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From Point Forecasts to Distributional Forecasts

- A forecast $\hat{y}_{T+h|T}$ is (usually) the mean of the conditional distribution: $Y_{T+h}|Y_1, \ldots Y_T$
- Most models produces Gaussian distributed forecast
- Because models assumes Gaussian residuals: remember that the residuals are the stochastic components that are determining the distribution of both the estimators and the forecasts
- The forecast distribution describes the probability to forecast any future value

Distributional Forecasts

Assuming the **residuals are uncorrelated** with variance σ^2, and with an estimate $\hat{\sigma^2}$

Model Typology

- **Mean:** $Y_{T+h|T} = \mathcal{N}(\overline{Y}, (1 + \frac{1}{T})\hat{\sigma^2})$
- **Naive:** $Y_{T+h|T} = \mathcal{N}(Y_T, h\hat{\sigma^2})$
- **Seasonal Naive:** $Y_{T+h|T} = \mathcal{N}(Y_{T+h-m(k+1)}, (k+1)\hat{\sigma^2})$
- **Drift:** $Y_{T+h|T} = \mathcal{N}(Y_T + \frac{h}{T-1}(Y_T - Y_1), h\frac{T+h}{T}\hat{\sigma^2})$

- where k is the integer part of $\frac{h-1}{m}$
- Note that when $h = 1$ and T is large, these all give the same approximate variance $\hat{\sigma^2}$

Prediction Intervals

Definition

A prediction interval gives **a region** within which we expect Y_{T+h} to lie with **a specified probability**

- Assuming that the forecasting errors are normally distributed, then a 95% prediction interval is:

$$Y_{T+h|T} \pm 1.96\hat{\sigma}_h$$

- where $\hat{\sigma}_h$ is the standard deviation of the h-step distribution
- when $h = 1$, $\hat{\sigma}_h$ can be estimated from the residuals

Prediction Interval Fit

```
brick_fc %>% hilo(level = 95)
```

```
## # A tsibble: 80 x 5 [1Q]
## # Key:       .model [4]
##    .model         Quarter       Bricks .mean        `95%`
##    <chr>            <qtr>       <dist> <dbl>       <hilo>
##  1 Seasonal_naive 2005 Q3 N(428, 2336)   428 [333, 523]95
##  2 Seasonal_naive 2005 Q4 N(397, 2336)   397 [302, 492]95
##  3 Seasonal_naive 2006 Q1 N(355, 2336)   355 [260, 450]95
##  4 Seasonal_naive 2006 Q2 N(435, 2336)   435 [340, 530]95
##  5 Seasonal_naive 2006 Q3 N(428, 4672)   428 [294, 562]95
##  6 Seasonal_naive 2006 Q4 N(397, 4672)   397 [263, 531]95
##  7 Seasonal_naive 2007 Q1 N(355, 4672)   355 [221, 489]95
##  8 Seasonal_naive 2007 Q2 N(435, 4672)   435 [301, 569]95
##  9 Seasonal_naive 2007 Q3 N(428, 7008)   428 [264, 592]95
```

Why Prediction Intervals Matter

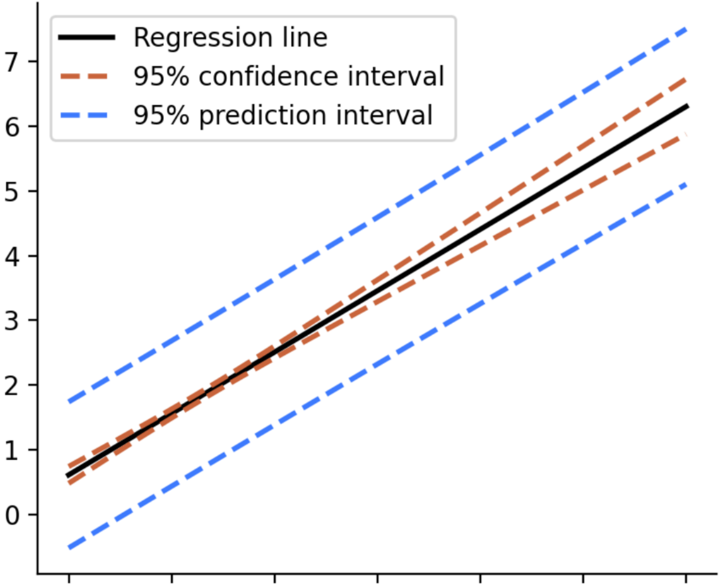
- **Point forecasts are often useless without a measure of uncertainty** (such as prediction intervals)
- Prediction intervals require a **stochastic model** (with random errors, etc.)
- For most models, prediction intervals get wider as the forecast horizon increases
- The degree of confidence (the probability) impacts the width of the prediction interval
- Usually too narrow due to unaccounted uncertainty: pay attention

Difference between Confidence Interval and Prediction Interval

- A confidence interval informs about where the true parameter of a model can be
  - **Confidence interval quantifies the uncertainty about the model**, or the distance between the model and reality
  - Confidence interval are associated with a wide range of parameters, values, etc.
  - They inform about how the model represents well the reality
  - Wide confidence intervals are associated with a less accurate model, and/or a very volatile model

- The prediction interval predicts in what range a future individual observation will fall
  - **Prediction interval quantifies the uncertainty about the future**, or the distance between today and the future
  - Prediction interval are not about the parameters of the model, but about the dependent variable (y_t)
  - The problem is that prediction intervals tend to neglect the uncertainty about the parameters used to generate the forecasts...

Difference between Confidence Interval and Prediction Interval



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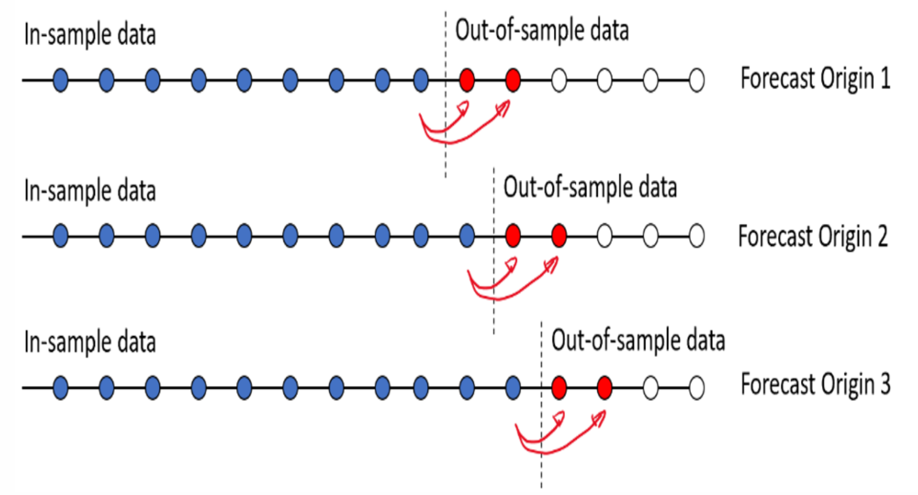
Fitting and Forecasting

Be careful

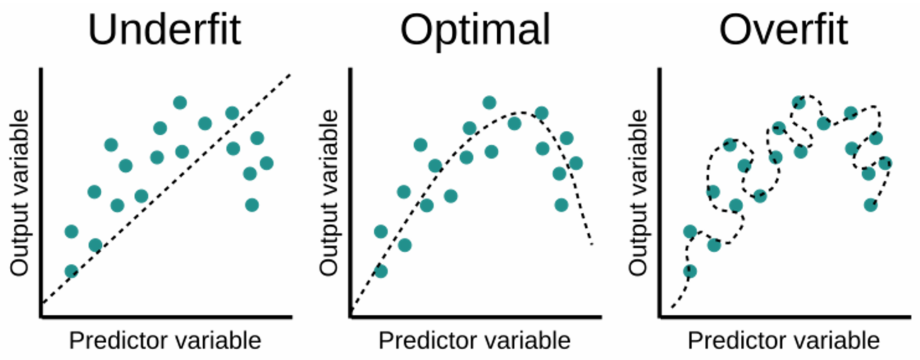
A model that fits the data well (in sample) might not necessarily forecast well

- A perfect in-sample fit can always be obtained by using a model with with enough parameters
- Over-fitting a model to data is just as bad as failing to identify a systematic pattern in the data
- Need to split the model between
- The test set must no be used to *any* aspect of model development or calculation of forecasts
- Forecast accuracy is only based on the test set

Train and Test Set



Underfit, Optimal, Overfit



Forecast Errors

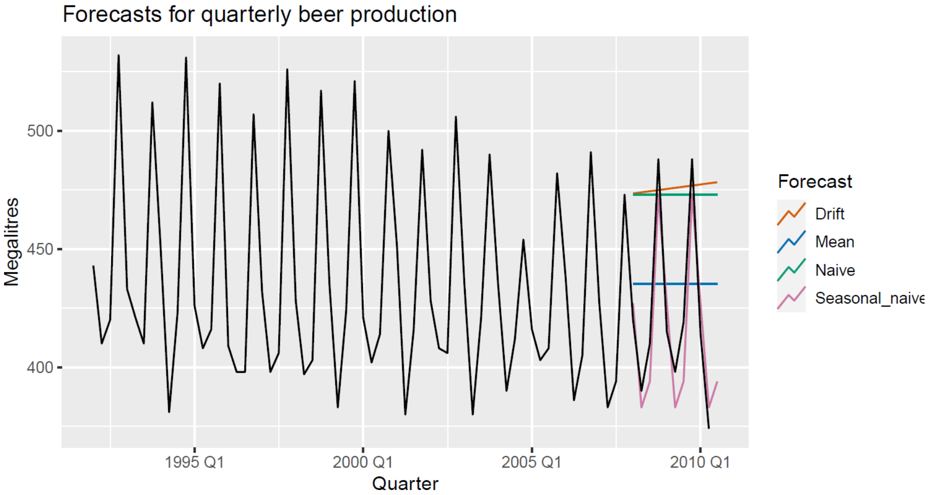
Definition: Forecast Errors

A forecast error is the difference between an observed value and its forecast

$$e_{T+h} = y_{T+h} - \hat{y}_{T+h}|Y_T, \ldots, Y_1$$

- The conditional set $Y_T, \ldots, Y_1$ should only be taken from the training dataset
- The true value y_{T+h} is taken from the test set
- Unlike residuals, forecast errors on the test involve multi-step forecasts
- These are the **true** forecast error, as the test data is not used to compute $\hat{y}_{T+h}$

Example: Forecasting Beer Production



Measures of Forecast Accuracy

Main Metrics

- **MAE**: mean absolute errors $\frac{1}{S}\sum_{s \in S} |e_{s,T+h}|$
- **MSE**: mean squared errors $\frac{1}{S}\sum_{s \in S} (e_{s,T+h})^2$
- **MAPE**: mean absolute percentage errors $\frac{1}{S} 100 * \sum_{s \in S} \frac{|e_{s,T+h}|}{|y_{s,t+h}|}$
- **RMSE**: root mean squared errors: $\sqrt{\frac{1}{S}\sum_{s \in S} (e_{s,T+h})^2}$

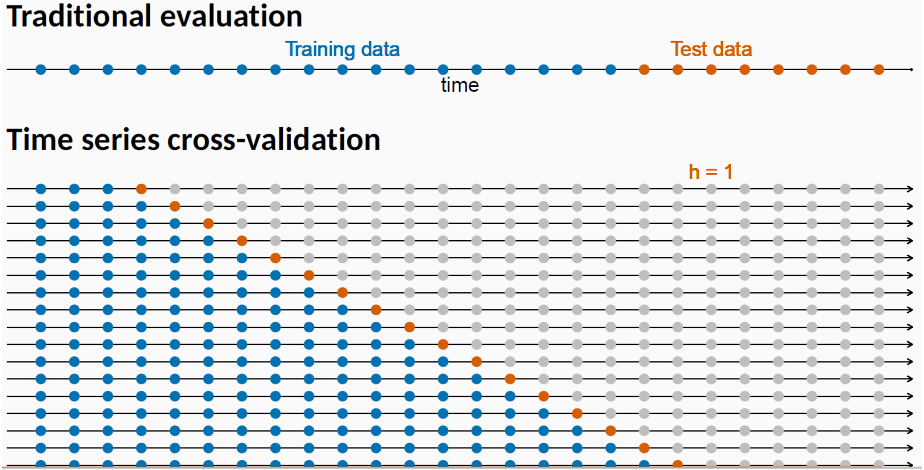
With:

- y_{T+h}: T+h observation, h being the horizon (h = 1, 2, ..., H)
- $\hat{y}_{T+h|T}$: the forecast based on data up to time T
- $e_{T+h} = y_{T+h} - \hat{y}_{T+h|T}$: The forecast errors
- S is the testing sample

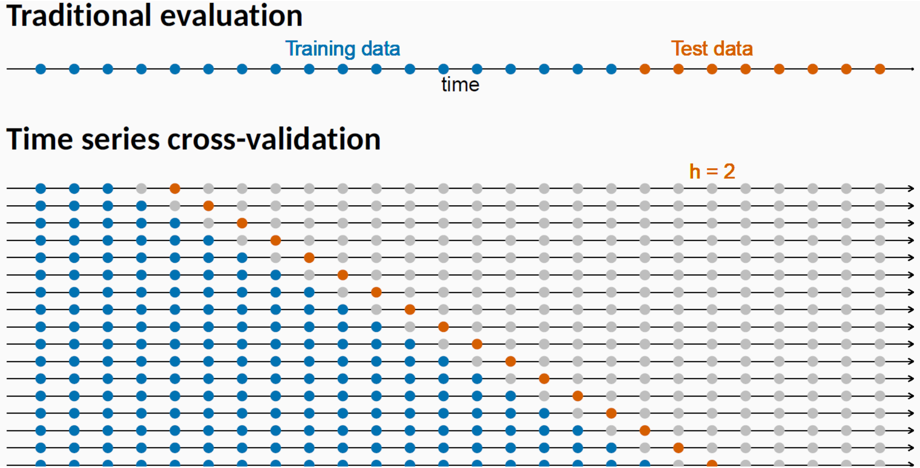
Note:

- MAE, MSE and RMSE are all **scale dependent**

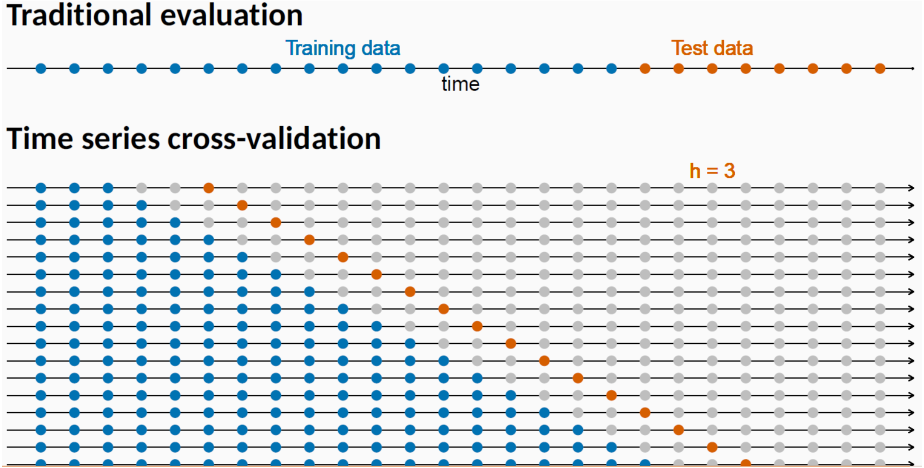
Time Series Cross-Validation



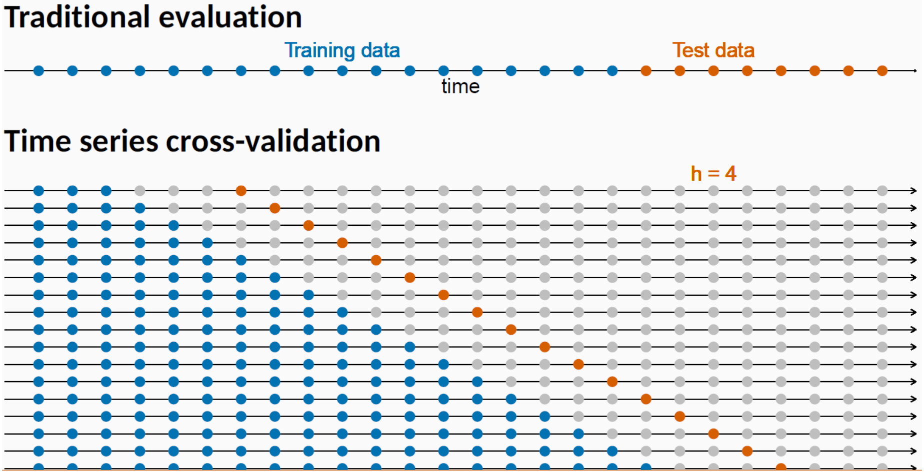
Time Series Cross-Validation



Time Series Cross-Validation



Time Series Cross-Validation



Time Series Cross-Validation