggers are transparent and relatively easy to understand and to communicate to the market while being part of a more global communication strategy from the central bank (see, for example, the newly issued Central Bank Transparency Code published by the IMF in 2020). FX interventions are performed on the wholesale market, where participants are aware of the concept of VaR. The general public might indeed be less familiar with the concept, but this is of little consequence from an operational standpoint. Besides, technical-based rules are objective and strengthen the independence of the central bank against political interference, as these rules are more difficult to bend.
- VaR-based FX interventions are financially sound for the central bank (Annex A). The central bank buys and sells FX with the same probability (if symmetric) as the extremes of the exchange rate distribution, and on the "good side" of the market (selling expensive and buying cheap), thereby using its resources efficiently.

4 Empirical Framework

4.1 Specification

A nonlinear univariate EGARCH-X model is used to estimate the forward-looking exchange rate Value at Risk $Q(r_t | t-1, \theta)$. More precisely, the VaR-based FX interventions model presented here uses an Exponential GARCH (EGARCH) model of volatility with exogenous regressors (EGARCH-X) to adequately capture nonlinearities and asymmetries. The advantage of the EGARCH-X model is that it is a relatively parsimonious model, as it incorporates nonlinearities and asymmetries with only two equations. GARCH models are also very stable, and easy to implement and to understand, compared with more complex forecasting models. Besides, the literature has shown that GARCH models have very decent forecasting performances in the case of FX markets (see, for example, [McMillan & Speight \(2012\)](#)).

That being said, other models (such as copulas, kernel regressions, time-varying coefficient models, and so on) could be used to estimate the conditional distribution in real time, and even intraday. The model presented here is a proof of concept and could be refined by the central bank for actual implementation and operational work. The EGARCH specification incor-

porates three components: drift, volatility, and the distribution of the error terms or innovations.

- **Drift:** $r_t = \textit{Intercept} + \rho r_{t-1} + \beta X_t + \varepsilon_t$ AR-X(p) for the average level of log-returns (r_t , the drift), and X_t a vector of exogenous regressors. The drift reflects the conditional exchange rate trend
- **Volatility:** $\log \sigma_t^2 = \omega + \beta g(r_{t-1})$ with $g(r_t) = \alpha r_t + \gamma(|r_t| - \mathbb{E}[|r_t|])$: where σ_t is the volatility
- **Errors term:** $\varepsilon_t = \sigma_t \epsilon_t$, $\epsilon_t \sim TSK(\nu, \lambda)$: the error term is parameterized for instance with a Tskew distribution (other distributions can be used, including Normal, T distribution and generalized error distributions).

The parameters in **bold** are estimated by the GARCH model.

The estimates use an EGARCH-X(1,1) with Tskew distributed innovations, estimated via Maximum Likelihood Estimation.² The public data and Python codes to replicate the results of this paper are available at <https://romainlafarguette.github.io/software/> GARCH models are a standard approach to estimating and forecasting volatility (Engle, 2001). The number and order of lags are chosen based on AIC/BIC criteria (Akaike Information Criterion/Bayesian Information Criterion). An alternative model is the JP Morgan RiskMetrics model (Zumbach, 2007), with exogeneous regressors and innovations distributed with generalized error distribution, also known as generalized Gaussian distribution (GGD). Annex B presents different specifications and alternative models as a robustness exercise.

Exogeneous regressors are incorporated into the model to capture market microstructures, interest rate parity, and international risk sentiment factors. The objective is to add more information in the estimation of the VaR, thereby refining the estimation of FX intervention triggers. This approach is advantageous because it combines the information of several intervention indicators in a single measure. Although our estimation is done at the daily level, we incorporated some proxies for intraday volatility and liquidity. The estimates include the following regressors:

- **The average exchange rate bid-ask spread over the day**, taken in absolute terms. The finance literature uses the bid-ask spread as a proxy of intraday market liquidity
- **The intraday spread (max–min over the day)**, which is another metric of intraday volatility

²We built a Python wrapper around the comprehensive "ARCH" Python package, developed by Kevin Sheppard from Oxford University (Sheppard, 2020)

- **The daily interest rate differential between local and foreign currencies** captures the interest rate parity arbitrage. We used the daily interest rate differential instead of the cross-currency basis swap, as the former has a longer available time series
- **The one-month forward exchange rate**, expressed as the first difference from the day before. As the previous variable controls for interest rate parity, the forward rate could capture the impact of the cost of hedging on the spot market
- **The first-difference variation of the VIX**, which captures global risk sentiment.³
- **The exchange rate of the US dollar vis-à-vis the euro**, to control for US dollar specific developments
- **Oil prices log returns**, as Mexico is an oil exporter and oil prices could impact the exchange rate

4.2 Estimation Results

As a proof of concept, the model is fitted against a real sample of Mexico FX data (against the US dollar) since 2001. The peso has been floating since 1994. Figure 1 presents the historical level, returns, and returns distribution of exchange rate. The bilateral spot exchange rate exhibits significant volatility in crisis times, such as the global financial crisis of 2008 and, more recently, the COVID-19 pandemic. The historical distribution shows that the returns are skewed to the right (depreciation). The model thus incorporates asymmetries to accurately capture the dynamics of this exchange rate series. Intervention data are available on the BM's website.⁴

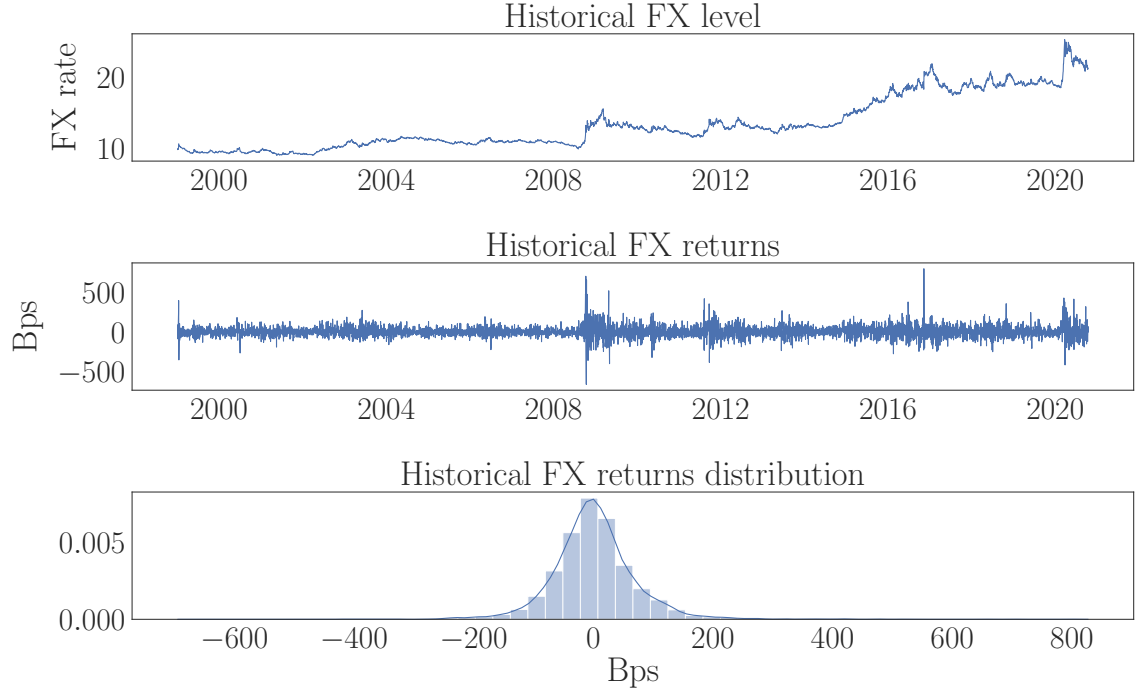
The signs of the coefficients are consistent with macroeconomic and finance theory (see Sarno & Taylor (2003) for an exposition of exchange rate economics). Table 1 presents the output of the GARCH estimation process, where we integrate progressively the coefficients under different specifications, starting from the micro to the macro factors. We have tried to insert year dummies to control for structural breaks, but most of the coefficients were insignificant.

- The bid-ask spread is positively correlated, suggesting that an increase in the bid-ask spread (market illiquidity) signals depreciation
- Forward points also contribute to predicting the movement in exchange rate log returns, with the same sign as expected

³We tried with the CBOE Volatility Index (VIX) in level, but the p-value was higher than in the first difference

⁴<https://www.banxico.org.mx/mercados/subastas-cambio-credito-banco.html>.

Figure 1: Mexican Peso against US Dollar



Sources: Bloomberg and authors' calculations

- A positive interest rate differential with the London Interbank Offered Rate (LIBOR) tends to appreciate the currency, due to carry-trade arbitrage
- The same results hold for the CBOE Volatility Index (VIX), as an increase in global risk sentiment tends to depreciate emerging market currencies
- Changes in the EUR/USD exchange rate have the expected impact on local currency returns and contribute to the explanatory power of the model. Similarly, with the oil price log returns, an increase in oil prices is associated with an appreciation of the local currency against the USD
- The model explains around 28 percent of the log-return variance

Once fitted, the GARCH model provides an in-sample estimation of the conditional volatility, σ_t^2 , over time, presented in Figure 2. Not surprisingly, the in-sample conditional volatility spikes during the global financial crisis and the COVID-19 crisis.

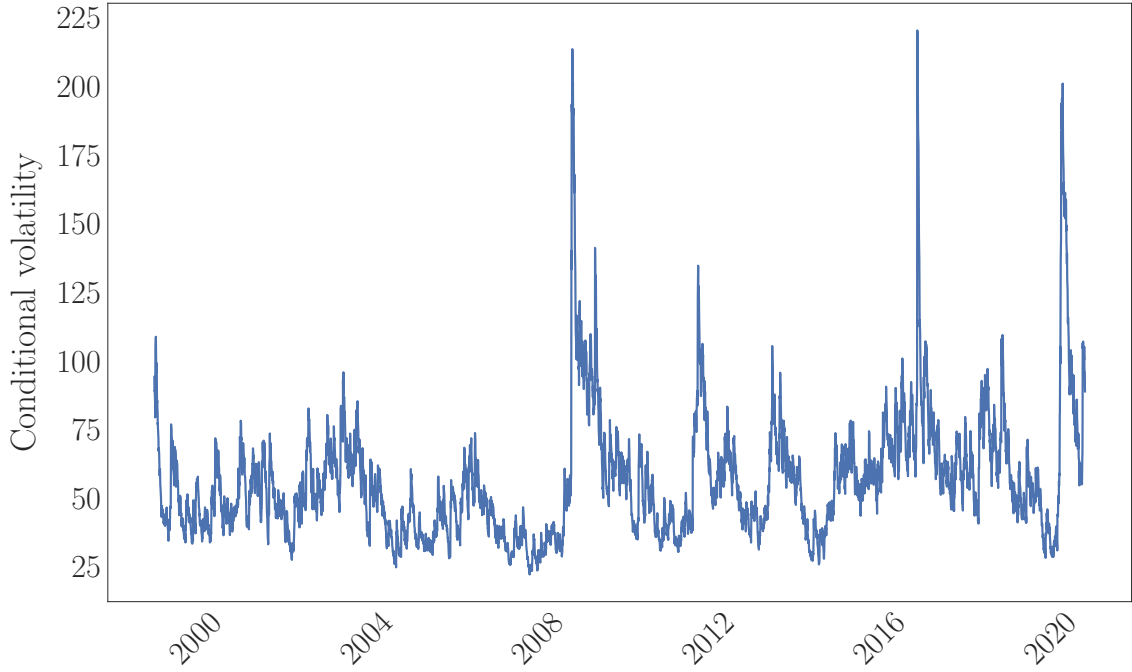
Figure 3 presents an out-of-sample plot of the conditional density, estimated through the fitted GARCH via expanding windows, over a 10-month timeframe (from January through end-October 2020). The plot of the conditional density shows that not only the conditional

Table 1: Results of the GARCH Estimates

	Microstructure	CIP	Dollar move	Risk Appetite	Baseline
Intercept	-2.34	-2.29	-1.74	-2.55	-1.63
Lag FX log returns	-0.07***	-0.08***	-0.08***	-0.08***	-0.08***
Bid ask abs	5.67	24.45	-33.58	-2.68	3.22
Min max abs	35.62	34.68	33.32	34.45*	26.2
Forward points first difference	23.29***	17.79***	26.33***	19.82***	19.44***
Interbank rate vs Libor		33.61***	39.43***	34.75***	33.86***
EURUSD log returns			-0.14***	-0.17***	-0.16***
VIX first diff				15.67***	15.37***
FX intervention dummy lag					2.23
Oil prices log returns					-0.02***
Omega	0.13***	0.13***	0.12***	0.11***	0.12***
Alpha	0.17***	0.17***	0.16***	0.16***	0.15***
Gamma	0.07***	0.06***	0.06***	0.05***	0.05***
Beta	0.98***	0.99***	0.99***	0.99***	0.99***
Nu	8.33***	8.67***	8.92***	8.71***	8.54***
Lambda	0.08*	0.07	0.09*	0.07*	0.08***
R2	5.8 %	6.7 %	10.4 %	27.3 %	27.6 %
R2 adjusted	5.8 %	6.6 %	10.3 %	27.2 %	27.5 %
Number of observations	5986	5986	5682	5682	5680
Significance *10%, **5%, ***1%					

Sources: authors' calculations

Figure 2: Conditional FX Volatility over Time



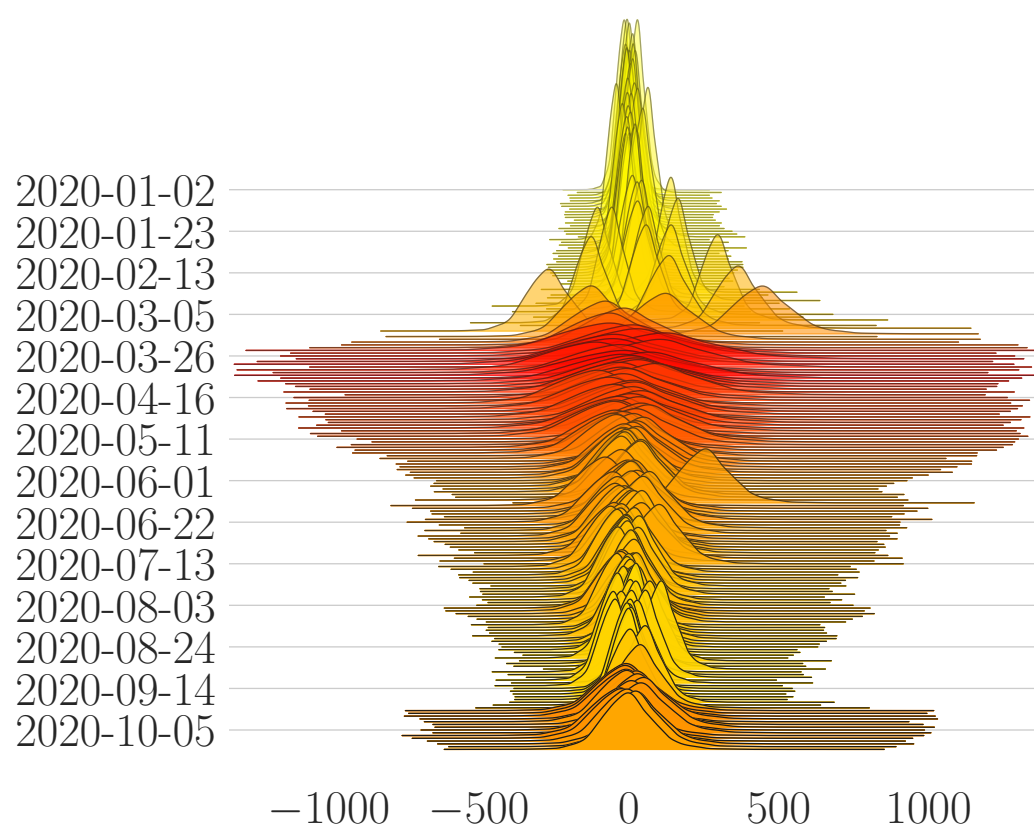
Sources: authors' calculations

volatility widens significantly during the COVID-19 crisis (in March 2020), but that the skewness varies also substantially.

Like other types of forecasting density models, the GARCH model can also be used to produce the so-called fan charts, as shown in Figure 4. The fan chart presents the forecasted quantiles of the conditional distribution across time and provides an intuitive way to understand the uncertainty surrounding the mean forecasts. In this case, the uncertainty is precisely captured by the GARCH model, both through the estimation of the conditional density and also via higher moments of the distribution, including the skewness.

Finally, the quality of the GARCH forecasting density model is evaluated out-of-sample. The correctness of the density model specification is assessed via a probability integral transform (PIT) test (Diebold & Tay (1998); Rossi & Sekhposyan (2019)). The PIT test is evaluated over four-month out-of-sample daily data, as presented in Figure 5. The PIT test outcome presented in Figure 5 indicates that the empirical distribution of the PIT is within the confidence band for all quantiles. This pattern suggests that the conditional density derived from the EGARCH-X models has a satisfactory out-of-sample accuracy and that it generates robust predictive distributions that capture well upside and downside risks. Annex B presents a series of alternative models (unconditional, quantile regressions, variation around GARCH, and so

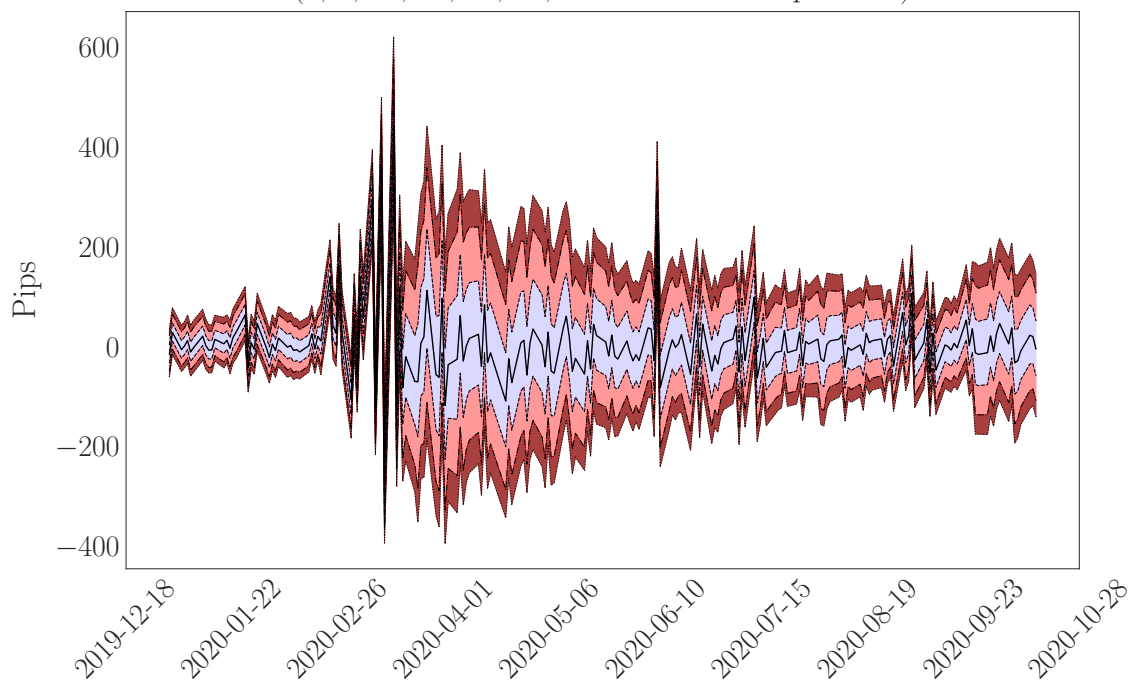
Figure 3: Out-of-Sample Conditional Density



Sources: authors' calculations

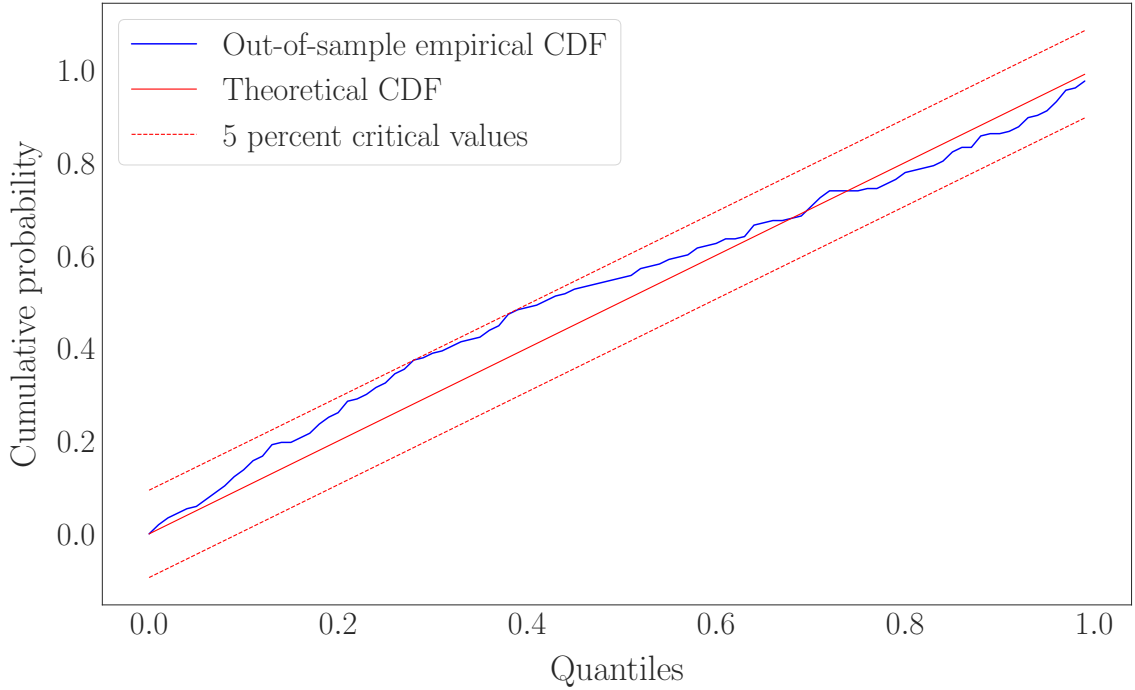
Figure 4: Out-of-Sample Fan Chart

Fan chart of predictive FX log returns
(1, 5, 10, 25, 75, 90, 95th conditional quantiles)



Sources: authors' calculations

Figure 5: Probability Integral Transform Test



Sources: authors' calculations

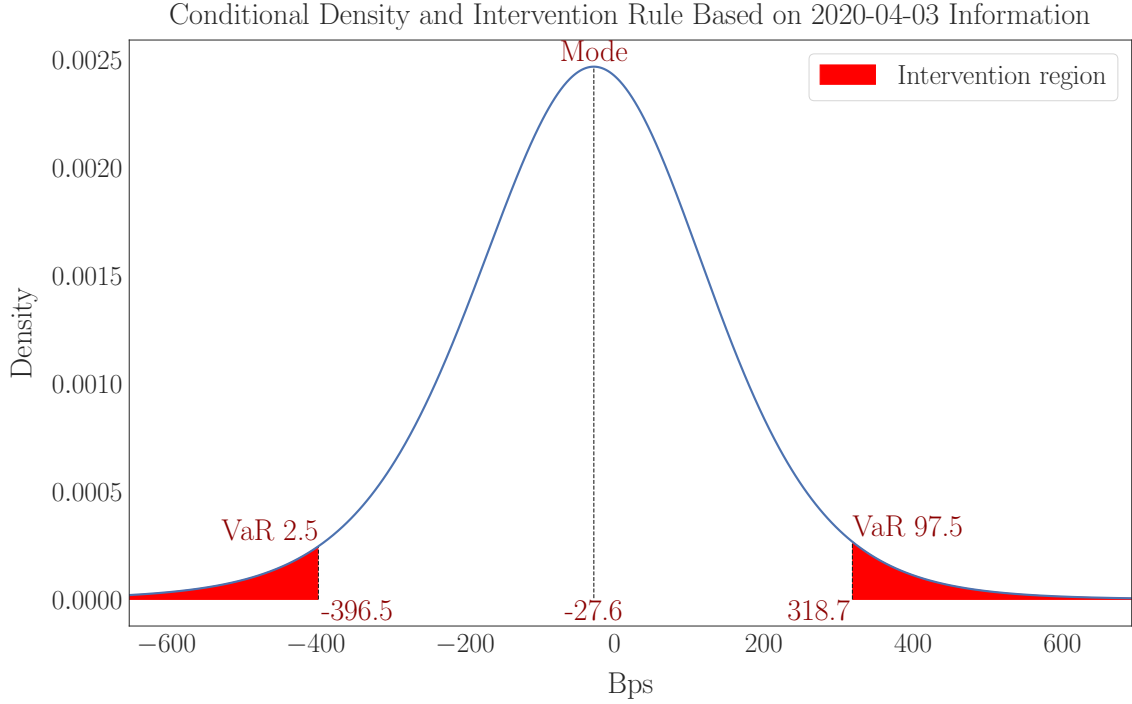
on) to benchmark the performance of our model, also using log score performance metrics against other models. The result of the tests is that the GARCH family of models dominates the alternative tested against an unconditional kernel density estimator and conditional density estimated from the quantile regression. Also, within the GARCH family (Gaussian, Tskew, GARCH versus EGARCH, and so on), there is no significant improvement in performance against our baseline model.

5 Operational Framework

5.1 Risk-Based Triggers

From the forecasted conditional VaR, it is straightforward to infer daily intervention regions for depreciation and appreciation, as explained in Section 3. In practical terms, at the end of each day, the central bank re-estimates the model and assesses the intervention regions for the day after. On that day, the central bank monitors the cumulated returns of the exchange rate—either depreciation or appreciation. Once the cumulated returns reach the intervention regions, the central bank buys or sells FX. These regions evolve every day as a function of market conditions. On the specific day presented in Figure 6, intervention happens only if the

Figure 6: VaR FX Intervention Rule Based on a Given Information Set



Sources: authors' calculations

exchange rate depreciates by more than 3.3 percent or appreciates by more than 3.7 percent, for a 5 percent ERaR (2.5 percent on each side). However, a new threshold would be computed every day, for each possible quantile, as shown on the fan chart (Figure 4).

Over time, the VaR-FXI rule can be represented by the region where the cumulative distribution function falls outside the central bank risk tolerance, for example, the 2.5th and 97.5th percentiles, as presented in Figure 7.

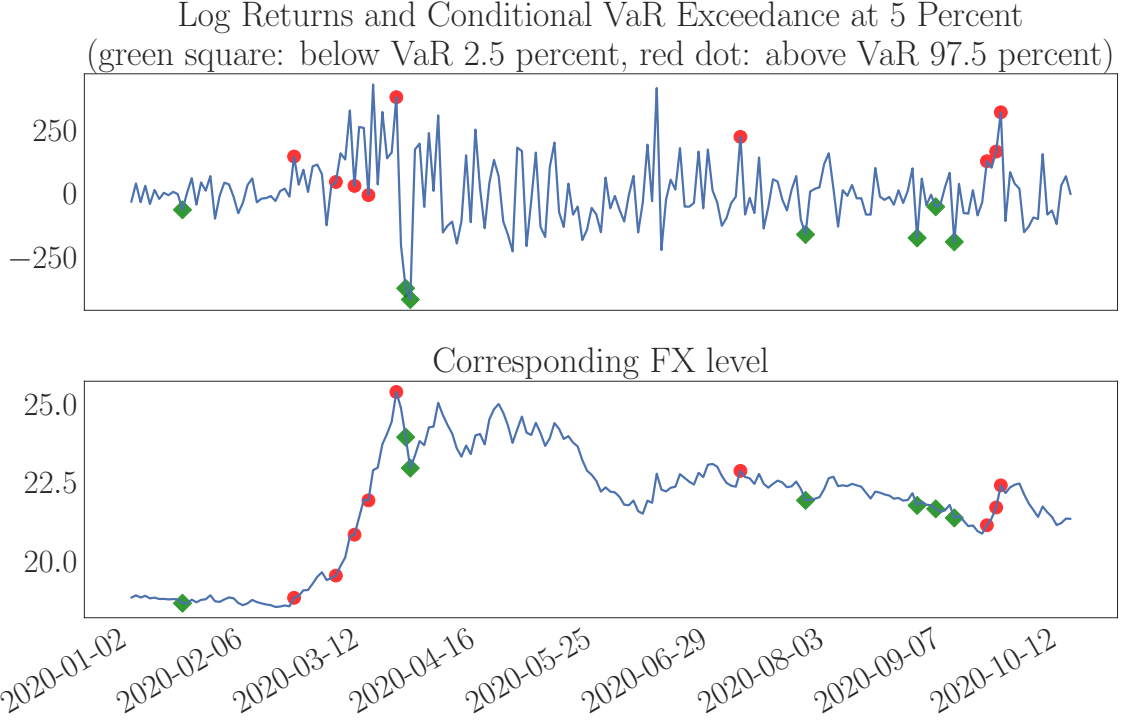
Had the BM followed the VaR FX intervention rule, it might have intervened in about 15 days over 10 months, from January through October 2020. Figure 9 presents the intervention days within the daily log-returns plot. It highlights with green dots (below ERaR 2.5th percentile) and red dots (above the ERaR 97.5th percentile) the occurrences falling in the intervention regions. The lower chart presents the corresponding FX level in which the central bank would have intervened. Fifteen days of intervention correspond to about 7.5 percent of the period, even when using an intervention region of 5 percent. The frequency of interventions would increase when volatility is unusually high, which was the case during the COVID-19 crisis. However, this exercise is purely counterfactual, and it is likely that after the first FX intervention, the FX volatility would have decreased, hence reducing the need for future interventions.

Figure 7: Conditional Cumulative Distribution Function and Intervention Thresholds



Sources: authors' calculations

Figure 8: Conditional VaR Exceedance, Out-of-Sample



Sources: authors' calculations

5.2 FX Intervention Calibration under Risk-Based Interventions

One of the major advantages of the VaR FX rule is that it provides certainty about the frequency of FX interventions over time. For example, with a risk tolerance of 2.5 percent both for depreciation and appreciation, the central bank will, on average, intervene 5 percent of the time. The central bank might temporarily deviate over a short period of time—depending on market conditions—but the VaR approach guarantees that over a sufficiently long period of time, the frequency is the same as the risk tolerance.

Another substantial benefit of the VaR rule is that it can be designed to be budget neutral for the central bank over the medium term. Should the central bank decide to operate with a symmetric rule (the same risk tolerance for appreciation and depreciation), it will intervene the same number of times on both sides, hence offsetting selling FX with buying FX.

Over the short term, the central bank may temporarily sell more FX than it buys, depending on market conditions. The fixed-frequency feature of the VaR can, therefore, be used

to determine the budget constraint of the central bank, that is, that the central bank has enough reserves to conduct the interventions in a credible and efficient way. A conservative and straightforward approach is to make sure that the maximum amount of daily intervention times the frequency on the depreciation side (the risk tolerance) is always less than the amount of FX reserves the central bank has for FX interventions. The maximum amount of daily intervention should be enough to have an impact on the exchange rate when the central bank intervenes, for example, by making sure it represents a significant share of the average daily market turnover across time.

6 The Case of Mexico

The Banco Mexico intervened via two types of auctions whose main difference was the reservation rate applied to each auction. In the first type, the BM operated an auction every day with a preannounced reservation rate, that is, the minimum rate for eligible bids. The auction with a minimum price was suspended in April 2010 and reintroduced with changes in November 2011 and suspended again after April 2013.⁵ On many days, the rule-based auction did not receive any demand, as the market was operating below the reservation rate. In the second type, the auction was organized when the BM found it opportune and did not have a reservation rate (an auction without a minimum rate). For some periods (for example, during 2015), daily interventions were preannounced without a minimum price. Size was also preannounced and later adjusted.

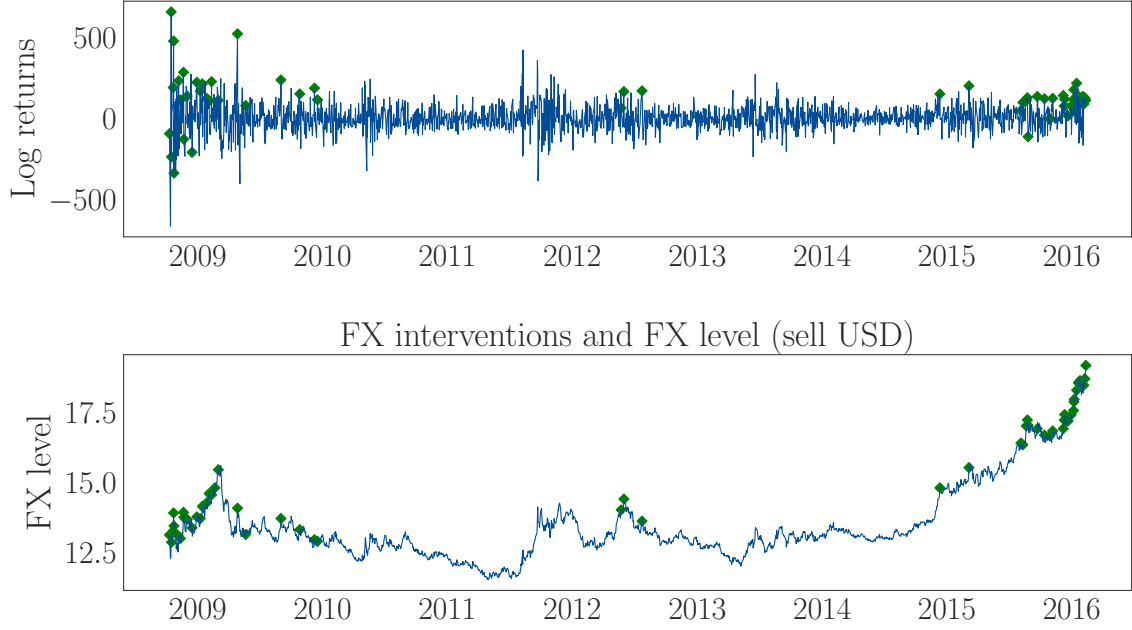
The BM may have intervened for more reasons than preventing disorderly market conditions. The exact rationale for intervention is outside the scope of the paper: we are focusing here on benchmarking the BM FX interventions against a risk framework, while recognizing that the BM could have had other objectives in mind. We are not assessing the efficiency, the relevance, or the rationale of the BM interventions.

6.1 Interventions with a Minimum Price

"Auctions with a minimum price" were triggered 31 times from October 2008 through November 2016. The auction reservation price was set at the previous day's exchange rate close (MEX/USD) multiplied by 1.02 (for example, 2 percent depreciation) from October 9, 2008, to December 8, 2014, by 1.015 from December 9, 2014, to November 23, 2015, and by 1.01 from November 24, 2015, to January 5, 2017. Auctions with a minimum price were suspended after January 5, 2017. Figure 9 presents the auctions with a minimum price in green, with the

⁵See <https://www.banxico.org.mx/markets/auctions-with-minimum-price-e.html#formoredetails>

Figure 9: FX Interventions Log Returns with Minimum Price



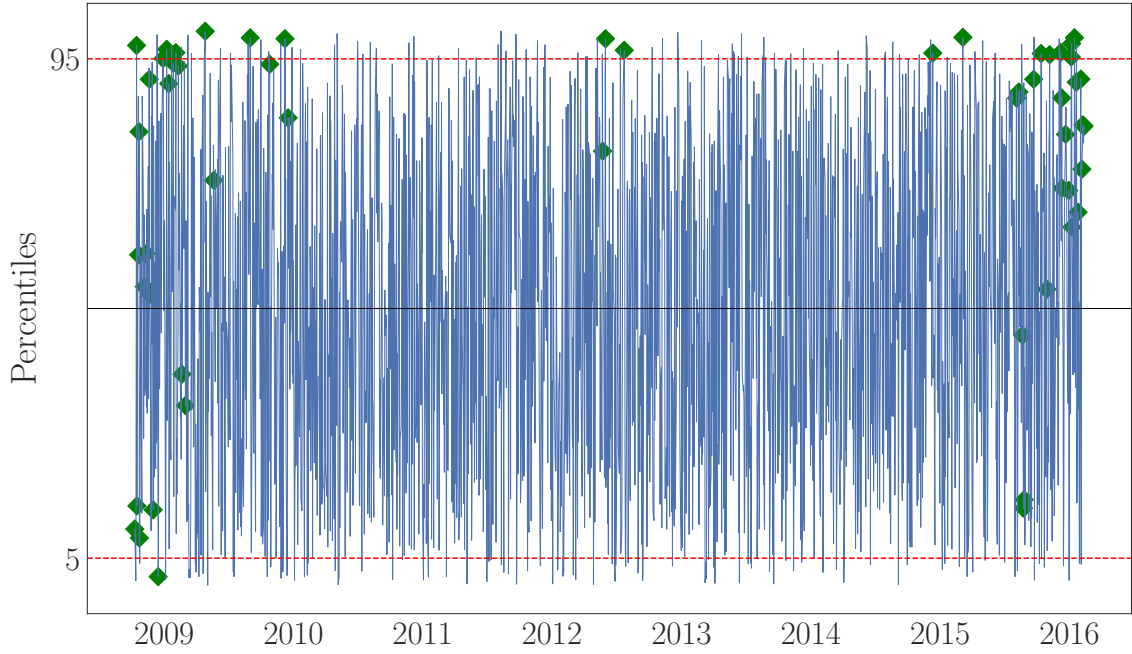
Sources: Banco Mexico and authors' calculations

corresponding FX daily log returns and level.

The VaR framework can be used by central banks to inform intervention decisions, as well as by external observers to assess central bank tolerance for exchange rate risk. First, the framework can be used to assess whether the central bank responded to exchange rate volatility and the degree to which it was tolerated. Second, the framework can be used to assess whether the interventions efficiently achieved the authorities' objectives.

The ex-ante conditional cumulative distribution function (CDF) estimated via the GARCH model is used to benchmark the Banco Mexico FX interventions. Although the central bank did not use the model to intervene, one can always assess the corresponding FX rate conditional quantile when the central bank has intervened. The outcome is presented in Figure 10. The back-testing exercise shows that most FX interventions have occurred on the top of the conditional distribution—where the depreciation was the largest—and often above the 95th percentile. A few outliers occur at the median or below, but these cases are rare. Overall, the interventions with the minimum price approach were often but not always equivalent to a risk framework: the central bank intervened most of the time when the pressure on the exchange rate was the largest, with exceptions.

Figure 10: Conditional CDF of FX Intervention with Minimum Price



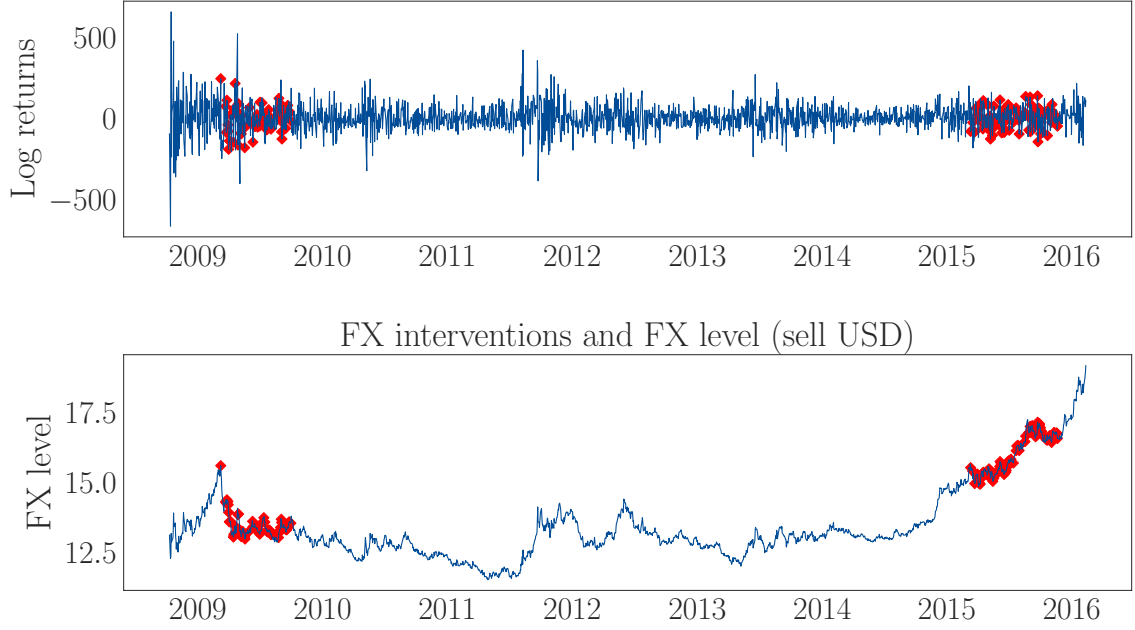
Sources: Banco Mexico and authors' calculations

6.2 Interventions without a Minimum Price

The BM organized 319 "auctions without a minimum price" from September 2009 through November 2015. Interventions without a minimum price overlapped with "auctions with a minimum price," which could constrain the volatility distribution even in the absence of actual intervention under the rule—the mere presence of the rule influences market participants' behavior. Figure 11 presents the FX interventions without a minimum price on the Mexican pesos in red, with the corresponding FX daily log returns and level.

As in the case of interventions with a minimum price, the conditional CDF can be used to benchmark the FX interventions relative to the risk of the day, as presented in Figure 12. While interventions with a minimum price exhibited a pattern broadly consistent with large depreciation pressures on the exchange rate, the interventions without a minimum price have no clear risk patterns. Such interventions had occurred across the full conditional distribution, even when the pressures were not on the depreciation side but, on the contrary, the appreciation side. In this case, the fact that the central bank was selling USD could have, therefore, amplified the currency appreciation. This pattern likely suggests that the BM rationales for FX intervention under the no-minimum-price rule are likely broader than just the volatility of the exchange rate. For example, during certain periods, the motivation for interventions without a minimum price was to prevent excessive accumulation of foreign reserves, explaining that those interventions

Figure 11: FX Interventions without a Minimum Price on the Mexican Peso/US Dollar



Sources: Banco Mexico and authors' calculations

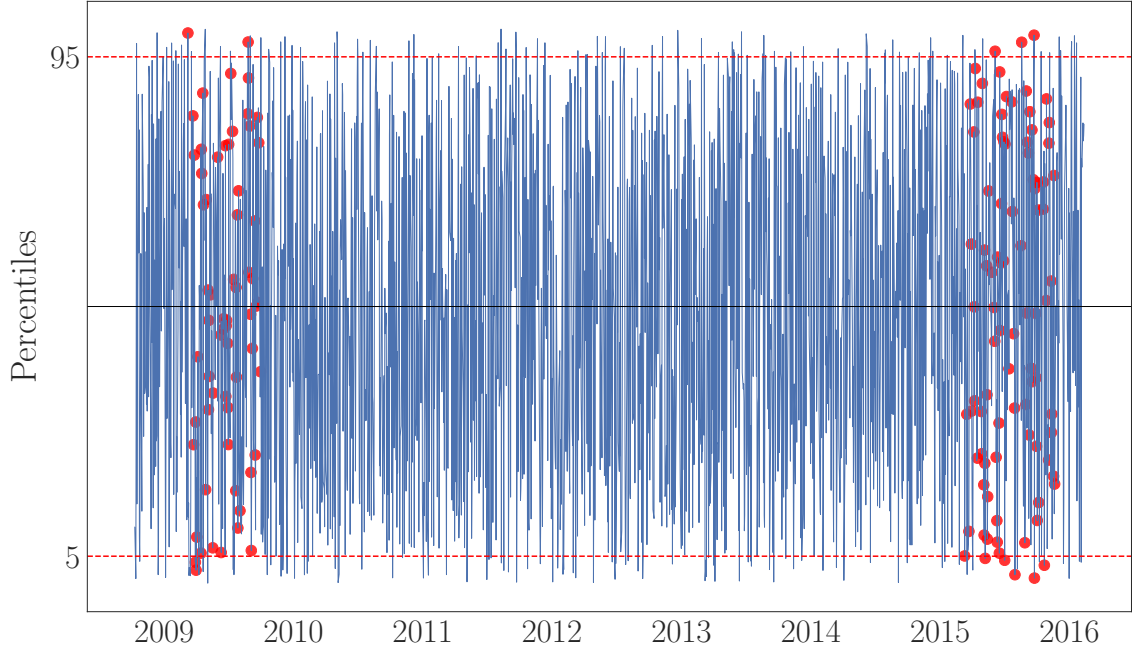
were not related to rate development.

7 Conclusion

FX interventions in the foreign exchange market would benefit from a risk-based approach. Central banks use quantitative tools such as VaR to measure the risk they are taking when they invest their foreign reserves or accept assets as collateral for their loans. In this paper, we argue that central banks could also use this tool when they intervene to support the FX market, as intervention is tantamount to removing some exchange rate risk from the market. A risk-based approach has very convincing advantages compared with other types of rules, such as fixed-volatility, or interventions without a minimum price, because it allows the central bank to control exactly the exchange rate risk to which the economy is exposed. It adds also the benefits of being budget neutral over the medium term, forward looking, and combining several market indicators into one.

The back-testing based on Mexico data is supportive of rule-based FX intervention, as far as the risk dimension is concerned. Our framework suggests that fixed rules are broadly consistent with a high risk level. On the contrary, interventions without a minimum price appear to be driven by other rationales than exchange rate volatility and risk.

Figure 12: FX Interventions without a Minimum Price on the Mexican Peso/US Dollar



Sources: Banco Mexico and authors' calculations

No central bank is currently known to use the VaR rule, because it comes across as difficult to communicate or because policymakers prefer to remain discrete about the intervention decision. We argue that transparency could help to alleviate communication challenges because VaR-based interventions provide objective and simple triggers in terms of a bilateral exchange rate. However, our VaR rule should be part of a broader communication and operationalization strategy of the central bank in order to guarantee a smooth and efficient implementation.

While policymakers are entitled to maintain full discretion over policymaking, the paper argues that using VaR as an input would contribute to a better informed decision, by anchoring the decision to intervene based on policymaker risk tolerance and structural weaknesses of the economy, for example, unhedged exposures. In summary, the VaR rule, while not currently widely used, could be considered as one option to improve the rules that central banks use.

The model of this paper may have several policy implications. The VaR rule would be particularly helpful for central banks concerned about financial stability risk arising from the exchange rate, but committed to a floating exchange rate arrangement in the context of an inflation targeting monetary policy framework. The central banks could accompany the transition to exchange rate flexibility by formalizing the commitment to the float in a rule and by

gradually adapting interventions to allow for more exchange rate volatility. Finally, the VaR rule could be used to foster the development of hedging markets by keeping a fixed amount of risk in the market (usually most of it) while absorbing some.

Looking beyond interventions in the spot FX market, the same model and method can be used to measure the risk transfers from any markets that the central banks deem useful to intervene in to support financial stability. In particular, VaR models can be used to measure risk based on spreads such as (1) between actual and theoretical forward rates for the exchange rate hedging market; (2) between onshore and offshore USD interest rates for local FX funding market; and (3) between risky and risk-free rates for the local currency-denominated fixed income market.

One issue that remains to be addressed is how to determine the risk tolerance in VaR. The paper quotes the factors underpinning it in broad terms, that is, direct and indirect exposures to exchange rate risk. However, it is, so far, left to the policymakers' judgment to translate those vulnerabilities into a risk tolerance level (a target VaR). A method that estimates the extent of the vulnerabilities and combined them with their impacts on macrofinancial stability would be most useful.

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A Comparative Static Financial Performances

This appendix presents a comparative static evaluation of the financial performance of the three different intervention strategies studied in this paper: the strategy without a minimum price, the strategy with a minimum price (rule-based), and the VaR rule strategy. We selected a one-year period of intervention, between October 2015 and October 2016. The frequency of intervention under the no minimum price and the minimum price strategies during this time was the same (18 interventions each), with similar volumes (USD 3.32 billion and USD 3.6 billion, respectively). Then, we constructed a series of interventions following the VaR rule at a similar frequency (18 interventions), corresponding to a 10 percent intervention threshold, only on the selling side to match the other strategies. We calibrated the intervention amount of the VaR rule to be at the average under the no minimum price strategy so that the total amount of intervention under the VaR rule exactly matched the intervention amount under the no minimum price strategy.

This financial benchmarking is purely a comparative exercise. It might be that some interventions without a minimum price complemented interventions with a minimum price and were executed at a less favorable time. The comparison is not taking such a selection effect into account and should not be interpreted as definite proof of the financial performance of one strategy against another. Under this caveat, the benchmarking shows that the rule-based strategies (either minimum price or VaR based) outperform the minimum price strategy, as presented in the table below.

Table 2 reports that on average, under the no minimum price rule, the central bank was selling USD when the local currency was slightly appreciating against the USD (by -6 bps; the negative value in this quotation means appreciation). Under the minimum price and the VaR rule schemes, on the contrary, the central bank was selling USD during a more favorable time. By design, the VaR rule is selecting days with particularly large depreciation, in the order of 160 bps. Likewise, the minimum price and VaR rule scheme were executed at much better rates in terms of level. While the VaR rule is designed to target the tails of the conditional distribution of the log returns, it does not necessarily guarantee that the intervention will happen at the most favorable exchange rate level. Our simulation shows that on average, it does nevertheless trigger interventions under better terms than for the two other schemes.

Looking at the performance against the strategy without a minimum price, computed against the gain realized by selling on average the same volume of USD at different points in time, the benchmarking exercise indicates that the minimum price intervention would outperform the strategy without a minimum price by 7 percent, while the VaR rule would outperform it by 8.3 percent.

Table 2: Financial Performances with and without Minimum Price

	No minimum price	Minimum price	VaR rule
Daily variation bps	-6.1	108.8	164.8
Average exchange rate	16.6	17.8	18
FX Performance against discretionary	0 %	6.9 %	8.3 %
Total volume bn USD	3.32	3.6	3.32
Number of interventions	18	18	18

Sources: authors' calculations

Table 3: Results of the Out-of-Sample Benchmark Tests

	PIT	Logscore diff against Baseline	Diff pvalue
Baseline	Pass		
Unconditional	Fail	-6.36	0
Quantile Reg	Pass	-2.09	0.02
Gaussian EGARCH	Fail	1.235	0.892
TSkew GARCH	Fail	1.536	0.937
Gaussian GARCH	Fail	1.86	0.968

Sources: authors' calculations

B Out-of-Sample Benchmarking

This appendix presents a series of alternative models to estimate the conditional density of foreign exchange log returns over time. Our baseline model relies on an EGARCH(1,1) with a Tskew parametrization of the error terms' distribution. The conditional benchmark models use the same sets of variables as our baseline model, presented in Section 4.1, and the same training/testing set.

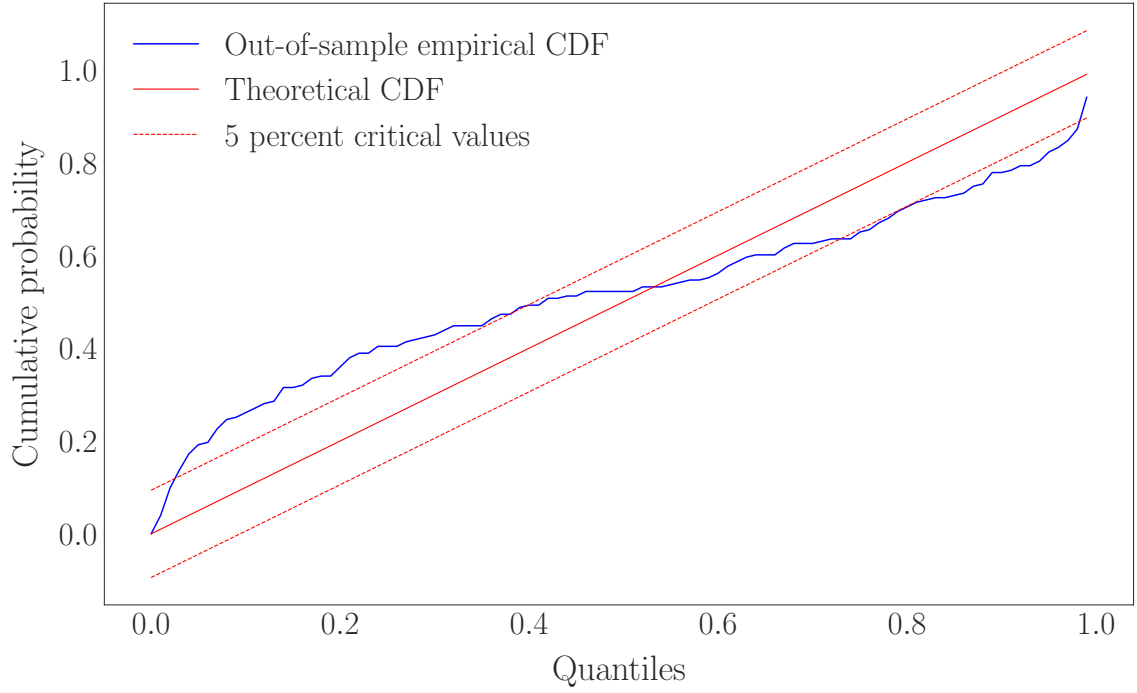
We conduct benchmarking against:

- Different parametric variations of the GARCH model
 - EGARCH(1,1) with Gaussian errors
 - GARCH(1,1) with Tskew
 - GARCH(1,1) with Gaussian errors
- Density estimation via quantile regressions. The full-fledged density is obtained via quantile interpolation and resampling, following the rearrangement procedure of [Chernozhukov et al. \(2010\)](#).
- Unconditional distribution, estimated via Gaussian KDE on historical data

We assess the performance metrics of these models out-of-sample against our baseline model using (1) PIT tests and (2) a density log score, as explained in [Diks et al. \(2011\)](#); we use the uncensored version of the log score.

We summarize the results in Table 3. We define "pass the PIT test" as: if the estimated invert CDF distribution lies within the uniform distribution +/- 5 percent interval confidence bands, as defined in [Rossi & Sekhposyan \(2019\)](#), and "fail" otherwise. Regarding the density log score, we provide both the test statistic (the average difference of the log score across time) and the p-value of the test statistic, as explained in [Diks et al. \(2011\)](#). A positive test statistic indicates that the model outperforms the baseline model, while the p-value tested the null hypothesis that the log scores of the two models are the same.

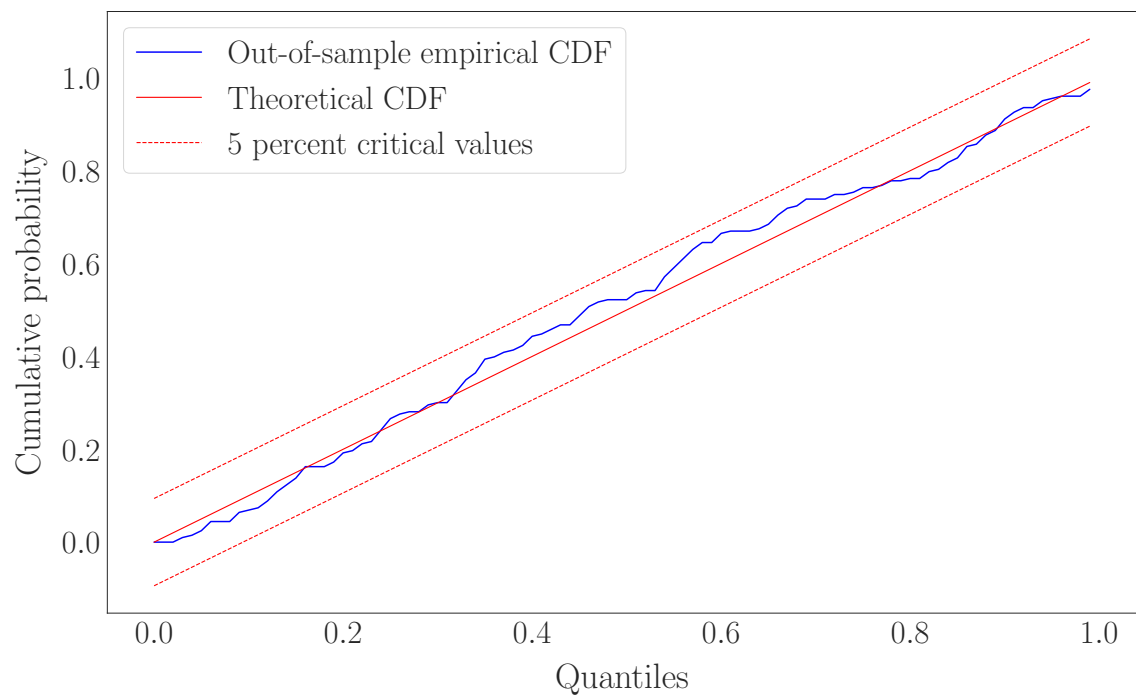
Figure 13: Unconditional Distribution PIT Test, Out-of-Sample



Sources: authors' calculations

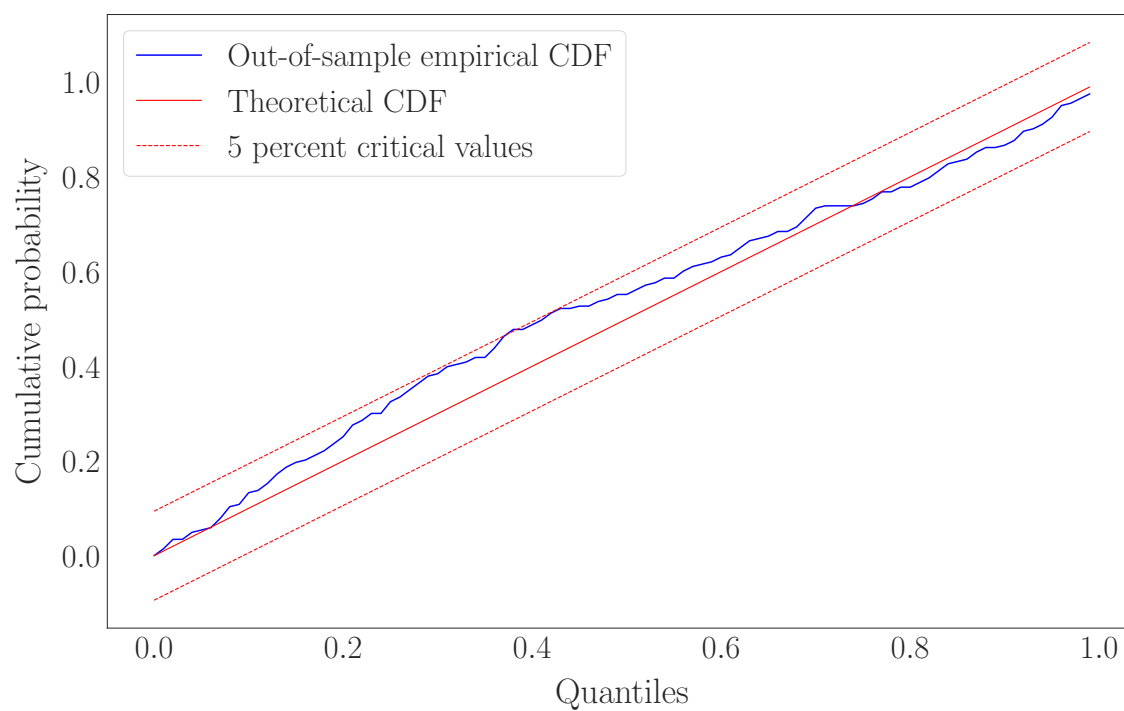
As Table 3 shows, the baseline model has a better log score than the unconditional distribution and the daily distribution fitted from the quantile regressions model with resampling, and these two models both fail the PIT test. However, the difference against variation of the GARCH model is less clear cut: while the other models outperform the baseline in terms of log score, the difference is not significant and they also fail the PIT test (albeit only by a very few numbers of percentiles). Therefore, while it is clear that the GARCH specification is appropriate to model FX returns—in line with the literature—variations around the type of GARCH and implied distributions are statistically equivalent.

Figure 14: Quantile Regression Distribution PIT Test, Out-of-Sample



Sources: authors' calculations

Figure 15: GARCH TSkew Benchmark PIT Test, Out-of-Sample



Sources: authors' calculations