Untitled

June 4, 2018

```
In [64]: import matplotlib.pyplot as plt
         import numpy as np
         import math
         from scipy.stats import chi2
         import scipy.stats as sts
```

```
In [65]: def fx(x):
             return 1 / (x + 3)

         def F(y):
             return -1 / 10 * (1 / y - 13)

         def g(y):
             return 0.1 / (y ** 2)
```

```
In [83]: def generate_Y_v(n):
             a = 0
             b = 10
             # print('M', (b+a)/2)
             # print('D', round(((b-a)**2)/12, 4))
             E = []
             for i in range(n):
                 E.append(np.random.uniform(0, 1))

             X = []
             Y_v = []
             for i in range(n):
                 X.append(E[i] * (b - a) + a)
                 Y_v.append(fx(X[i]))
             Y_v = sorted(Y_v)
             return Y_v
```

0.0.1

```
In [84]: def math_interval_unknown(Y_v, a):
             n = len(Y_v)
             M = sum(Y_v) / n #
             D = 0
             for y in Y_v:          #
                 D += (M - y) ** 2 #
             D /= n - 1             #

             #      (1 - a/2)
             u = sts.norm.ppf(1 - a / 2.0)
             print('m_interval: ({l}, {r})'.format(l=(M - math.sqrt(D / (n - 1)) * u),
                                                   r=(M + math.sqrt(D / (n - 1)) * u)))
```

```
In [123]:     Y_v = generate_Y_v(20)

              alpha = [0.1, 0.01, 0.05, 0.001]
              for _alpha in alpha:
                  print('alpha = {}, unknown D:'.format(_alpha))
                  math_interval_unknown(Y_v, _alpha)
                  print()
```

```
alpha = 0.1, unknown D:
m_interval: (0.12722565624744328, 0.17649576562191568)

alpha = 0.01, unknown D:
m_interval: (0.11328238717494178, 0.1904390346944172)

alpha = 0.05, unknown D:
m_interval: (0.12250623272883113, 0.18121518914052784)

alpha = 0.001, unknown D:
m_interval: (0.1025783282035456, 0.20114309366581337)
```

0.0.2

```
In [86]: def math_interval_known(Y_v, a):
             n = len(Y_v)
             M = sum(Y_v) / n
             D = 0.004
             #      (1 - a/2)
             u = sts.norm.ppf(1 - a / 2.0)
             print('m_interval: ({l}, {r})'.format(l=(M - math.sqrt(D / (n - 1)) * u),
                                                   r=(M + math.sqrt(D / (n - 1)) * u)))
```

```
In [87]:     Y_v = generate_Y_v(20)

             alpha = [0.1, 0.01, 0.05, 0.001]
             for _alpha in alpha:
                 print('alpha = {}, known D:'.format(_alpha))
                 math_interval_known(Y_v, _alpha)
                 print()
```

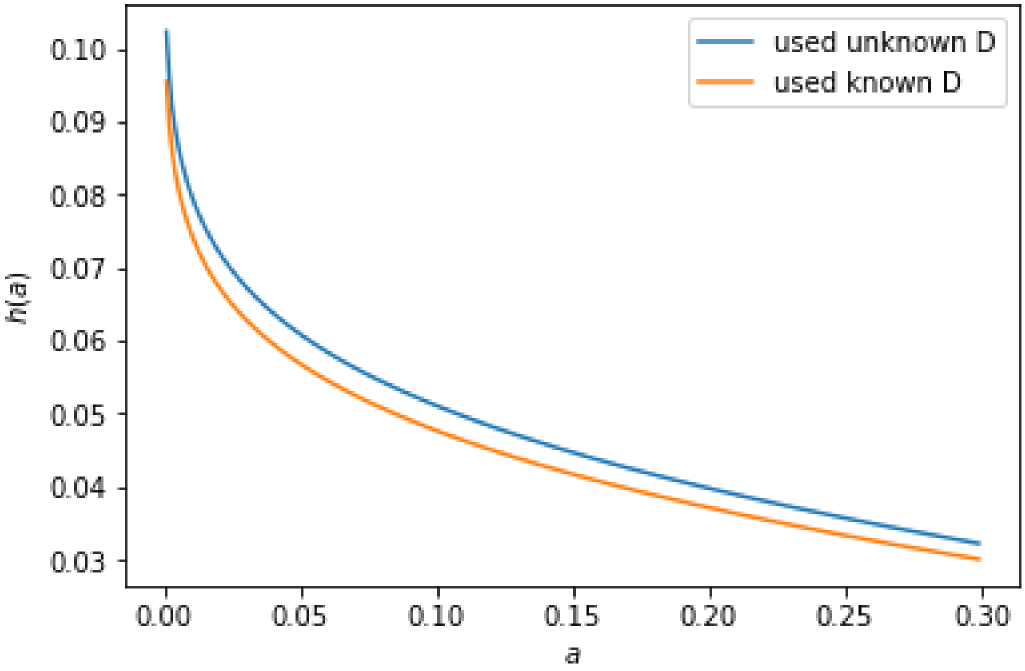
```
alpha = 0.1, known D:
m_interval: (0.09512407723622843, 0.14285616688665267)

alpha = 0.01, known D:
m_interval: (0.08161606238019777, 0.15636418174268335)

alpha = 0.05, known D:
m_interval: (0.09055197562434464, 0.14742826849853646)

alpha = 0.001, known D:
m_interval: (0.07124614218045025, 0.16673410194243085)
```

0.0.3

```
In [88]: def plot_m_unknown(Y_v):
             n = len(Y_v)
             M = sum(Y_v) / n
             D = 0
             for y in Y_v:
                 D += (M - y) ** 2
             D /= n - 1
             a = np.arange(0,0.3,0.001)
             h = [2 * math.sqrt(D / (n-1))*sts.norm.ppf(1 - x/2.0) for x in a]
             plt.plot(a,h, label='used unknown D')
             plt.ylabel('$h(a)$')
             plt.xlabel('$a$')
             plt.legend()
```

```
In [89]: def plot_m_known(Y_v):
             n = len(Y_v)
             D = 0.004
             a = np.arange(0,0.3,0.001)
             h = [2*math.sqrt(D/(n-1))*sts.norm.ppf(1 - x/2.0) for x in a]
             plt.plot(a,h, label='used known D')
             plt.ylabel('$h(a)$')
             plt.xlabel('$a$')
             plt.legend()
```

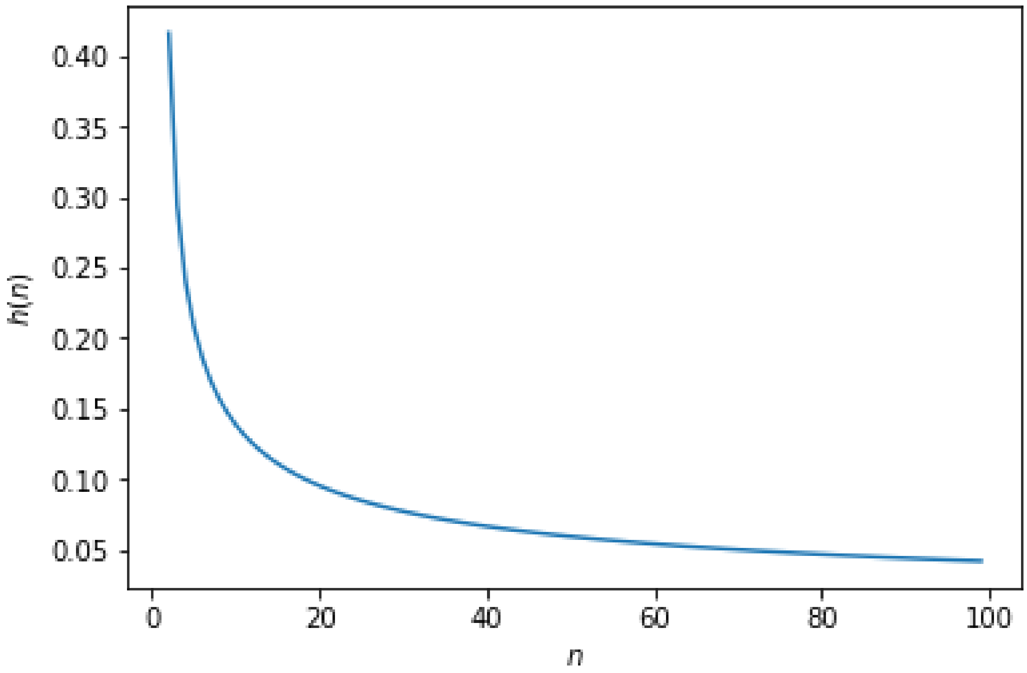
```
In [94]:     n = 20
             Y_v = generate_Y_v(n)
             plot_m_unknown(Y_v)
             plot_m_known(Y_v)
             plt.show()
```



0.0.4

```
In [95]: def plot_h_n(a):
             D = 0.004
             n = np.arange(2, 100, 1)
             h = [2*math.sqrt(D / (x-1))*sts.norm.ppf(1 - a/2.0) for x in n]
             plt.plot(n, h)
             plt.ylabel('$h(n)$')
             plt.xlabel('$n$')
```

```
In [96]:     n = 20
             plot_h_n(0.001)
             plt.show()
```

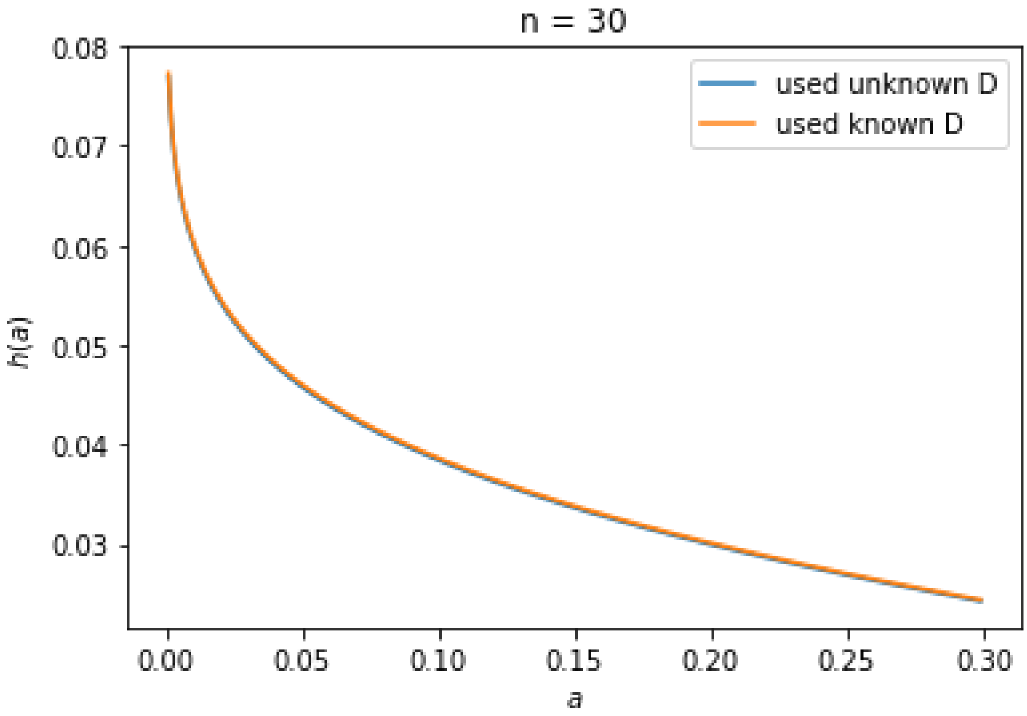


0.0.5 30, 50, 70, 100, 150

```
In [97]:     Y_v = generate_Y_v(30)

             alpha = [0.1, 0.01, 0.05, 0.001]
             for _alpha in alpha:
                 print('alpha = {}, unknown D:'.format(_alpha))
                 math_interval_unknown(Y_v, _alpha)
                 print()
             for _alpha in alpha:
                 print('alpha = {}, known D:'.format(_alpha))
                 math_interval_known(Y_v, _alpha)
                 print()
             plt.title('n = 30')
             plot_m_unknown(Y_v)
             plot_m_known(Y_v)
             plt.show()
```

```
alpha = 0.1, unknown D:
m_interval: (0.1208387002697719, 0.15926820914376094)

alpha = 0.01, unknown D:
m_interval: (0.10996328332109884, 0.170143626092434)

alpha = 0.05, unknown D:
```

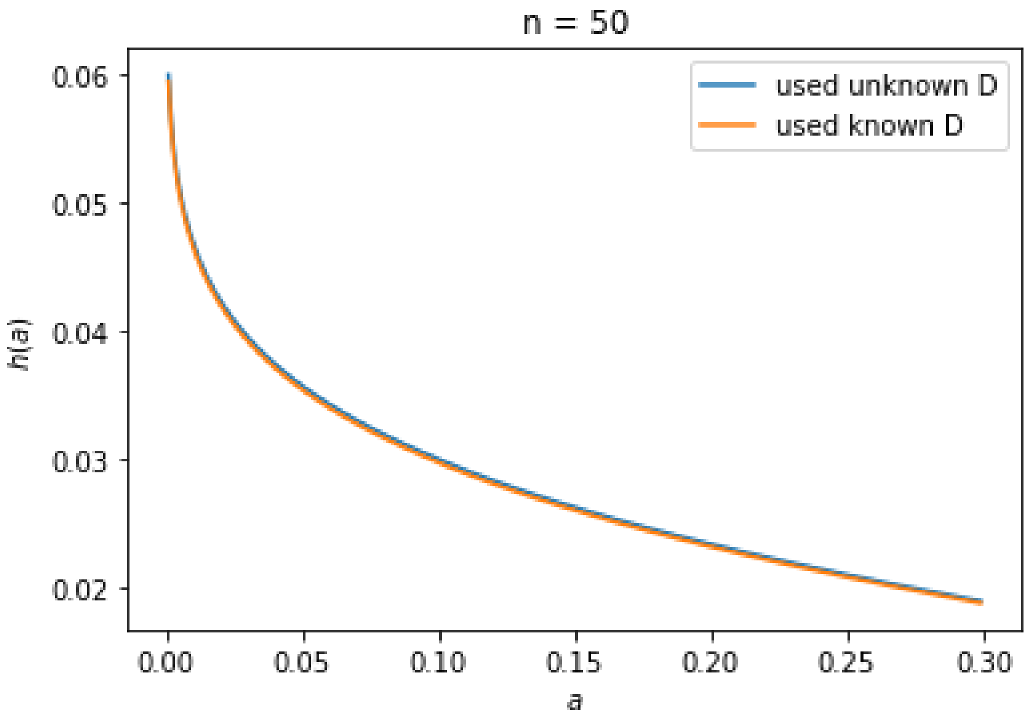
```
m_interval: (0.11715766261243343, 0.1629492468010994)

alpha = 0.001, unknown D:
m_interval: (0.10161437290763628, 0.17849253650589655)

alpha = 0.1, known D:
m_interval: (0.12073562854900642, 0.1593712808645264)

alpha = 0.01, known D:
m_interval: (0.1098018737255186, 0.17030503568801425)

alpha = 0.05, known D:
m_interval: (0.11703484508191594, 0.1630720643316169)

alpha = 0.001, known D:
m_interval: (0.10140817811505265, 0.17869873129848018)
```



```
In [98]:     Y_v = generate_Y_v(50)

             alpha = [0.1, 0.01, 0.05, 0.001]
             for _alpha in alpha:
                 print('alpha = {}, unknown D:'.format(_alpha))
```

```
        math_interval_unknown(Y_v, _alpha)
        print()
    for _alpha in alpha:
        print('alpha = {} with known D:'.format(_alpha))
        math_interval_known(Y_v, _alpha)
        print()
    plt.title('n = 50')
    plot_m_unknown(Y_v)
    plot_m_known(Y_v)
    plt.show()
```

```
alpha = 0.1, unknown D:
m_interval: (0.1294335229113622, 0.15942002486306936)

alpha = 0.01, unknown D:
m_interval: (0.12094744746797437, 0.1679061003064572)

alpha = 0.05, unknown D:
m_interval: (0.12656121342497167, 0.1622923343494599)

alpha = 0.001, unknown D:
m_interval: (0.11443280219293742, 0.17442074558149415)

alpha = 0.1 with known D:
m_interval: (0.12956539137648557, 0.159288156397946)

alpha = 0.01 with known D:
m_interval: (0.12115395256430217, 0.1676995952101294)

alpha = 0.05 with known D:
m_interval: (0.12671834439279975, 0.16213520338163182)

alpha = 0.001 with known D:
m_interval: (0.11469660482097638, 0.17415694295345518)
```

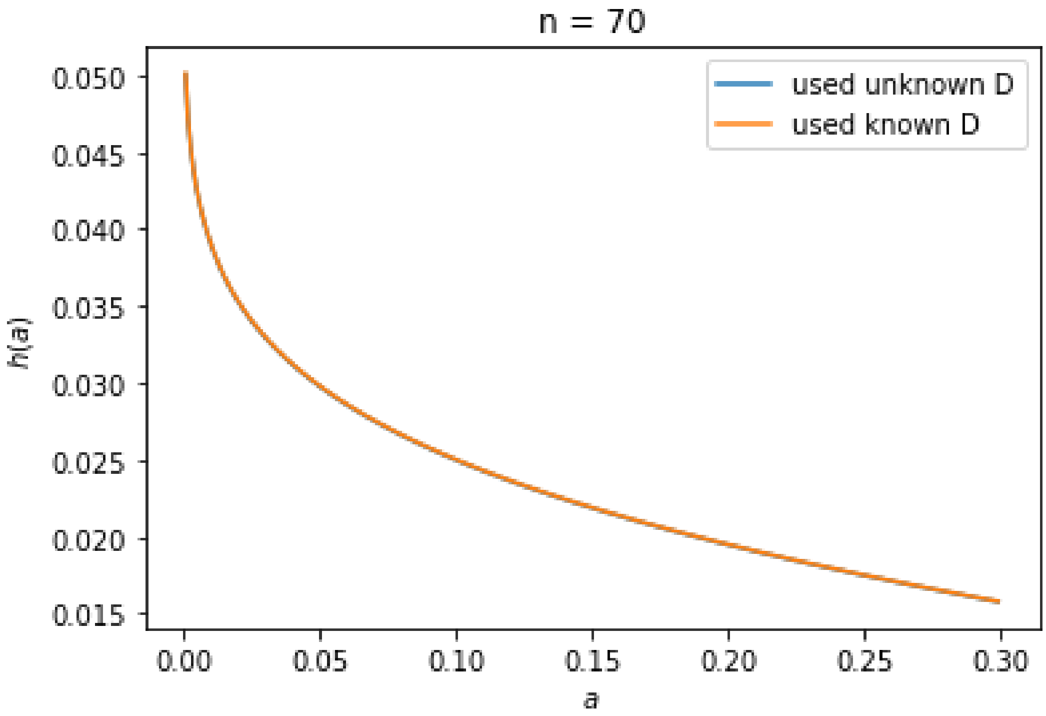


In [124]:

```
Y_v = generate_Y_v(70)

alpha = [0.1, 0.01, 0.05, 0.001]
for _alpha in alpha:
    print('M for alpha = {}, unknown D:'.format(_alpha))
    math_interval_unknown(Y_v, _alpha)
    print()
for _alpha in alpha:
    print('M for alpha = {} with known D:'.format(_alpha))
    math_interval_known(Y_v, _alpha)
    print()
plt.title('n = 70')
plot_m_unknown(Y_v)
plot_m_known(Y_v)
plt.show()
```

```
M for alpha = 0.1, unknown D:
m_interval: (0.13194672375230682, 0.1569616935094913)

M for alpha = 0.01, unknown D:
m_interval: (0.12486757457660753, 0.1640408426851906)

M for alpha = 0.05, unknown D:
m_interval: (0.12955062116418076, 0.15935779609761735)
```

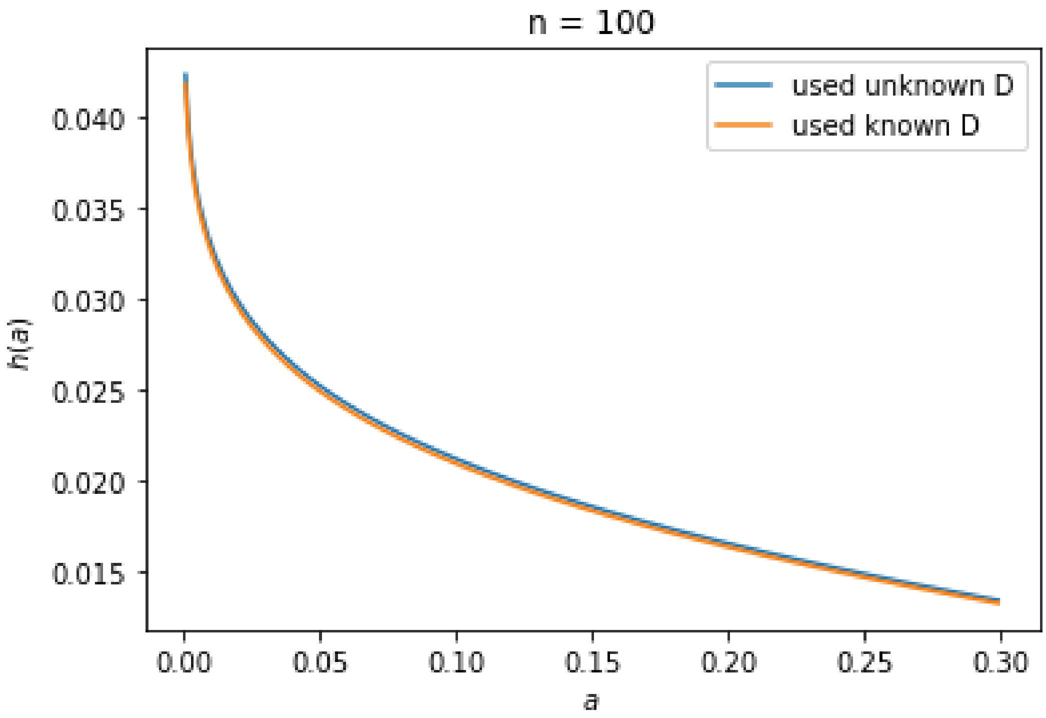
```
M for alpha = 0.001, unknown D:
m_interval: (0.11943300755721045, 0.16947540970458766)

M for alpha = 0.1 with known D:
m_interval: (0.13193050714978174, 0.15697791011201637)

M for alpha = 0.01 with known D:
m_interval: (0.12484217949019762, 0.1640662377716005)

M for alpha = 0.05 with known D:
m_interval: (0.12953129789043774, 0.15937711937136037)

M for alpha = 0.001 with known D:
m_interval: (0.11940056627293541, 0.1695078509888627)
```



```
In [101]:    Y_v = generate_Y_v(100)

             alpha = [0.1, 0.01, 0.05, 0.001]
             for _alpha in alpha:
                 print('alpha = {}, unknown D:'.format(_alpha))
                 math_interval_unknown(Y_v, _alpha)
```

```
                print()
            for _alpha in alpha:
                print('alpha = {} with known D:'.format(_alpha))
                math_interval_known(Y_v, _alpha)
                print()
            plt.title('n = 100')
            plot_m_unknown(Y_v)
            plot_m_known(Y_v)
            plt.show()
```

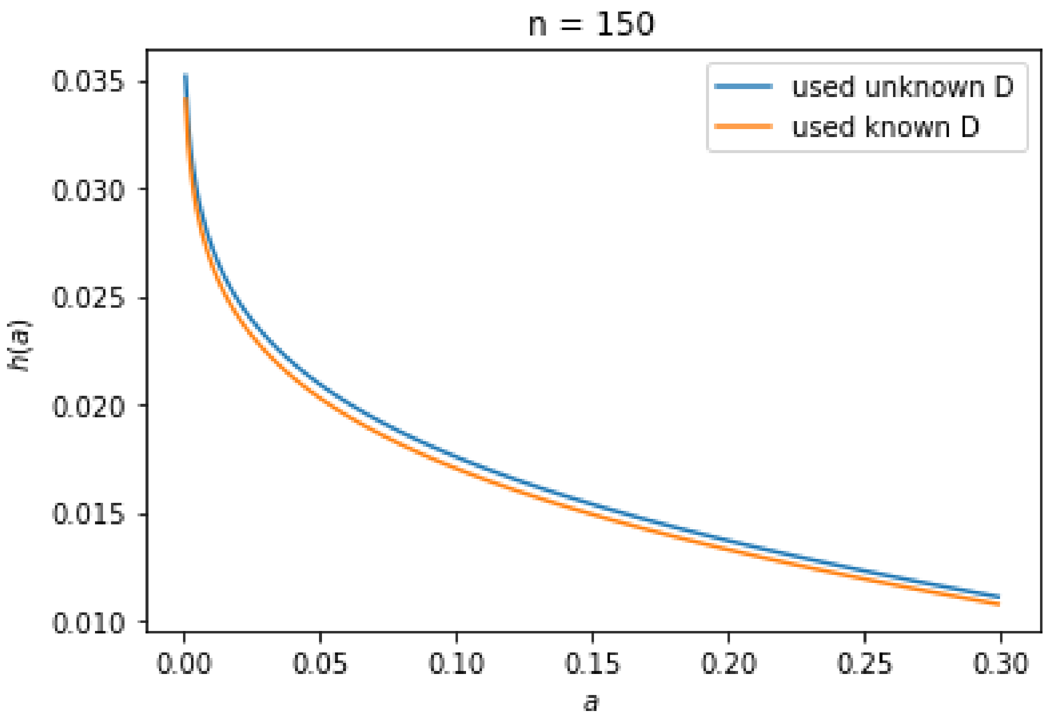
```
alpha = 0.1, unknown D:
m_interval: (0.1294459846368621, 0.15061174721652051)

alpha = 0.01, unknown D:
m_interval: (0.12345614766431084, 0.15660158418907177)

alpha = 0.05, unknown D:
m_interval: (0.12741858508415166, 0.15263914676923096)

alpha = 0.001, unknown D:
m_interval: (0.11885783088150666, 0.16119990097187598)

alpha = 0.1 with known D:
m_interval: (0.1295734899394703, 0.15048424191391233)

alpha = 0.01 with known D:
m_interval: (0.12365582007858195, 0.15640191177480067)

alpha = 0.05 with known D:
m_interval: (0.1275705170231735, 0.1524872148302091)

alpha = 0.001 with known D:
m_interval: (0.11911290501075714, 0.16094482684262548)
```



```
In [102]:     Y_v = generate_Y_v(150)

              alpha = [0.1, 0.01, 0.05, 0.001]
              for _alpha in alpha:
                  print('alpha = {}, unknown D:'.format(_alpha))
                  math_interval_unknown(Y_v, _alpha)
                  print()
              for _alpha in alpha:
                  print('alpha = {} with known D:'.format(_alpha))
                  math_interval_known(Y_v, _alpha)
                  print()
              plt.title('n = 150')
              plot_m_unknown(Y_v)
              plot_m_known(Y_v)
              plt.show()
```

```
alpha = 0.1, unknown D:
m_interval: (0.14646233303330583, 0.16404156104175896)

alpha = 0.01, unknown D:
m_interval: (0.1414874728326562, 0.1690164212424086)

alpha = 0.05, unknown D:
m_interval: (0.14477847596143328, 0.1657254181136315)
```

```
alpha = 0.001, unknown D:
m_interval: (0.13766833998186542, 0.17283555409319937)

alpha = 0.1 with known D:
m_interval: (0.14672950378244887, 0.16377439029261592)

alpha = 0.01 with known D:
m_interval: (0.1419058603617455, 0.1685980337133193)

alpha = 0.05 with known D:
m_interval: (0.14509682954467185, 0.16540706453039294)

alpha = 0.001 with known D:
m_interval: (0.13820281458648864, 0.17230107948857615)
```



0.0.6

```
In [103]: def d_interval_unknown(Y_v, a):
              n = len(Y_v)
              M = 0.146
              D = sum([(M - y)**2 for y in Y_v]) / (n-1)
```

```
            #Chi2
            print("d_interval: ({}, {})".format(D*(n-1)/sts.chi2.ppf(1-a/2, n-1), D*(n-1)/sts
```

```
In [104]: Y_v = generate_Y_v(20)
          alpha = [0.1, 0.01, 0.05, 0.001]
          for _alpha in alpha:
              print('alpha = {}, unknown M:'.format(_alpha))
              d_interval_unknown(Y_v, _alpha)
              print()
```

```
alpha = 0.1, unknown M:
d_interval: (0.0025746342175709345, 0.007671093838870055)

alpha = 0.01, unknown M:
d_interval: (0.0020115090072929317, 0.011339696139901196)

alpha = 0.05, unknown M:
d_interval: (0.002362345806084886, 0.008713682474948322)

alpha = 0.001, unknown M:
d_interval: (0.0016881290049082578, 0.015798693658348996)
```

```
In [105]: def d_interval_known(Y_v, a):
              n = len(Y_v)
              M = sum([x / n for x in Y_v])
              D = sum([(M - y)**2 for y in Y_v]) / (n-1)
              #Chi2
              print("d_interval: ({}, {})".format(D*(n-1)/sts.chi2.ppf(1-a/2, n-1), D*(n-1)/sts
```

```
In [106]: Y_v = generate_Y_v(20)
          alpha = [0.1, 0.01, 0.05, 0.001]
          for _alpha in alpha:
              print('alpha = {}, known M:'.format(_alpha))
              d_interval_known(Y_v, _alpha)
              print()
```

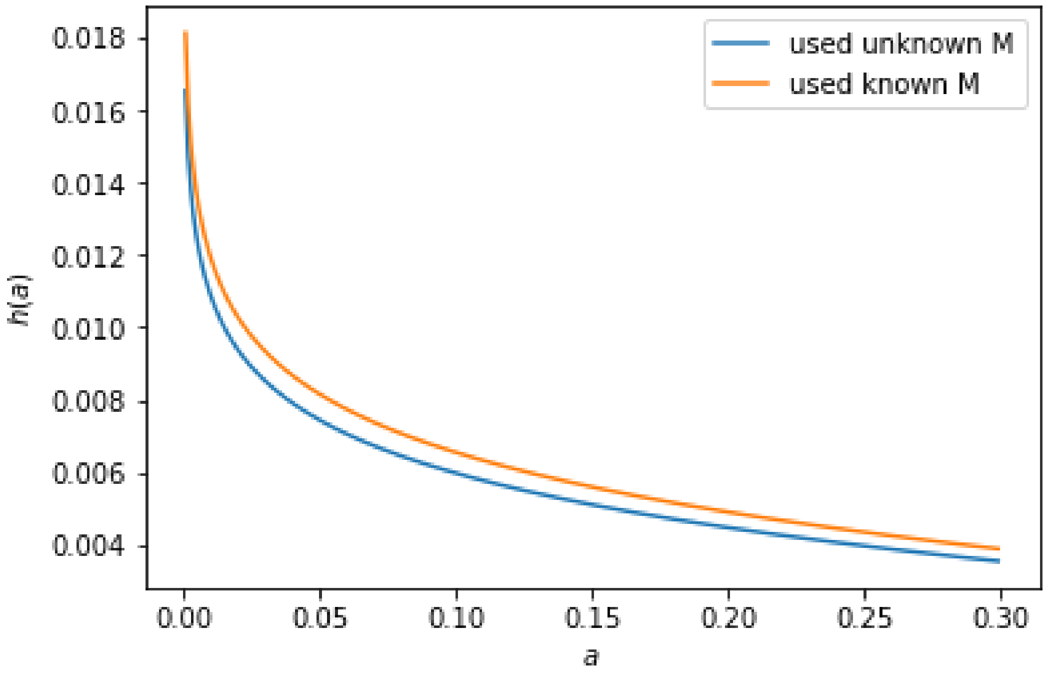
```
alpha = 0.1, known M:
d_interval: (0.001439627237057371, 0.004289353242139165)

alpha = 0.01, known M:
d_interval: (0.00112475128883249, 0.0063406814496383196)

alpha = 0.05, known M:
d_interval: (0.0013209244802924525, 0.00487232498519354)

alpha = 0.001, known M:
d_interval: (0.0009439307838553291, 0.008833965440707556)
```

0.0.7

```
In [107]: def plot_d_unknown(Y_v):
              n = len(Y_v)
              M = sum(Y_v) / n
              a = np.arange(0.001,0.3,0.001)
              D = sum([(M - y)**2 for y in Y_v]) / (n-1)
              h = [D*(n-1)/sts.chi2.ppf(x/2, n-1) - D*(n-1)/sts.chi2.ppf(1-x/2, n-1) for x in a
              plt.plot(a,h, label='used unknown M')
              plt.ylabel('$h(a)$')
              plt.xlabel('$a$')
              plt.legend()
```

```
In [110]: def plot_d_known(Y_v):
              n = len(Y_v)
              M = 0.14
              a = np.arange(0.001,0.3,0.001)
              D = sum([(M - y)**2 for y in Y_v]) / (n-1)
              h = [D*(n-1)/sts.chi2.ppf(x/2, n-1) - D*(n-1)/sts.chi2.ppf(1-x/2, n-1) for x in a
              plt.plot(a,h, label='used known M')
              plt.ylabel('$h(a)$')
              plt.xlabel('$a$')
              plt.legend()
```

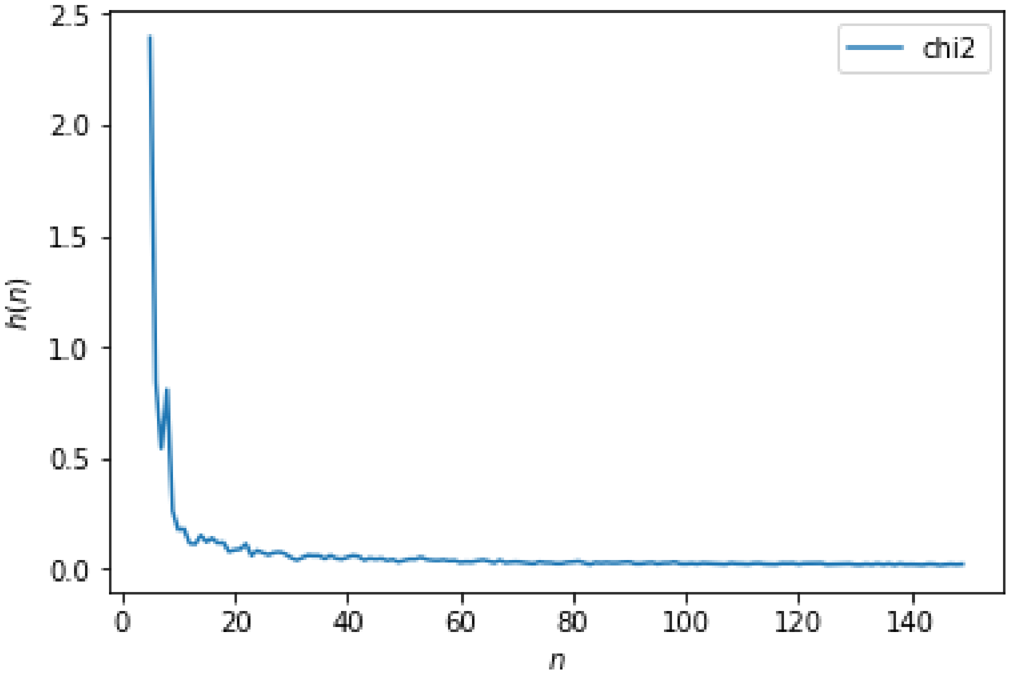
```
In [111]: n = 20
          Y_v = generate_Y_v(n)
          plot_d_unknown(Y_v)
          plot_d_known(Y_v)
          plt.show()
```



0.0.8

```
In [112]: def plot_d(a):
              M = 0
              n = np.arange(5,150,1)
              h = []
              for _n in n:
                  Y_v = generate_Y_v(_n)
                  D = sum([(x - M)**2 for x in Y_v]) / (_n-1)
                  h.append(D*(_n-1)/sts.chi2.ppf(a/2, _n-1) - D*(_n-1)/sts.chi2.ppf(1-a/2, _n-
              plt.plot(n, h, label='chi2')
              plt.ylabel('$h(n)$')
              plt.xlabel('$n$')
              plt.legend()
```

```
In [117]: plot_d(0.001)
          plt.show()
```

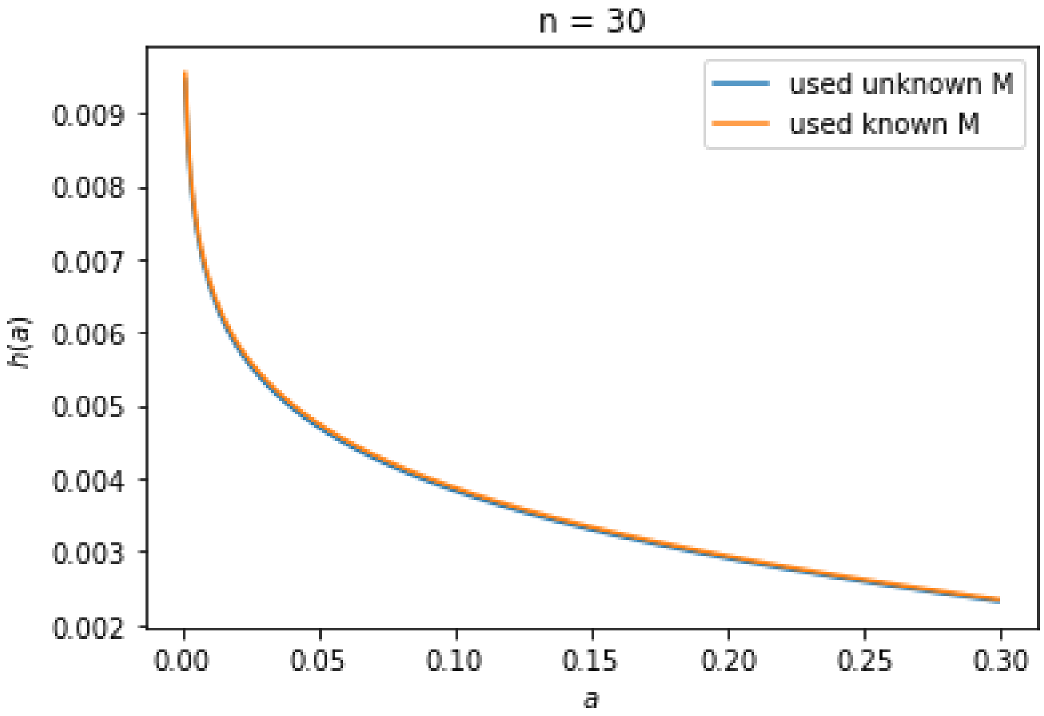


0.0.9 30, 50, 70, 100, 150

```
In [118]:     Y_v = generate_Y_v(30)

              alpha = [0.1, 0.01, 0.05, 0.001]
              for _alpha in alpha:
                  print('alpha = {}, unknown M:'.format(_alpha))
                  d_interval_unknown(Y_v, _alpha)
                  print()
              for _alpha in alpha:
                  print('alpha = {}, known M:'.format(_alpha))
                  d_interval_known(Y_v, _alpha)
                  print()
              plt.title('n = 30')
              plot_d_unknown(Y_v)
              plot_d_known(Y_v)
              plt.show()
```

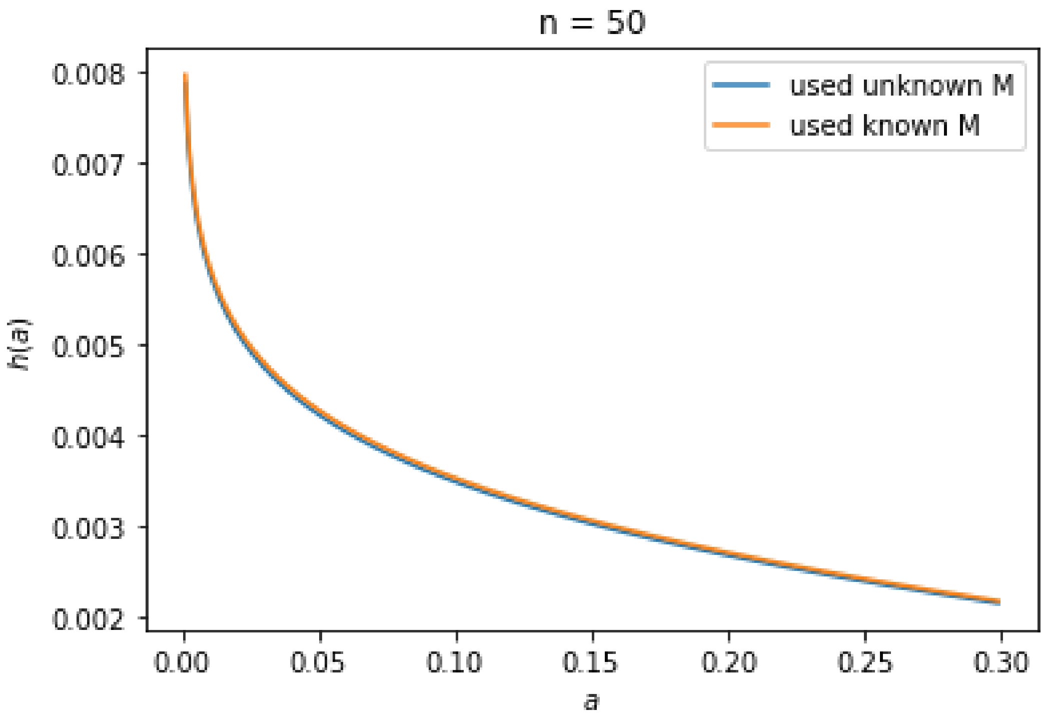
```
alpha = 0.1, unknown M:
d_interval: (0.0027297103998635427, 0.006560074283685622)

alpha = 0.01, unknown M:
d_interval: (0.0022196775831938016, 0.008853508072652713)

alpha = 0.05, unknown M:
```

```
d_interval: (0.0025407346889780684, 0.007239214718258543)

alpha = 0.001, unknown M:
d_interval: (0.0019127170902154991, 0.011359211110623001)

alpha = 0.1, known M:
d_interval: (0.0027295933543626466, 0.0065597929984692535)

alpha = 0.01, known M:
d_interval: (0.0022195824070628226, 0.00885312844876034)

alpha = 0.05, known M:
d_interval: (0.0025406257464454358, 0.007238904312615207)

alpha = 0.001, known M:
d_interval: (0.0019126350760465575, 0.011358724046297683)
```



In [119]:
```
Y_v = generate_Y_v(50)

alpha = [0.1, 0.01, 0.05, 0.001]
for _alpha in alpha:
    print('alpha = {}, unknown M:'.format(_alpha))
    d_interval_unknown(Y_v, _alpha)
    print()
for _alpha in alpha:
    print('alpha = {}, known M:'.format(_alpha))
    d_interval_known(Y_v, _alpha)
    print()
plt.title('n = 50')
plot_d_unknown(Y_v)
plot_d_known(Y_v)
plt.show()
```

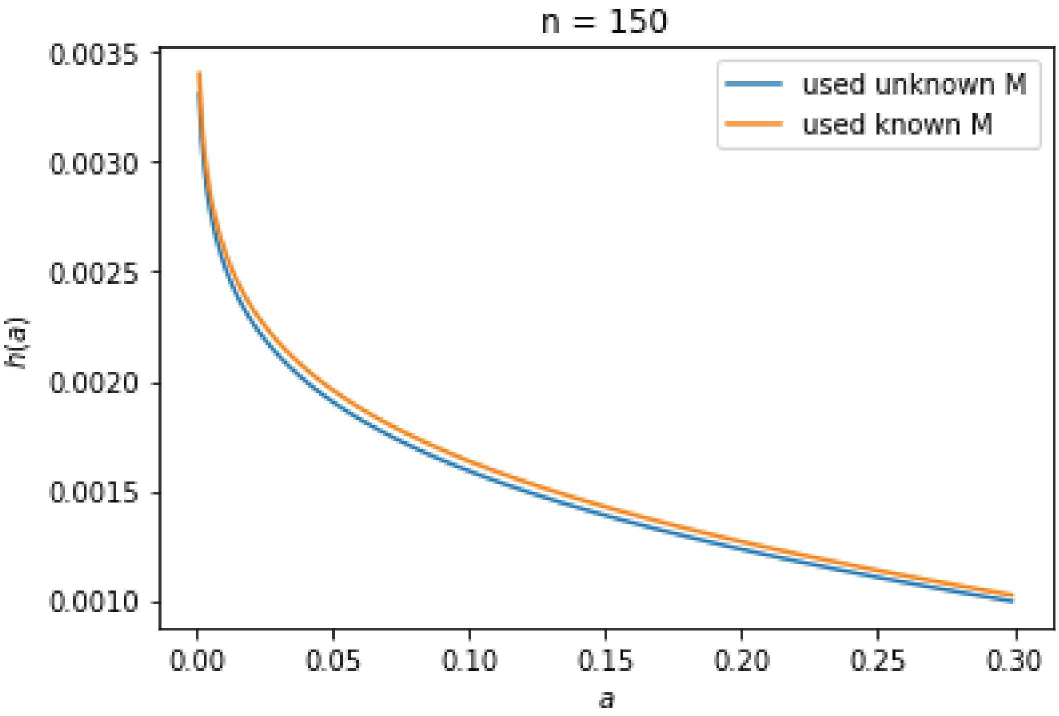
```
alpha = 0.1, unknown M:
d_interval: (0.0036452572566754923, 0.007127004509907403)

alpha = 0.01, unknown M:
d_interval: (0.0030911319490782895, 0.00887439331297036)

alpha = 0.05, unknown M:
d_interval: (0.003443650380045785, 0.007663510738552748)

alpha = 0.001, unknown M:
d_interval: (0.002740791221091033, 0.010611191484073065)

alpha = 0.1, known M:
d_interval: (0.0036433696934340264, 0.0071233140511037)

alpha = 0.01, known M:
d_interval: (0.0030895313193748807, 0.008869798032739535)

alpha = 0.05, known M:
d_interval: (0.003441867211556941, 0.00765954246960707)

alpha = 0.001, known M:
d_interval: (0.0027393720025291634, 0.010605696866386806)
```



```
In [120]:     Y_v = generate_Y_v(150)

              alpha = [0.1, 0.01, 0.05, 0.001]
              for _alpha in alpha:
                  print('alpha = {}, unknown M:'.format(_alpha))
                  d_interval_unknown(Y_v, _alpha)
                  print()
              for _alpha in alpha:
                  print('alpha = {}, known M:'.format(_alpha))
                  d_interval_known(Y_v, _alpha)
                  print()
              plt.title('n = 150')
              plot_d_unknown(Y_v)
              plot_d_known(Y_v)
              plt.show()
```

```
alpha = 0.1, unknown M:
d_interval: (0.0034354410662144З, 0.005034820470179231)

alpha = 0.01, unknown M:
d_interval: (0.003109234898724331, 0.005662287889912709)

alpha = 0.05, unknown M:
d_interval: (0.0033200824105958766, 0.005236434441960248)
```

```
alpha = 0.001, unknown M:
d_interval: (0.002886575709432217, 0.006215495576046985)

alpha = 0.1, known M:
d_interval: (0.0034166404101570107, 0.005007267114977886)

alpha = 0.01, known M:
d_interval: (0.0030922194253670617, 0.005631300681846979)

alpha = 0.05, known M:
d_interval: (0.0033019130616590655, 0.0052077777422780145)

alpha = 0.001, known M:
d_interval: (0.002870778751763391, 0.0061814809059363915)
```


n = 150
h(a)
a
used unknown M
used known M