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> f := x → {
    3 · x      x ≥ 0 and x < 1
    -3 · (x - 2) x ≥ 1 and x < 2
f := x → piecewise(0 ≤ x and x < 1, 3 x, 1 ≤ x and x < 2, -3 x + 6)

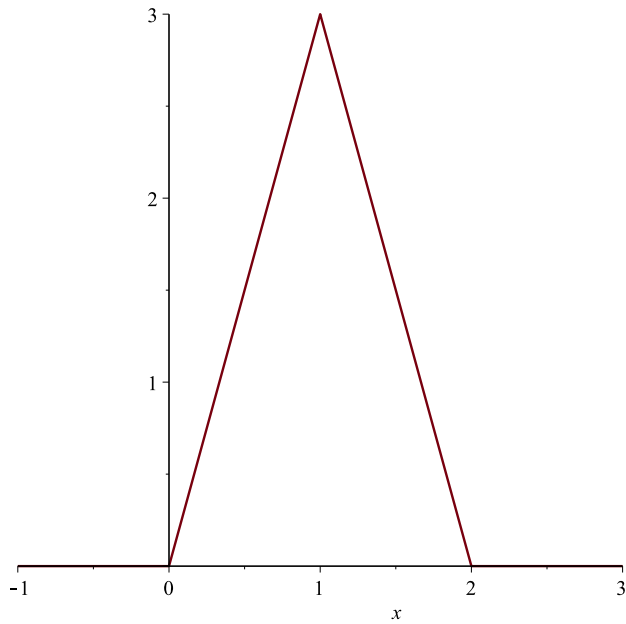
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(1)

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> plot(f(x), discount = true, x = -1 .. 3)

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>
> a0 := simplify(int(f(x), x = 0 .. 2))
                                     a0 := 3/4

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(2)

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> an := simplify(int(f(x) · cos(n · Pi · x), x = 0 .. 2)) assuming n :: posint
                                     an := 3/2 * (-1 + (-1)^n) / (n^2 * pi^2)

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(3)

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> bn := simplify(int(f(x) · sin(n · Pi · x), x = 0 .. 2)) assuming n :: posint
                                     bn := 0

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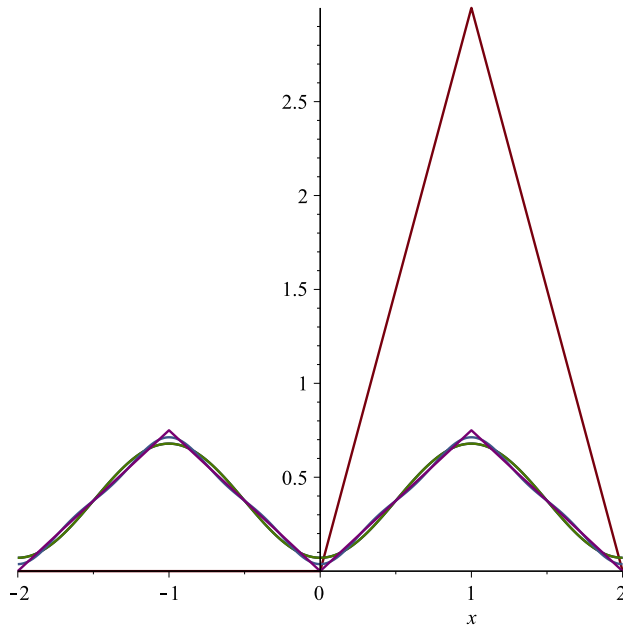
(4)

$$> S := (x, k) \rightarrow \frac{a0}{2} + \sum_{n=1}^k (an \cdot \cos(n \cdot \text{Pi} \cdot x) + bn \cdot \sin(n \cdot \text{Pi} \cdot x))$$

$$S := (x, k) \rightarrow \frac{1}{2} a0 + \sum_{n=1}^k (an \cos(n \pi x) + bn \sin(n \pi x))$$

(5)

> plot([f(x), S(x, 1), S(x, 2), S(x, 3), S(x, infinity)], *discont = true*, x = -2..2)



$$> \left[ \frac{1}{4} \int_0^2 f(x) \, dx \right] = a0$$

$$\frac{1}{4} \int_0^2 f(x) \, dx = \frac{3}{4}$$

(6)

$$> \left[ \frac{1}{4} \int_0^2 f(x) \cos(nx) \, dx \right] = an$$

$$\frac{1}{4} \int_0^2 f(x) \cos(nx) \, dx = \frac{3}{2} \frac{-1 + (-1)^n}{n^2 \pi^2}$$

(7)

$$> \text{'}\frac{1}{4}\int_0^2 f(x)\sin(nx) \, dx\text{'}=bn$$

$$\frac{1}{4}\int_0^2 f(x)\sin(nx) \, dx=0 \quad (8)$$

$$> S := (x, k) \rightarrow \frac{6}{2} + \sum_{n=1}^k \left( -\frac{2}{n\pi} \cdot \sin(n \cdot \text{Pi} \cdot x) \right)$$

$$S := (x, k) \rightarrow \frac{3}{e} + \sum_{n=1}^k \left( -\frac{2 \sin(n \pi x)}{n \pi} \right) \quad (9)$$

$$> g := (x) \rightarrow \text{piecewise}(\text{and}(0 \leq x, x < 1), 3x, \text{and}(1 \leq x, x < 2), -3x + 6, \text{and}(-2 \leq x, x < -1), 3x + 6, \text{and}(-1 \leq x, x < 0), -3x)$$

$$g := x \rightarrow \text{piecewise}(\text{and}(0 \leq x, x < 1), 3x, \text{and}(1 \leq x, x < 2), -3x + 6, \text{and}(-2 \leq x, x < -1), 3x + 6, \text{and}(-1 \leq x, x < 0), -3x) \quad (10)$$

$$> a0l := \frac{\text{simplify}(\text{int}(g(x), x=-2..2))}{2}$$

$$a0l := 3 \quad (11)$$

$$> anl := \frac{\text{simplify}(\text{int}(g(x) \cdot \cos(n \cdot \text{Pi} \cdot x), x=-2..2)) \text{ assuming } n :: \text{posint}}{2}$$

$$anl := \frac{6(-1 + (-1)^n)}{n^2 \pi^2} \quad (12)$$

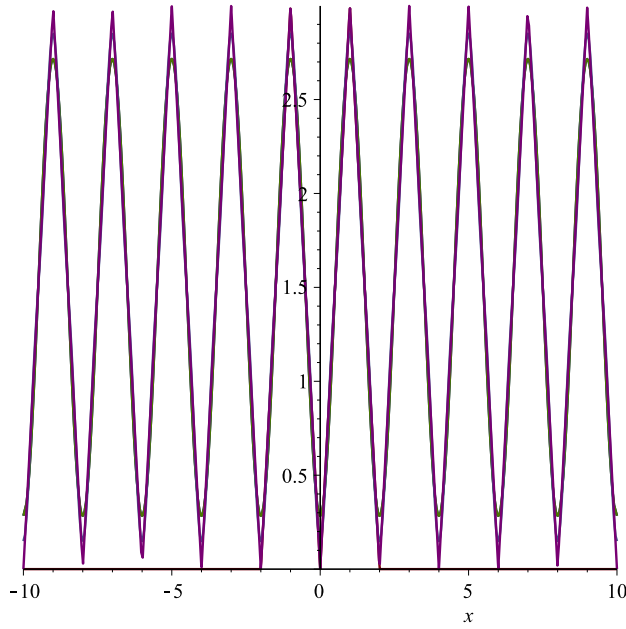
$$> bn1 := \frac{\text{simplify}(\text{int}(g(x) \cdot \sin(n \cdot \text{Pi} \cdot x), x=-2..2)) \text{ assuming } n :: \text{posint}}{2}$$

$$bn1 := 0 \quad (13)$$

$$> Sl := (x, k) \rightarrow \frac{a0l}{2} + \sum_{n=1}^k (anl \cdot \cos(n \cdot \text{Pi} \cdot x) + bn1 \cdot \sin(n \cdot \text{Pi} \cdot x))$$

$$Sl := (x, k) \rightarrow \frac{1}{2} a0l + \sum_{n=1}^k (anl \cos(n \pi x) + bn1 \sin(n \pi x)) \quad (14)$$

$$> \text{plot}([g(x), Sl(x, 1), Sl(x, 2), Sl(x, 3), Sl(x, \text{infinity})], \text{discont}=\text{true}, x=-10..10)$$



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>
> h := (x) → piecewise( and( -1 ≤ x, x < 0), 3 x, and( 0 ≤ x, x < 1), 3 x, and( 1 ≤ x, x < 2),
-3 x + 6, and( -2 ≤ x, x < -1), -3 ( x + 2 ) )
h := x → piecewise( and( -1 ≤ x, x < 0), 3 x, and( 0 ≤ x, x < 1), 3 x, and( 1 ≤ x, x < 2), -3 x
+ 6, and( -2 ≤ x, x < -1), -3 x - 6)

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> a02 :=  $\frac{\text{simplify}(\text{int}(h(x), x=-2..2))}{4}$ 
a02 := 0

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> an2 :=  $\frac{\text{simplify}\left(\text{int}\left(h(x) \cdot \frac{\cos(n \cdot \text{Pi} \cdot x)}{2}, x=-2..2\right)\right)}{4}$  assuming n :: posint
an2 := 0

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> bn2 :=  $\frac{\text{simplify}\left(\text{int}\left(h(x) \cdot \frac{\sin(n \cdot \text{Pi} \cdot x)}{2}, x=-2..2\right)\right)}{4}$  assuming n :: posint
bn2 := 0

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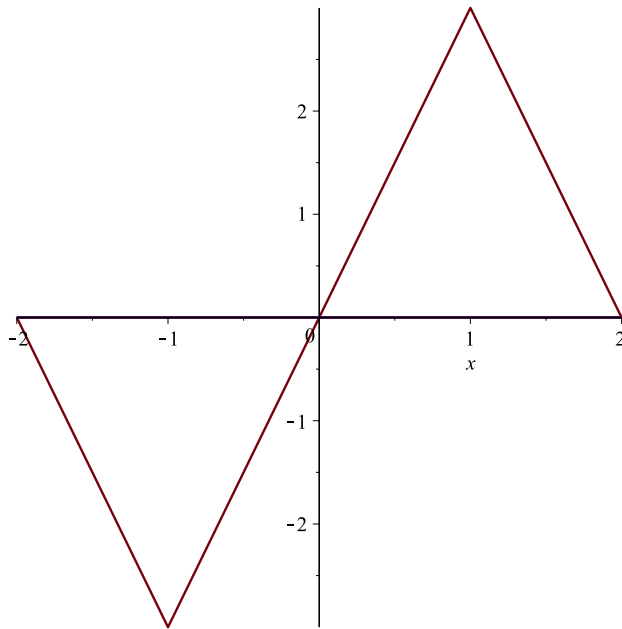
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> S2 := (x, k) →  $\frac{a02}{2} + \sum_{n=1}^k \left( an2 \cdot \frac{\cos(n \cdot \text{Pi} \cdot x)}{2} + bn2 \cdot \frac{\sin(n \cdot \text{Pi} \cdot x)}{2} \right)$ 

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$$S2 := (x, k) \rightarrow \frac{1}{2} a02 + \sum_{n=1}^k \left( \frac{1}{2} an2 \cos(n \pi x) + \frac{1}{2} bn2 \sin(n \pi x) \right) \quad (19)$$

> plot([ h(x), S2(x, 1), S2(x, 2), S2(x, 3), S2(x, infinity) ], *discont = true*, x = -2 .. 2)



>

> '  $\frac{1}{4} \int_0^2 g(x) \, dx = a01$

$$\frac{1}{4} \int_0^2 g(x) \, dx = \frac{3}{2} \quad (20)$$

> '  $\frac{1}{4} \int_{-2}^2 g(x) \cos(nx) \, dx = an1$

$$\frac{1}{4} \int_{-2}^2 g(x) \cos(nx) \, dx = \frac{3}{4} \frac{-1 + (-1)^n}{n^2 \pi^2} \quad (21)$$

$$\begin{aligned} &> \left[ \frac{1}{4} \int_{-2}^2 g(x) \sin(nx) \, dx \right] = bn1 \\ & \frac{1}{4} \int_{-2}^2 g(x) \sin(nx) \, dx = 0 \end{aligned} \quad (22)$$

$$\begin{aligned} &> SI := (x, k) \rightarrow \frac{3}{2} + \sum_{n=i}^k \left( \frac{3 \left( -1 + (-1)^n \right)}{n^2 \pi^2} \cdot \cos(n \cdot \text{Pi} \cdot x) \right) \\ & SI := (x, k) \rightarrow \frac{3}{4} + \sum_{n=i}^k \frac{(-3 + 3 \left( -1 \right)^n) \cos(n \pi x)}{n^2 \pi^2} \end{aligned} \quad (23)$$

$$\begin{aligned} &> \left[ \frac{1}{4} \int_0^2 h(x) \, dx \right] = a02 \\ & \frac{1}{4} \int_0^2 h(x) \, dx = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} &> \left[ \frac{1}{4} \int_{-2}^2 h(x) \cos(nx) \, dx \right] = an2 \\ & \frac{1}{4} \int_{-2}^2 h(x) \cos(nx) \, dx = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} &> \left[ \frac{1}{4} \int_{-2}^2 h(x) \sin(nx) \, dx \right] = bn2 \\ & \frac{1}{4} \int_{-2}^2 h(x) \sin(nx) \, dx = 0 \end{aligned} \quad (26)$$

$$\begin{aligned} &> S2 := (x, k) \rightarrow \frac{3}{2} + \sum_{n=i}^k \left( \frac{3 \left( -1 + (-1)^n \right)}{n^2 \pi^2} \cdot \cos(n \cdot \text{Pi} \cdot x) \right) \\ & S2 := (x, k) \rightarrow \frac{3}{4} + \sum_{n=i}^k \frac{(-3 + 3 \left( -1 \right)^n) \cos(n \pi x)}{n^2 \pi^2} \end{aligned} \quad (27)$$

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