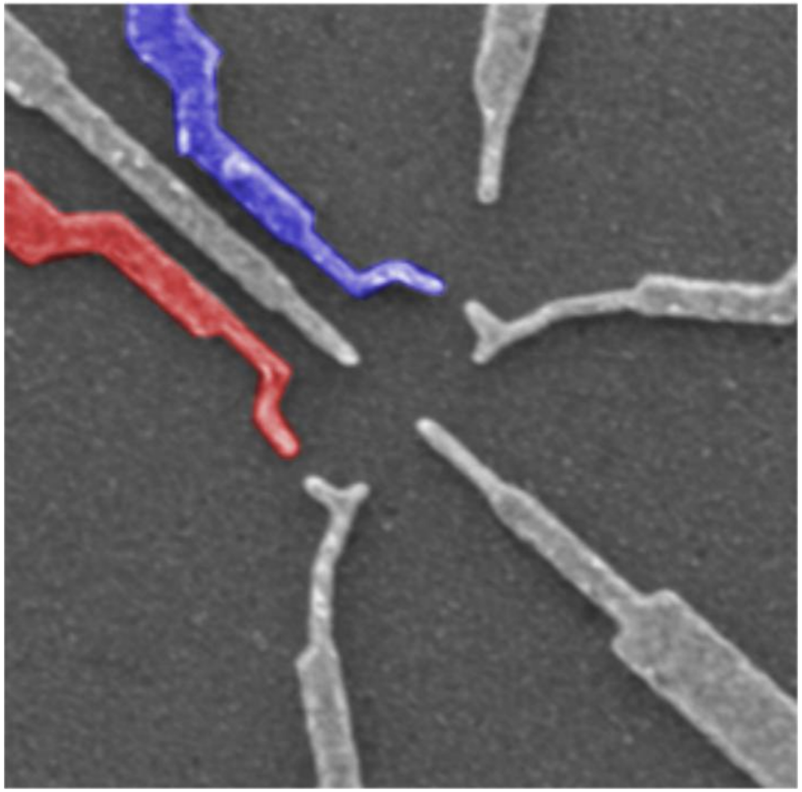
From SWAP to C-phase gate regime in single spin qubits

GDR Workshop – IQFA
28-30 November 2012
Institut Néel, Grenoble

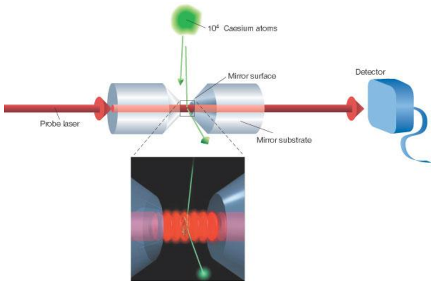


R. THALINEAU, A. D. WIECK, C. BAUERLE, T. MEUNIER
Quantum coherence group

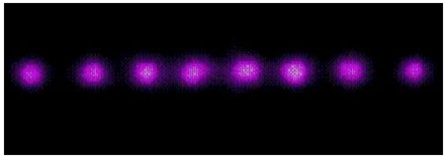
An electron spin as a qubit candidate

<u>Less interaction with the environment</u>

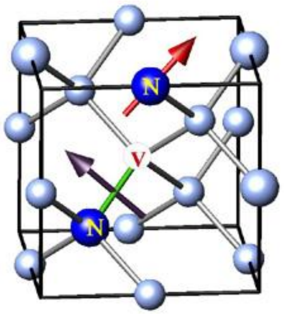
Neutral atoms



Ion traps



NV center



An electron spin as a qubit candidate

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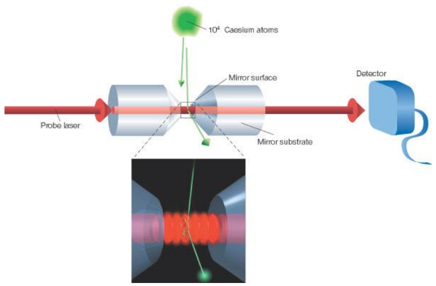

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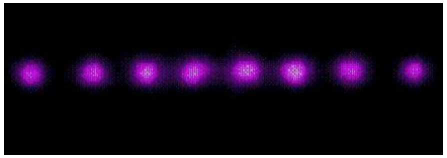

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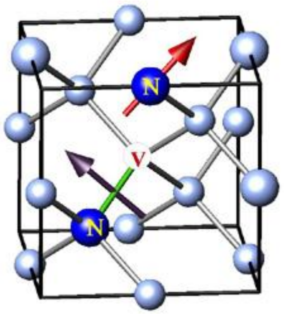
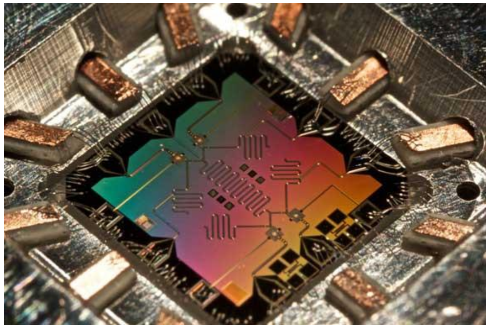


<u>Scalability potential</u>

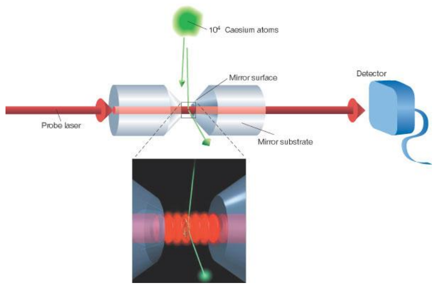


Superconducting qubits

An electron spin as a qubit candidate

Less interaction with the environment

Neutral atoms



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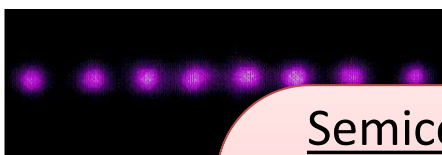
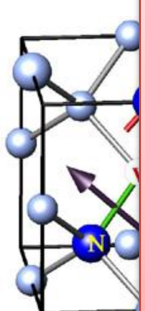


NV center



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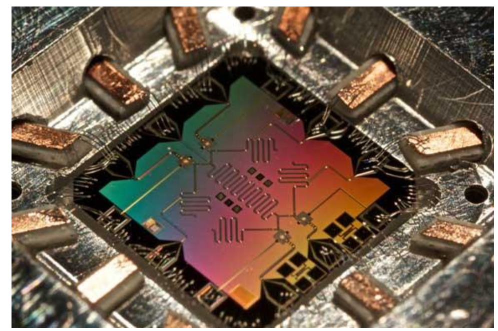
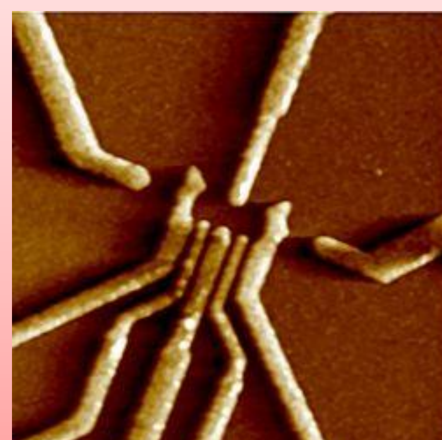
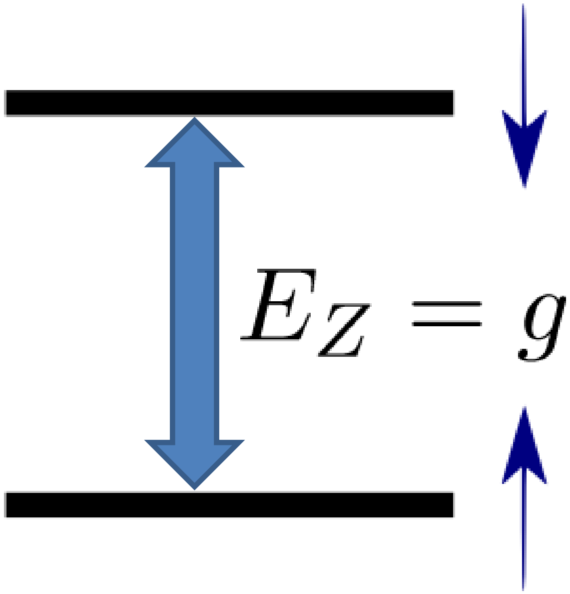


Superconducting qubits

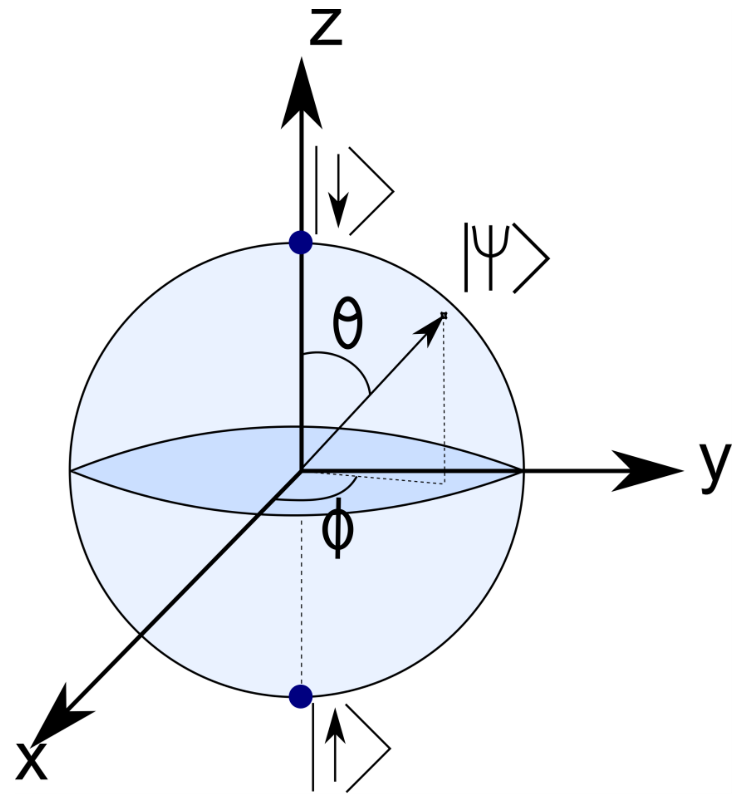
Semiconductor spin qubits



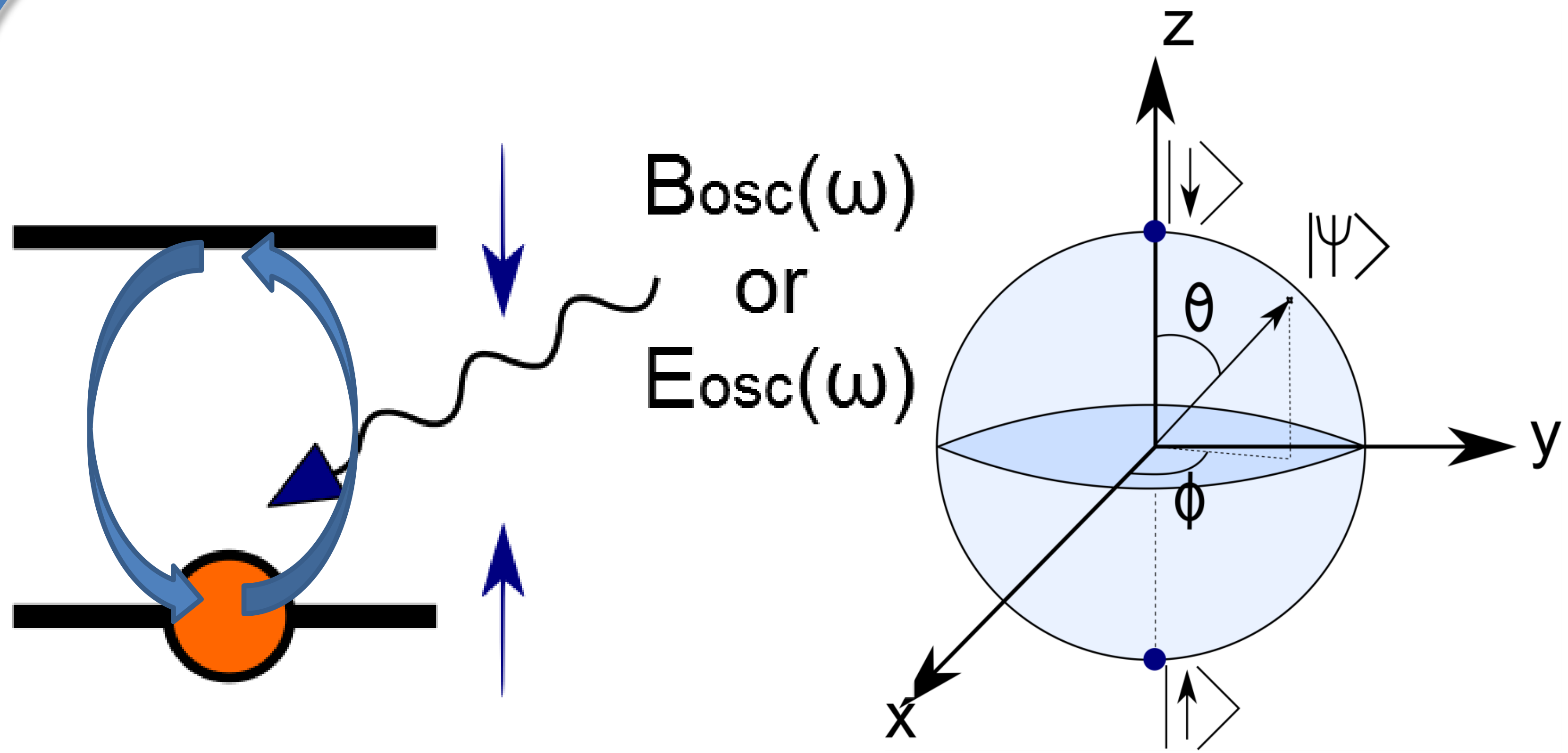
A single electron spin qubit



$$E_Z = g^* \mu_B B_z$$



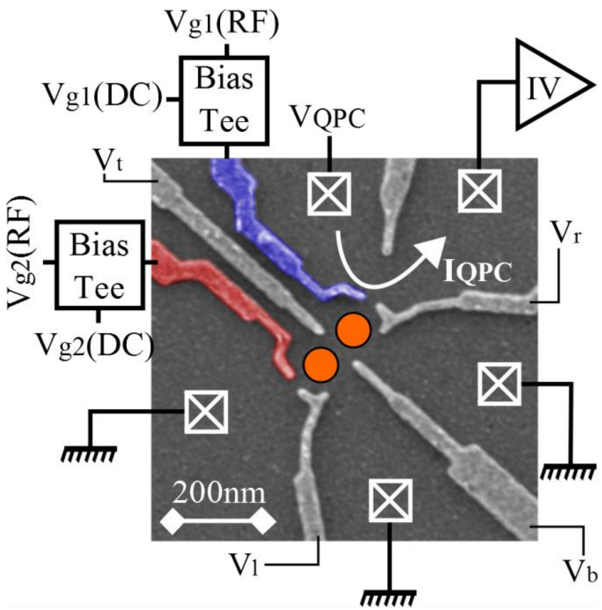
A single electron spin qubit



Interaction between two qubits

Double quantum dot :

two single spin qubits tunnel coupled



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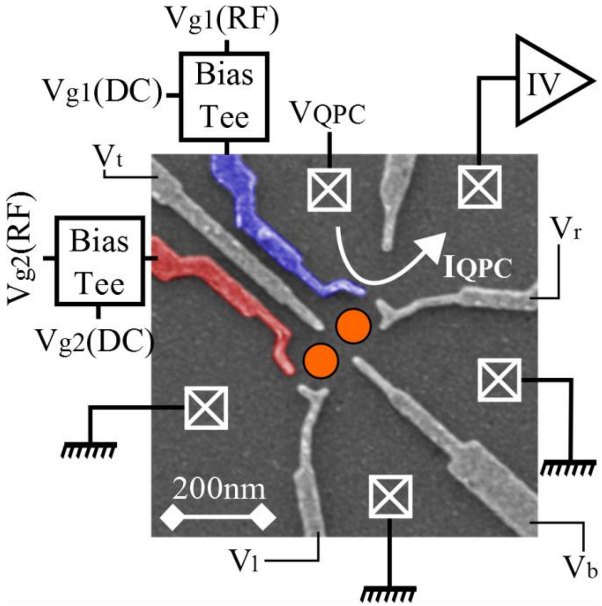
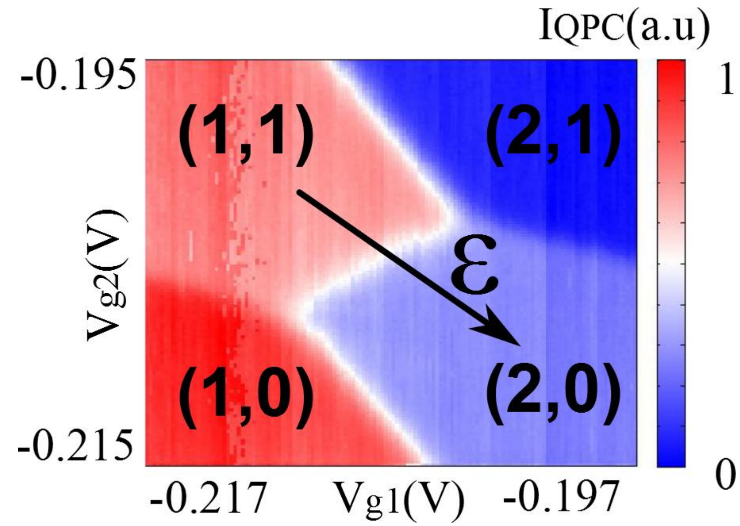


A single electron can be isolated in each quantum dot



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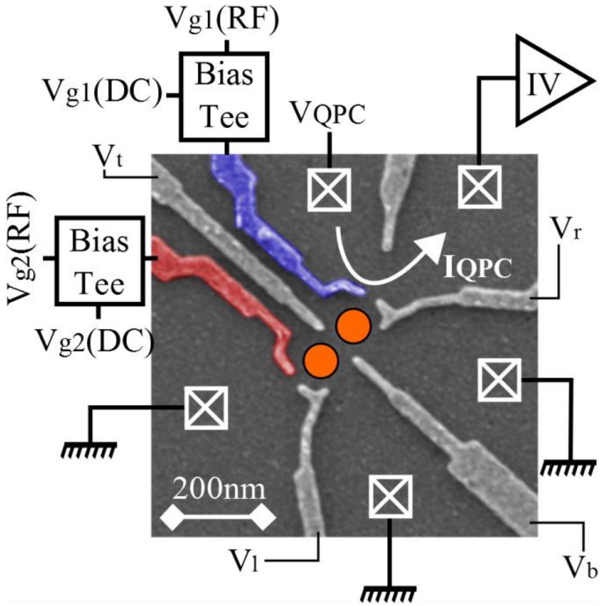

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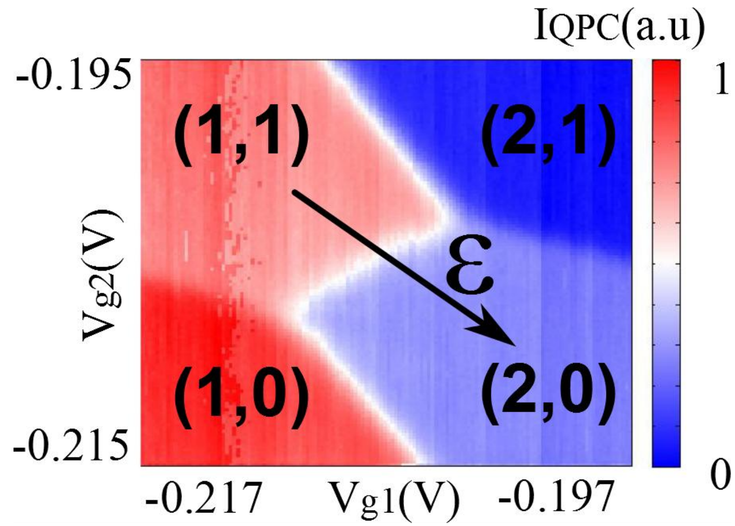
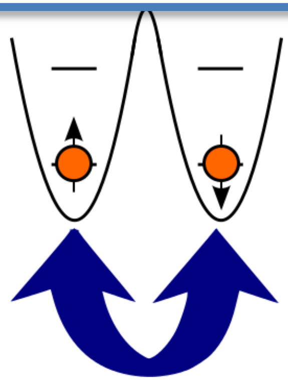


How to make them interact ?



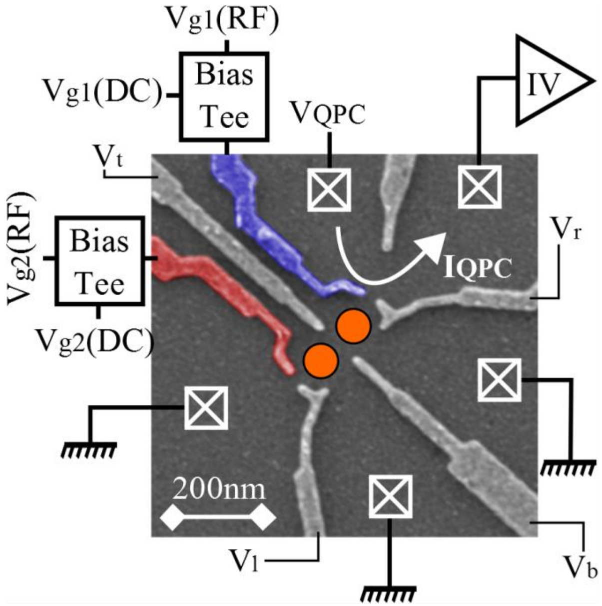
Dipolar interaction

very weak at 100nm (~25peV~5kHz)

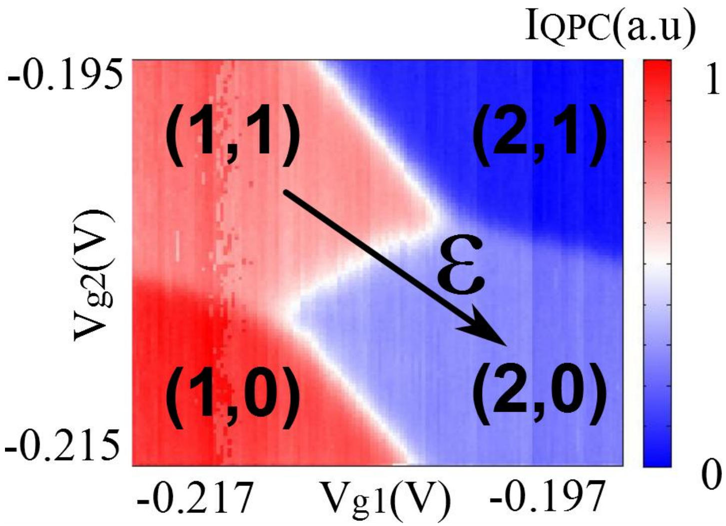
Interaction between two qubits

Double quantum dot :

two single spin qubits tunnel coupled



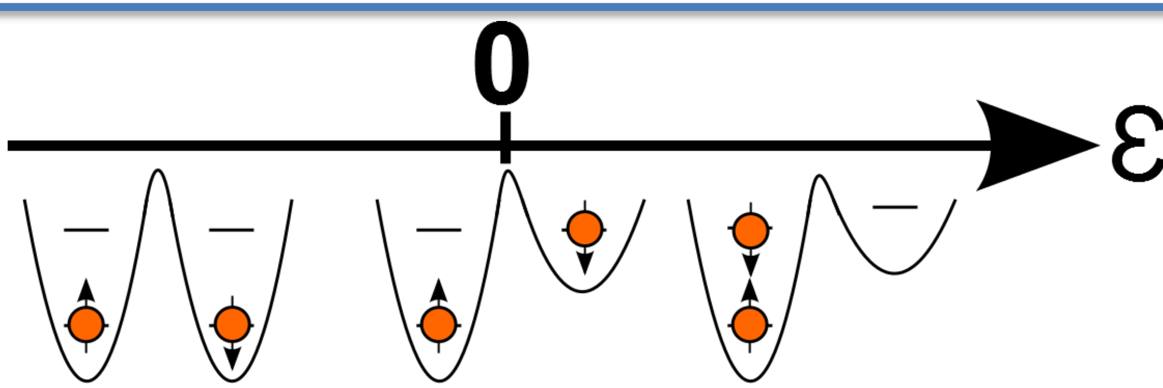
A single electron can be isolated in each quantum dot



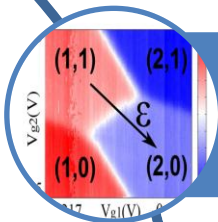
Controlled exchange interaction :

Tunnel coupling

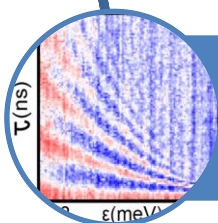
J=0.1-20µeV



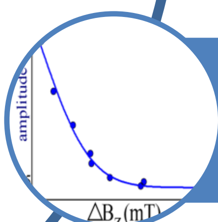
Outline



Two electron spin states



Two qubit gate : SWAP



From SWAP to C-phase gate

Two electron spin states

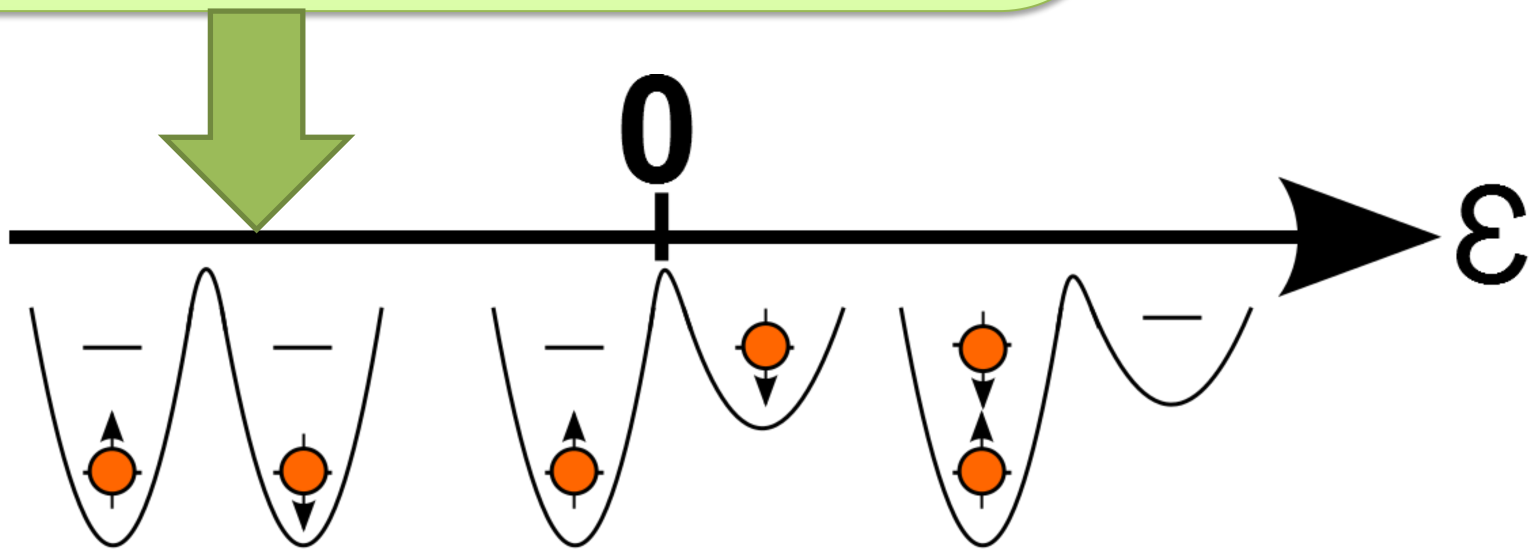
Manipulation region

$$|S^{(1,1)}\rangle = 1/\sqrt{2}\,(|\uparrow,\downarrow\rangle - |\downarrow,\uparrow\rangle)$$

$$|{T_+}^{(1,1)}\rangle = |\uparrow,\uparrow\rangle$$

$$|{T_0}^{(1,1)}\rangle = 1/\sqrt{2}\,(|\uparrow,\downarrow\rangle + |\downarrow,\uparrow\rangle)$$

$$|{T_-}^{(1,1)}\rangle = |\downarrow,\downarrow\rangle$$



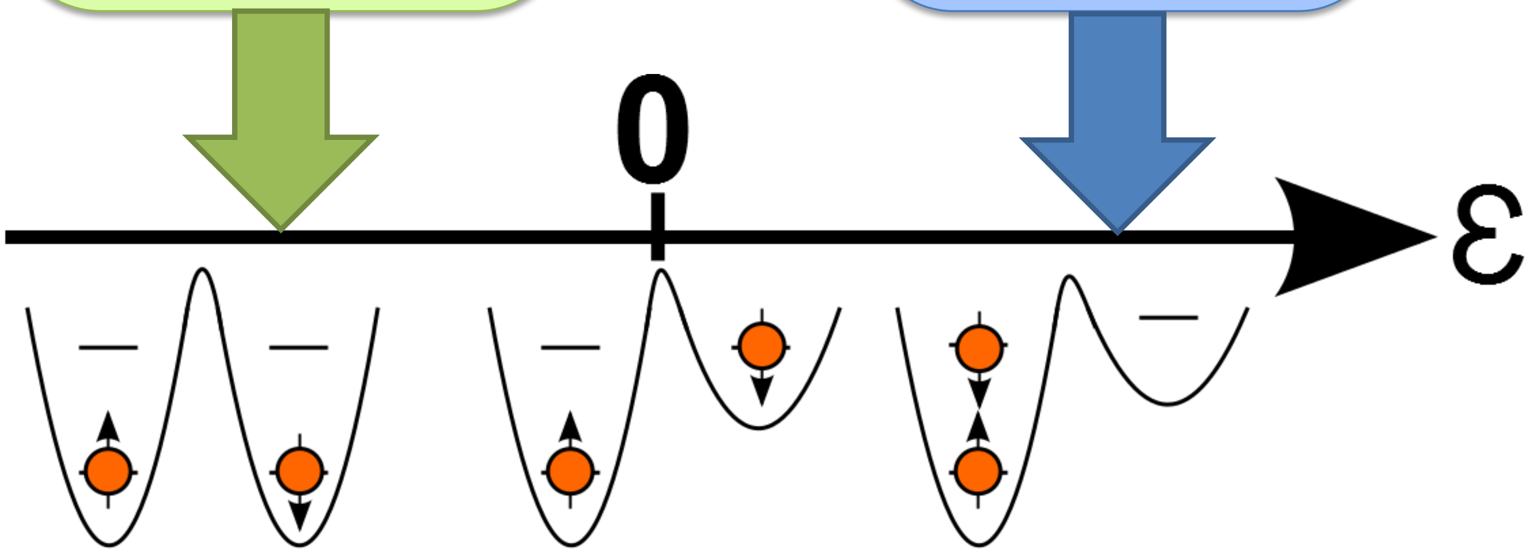
Two electron spin states

Region (1,1) :

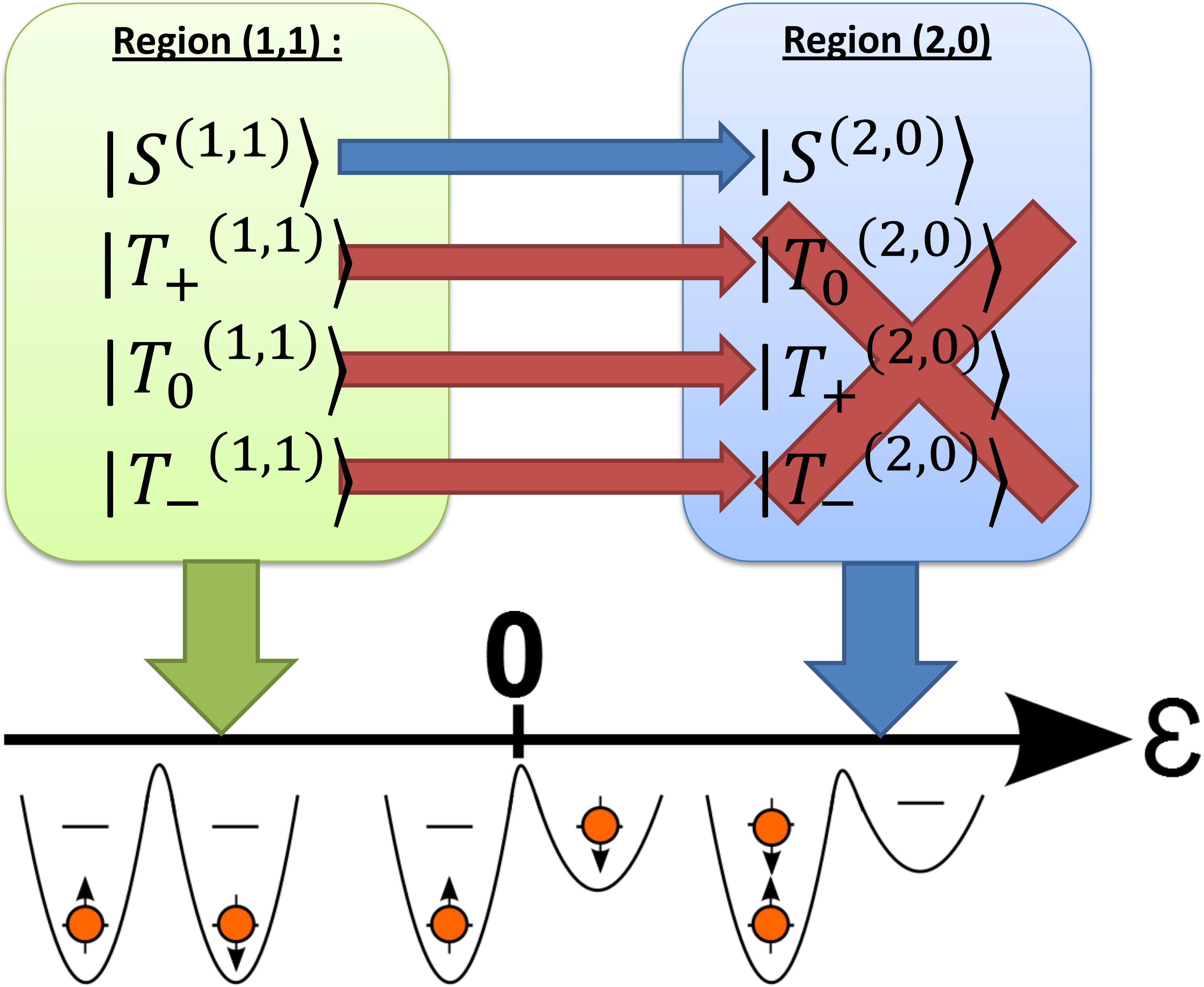
$$|S^{(1,1)}\rangle$$
$$|T_+{}^{(1,1)}\rangle$$
$$|T_0{}^{(1,1)}\rangle$$
$$|T_-{}^{(1,1)}\rangle$$

Region (2,0)

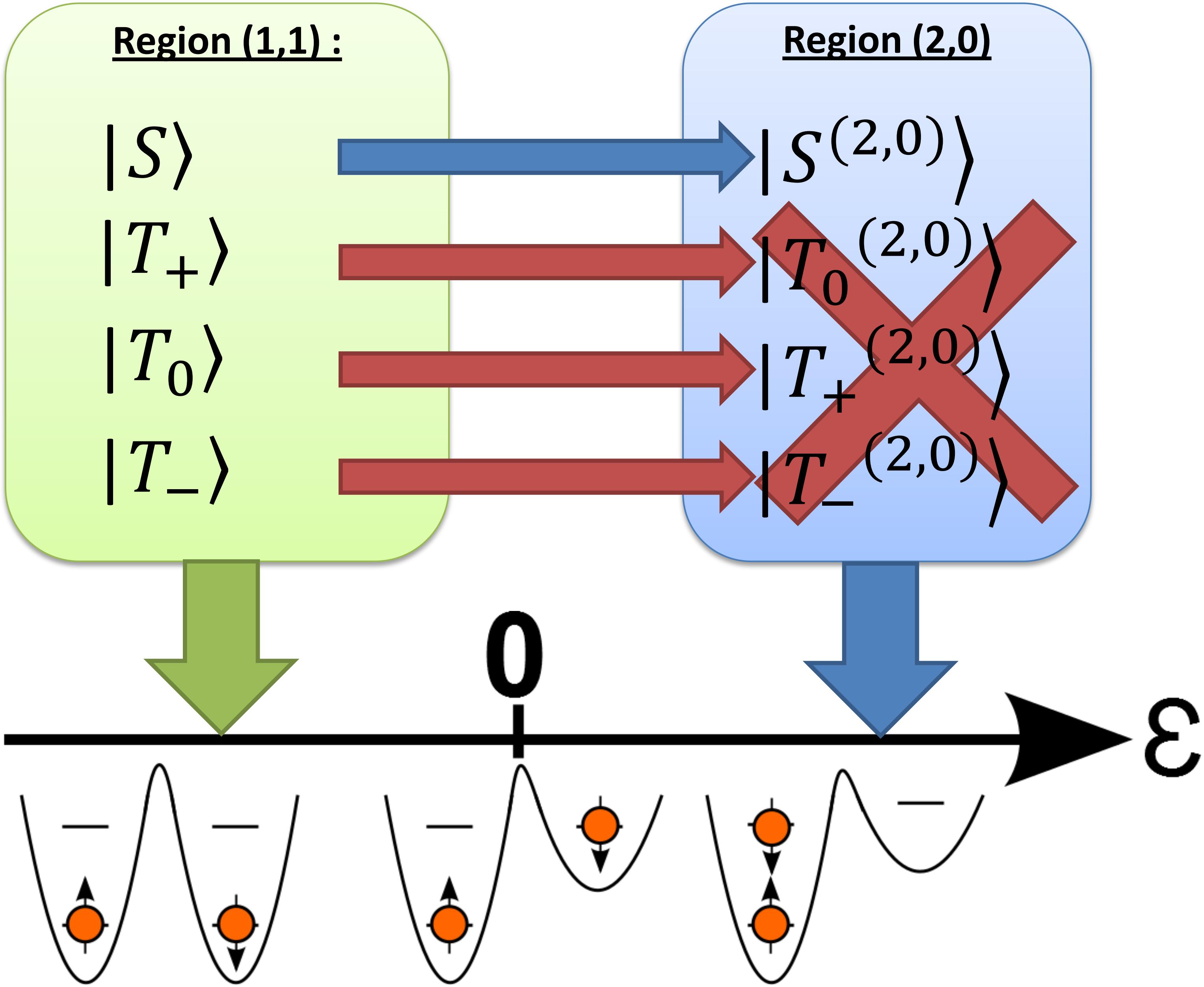
$$|S^{(2,0)}\rangle$$
$$\cancel{|T_0{}^{(2,0)}\rangle}$$
$$\cancel{|T_+{}^{(2,0)}\rangle}$$
$$\cancel{|T_-{}^{(2,0)}\rangle}$$



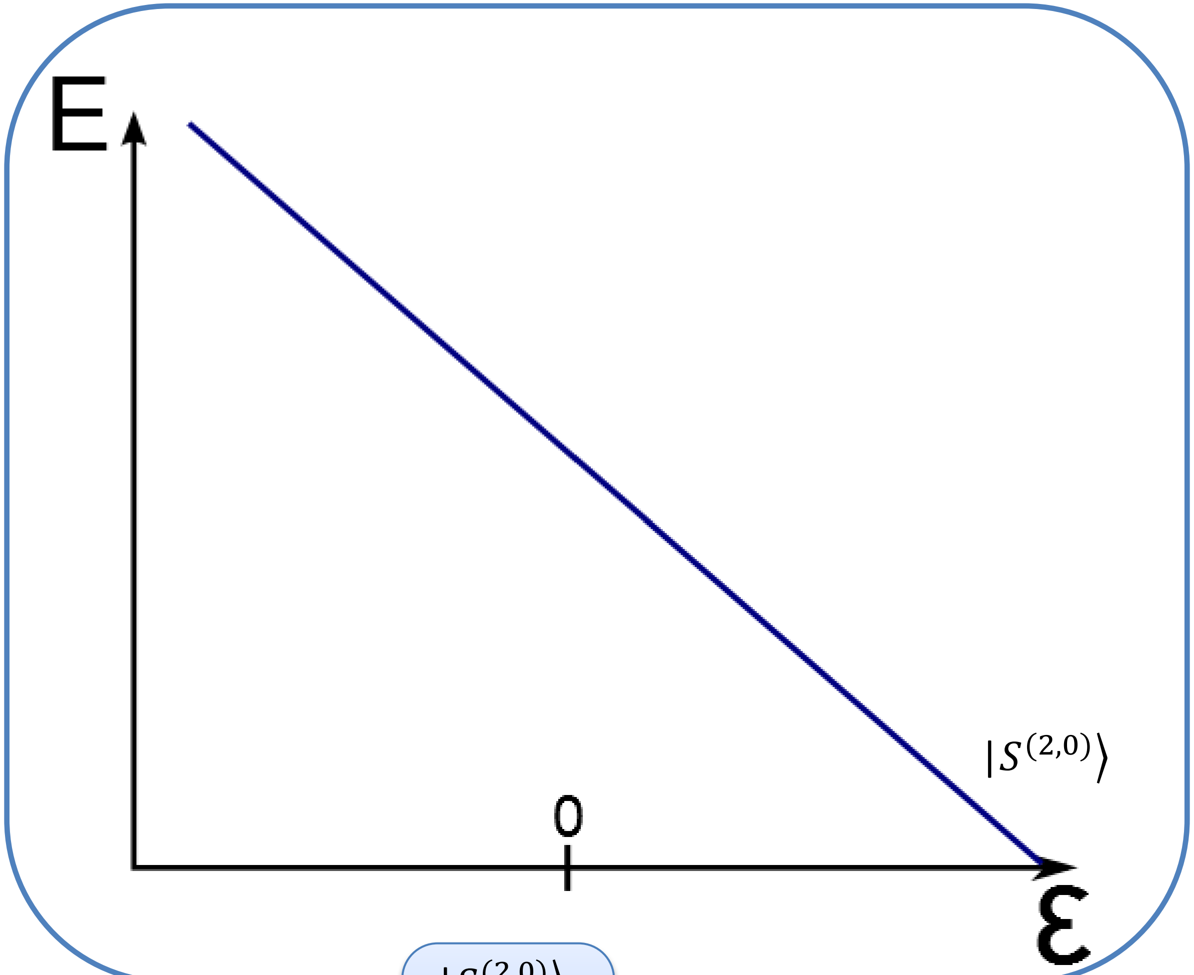
Two electron spin states

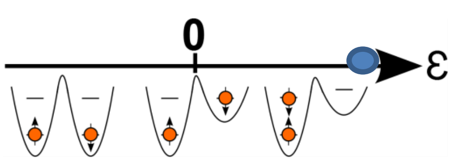


Two electron spin states



Two electron spin states





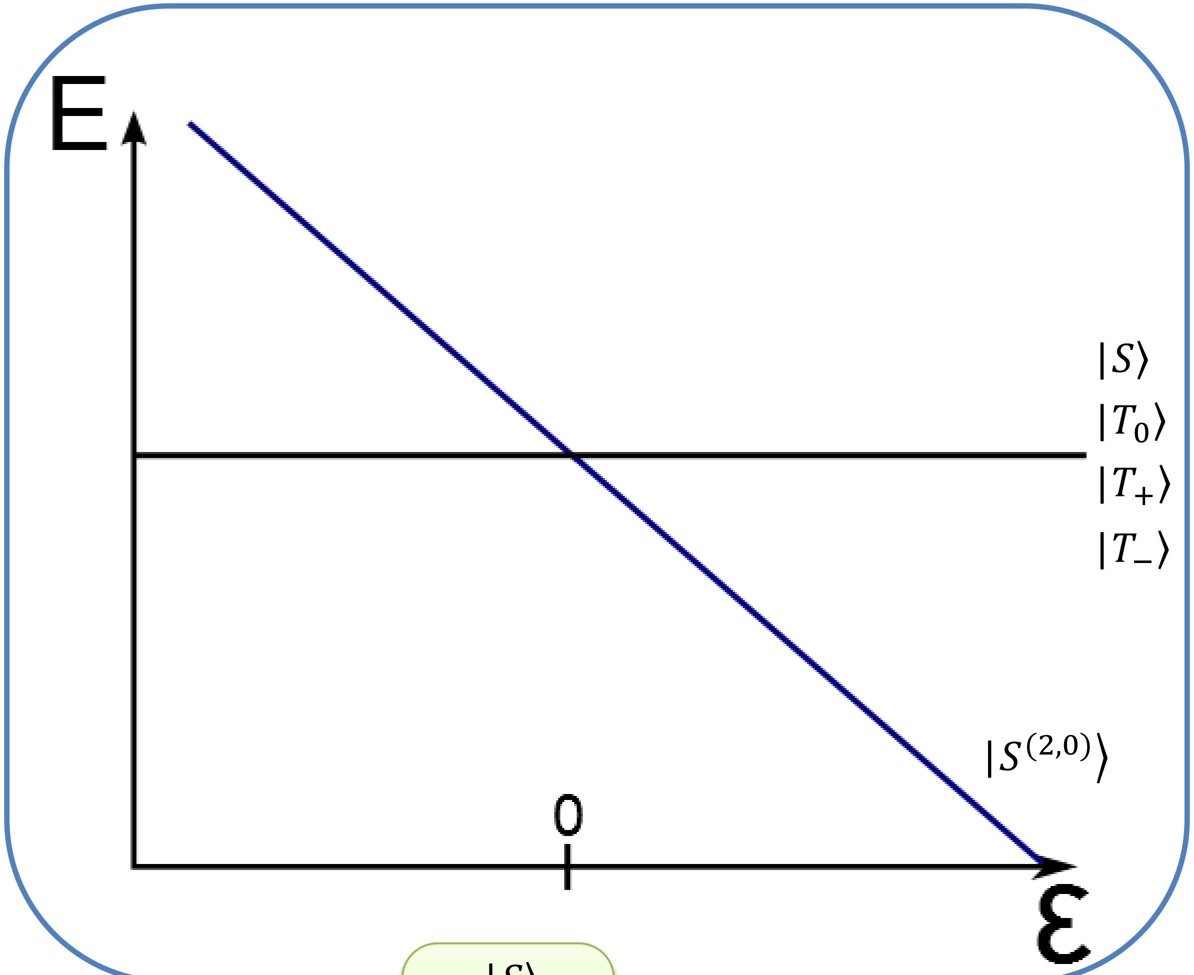
$|S^{(2,0)}\rangle$

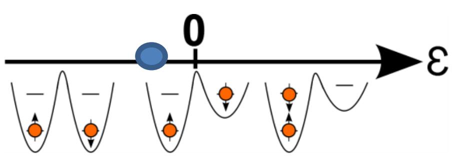
$|T_0^{(2,0)}\rangle$

$|T_+^{(2,0)}\rangle$

$|T_-^{(2,0)}\rangle$

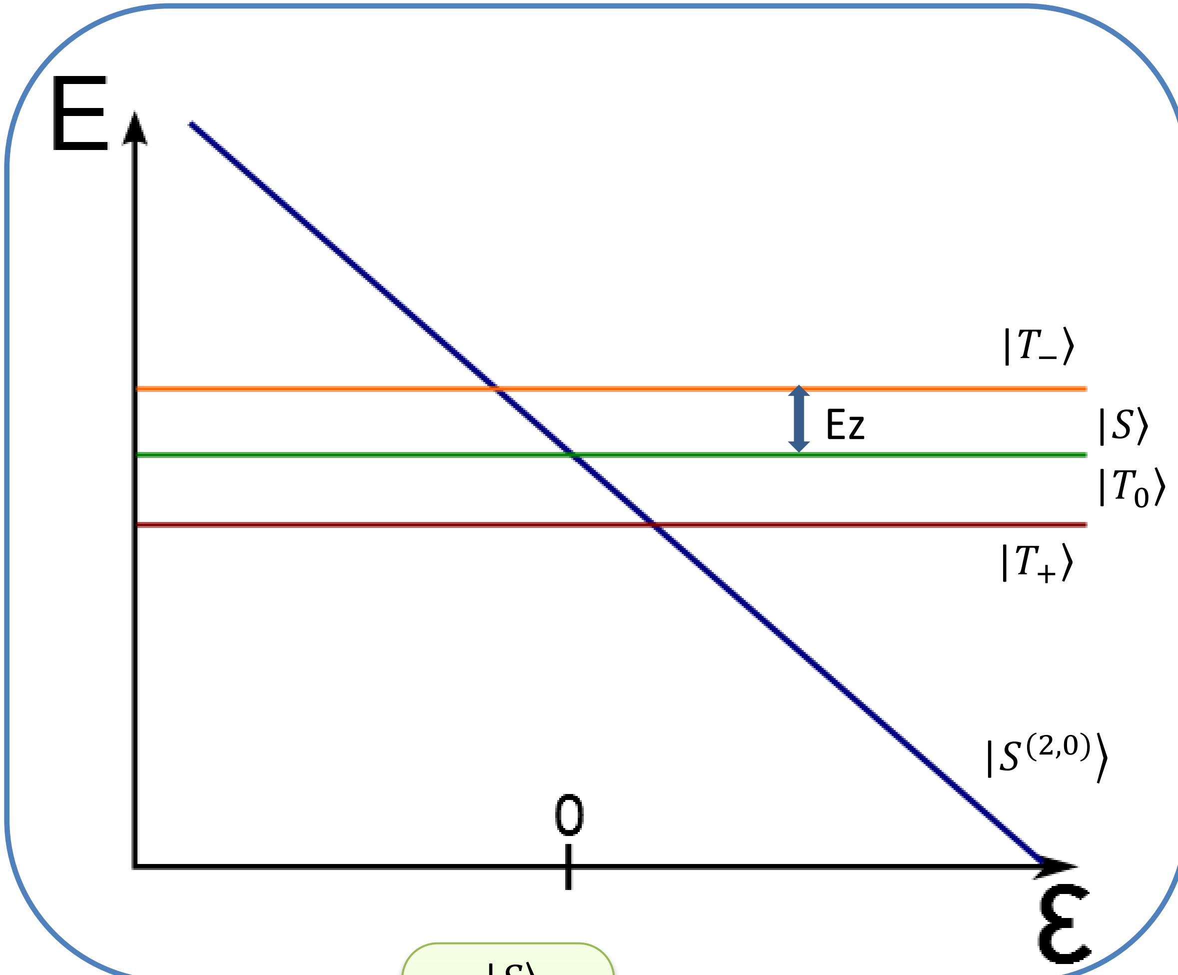
Two electron spin states



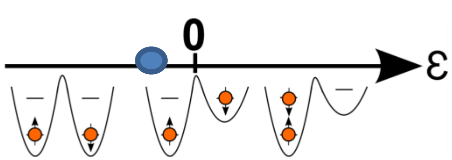


$|S\rangle$
$|T_0\rangle$
$|T_+\rangle$
$|T_-\rangle$

Two electron spin states



1) A magnetic field splits in energy the triplets states :
Bext=100mT ⇨ 2.5µeV



$|S\rangle$
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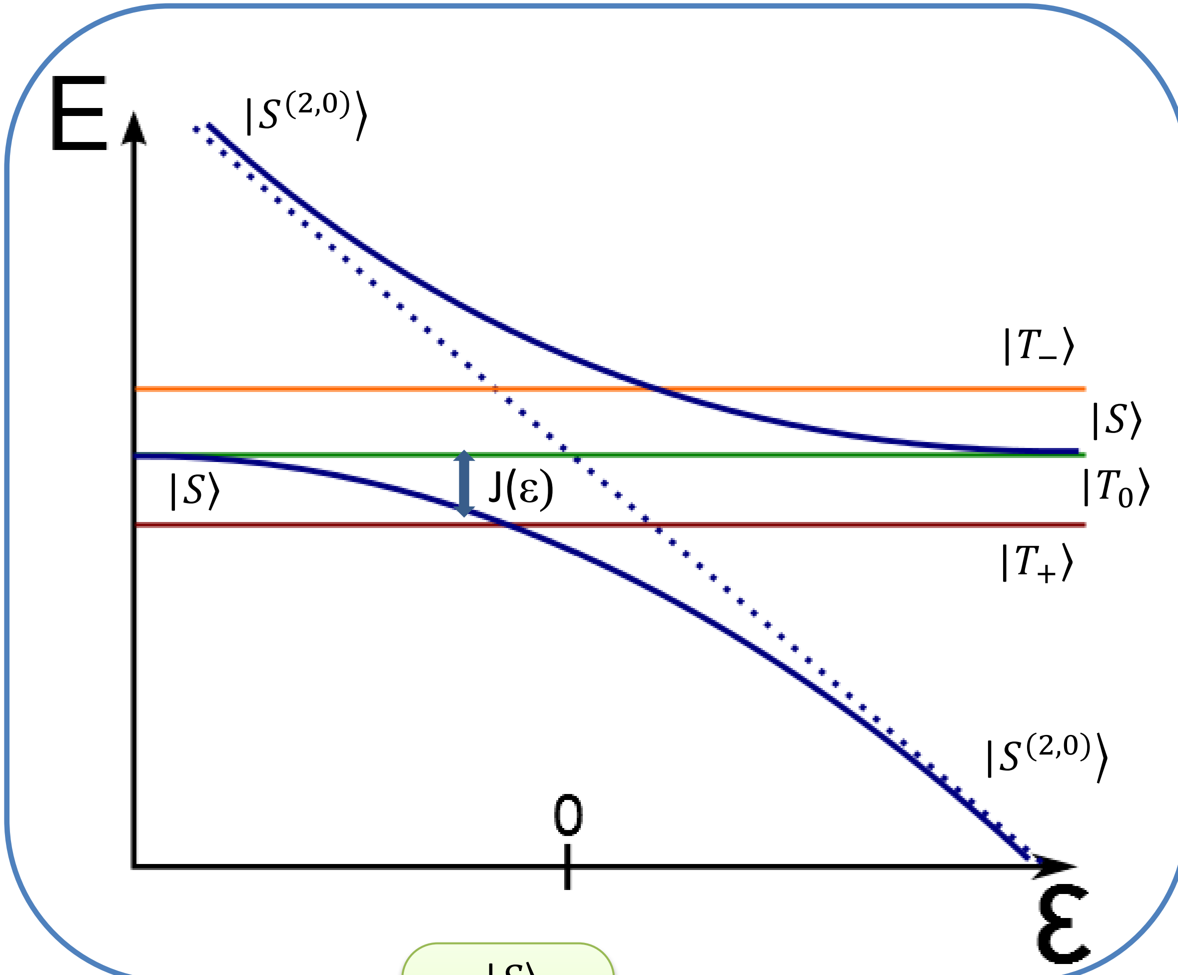
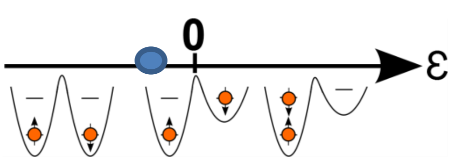
Two electron spin states



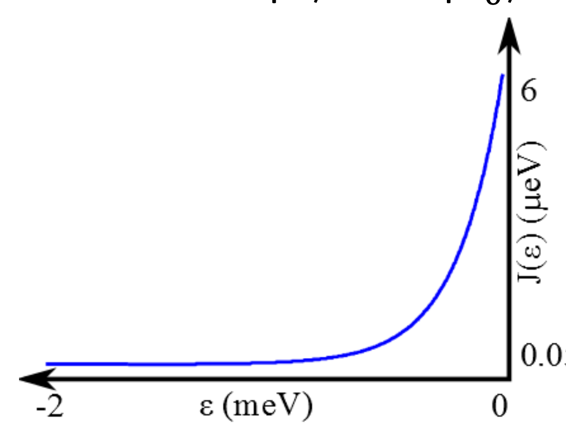
$|S\rangle$
$|T_0\rangle$
$|T_+\rangle$
$|T_-\rangle$



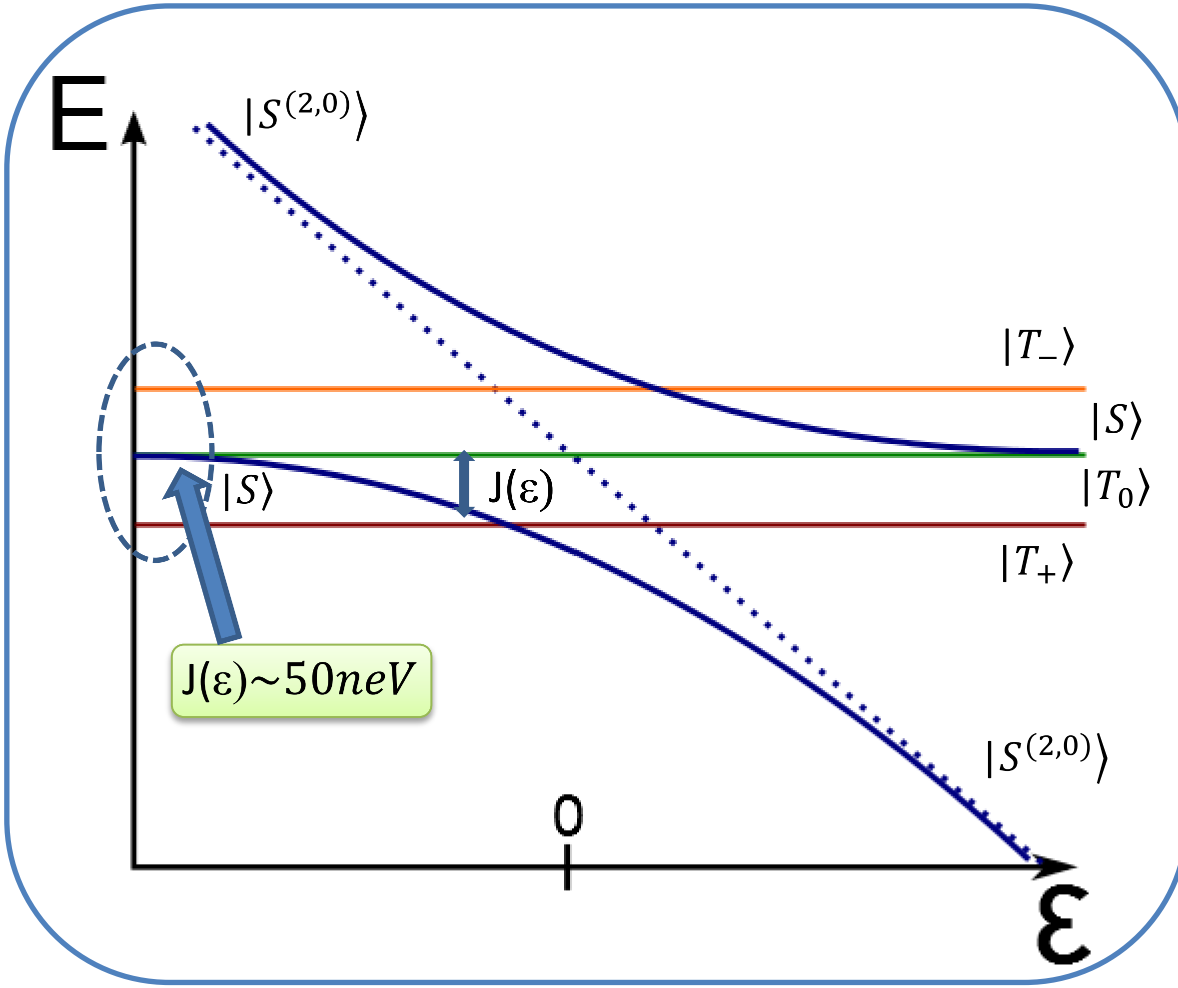
1) A magnetic field splits in energy the triplets states :
Bext=100mT ⇒ 2.5µeV

2) Tunnelling couples the two singlet states
t=6µeV

Exchange energy between $|S\rangle$ and $|T_0\rangle$



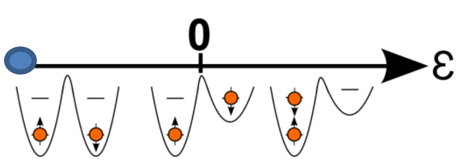
Two electron spin states



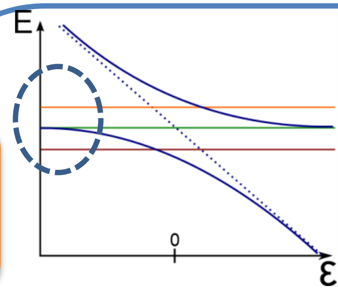
1) A magnetic field splits in energy the triplets states : Bext=100mT ⟹ 2.5µeV

2) Tunnelling couples the two singlet states
t=6µeV

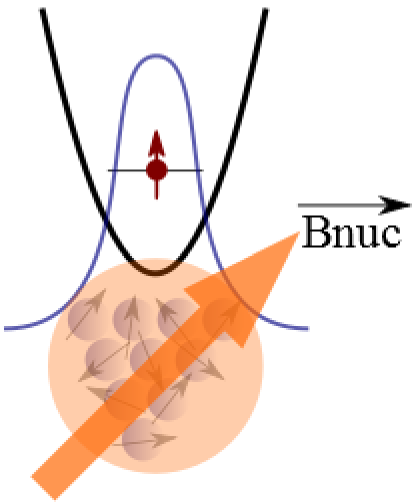
3) For ε≪0, the exchange interaction vanishes : the spin are now independant



Two electron spin states



Hyperfine interaction with the host nuclear spins



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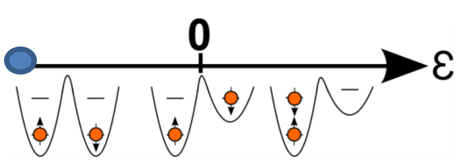
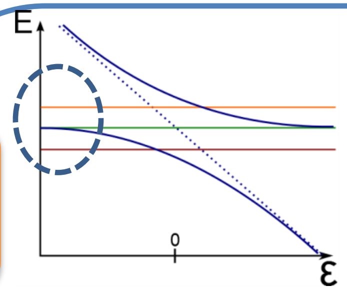
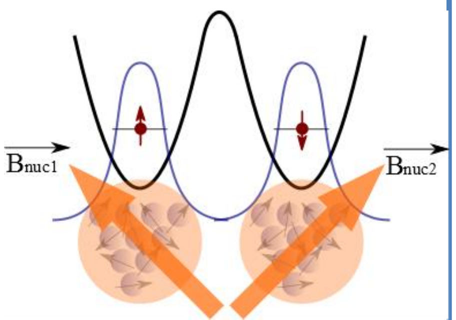


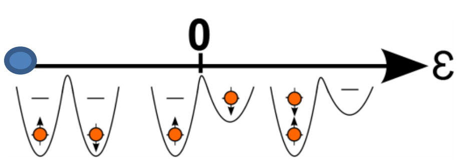
Two electron spin states

Hyperfine interaction with the host nuclear spins





ΔBz



1) A magnetic field splits in energy the triplets states : Bext=100mT ⟹ 2.5μeV

2) Tunnelling couples the two singlet states t=6μeV

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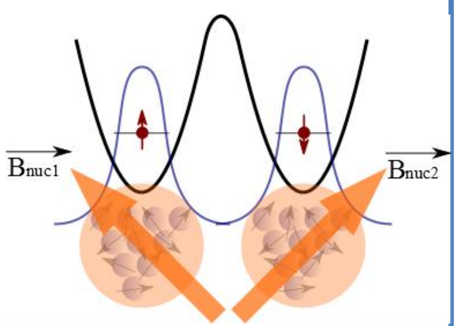
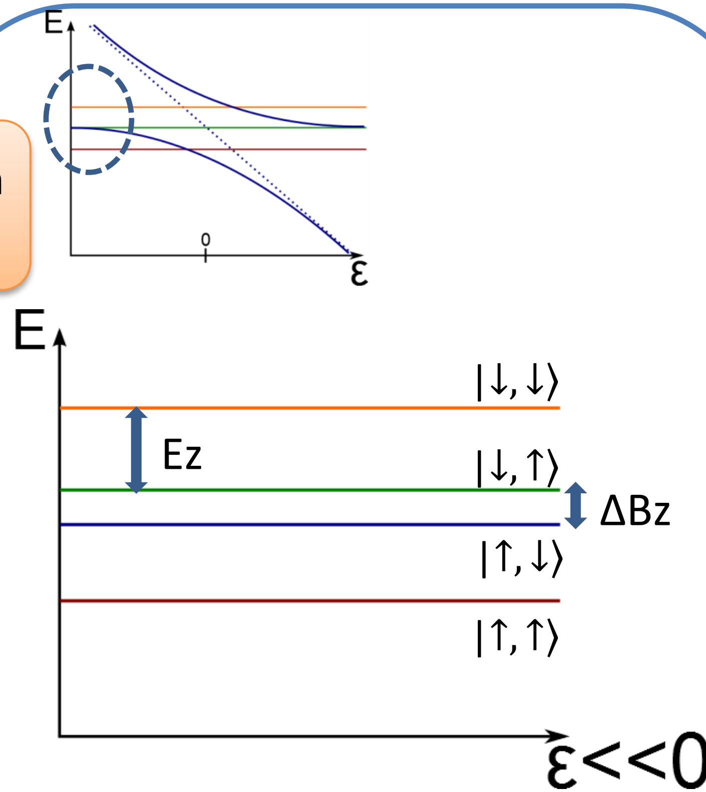
4) Non uniformity of the nuclear magnetic field : ΔBz~4mT ⟹ 0,1μeV

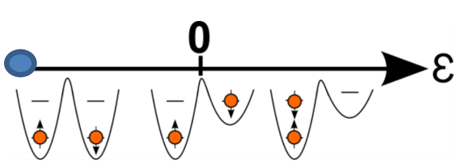
Two electron spin states

Hyperfine interaction with the host nuclear spins



ΔBz





|↑,↑⟩
|↑,↓⟩
|↓,↑⟩
|↓,↓⟩

1) A magnetic field splits in energy the triplets states : Bext=100mT ⇒ 2.5µeV

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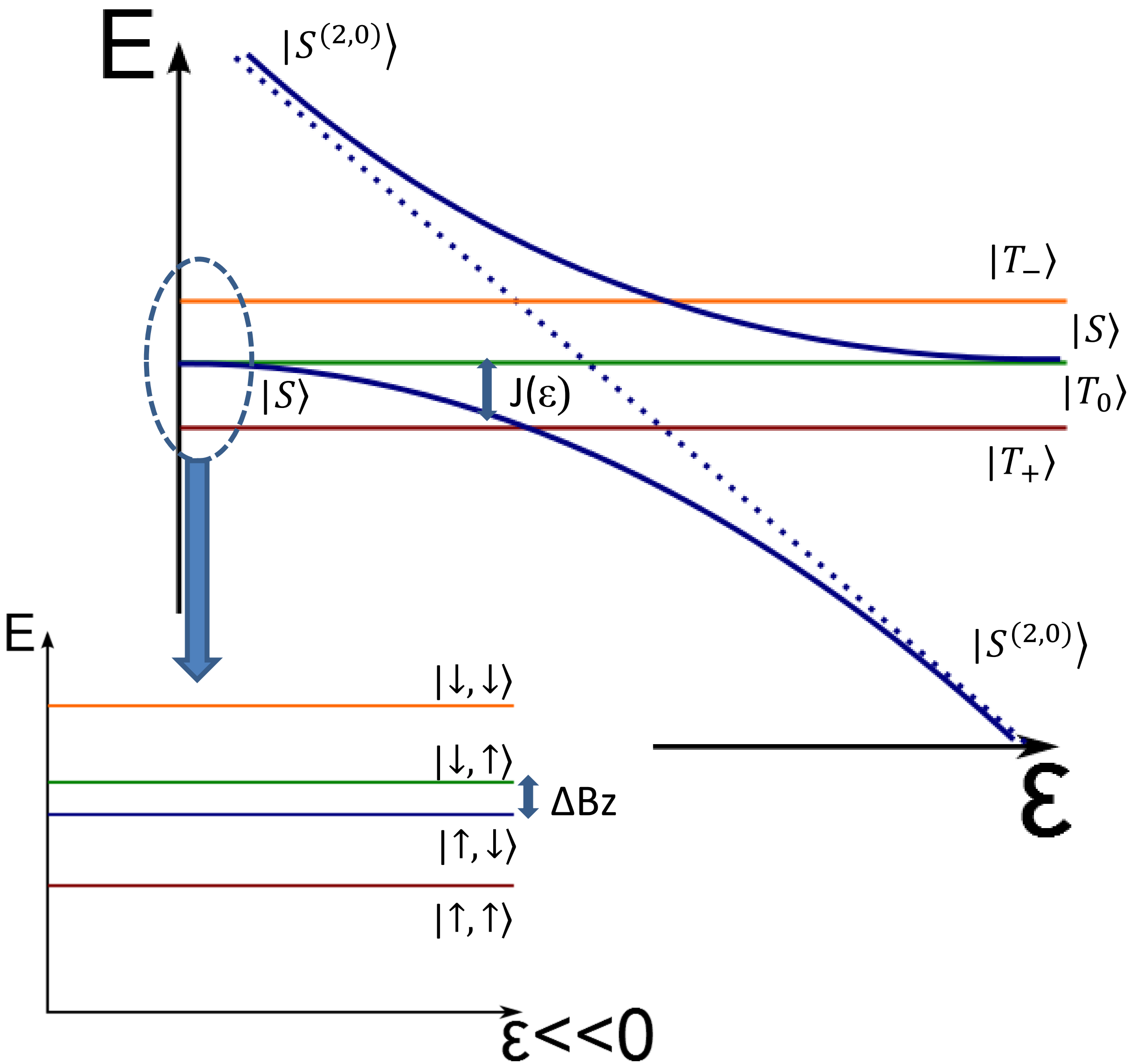
Two electron spin states



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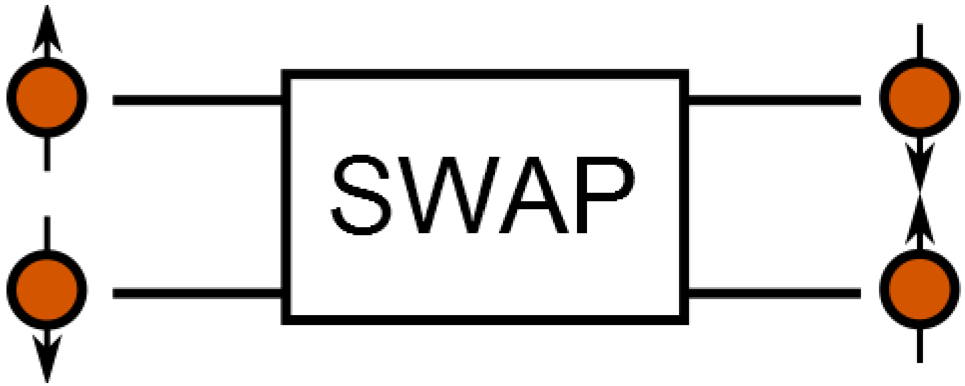
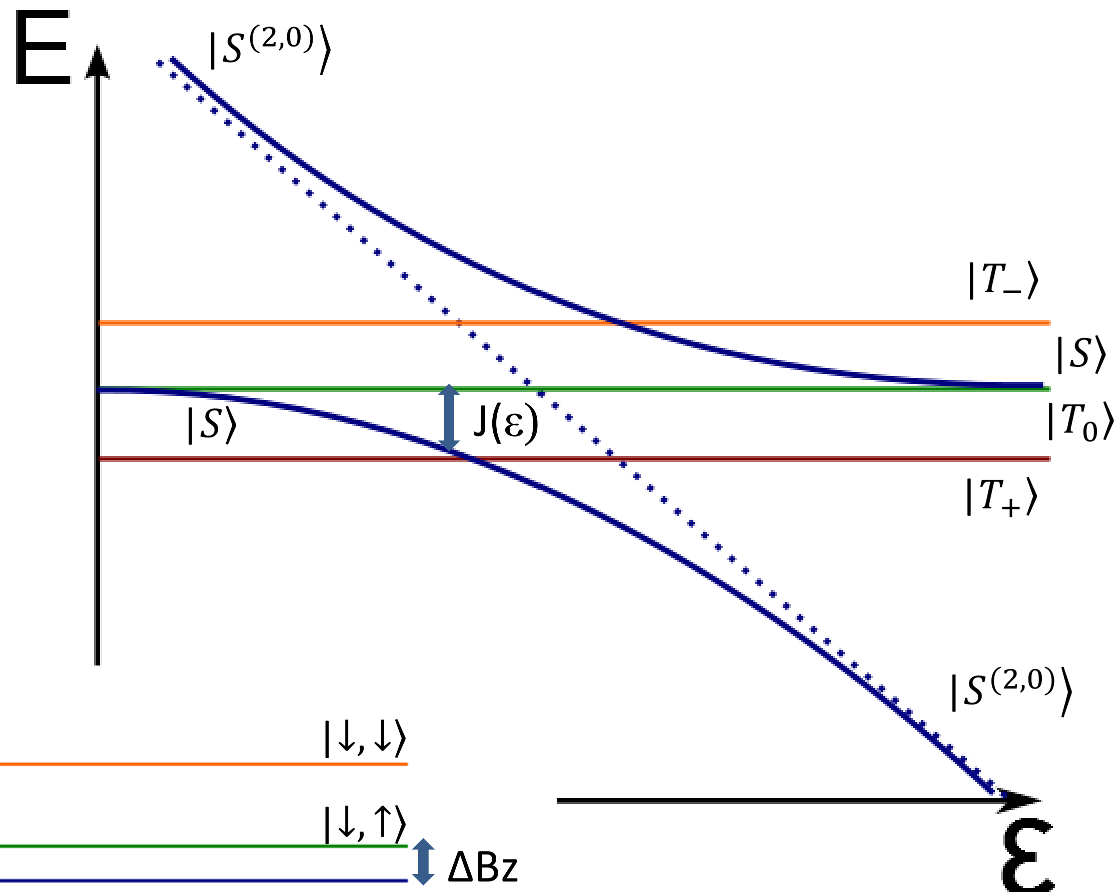
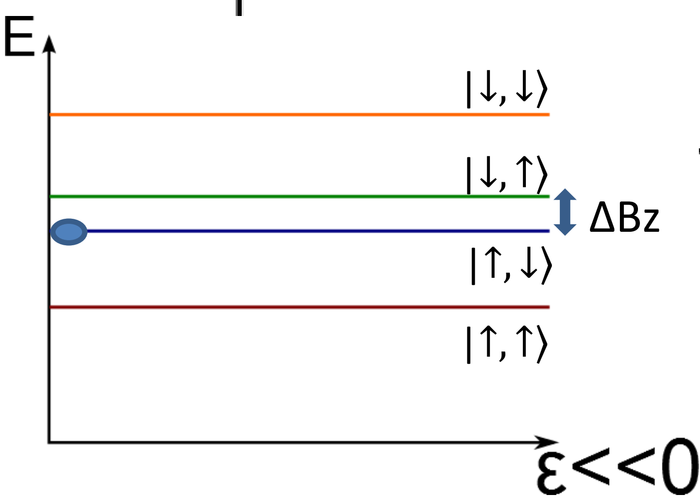
Two qubit gate : SWAP

<u>SWAP :</u> two qubit gate which exchange the state of two qubits



Two qubit gate : SWAP

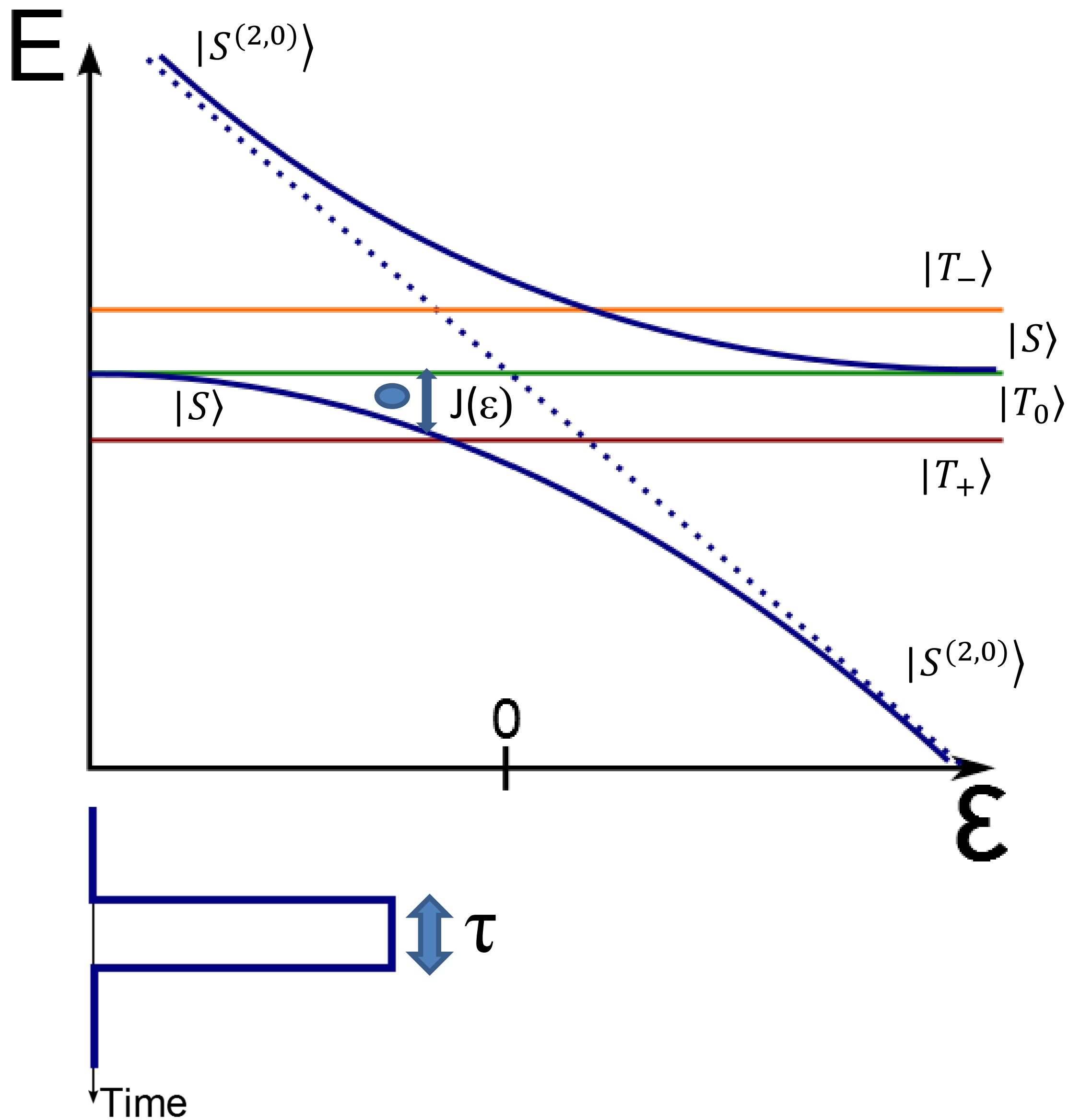




<u>Initialization in the two qubit subspace</u>

$|\Psi(\tau=0)\rangle = |\uparrow,\downarrow\rangle$

Two qubit gate : SWAP



<u>Initialization in the two qubit subspace</u>

$|\Psi(\tau=0)\rangle = |\uparrow,\downarrow\rangle$

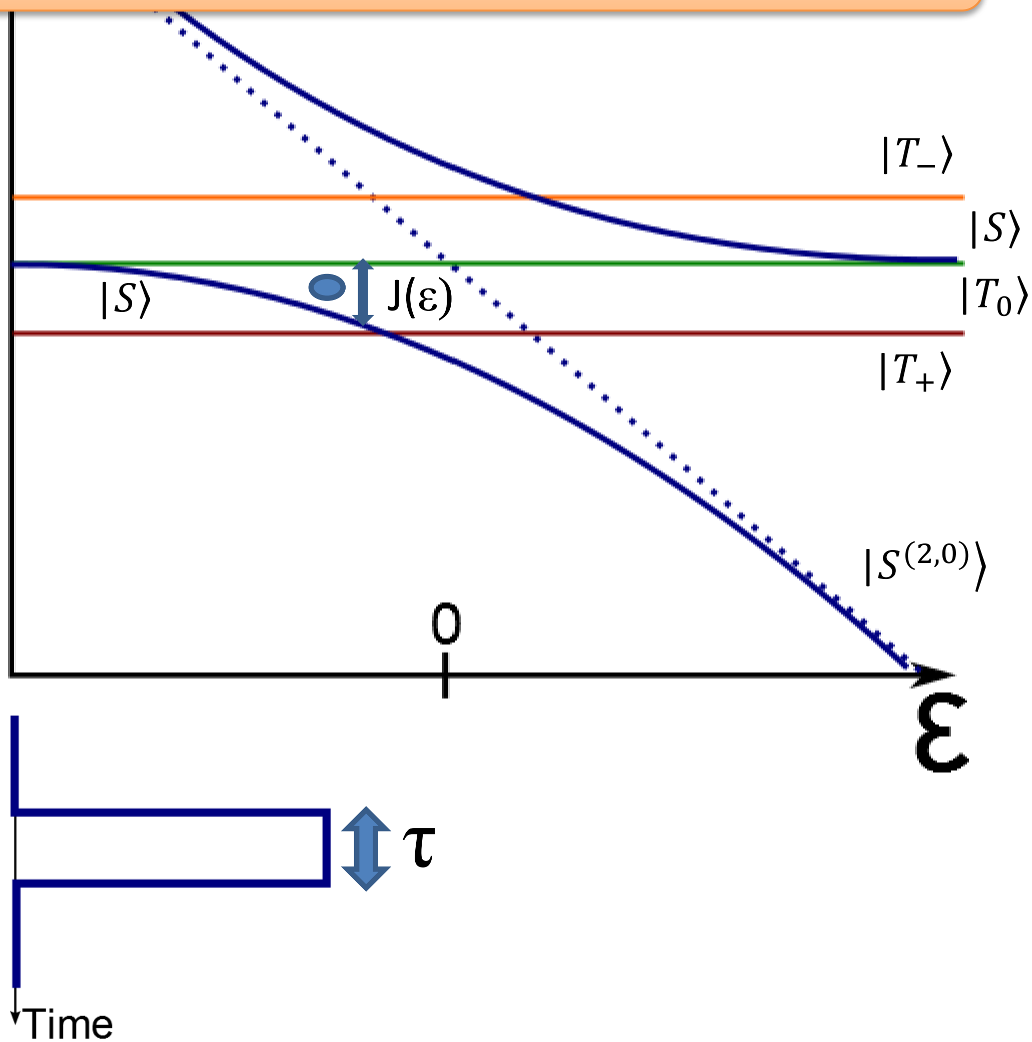
<u>Manipulation :</u>

Non adiabatic pulse with respect to ΔBz

$|\Psi(\tau=0)\rangle = |\uparrow,\downarrow\rangle = 1/\sqrt{2}\,(|S\rangle + |T_0\rangle)$

Two qubit gate : SWAP

$|\Psi(\tau)\rangle = \cos(J(\varepsilon)\,\tau/2\hbar)\,|\uparrow,\downarrow\rangle + i\,\sin(J(\varepsilon)\,\tau/2\hbar)\,|\downarrow,\uparrow\rangle$



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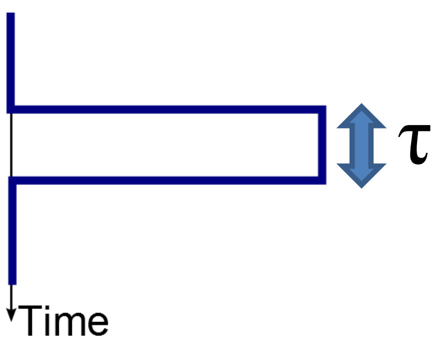
$|\Psi(\tau=0)\rangle = |\uparrow,\downarrow\rangle = 1/\sqrt{2}\,(|S\rangle + |T_0\rangle)$

Two qubit gate : SWAP

$|\Psi(\tau)\rangle = \cos(J(\varepsilon)\,\tau/2\hbar)\,|\uparrow,\downarrow\rangle + i\,sin\,(J(\varepsilon)\,\tau/2\hbar)\,|\downarrow,\uparrow\rangle$

$$\frac{J(\varepsilon)\,\tau}{2\hbar} = \frac{\pi}{2}, \qquad |\Psi(\tau)\rangle = i\,|\downarrow,\uparrow\rangle$$

SWAP gate



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$|\Psi(\tau = 0)\rangle = |\uparrow,\downarrow\rangle$

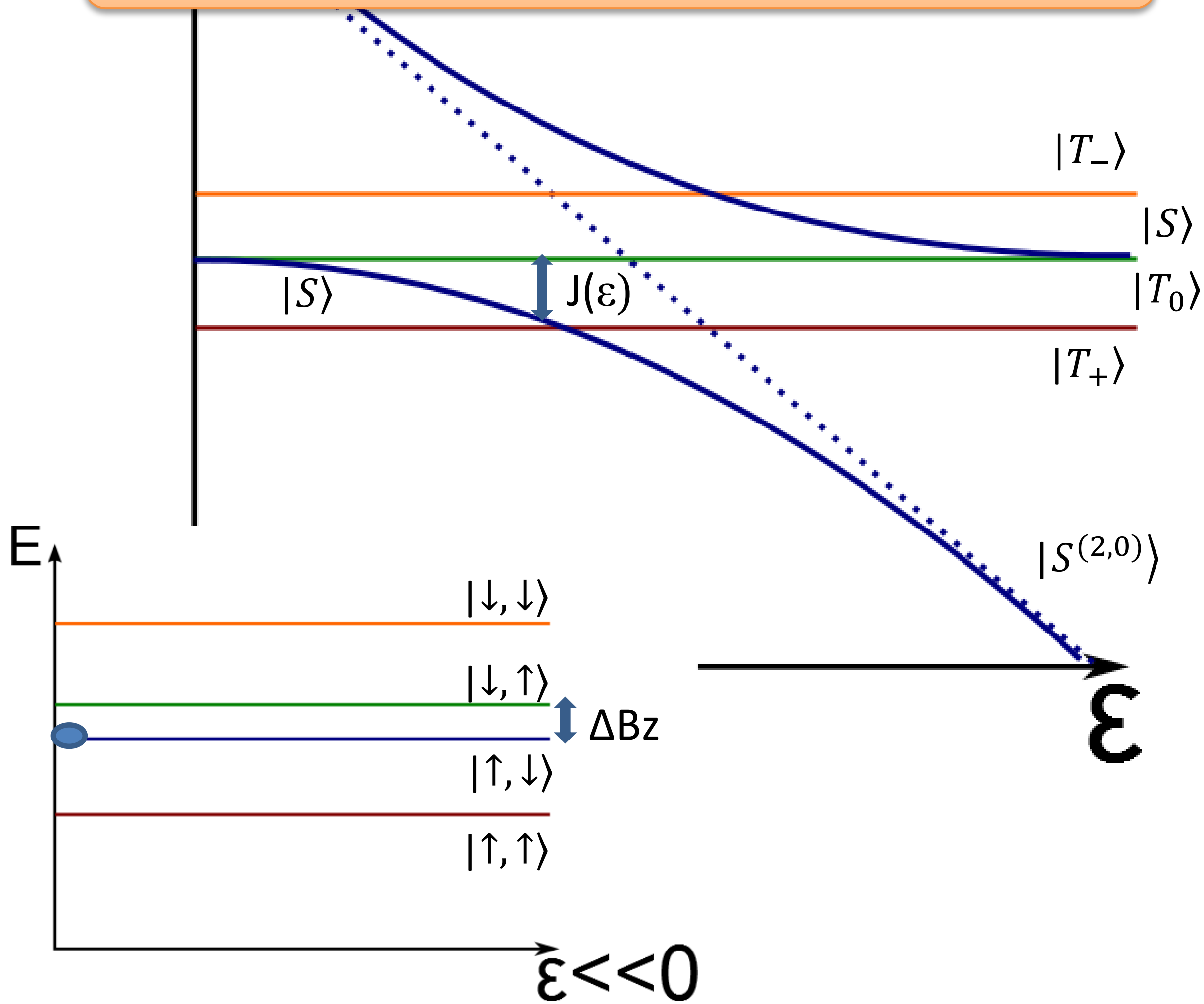
<u>Manipulation :</u>

Non adiabatic pulse with respect to ΔBz

$|\Psi(\tau = 0)\rangle = |\uparrow,\downarrow\rangle = 1/\sqrt{2}\,(|S\rangle + |T_0\rangle)$

Two qubit gate : SWAP

$|\Psi(\tau)\rangle = \cos(J(\varepsilon)\,\tau/2\hbar)\,|\uparrow,\downarrow\rangle + i\,\sin(J(\varepsilon)\,\tau/2\hbar)\,|\downarrow,\uparrow\rangle$



Initialization in the two qubit subspace

$|\Psi(\tau=0)\rangle = |\uparrow,\downarrow\rangle$

Manipulation :

Non adiabatic pulse with respect to ΔBz

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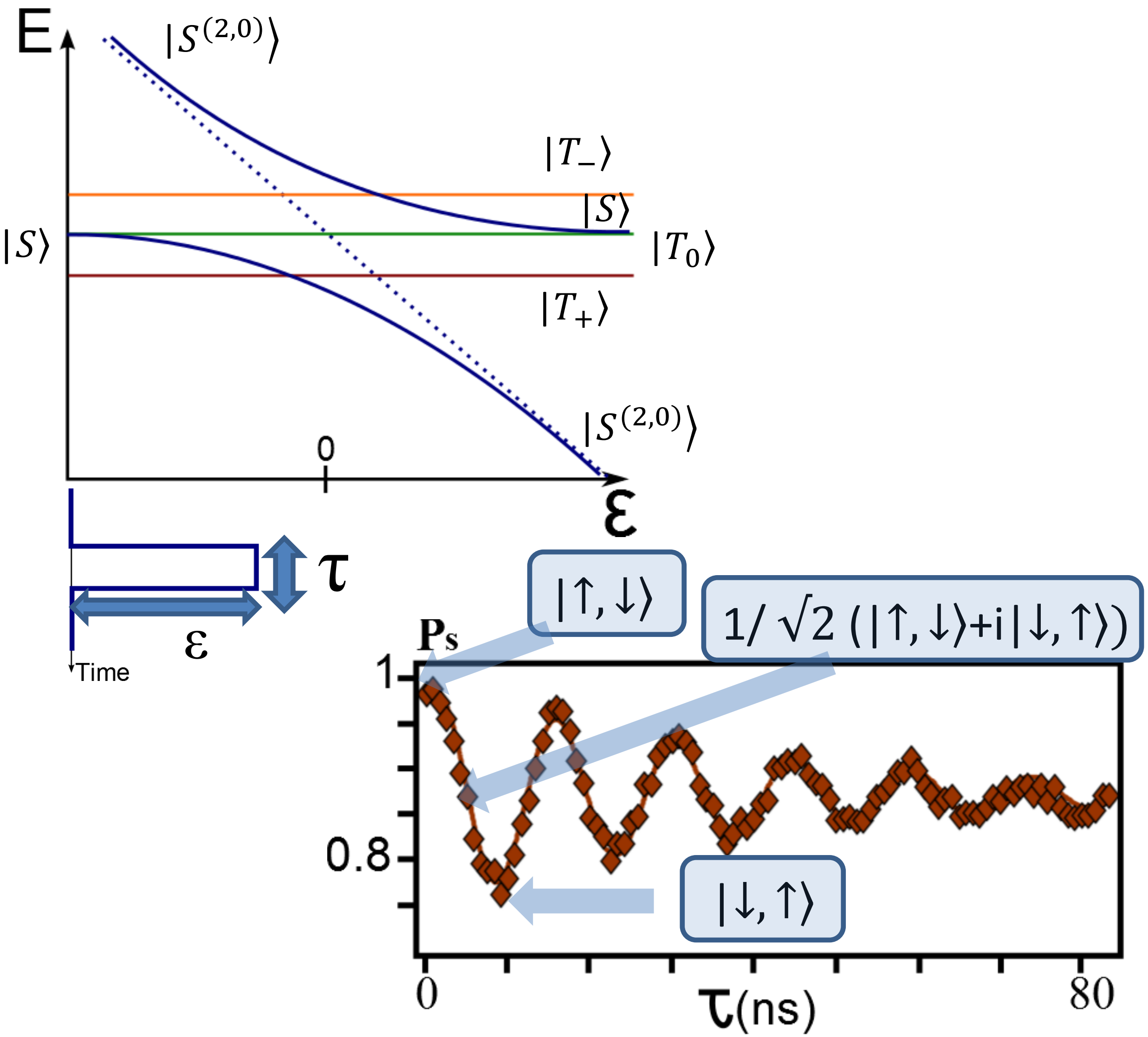

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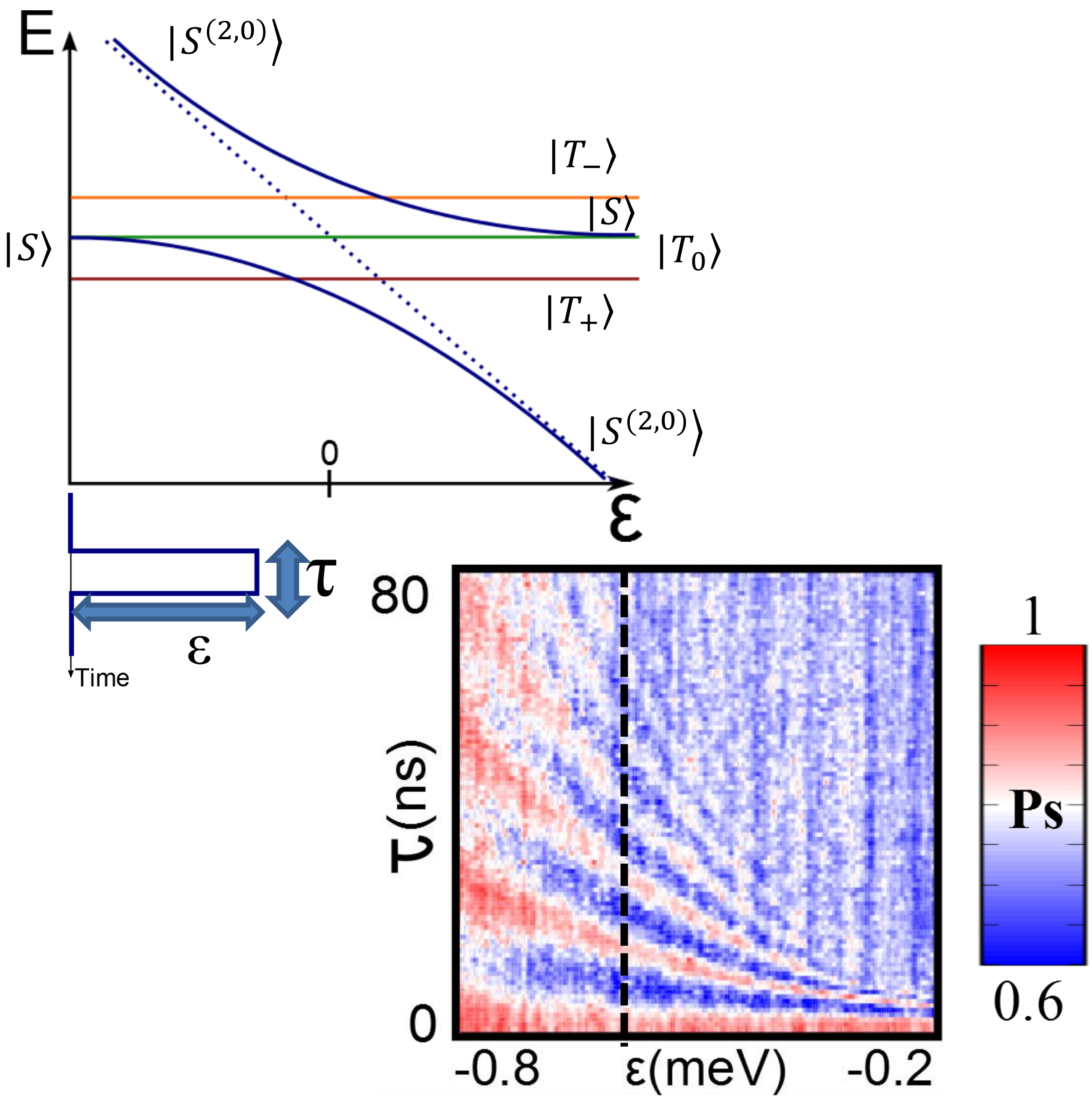

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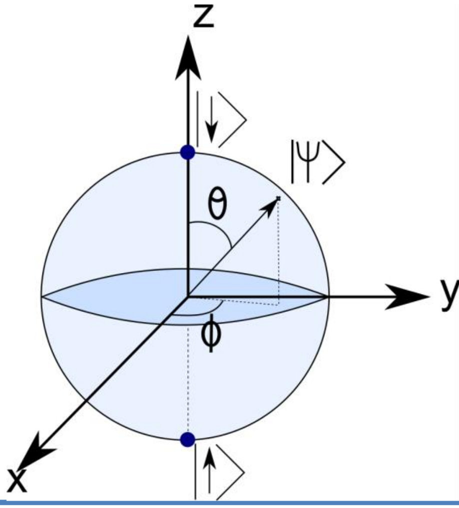
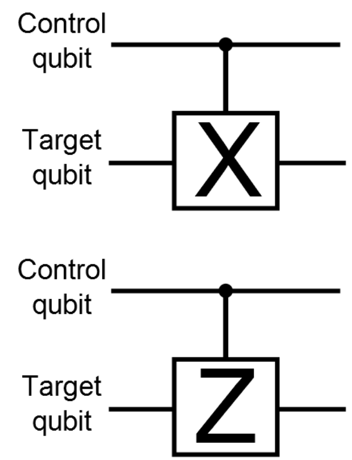
From SWAP to C-phase gate regime

Natural building block for quantum algorithm

Single qubit rotations



Two qubit controlled gates



From SWAP to C-phase gate regime

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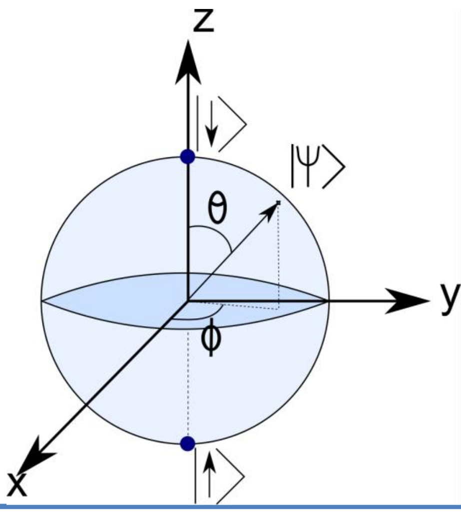
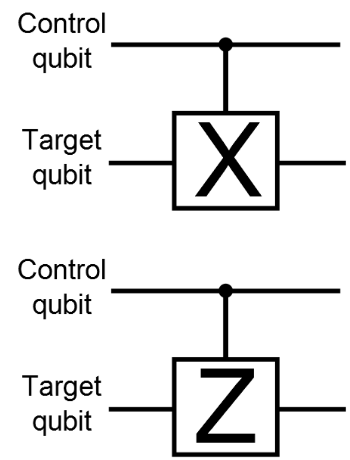
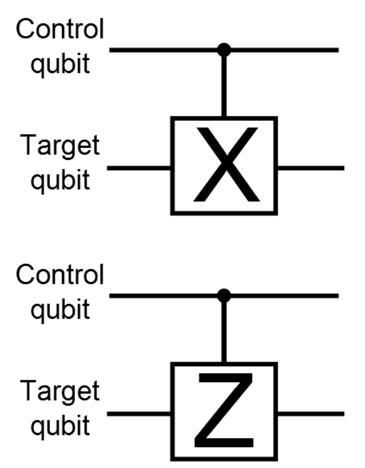


Two qubit controlled gates



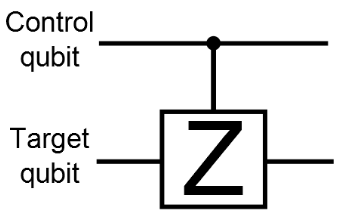
SWAP gate

5 steps



From SWAP to C-phase gate regime

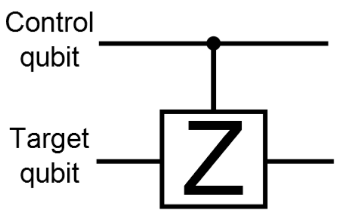
C-phase



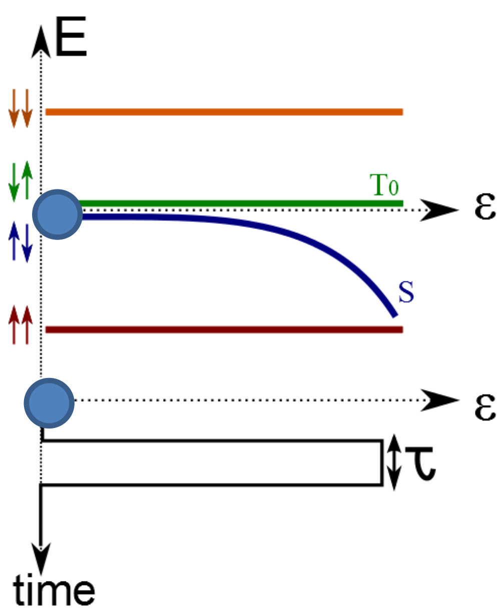
$$U = \begin{array}{c} \begin{array}{cccc} |\uparrow,\uparrow\rangle & |\uparrow,\downarrow\rangle & |\downarrow,\uparrow\rangle & |\downarrow,\downarrow\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array} \Rightarrow U_1 = \begin{array}{c} \begin{array}{cccc} |\uparrow,\uparrow\rangle & |\uparrow,\downarrow\rangle & |\downarrow,\uparrow\rangle & |\downarrow,\downarrow\rangle \end{array} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\phi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$

From SWAP to C-phase gate regime

C-phase



$$U = \begin{array}{c} \begin{matrix} |\uparrow,\uparrow\rangle & |\uparrow,\downarrow\rangle & |\downarrow,\uparrow\rangle & |\downarrow,\downarrow\rangle \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \end{array} \Rightarrow U_1 = \begin{array}{c} \begin{matrix} |\uparrow,\uparrow\rangle & |\uparrow,\downarrow\rangle & |\downarrow,\uparrow\rangle & |\downarrow,\downarrow\rangle \end{matrix} \\ \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\phi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \end{array}$$



<u>Adiabatic pulse vs ΔBz</u>

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{-i\phi} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

From SWAP to C-phase gate regime

Adiabatic pulse vs ΔBz

$$P_{LZ} = e^{-2\pi \frac{\Delta B_z^2}{\hbar \frac{dE}{dt}}}$$

ΔBz~4mT

dE/dt~10^4 eV/s

$$P_{LZ} \sim 1$$

From SWAP to C-phase gate regime

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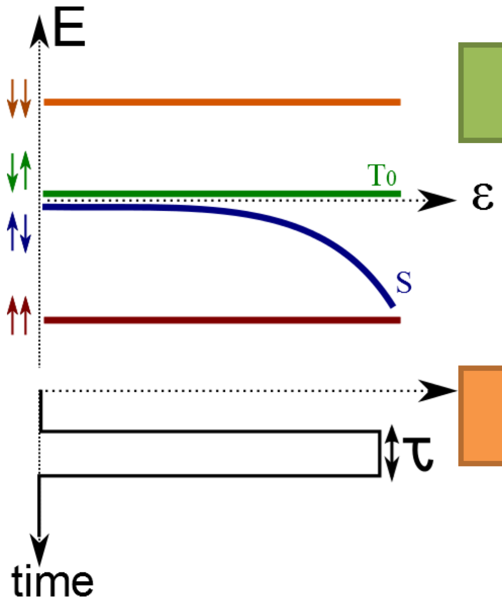
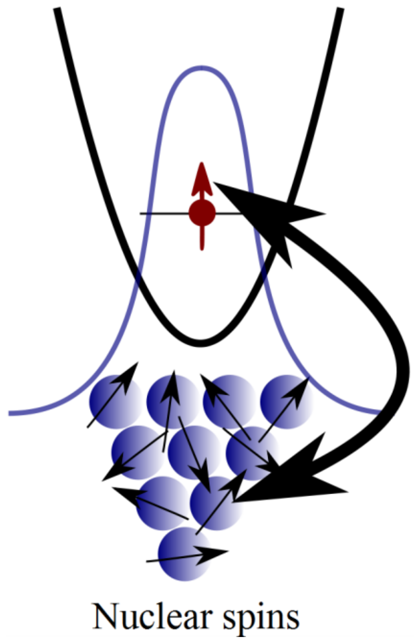
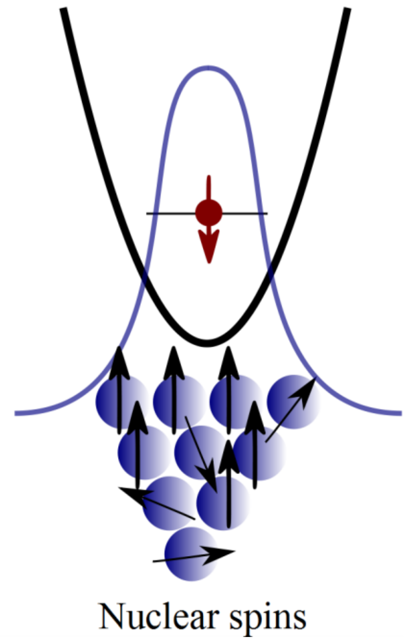


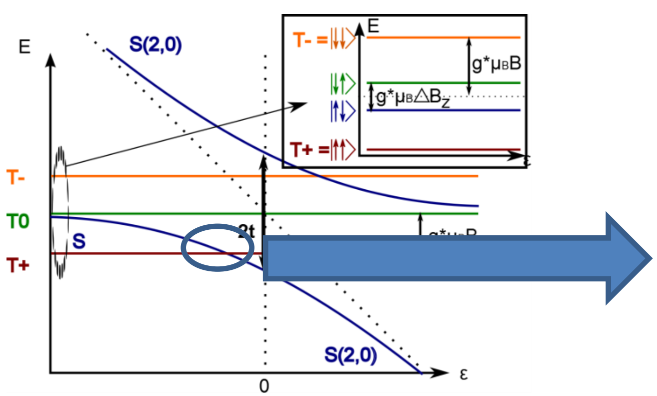
Weak gradient ΔBz → **SWAP**

Strong gradient ΔBz → **C-phase**

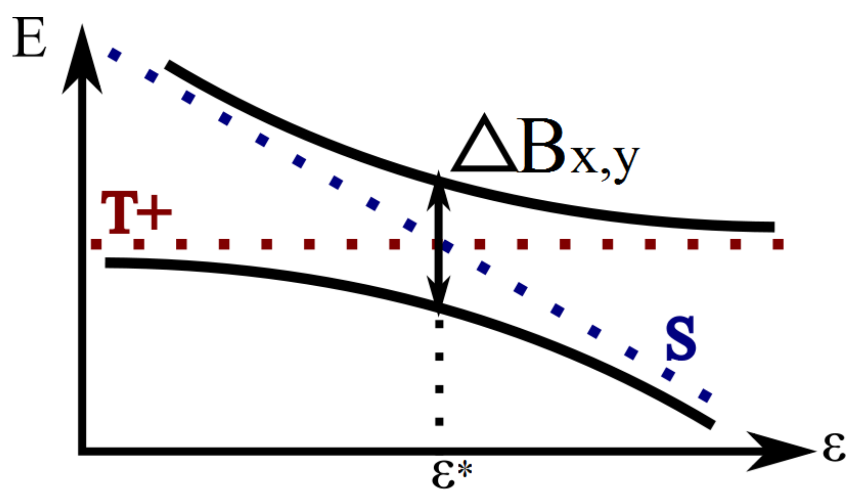
Dynamical nuclear polarization



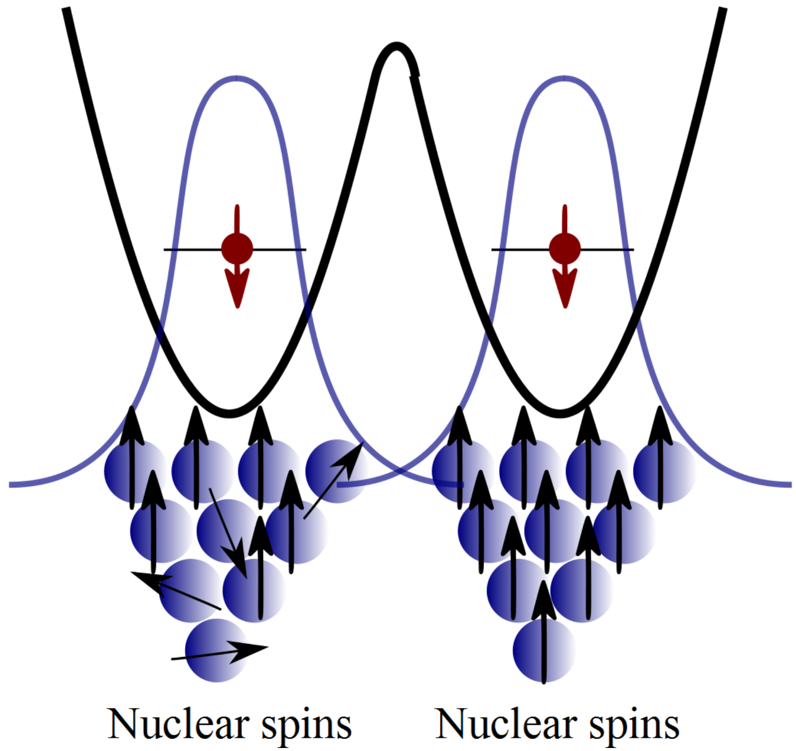


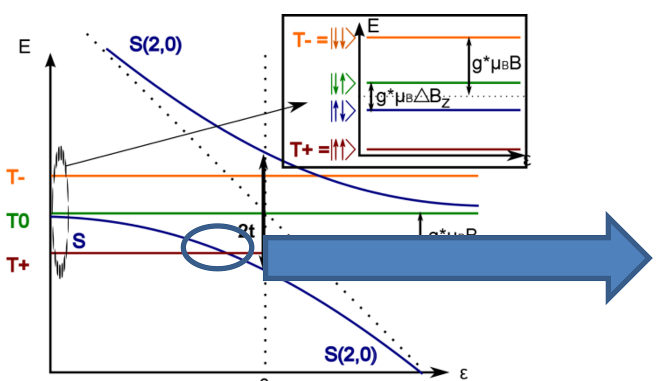


Petta et al, PRL(2008)



Dynamical nuclear polarization





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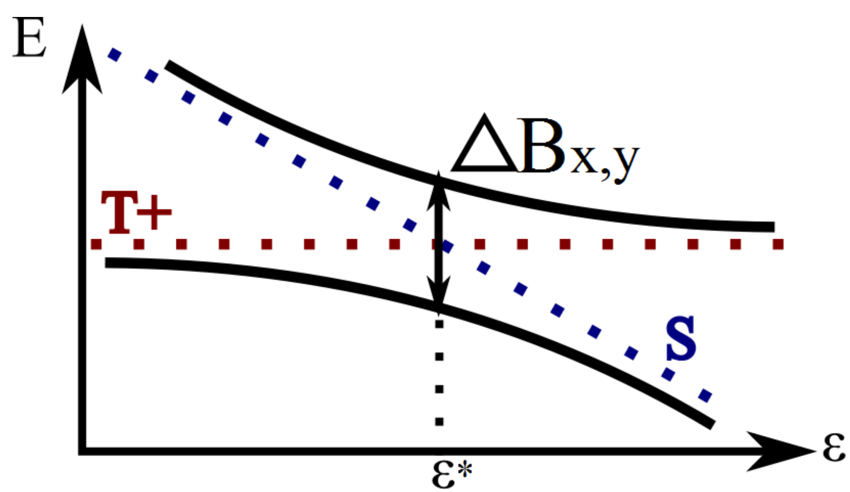
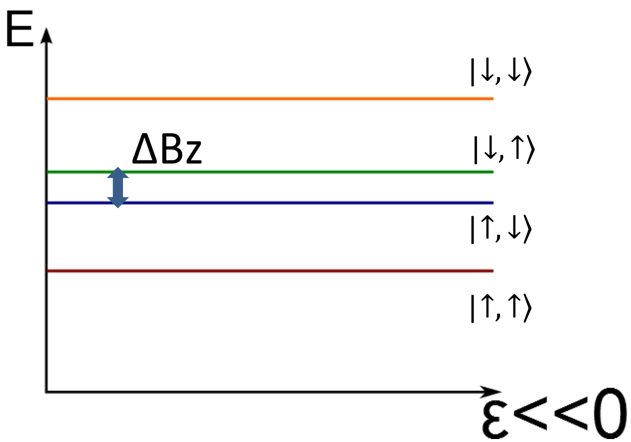
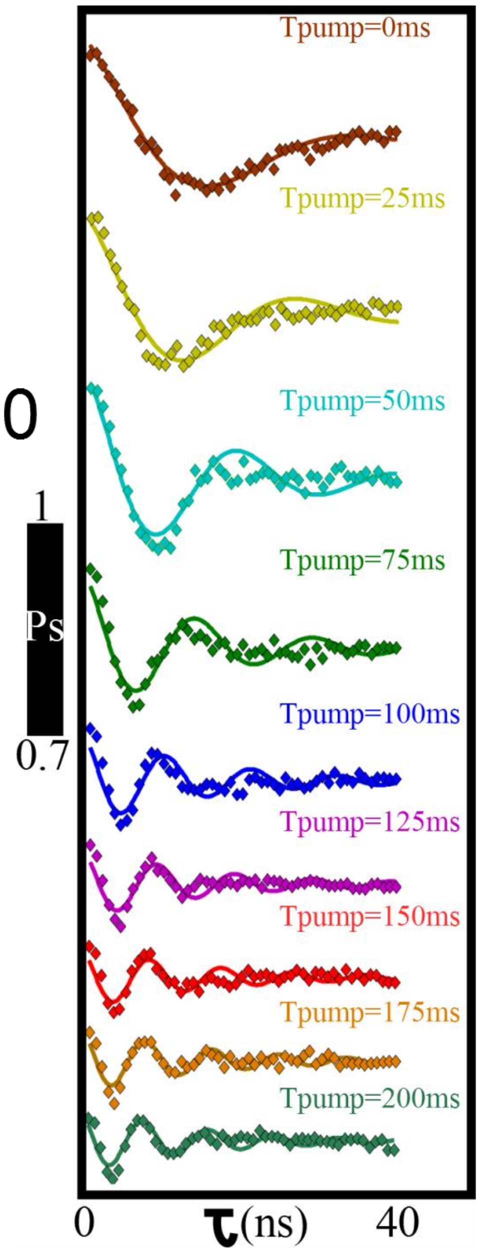


Dynamical nuclear polarization

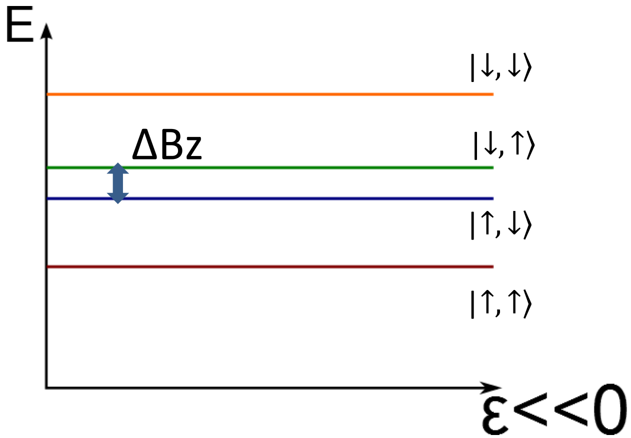


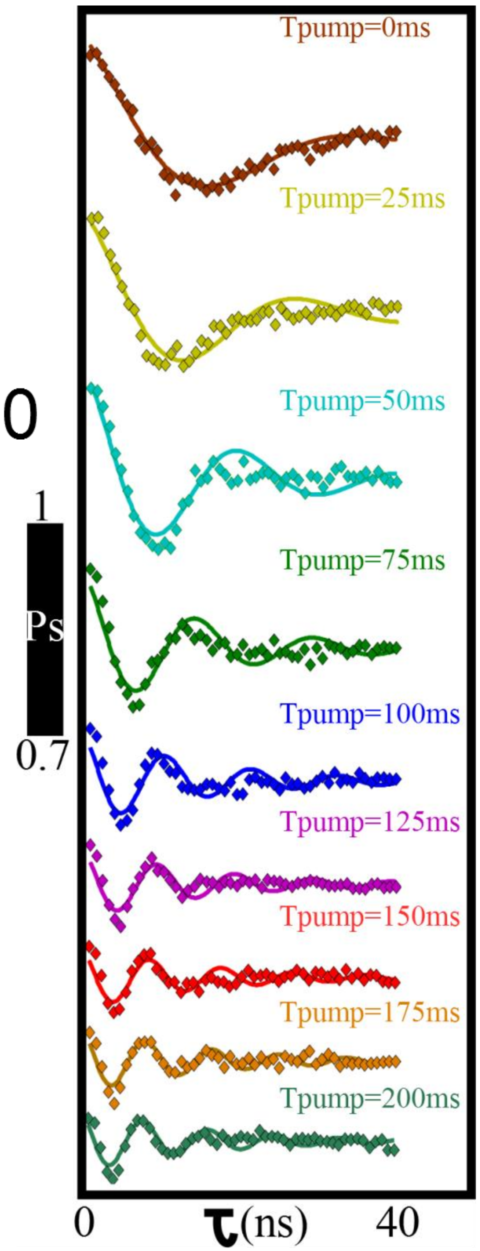


$$|\Psi(\tau)\rangle = \frac{1}{\sqrt{2}}(|\uparrow,\downarrow\rangle e^{i\frac{g^*\mu_B \Delta B_z.\tau}{2\hbar}} - |\downarrow,\uparrow\rangle e^{-i\frac{g^*\mu_B \Delta B_z.\tau}{2\hbar}})$$

Foletti et al, Nat phys (2009)

Dynamical nuclear polarization





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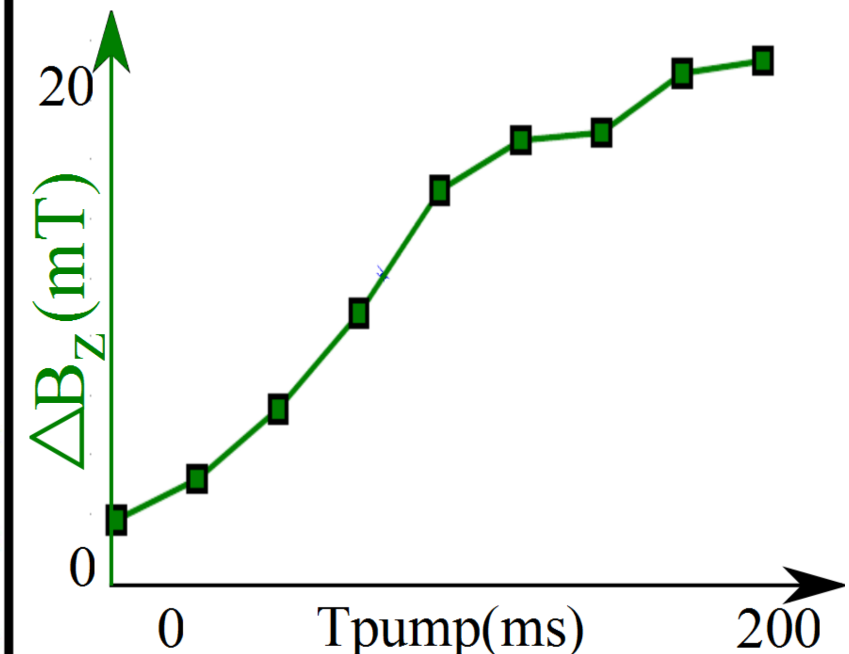
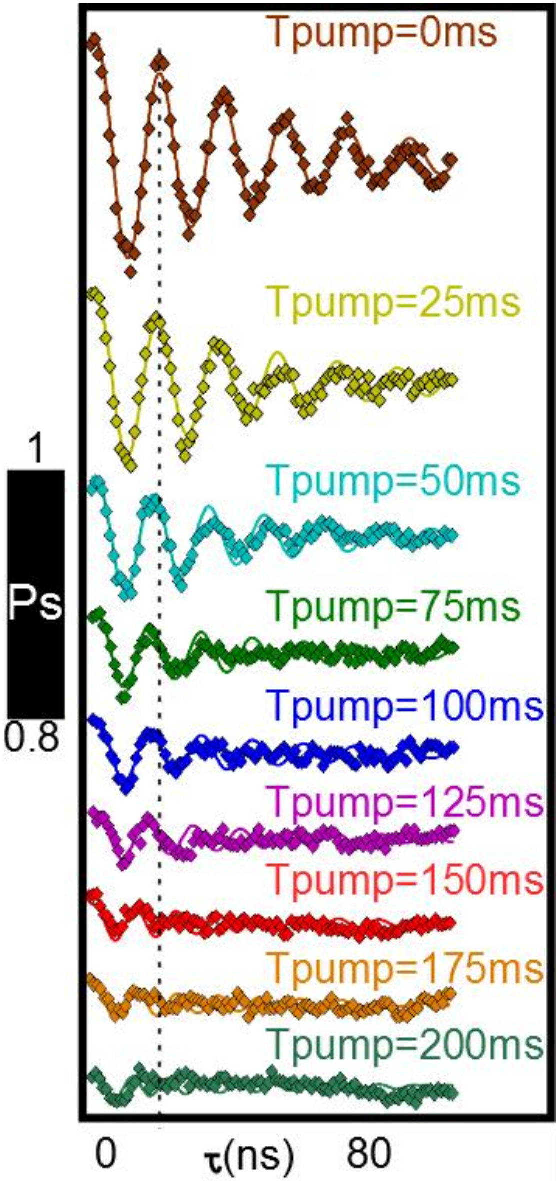


SWAP in presence of a finite gradient ΔBz

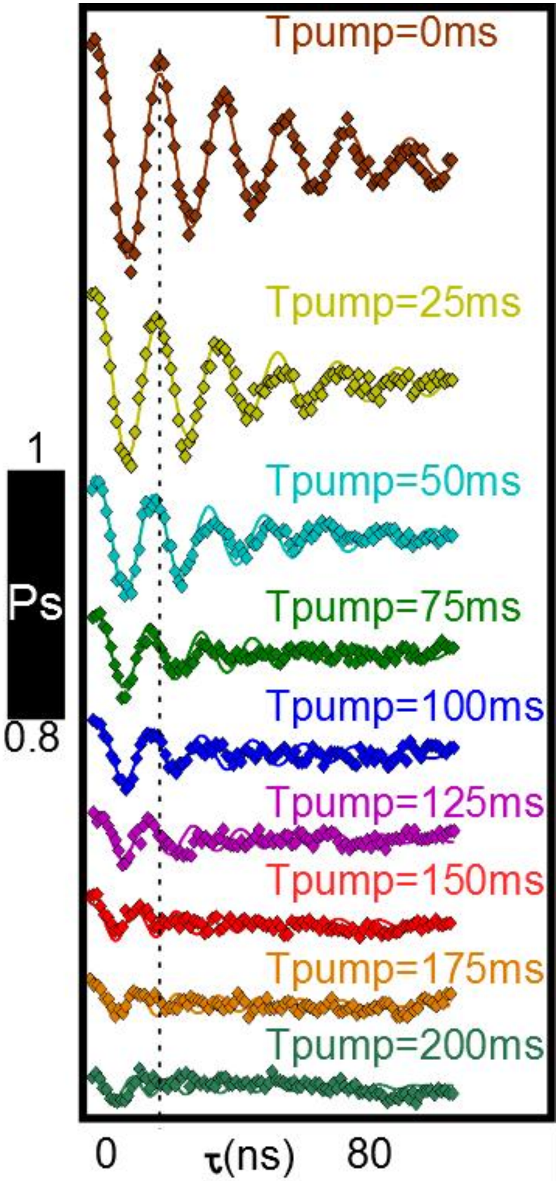


$$|\Psi(\tau=0)\rangle = |\uparrow,\downarrow\rangle$$

$$|\Psi(\tau)\rangle = |\uparrow,\downarrow\rangle = 1/\sqrt{2}\,(|S\rangle e^{iJ(\varepsilon)\tau/(2h)} + |T_0\rangle e^{-iJ(\varepsilon)\tau/(2h)})$$

The amplitude decreases for increasing gradient **ΔBz**

SWAP in presence of a finite gradient ΔBz



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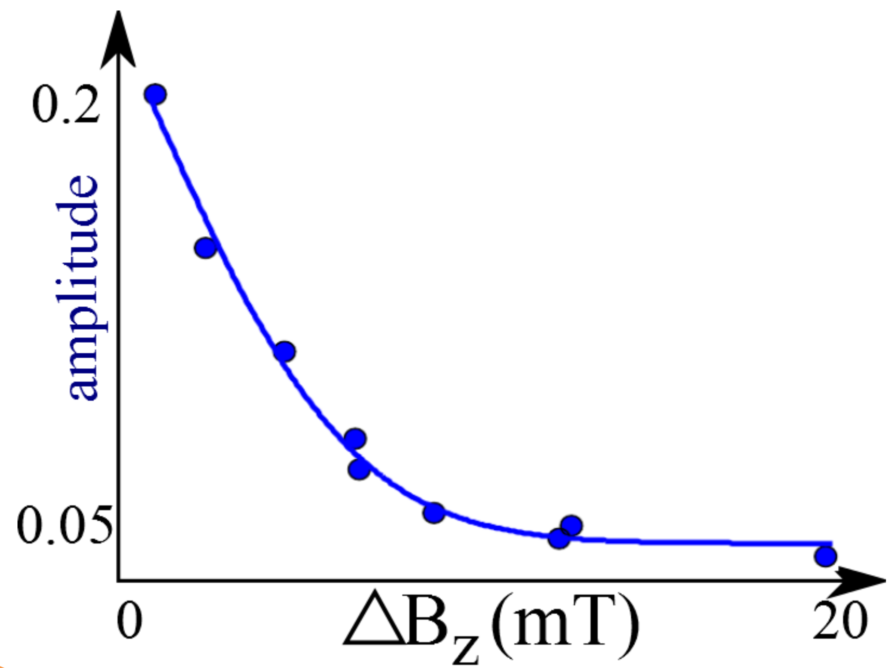
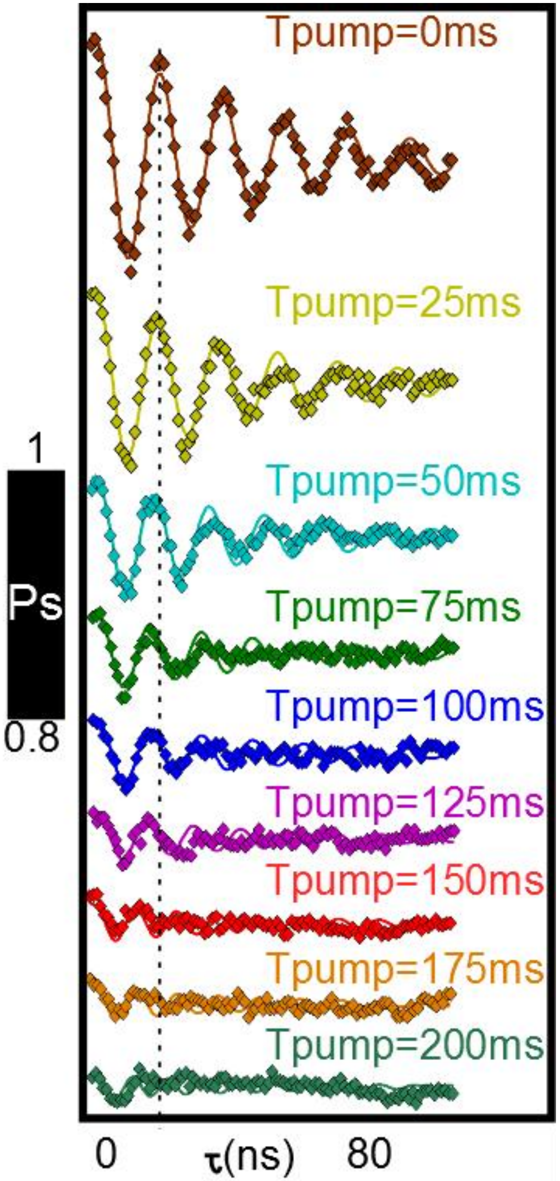
The amplitude decreases for increasing gradient **ΔBz**

Exponential dependence of the amplitude with respect to the gradient **ΔBz**



SWAP in presence of a finite gradient ΔBz



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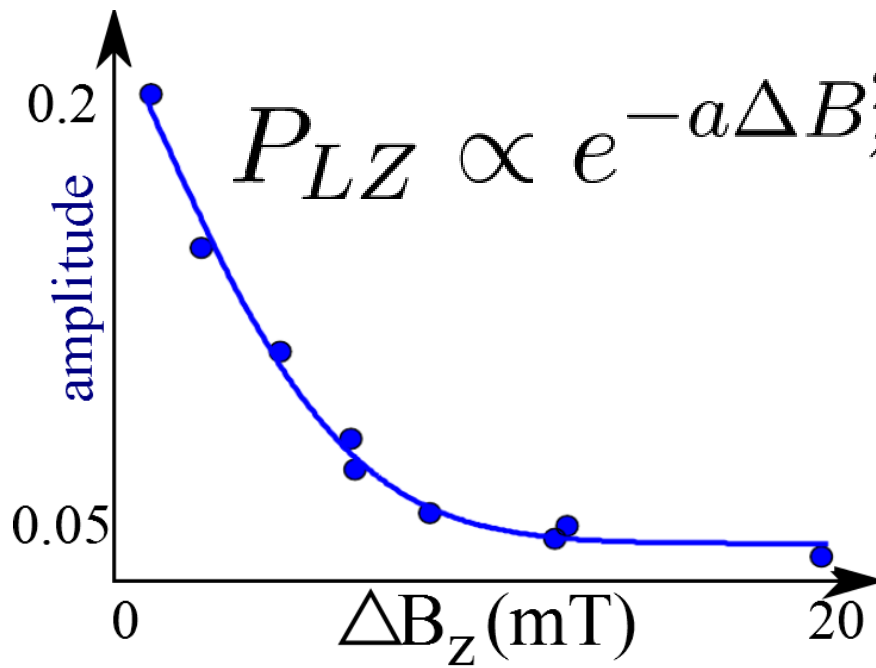
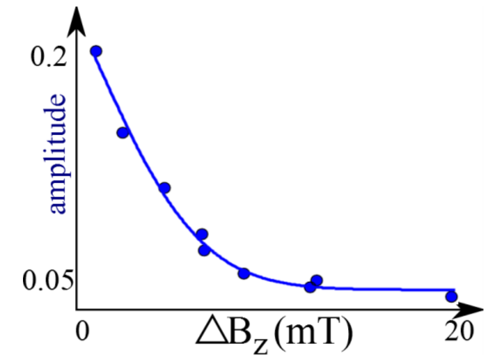


Conclusion and perspectives

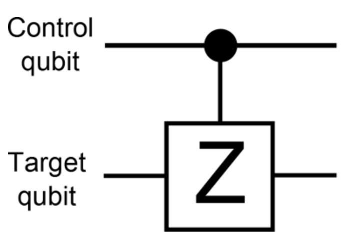
Demonstration of the feasability of a Cphase gate with single spin qubit



Estimation of the C-phase gate duration :

τ~80ns

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & e^{i\phi_1} & 0 & 0 \\ 0 & 0 & e^{i\phi_2} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



We need to prove the entanglement :
Single shot readout of each qubit

Thank you for your attention

Christopher BAUERLE
Tristan MEUNIER
Andreas Wieck (Bochum university)

PhD students :

Tobias BAUTZE
Benoit BERTRAND
Hanno FLENTJE
Sylvain Hermelin (now at Geneva)
Grégoire ROUSSELY

Any questions ?