Let $I = \{1, \ldots, i\}$ for $i \leqslant \operatorname{rank}(\mathsf{A})$. Then

$$\det(\mathsf{D_x}\,\mathsf{V_u}\,\mathsf{A}\,\mathsf{W_v}\,\mathsf{D_y})_{I,I} = \sum_{\substack{|J|=i\\|K|=i}} \det(\mathsf{D_x}\,\mathsf{V_u})_{I,J} \det(\mathsf{A})_{J,K} \det(\mathsf{W_v}\,\mathsf{D_y})_{K,I}$$

but $\det(\mathsf{D_x}\,\mathsf{V_u})_{I,J} = x_1 \cdots x_i \det(\mathsf{V_u})_{I,J}$. So the monomial $x_1 \cdots x_i$ do not depend on J and is not unique in the sum.

In fact, Pan uses the following regularization in his book $\mathsf{A}'' = \mathsf{C_{a,u}}\,\mathsf{D_x}\,\mathsf{A}'\,\mathsf{D_y}\,\mathsf{C_{v,b}}$ since

$$\det(\mathsf{C_{a,u}}\,\mathsf{D_x})_{I,J} = x_{j_1} \cdots x_{j_i} \det(\mathsf{C_{a,u}})_{I,J}.$$