Week 2: Multiple Regression, Training Exercises

Coursera/Erasumus U., Econometric Methods and Applications

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Training Exercise 2.3

Questions

In economic and business applications, the variables $(x_{1i}, \ldots, x_{ki})$ usually do not have natural measurement units. Personal income, for example, can be measured in units or thousands of local currency or US dollars, and per month or per year. A change of measurement scale of the j-th variable corresponds to a transformation $\tilde{x_{ji}} = a_j x_{ji}$ (with a_j fixed for $i = 1, \ldots, n$). Let $A = diag(a_1, \ldots, a_k)$ and let $\tilde{X} = XA$. We even allow for non-diagonal A and define $\tilde{X} = XA$ where A is any invertible (k x k) matrix. As before, let $\hat{y} = Xb$ be the predicted values of y.

(a) Prove that $\hat{y}$, e, s^2, and R^2 do not depend on A (that is, are invariant under linear transformations).

(b) Prove that $\tilde{b} = A^{-1}b$ and provide and intuitve interpretation.

Answers

(a) Prove that $\hat{y}$, e, s^2, and R^2 do not depend on A (that is, are invariant under linear transformations).

$$\begin{aligned}\tilde{X} &= XA \\ H &= X(X'X)^{-1}X'\end{aligned}$$

After transformation:

$$\begin{aligned}\tilde{H} &= \tilde{X}(\tilde{X}'\tilde{X})^{-1}\tilde{X}' \\ &= XA(A'X'XA)^{-1}A'X' \\ &= XAA^{-1}(X'X)^{-1}(A')^{-1}A'X' \\ &= X(X'X)^{-1}X' \\ &= H\end{aligned}$$

$$\begin{gathered}\tilde{\hat{y}} = \tilde{H}y = Hy = \hat{y} \\ \tilde{e} = \tilde{M}y = (I - \tilde{H})y = (I - H)y = My = e \\ \tilde{s}^2 = \tilde{e}'\tilde{e}/(n-u) = e'e/(n-u) = s^2 \\ \tilde{R^2} = (corr(y, \tilde{\hat{y}}))^2 = (corr(y, \hat{y}))^2 = R^2\end{gathered}$$

(b) Prove that $\tilde{b} = A^{-1}b$ and provide and intuitve interpretation.

$$\begin{aligned}\tilde{b} &= (\tilde{X}'\tilde{X}^{-1}\tilde{X}'y) \\ &= (A'X'XA)^{-1}A'X'y \\ &= A^{-1}(X'X)^{-1}(A^1)^{-1}A'X'y \\ &= A^{-1}(X'X)^{-1}X'y \\ &= A^{-1}b\end{aligned}$$

$$\begin{aligned}y &= X\beta + \varepsilon \\ &= XAA^{-1}\beta + \varepsilon \\ &= \tilde{X}(A^{-1}\beta) + \varepsilon\end{aligned}$$

$$\begin{aligned}\beta &\to A^{-1}\beta \\ b &\to A^{-1}b\end{aligned}$$