Designing, Visualizing and Understanding Deep Neural Networks

Lecture 6: Convolutional Networks II

CS 182/282A Spring 2019

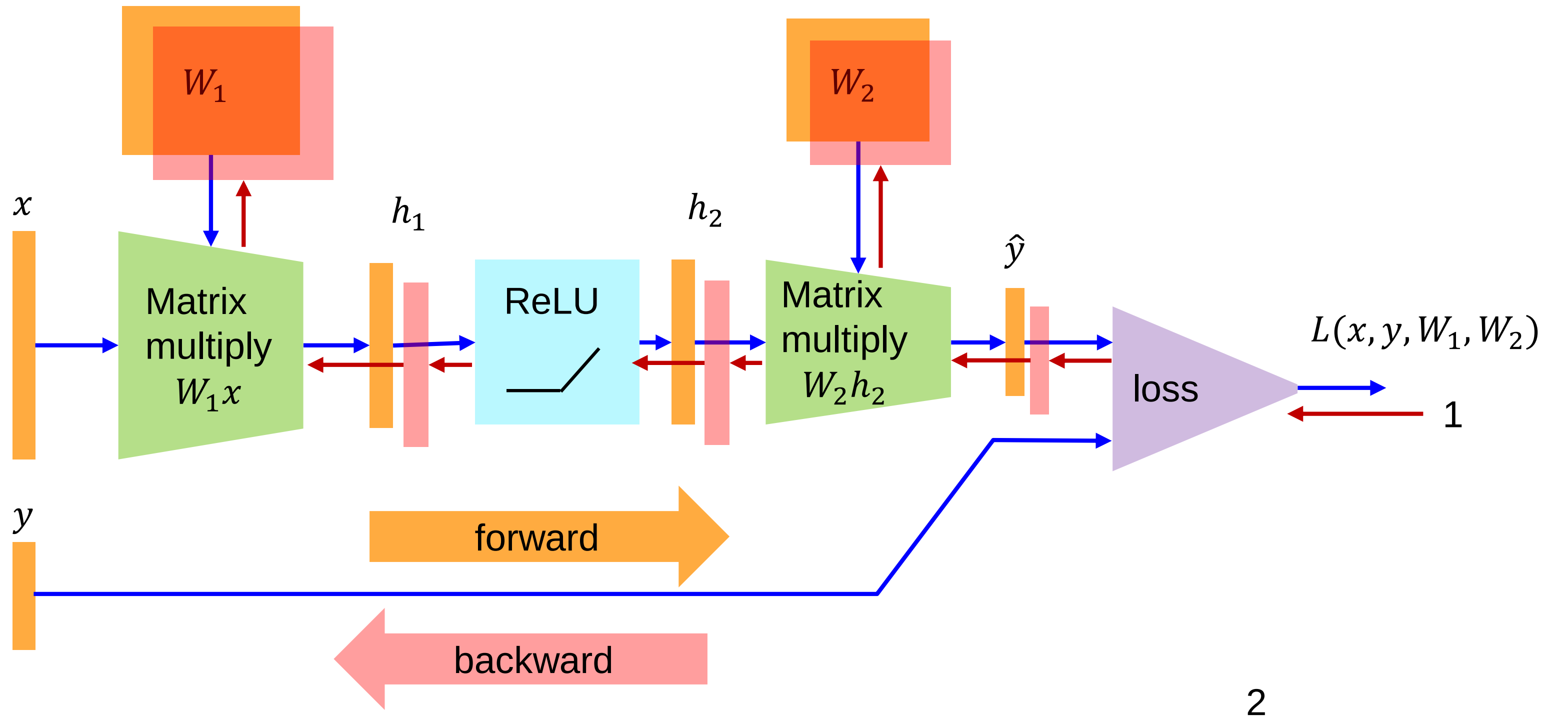
John Canny

Slides originated from Efros, Karpathy, Ransato, Seitz, and Palmer

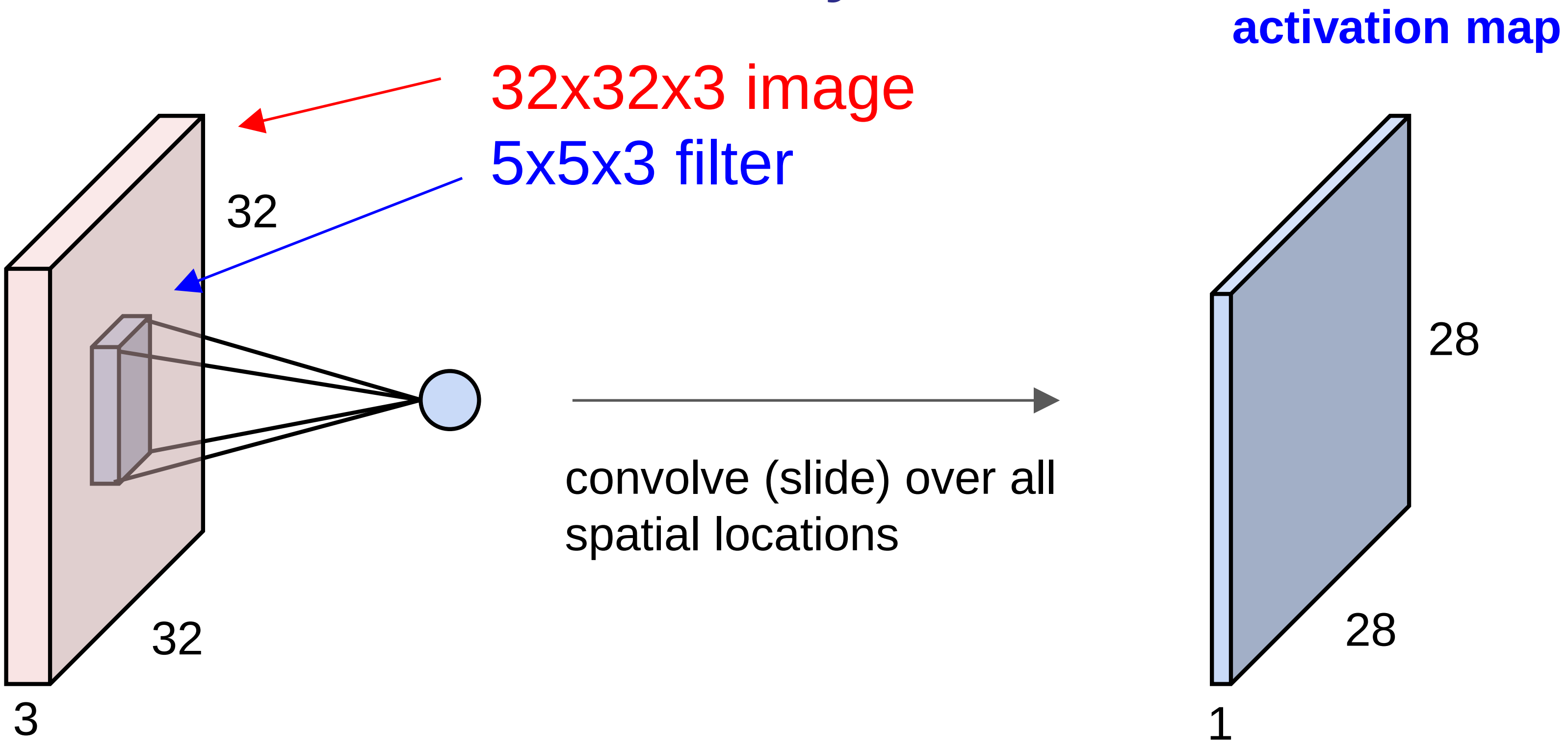
Last Time: Backpropagation

Backprop Efficiency: matrix-vector multiply only and common subexpressions.

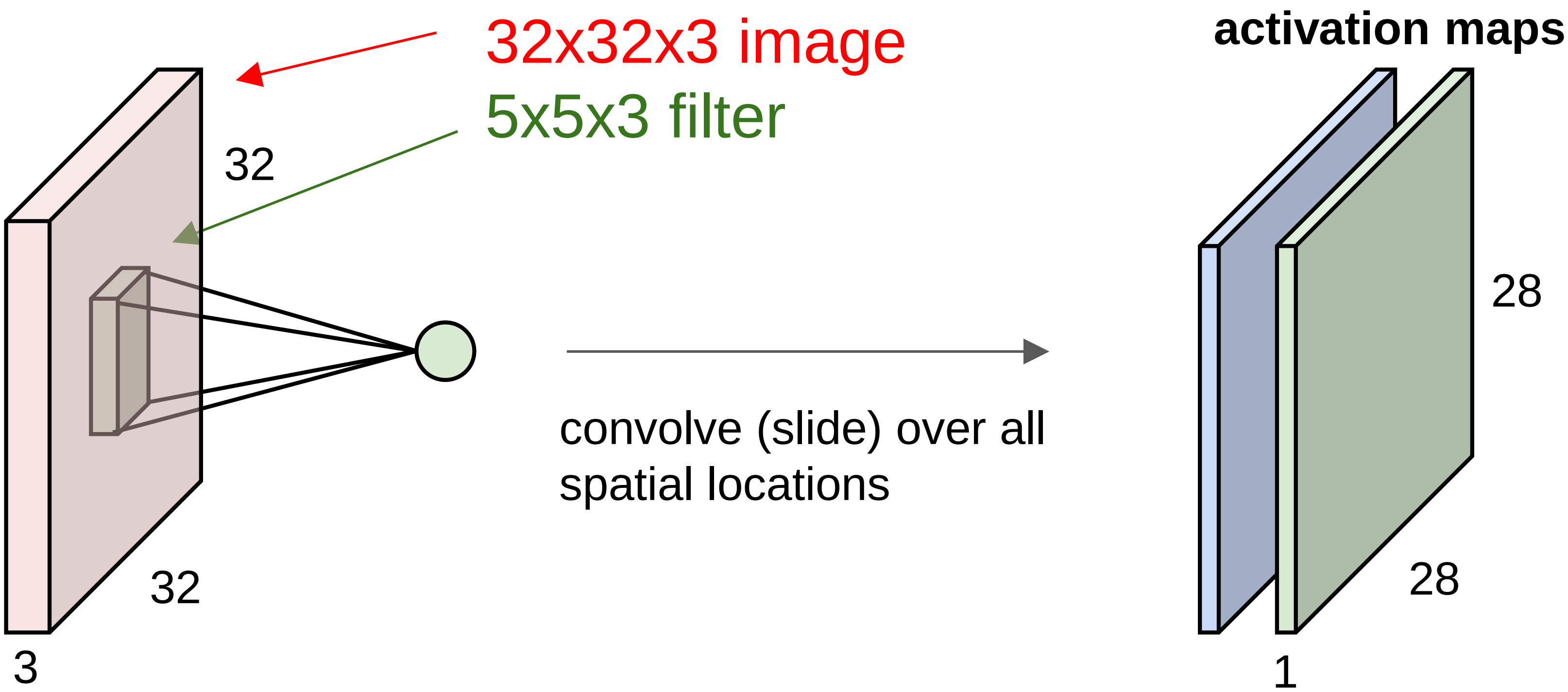
Example: a real neural network (Tensorflow style):



Last Time: Convolution Layers



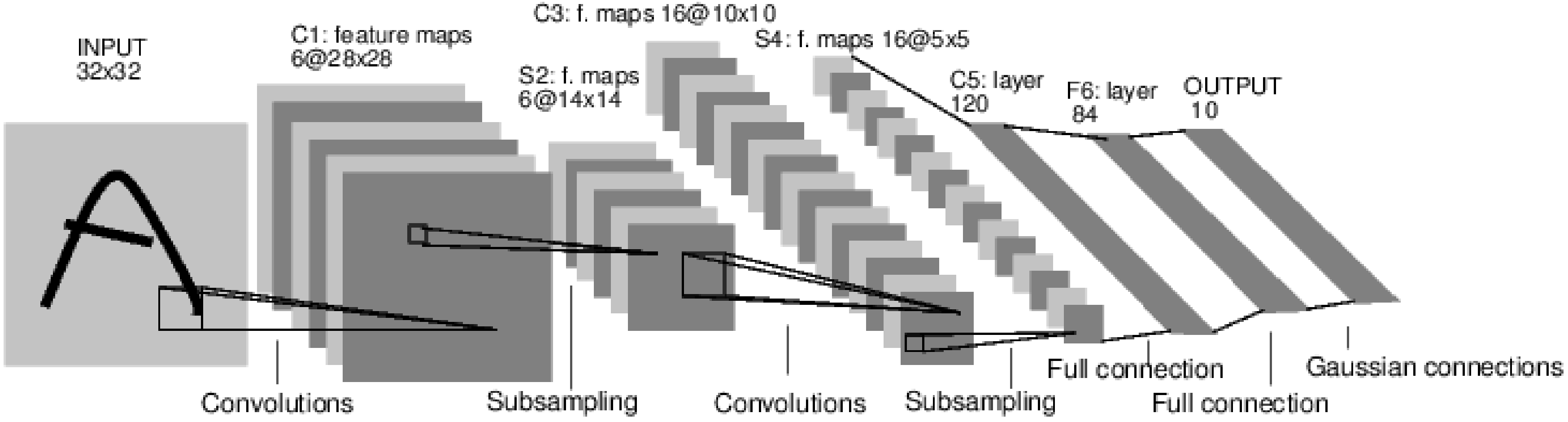
Last Time: Convolution Layers



Projects ! The Project Proposal page is up

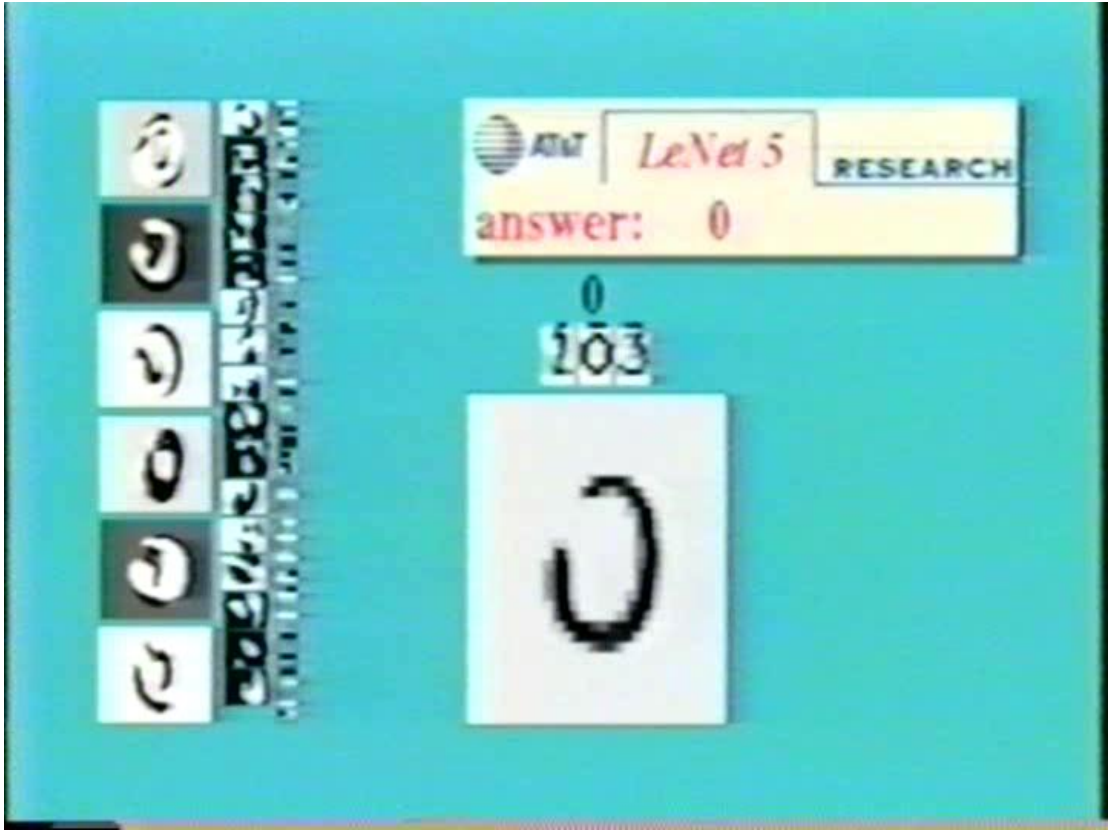
This Time: Case Study: LeNet-5

[LeCun et al., 1998]



Conv filters were 5x5, applied at stride 1
Subsampling (Pooling) layers were 2x2 applied at stride 2
i.e. architecture is [CONV-POOL-CONV-POOL-CONV-FC]

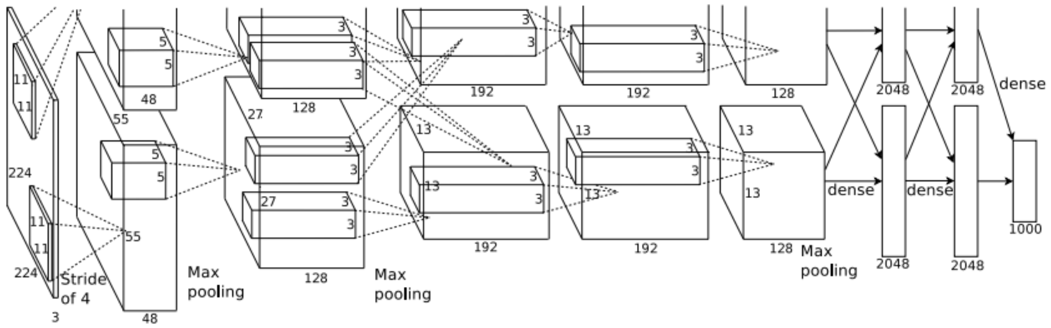
Handwritten digit classification



[Courtesy of Yann LeCun]

Case Study: AlexNet

[Krizhevsky et al. 2012]



Input: 227x227x3 images

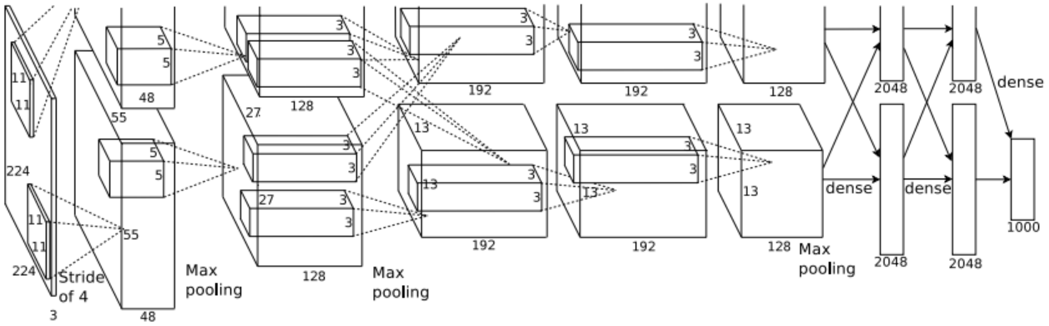
First layer (CONV1): 96 11x11 filters applied at stride 4

=>

Q: what is the output volume size? Hint: (227-11)/4+1 = 55

Case Study: AlexNet

[Krizhevsky et al. 2012]



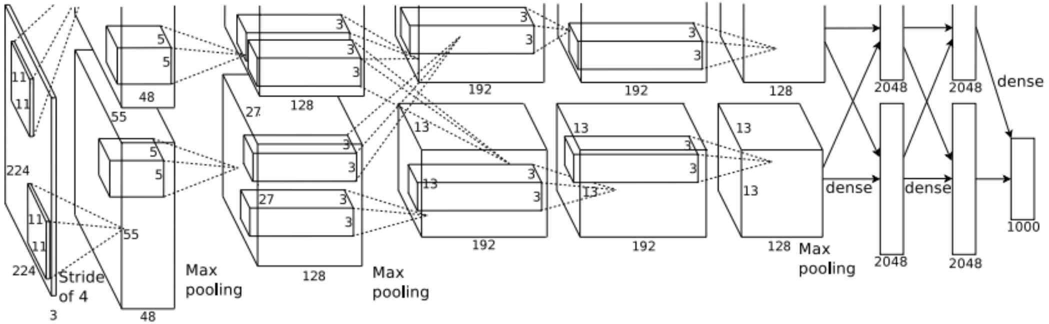
Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**

Q: What is the total number of parameters in this layer?

Case Study: AlexNet

[Krizhevsky et al. 2012]

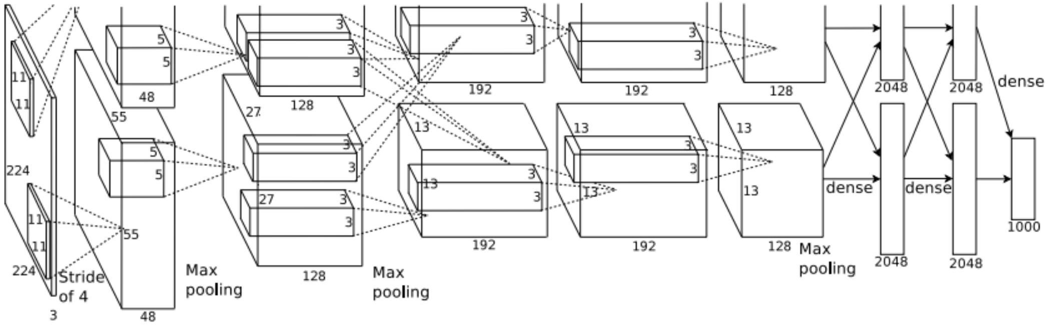


Input: 227x227x3 images

First layer (CONV1): 96 11x11 filters applied at stride 4
=>
Output volume **[55x55x96]**
Parameters: (11*11*3)*96 = **35K**

Case Study: AlexNet

[Krizhevsky et al. 2012]



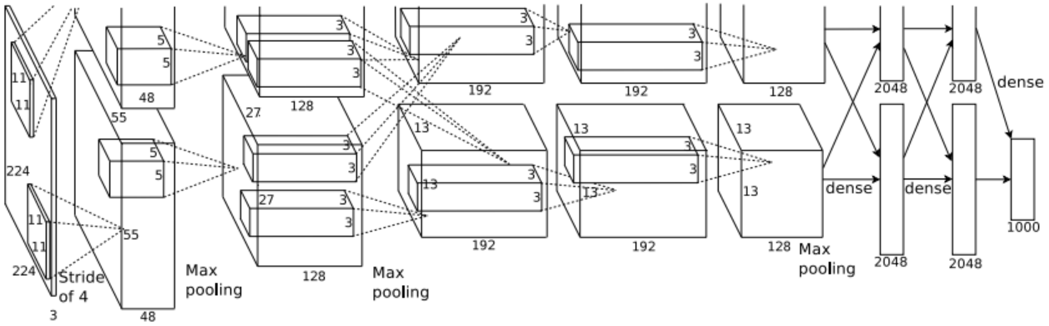
Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2

Q: what is the output volume size? Hint: (55-3)/2+1 = 27

Case Study: AlexNet

[Krizhevsky et al. 2012]



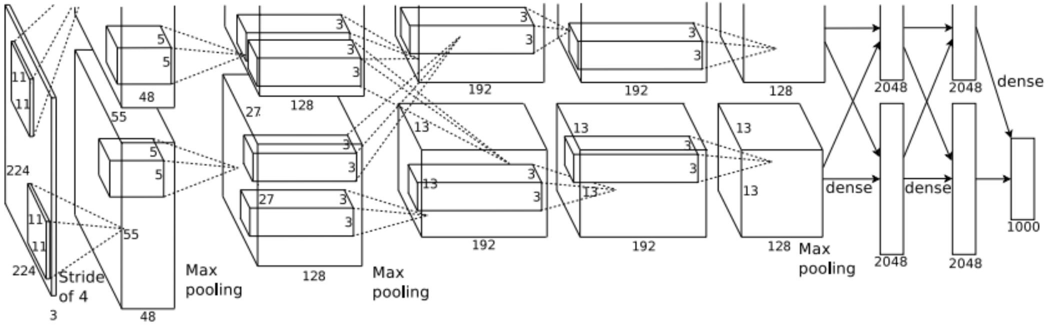
Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96

Q: what is the number of parameters in this layer?

Case Study: AlexNet

[Krizhevsky et al. 2012]

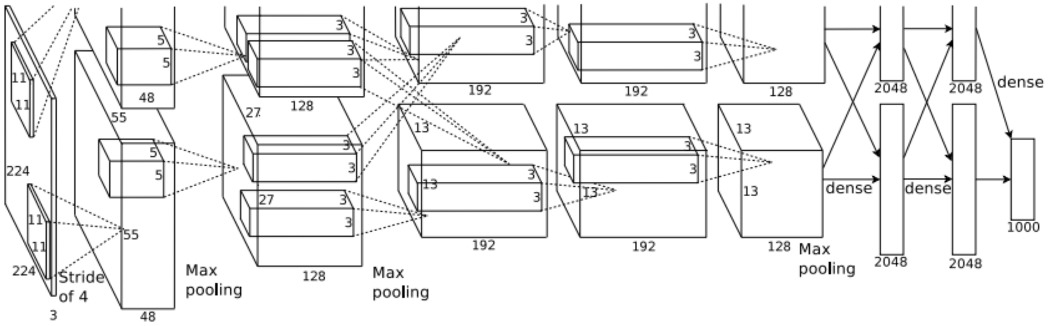


Input: 227x227x3 images
After CONV1: 55x55x96

Second layer (POOL1): 3x3 filters applied at stride 2
Output volume: 27x27x96
Parameters: 0!

Case Study: AlexNet

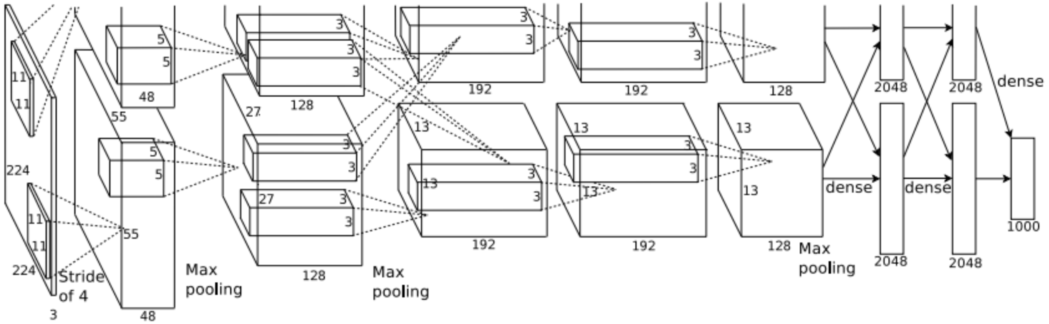
[Krizhevsky et al. 2012]



Input: 227x227x3 images
After CONV1: 55x55x96
After POOL1: 27x27x96
...

Case Study: AlexNet

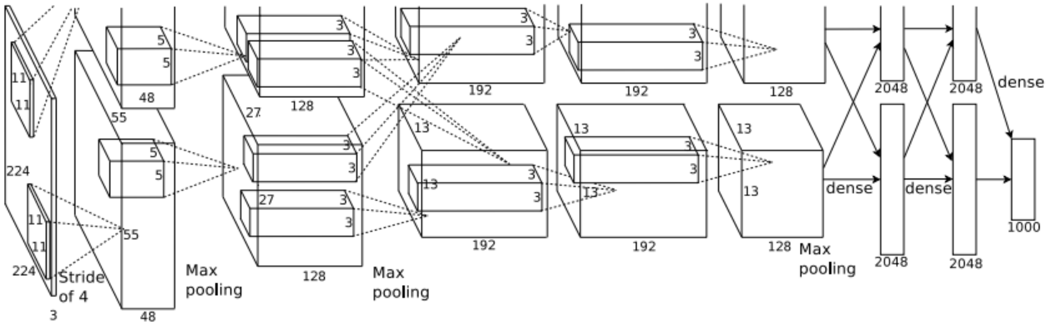
[Krizhevsky et al. 2012]



Full (simplified) AlexNet architecture:
[227x227x3] INPUT
[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
[27x27x96] NORM1: Normalization layer
[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Case Study: AlexNet

[Krizhevsky et al. 2012]



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[55x55x96] CONV1: 96 11x11 filters at stride 4, pad 0
[27x27x96] MAX POOL1: 3x3 filters at stride 2
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[27x27x256] CONV2: 256 5x5 filters at stride 1, pad 2
[13x13x256] MAX POOL2: 3x3 filters at stride 2
[13x13x256] NORM2: Normalization layer
[13x13x384] CONV3: 384 3x3 filters at stride 1, pad 1
[13x13x384] CONV4: 384 3x3 filters at stride 1, pad 1
[13x13x256] CONV5: 256 3x3 filters at stride 1, pad 1
[6x6x256] MAX POOL3: 3x3 filters at stride 2
[4096] FC6: 4096 neurons
[4096] FC7: 4096 neurons
[1000] FC8: 1000 neurons (class scores)

Details/Retrospectives:
- first use of ReLU
- used LRNorm layers (not common anymore)
- heavy data augmentation
- dropout 0.5
- batch size 128
- SGD Momentum 0.9
- Learning rate 1e-2, reduced by 10 manually when val accuracy plateaus
- L2 weight decay 5e-4
- 7 CNN ensemble: 18.2% -> 15.4%

Slide based on cs231n by Fei-Fei Li & Andrej Karpathy & Justin Johnson

“You need a lot of a data if you want to train/use CNNs”

Transfer Learning

"You need a lot of a data if you want to train/use CNNs"

NOT ALWAYS

Transfer Learning with CNNs

1. Train on Imagenet

Transfer Learning with CNNs

1. Train on Imagenet

2. If small dataset: fix all weights (treat CNN as fixed feature extractor), retrain only the classifier

i.e. swap the Softmax layer at the end

Slide based on cs231n by Fei-Fei Li & Andrej Karpathy & Justin Johnson

Transfer Learning with CNNs

1. Train on Imagenet

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i.e. swap the Softmax layer at the end

3. If you have medium sized dataset, **“finetune”** instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.

Transfer Learning with CNNs

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3. If you have medium sized dataset, **"finetune"** instead: use the old weights as initialization, train the full network or only some of the higher layers

retrain bigger portion of the network, or even all of it.

tip: use only ~1/10th of the original learning rate in finetuning to player, and ~1/100th on intermediate layers

Slide based on cs231n by Fei-Fei Li & Andrej Karpathy & Justin Johnson

Case Study: VGGNet

[Simonyan and Zisserman, 2014]

Only 3x3 CONV stride 1, pad 1
and 2x2 MAX POOL stride 2

best model

11.2% top 5 error in ILSVRC 2013
->
7.3% top 5 error

ConvNet Configuration					
A	A-LRN	B	C	D	E
11 weight layers	11 weight layers	13 weight layers	16 weight layers	16 weight layers	19 weight layers
input (224 × 224 RGB image)					
conv3-64	conv3-64 **LRN**	conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	conv3-64 conv3-64
maxpool					
conv3-128	conv3-128	conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	conv3-128 conv3-128
maxpool					
conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	conv3-256 conv3-256 conv3-256 **conv3-256**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	conv3-512 conv3-512 conv3-512 **conv3-512**
maxpool					
FC-4096					
FC-4096					
FC-1000					
soft-max					

Table 2: **Number of parameters** (in millions).

Network	A,A-LRN	B	C	D	E
Number of parameters	133	133	134	138	144

(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*3)*64 = 1,728
CONV3-64: [224x224x64] memory: 224*224*64=3.2M params: (3*3*64)*64 = 36,864
POOL2: [112x112x64] memory: 112*112*64=800K params: 0
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*64)*128 = 73,728
CONV3-128: [112x112x128] memory: 112*112*128=1.6M params: (3*3*128)*128 = 147,456
POOL2: [56x56x128] memory: 56*56*128=400K params: 0
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*128)*256 = 294,912
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
CONV3-256: [56x56x256] memory: 56*56*256=800K params: (3*3*256)*256 = 589,824
POOL2: [28x28x256] memory: 28*28*256=200K params: 0
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*256)*512 = 1,179,648
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
CONV3-512: [28x28x512] memory: 28*28*512=400K params: (3*3*512)*512 = 2,359,296
POOL2: [14x14x512] memory: 14*14*512=100K params: 0
CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
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CONV3-512: [14x14x512] memory: 14*14*512=100K params: (3*3*512)*512 = 2,359,296
POOL2: [7x7x512] memory: 7*7*512=25K params: 0
FC: [1x1x4096] memory: 4096 params: 7*7*512*4096 = 102,760,448
FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

ConvNet Configuration			
B	C	D	
13 weight layers	16 weight layers	16 weight layers	19
put (224 × 224 RGB image)			
conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	co co
maxpool			
conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	co co
maxpool			
conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	co co co **co**
maxpool			
conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	co co co **co**
maxpool			
conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	co co co **co**
maxpool			
FC-4096			
FC-4096			
FC-1000			
soft-max			

(not counting biases)

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FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

TOTAL memory: 24M * 4 bytes ~= 93MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters

ConvNet Configuration			
B	C	D	
13 weight layers	16 weight layers	16 weight layers	19
put (224 × 224 RGB image)			
conv3-64 **conv3-64**	conv3-64 conv3-64	conv3-64 conv3-64	cc cc
maxpool			
conv3-128 **conv3-128**	conv3-128 conv3-128	conv3-128 conv3-128	co co
maxpool			
conv3-256 conv3-256	conv3-256 conv3-256 **conv1-256**	conv3-256 conv3-256 **conv3-256**	co co co **co**
maxpool			
conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	co co co **co**
maxpool			
conv3-512 conv3-512	conv3-512 conv3-512 **conv1-512**	conv3-512 conv3-512 **conv3-512**	co co co **co**
maxpool			
FC-4096			
FC-4096			
FC-1000			
soft-max			

(not counting biases)

INPUT: [224x224x3] memory: 224*224*3=150K params: 0
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FC: [1x1x4096] memory: 4096 params: 4096*4096 = 16,777,216
FC: [1x1x1000] memory: 1000 params: 4096*1000 = 4,096,000

Note:

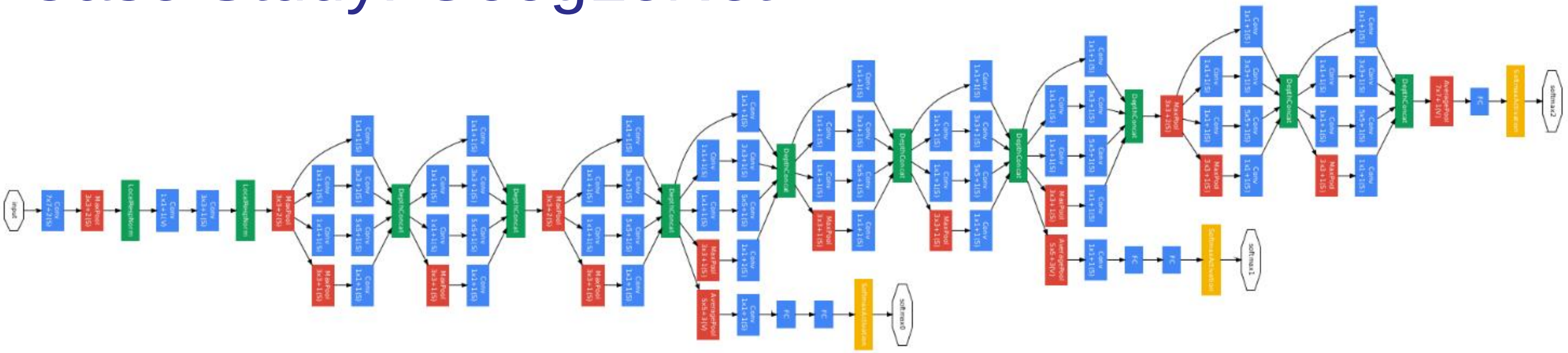
Most memory is in early CONV

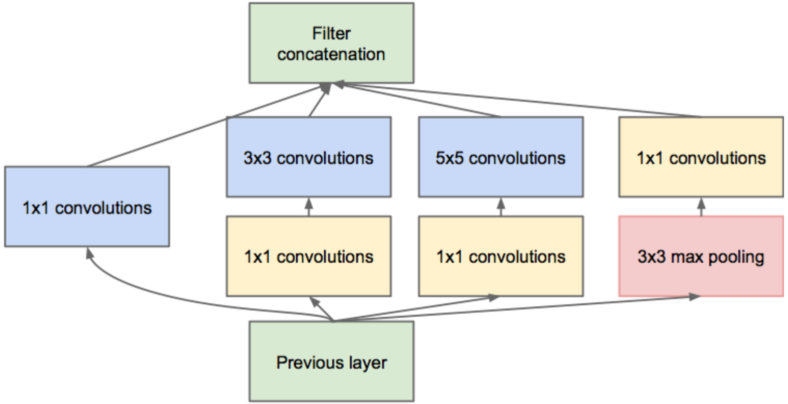
Most params are in late FC

TOTAL memory: 24M * 4 bytes ~= 93MB / image (only forward! ~*2 for bwd)
TOTAL params: 138M parameters

Case Study: GoogLeNet

[Szegedy et al., 2014]



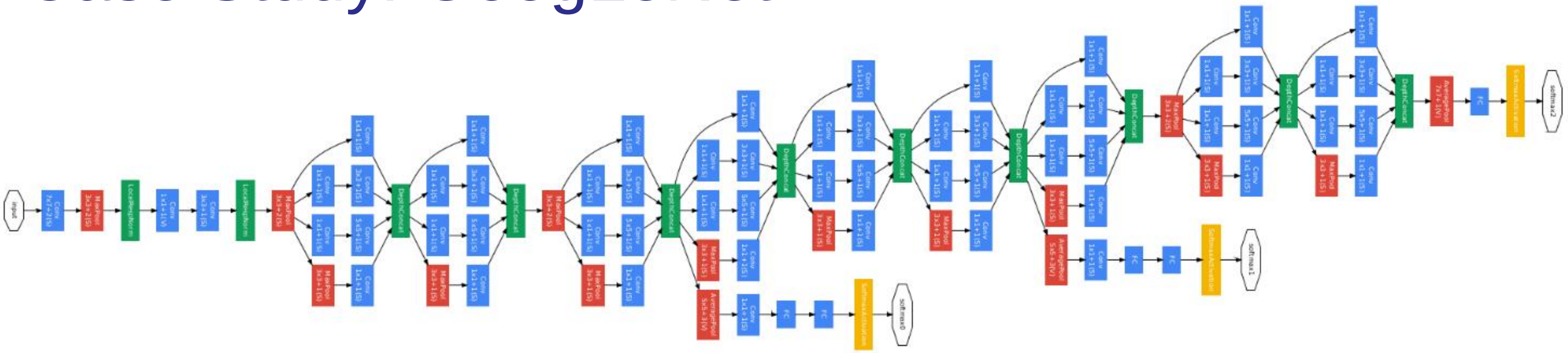


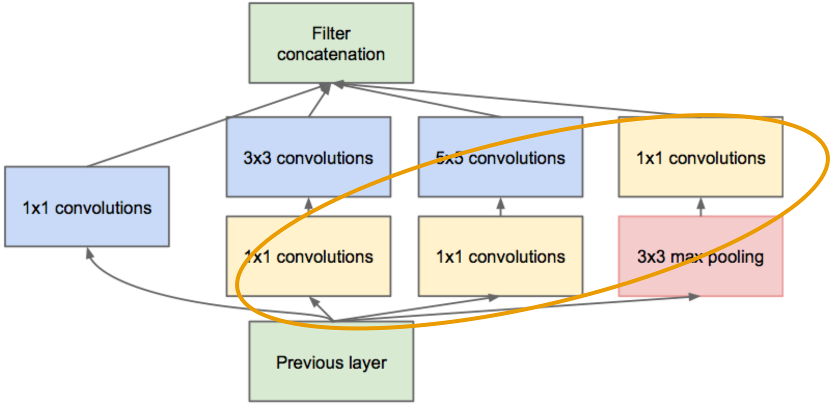
Inception module

ILSVRC 2014 winner (6.7% top 5 error)

Case Study: GoogLeNet

[Szegedy et al., 2014]



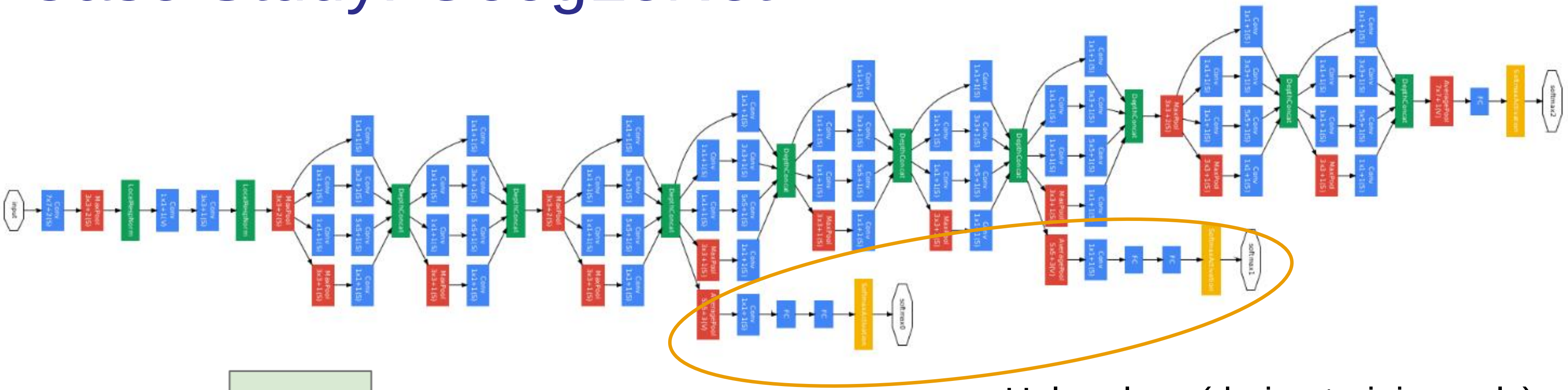


1x1 dimension reduction layers
(reduce compute bottlenecks)

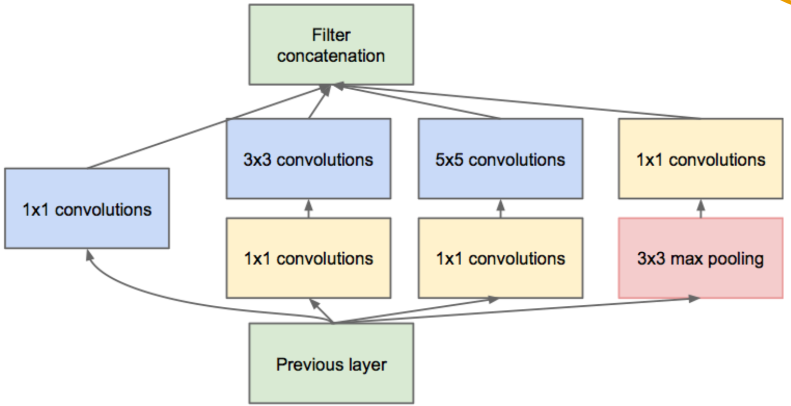
Inception module

Case Study: GoogLeNet

[Szegedy et al., 2014]



Helper loss (during training only)



Inception module

Case Study: GoogLeNet

type	patch size/ stride	output size	depth	#1×1	#3×3 reduce	#3×3	#5×5 reduce	#5×5	pool proj	params	ops
convolution	7×7/2	112×112×64	1							2.7K	34M
max pool	3×3/2	56×56×64	0								
convolution	3×3/1	56×56×192	2		64	192				112K	360M
max pool	3×3/2	28×28×192	0								
inception (3a)		28×28×256	2	64	96	128	16	32	32	159K	128M
inception (3b)		28×28×480	2	128	128	192	32	96	64	380K	304M
max pool	3×3/2	14×14×480	0								
inception (4a)		14×14×512	2	192	96	208	16	48	64	364K	73M
inception (4b)		14×14×512	2	160	112	224	24	64	64	437K	88M
inception (4c)		14×14×512	2	128	128	256	24	64	64	463K	100M
inception (4d)		14×14×528	2	112	144	288	32	64	64	580K	119M
inception (4e)		14×14×832	2	256	160	320	32	128	128	840K	170M
max pool	3×3/2	7×7×832	0								
inception (5a)		7×7×832	2	256	160	320	32	128	128	1072K	54M
inception (5b)		7×7×1024	2	384	192	384	48	128	128	1388K	71M
avg pool	7×7/1	1×1×1024	0								
dropout (40%)		1×1×1024	0								
linear		1×1×1000	1							1000K	1M
softmax		1×1×1000	0								

Fun features:

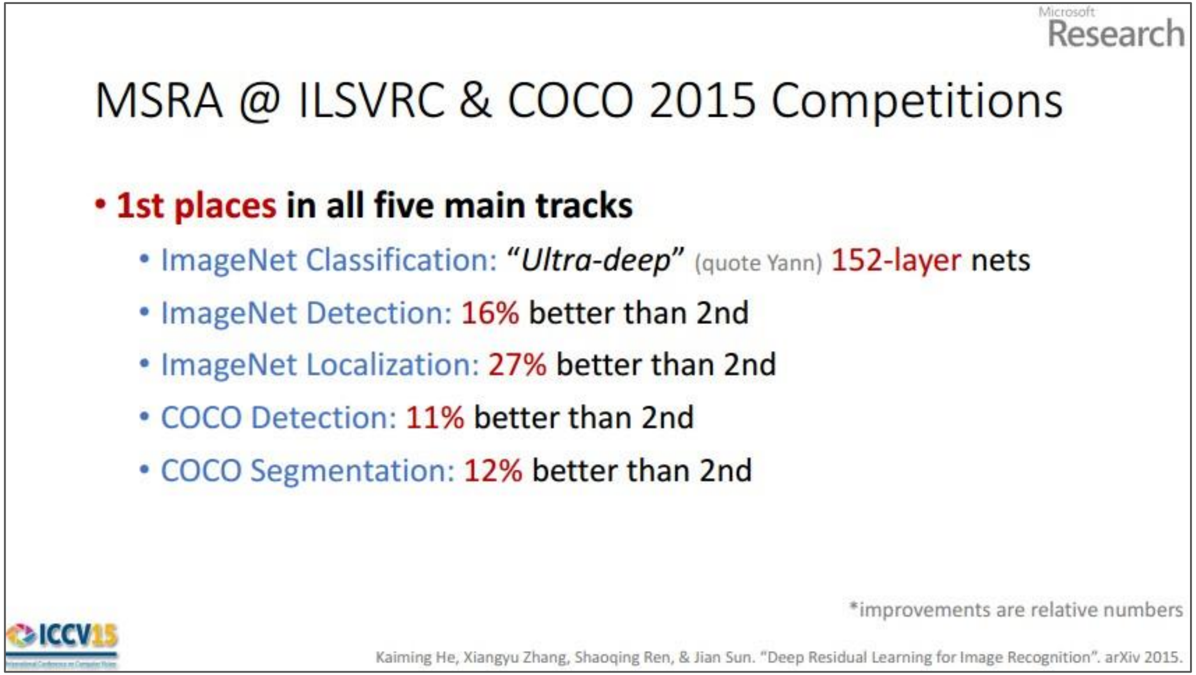
- Only 5 million params!
(Removes FC layers completely)

Compared to AlexNet:
- 12X less params
- 2x more compute
- 6.67% (vs. 16.4%)

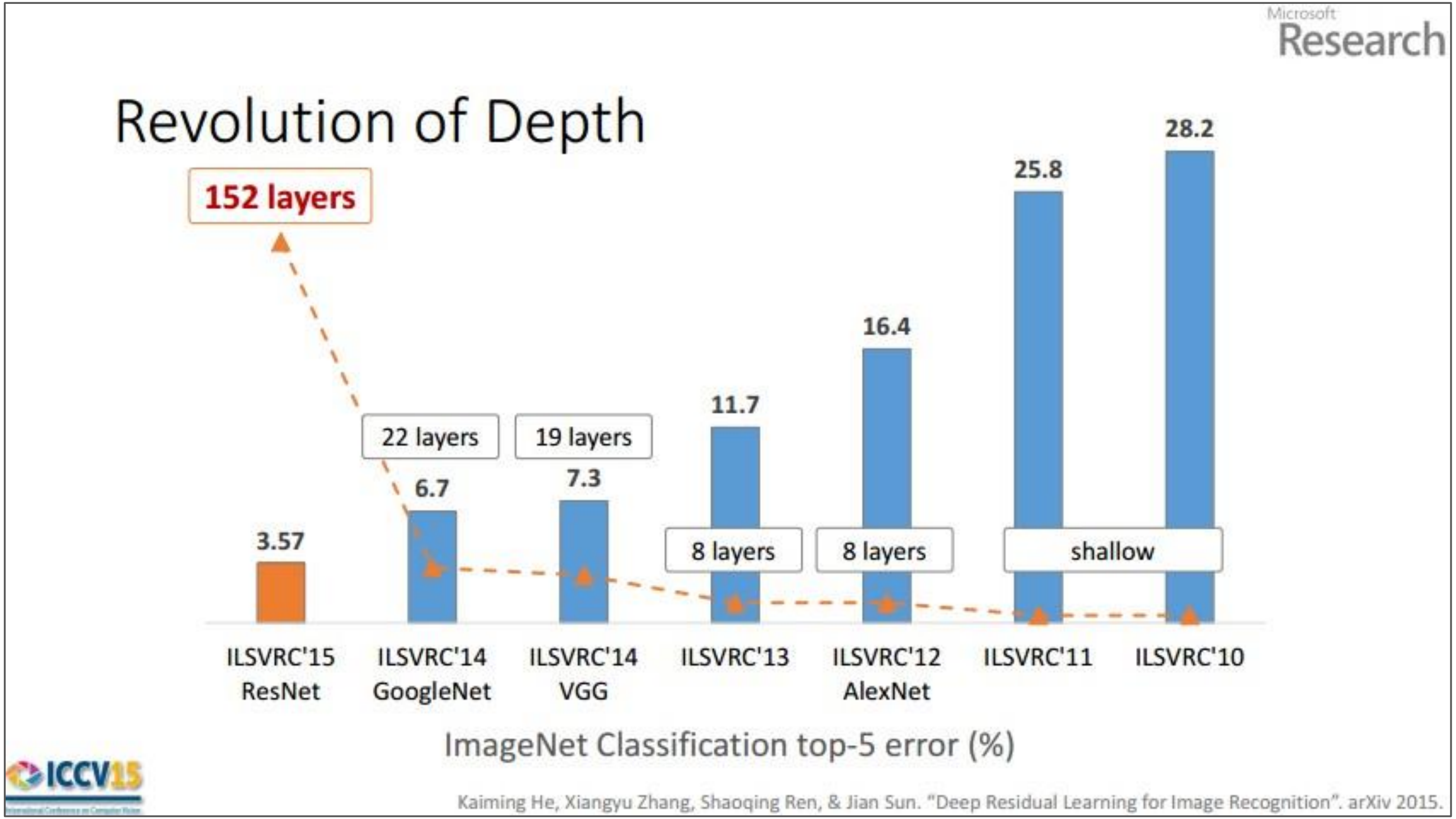
Case Study: ResNet

[He et al., 2015]

ILSVRC 2015 winner (3.6% top 5 error)

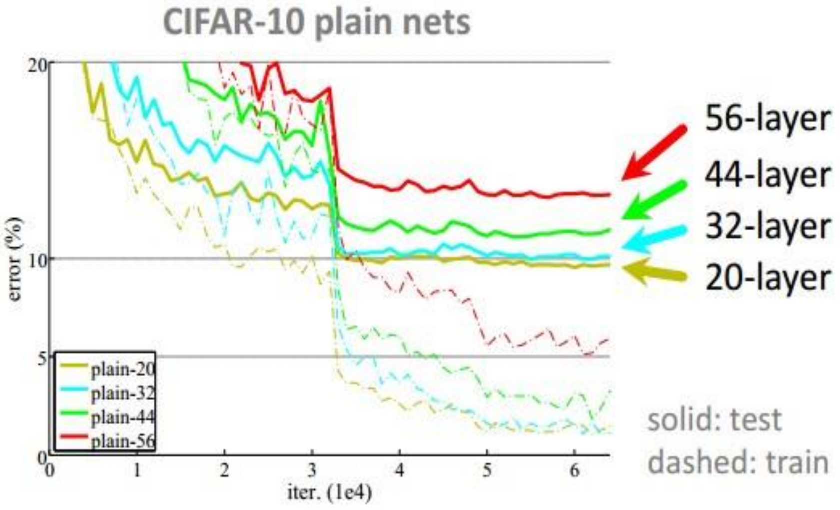


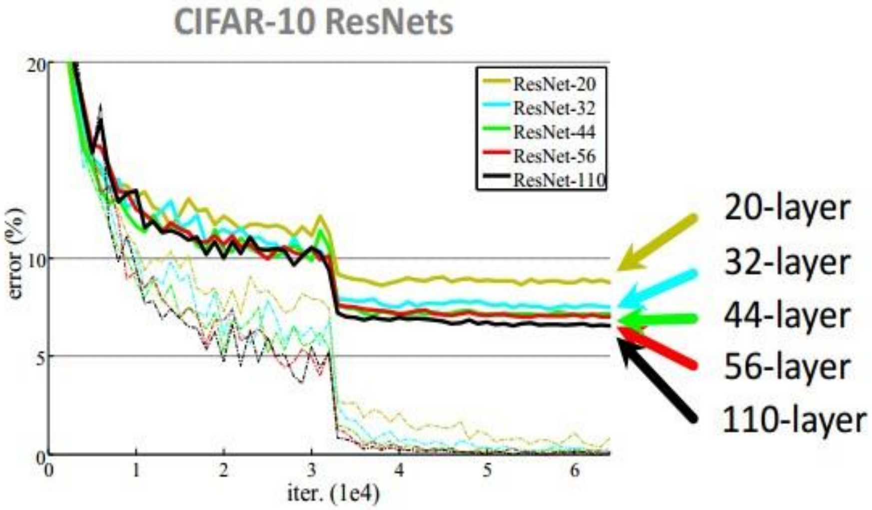
Slide from Kaiming He's recent presentation https://www.youtube.com/watch?v=1PGLj-uKT1w



(slide from Kaiming He's recent presentation)

CIFAR-10 experiments

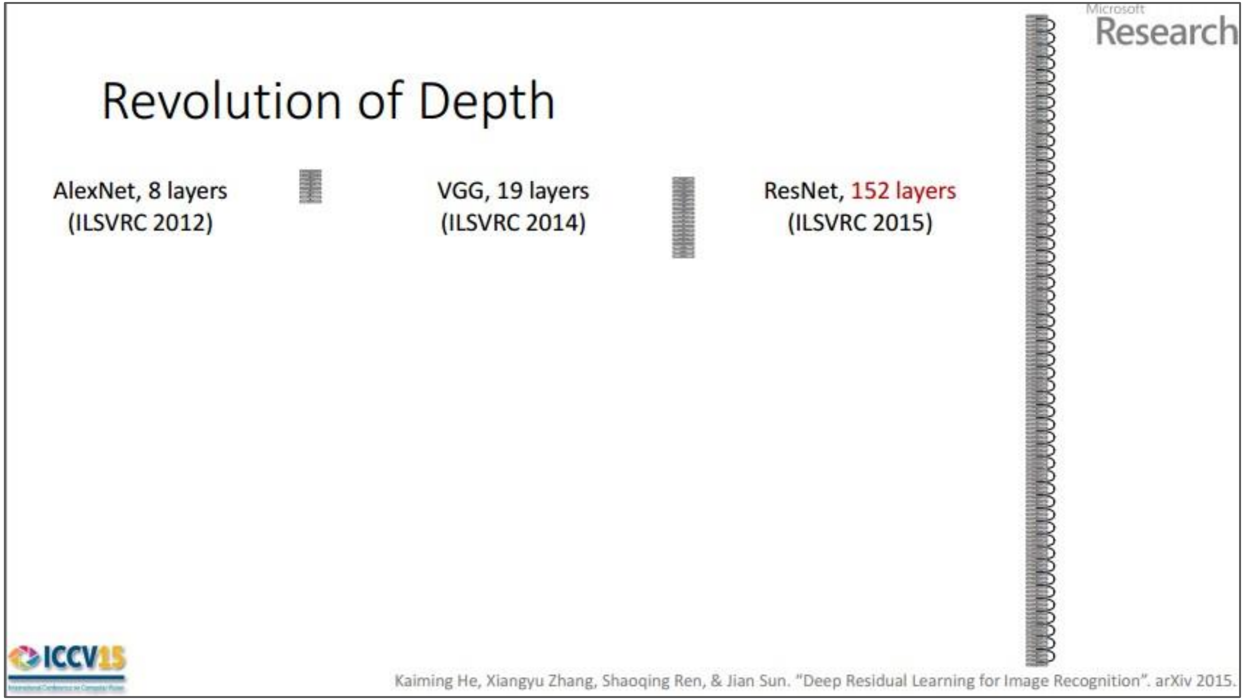




Case Study: ResNet

[He et al., 2015]

ILSVRC 2015 winner (3.6% top 5 error)



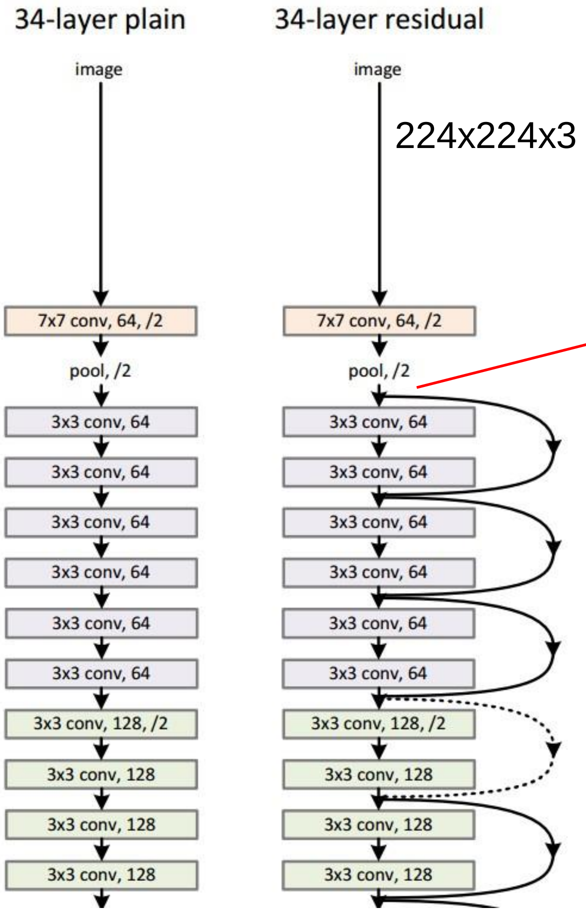
(slide from Kaiming He's recent presentation)

2-3 weeks of training on 8 GPU machine

at runtime: faster than a VGGNet! (even though it has 8x more layers)

Case Study: ResNet

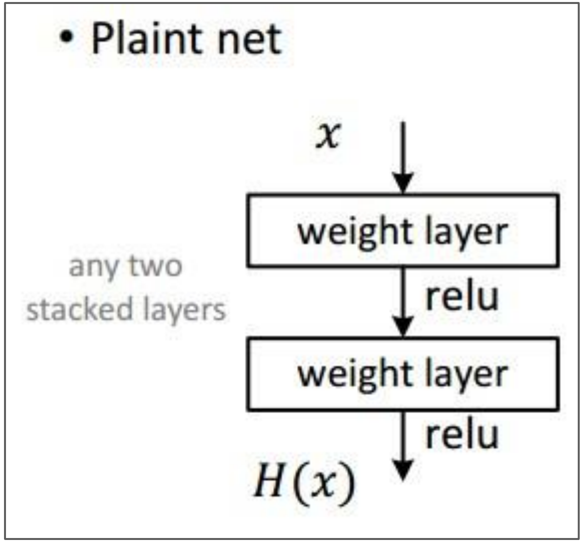
[He et al., 2015]

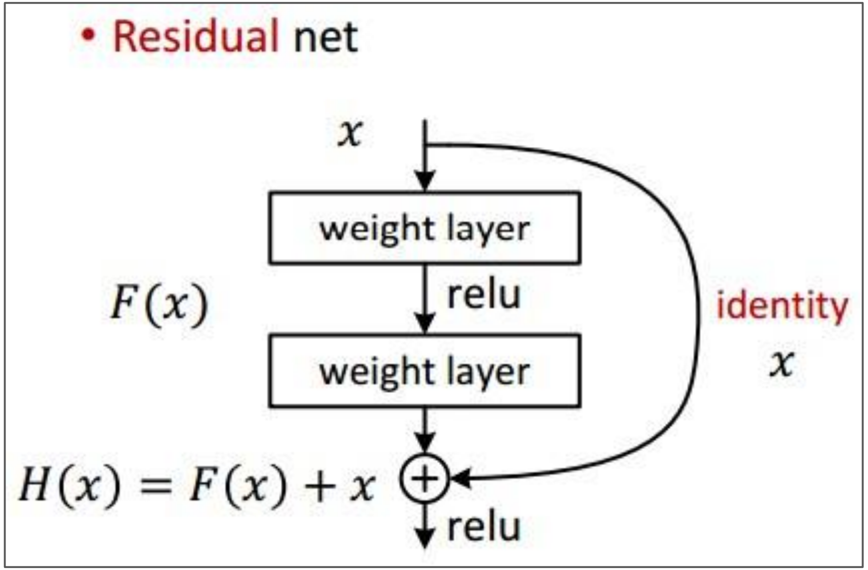


spatial dimension only 56x56!

Case Study: ResNet

[He et al., 2015]





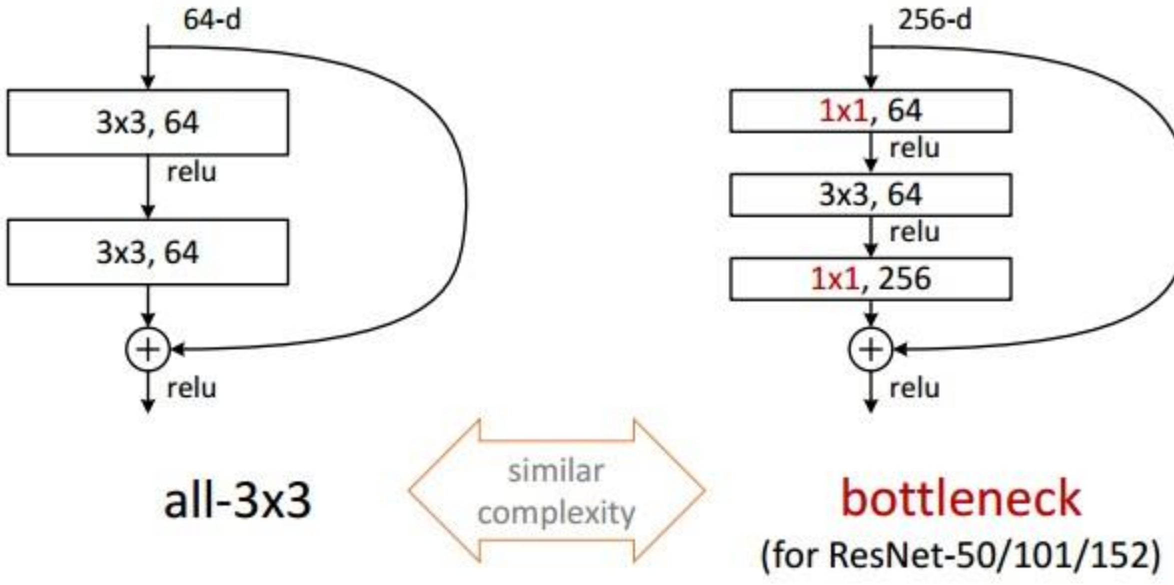
Case Study: ResNet

[He et al., 2015]

- Batch Normalization after every CONV layer
- Xavier/2 initialization from He et al.
- SGD + Momentum (0.9)
- Learning rate: 0.1, divided by 10 when validation error plateaus
- Mini-batch size 256
- Weight decay of 1e-5
- No dropout used

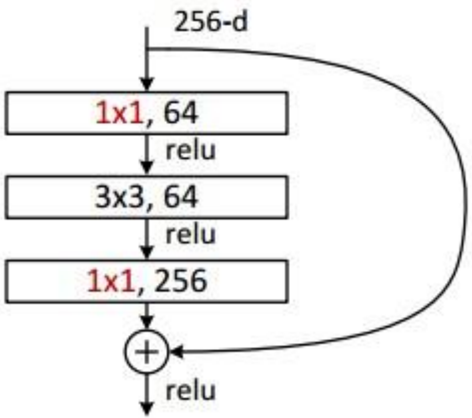
Case Study: ResNet

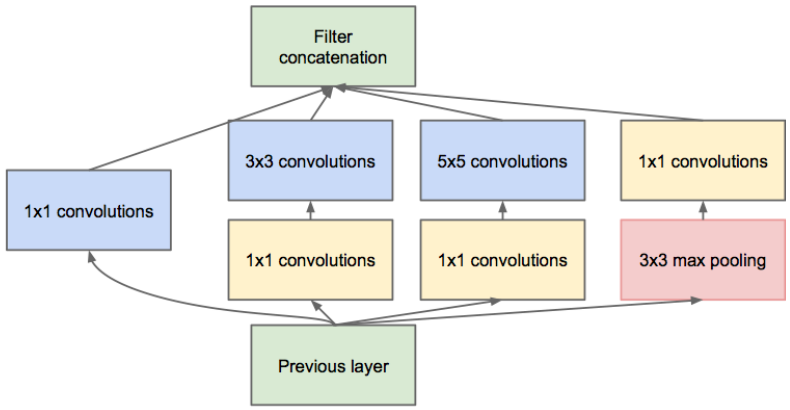
[He et al., 2015]



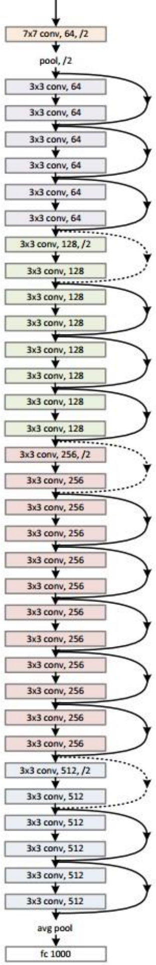
Case Study: ResNet

[He et al., 2015]





(this trick is also used in GoogLeNet)



Case Study: ResNet

[He et al., 2015]

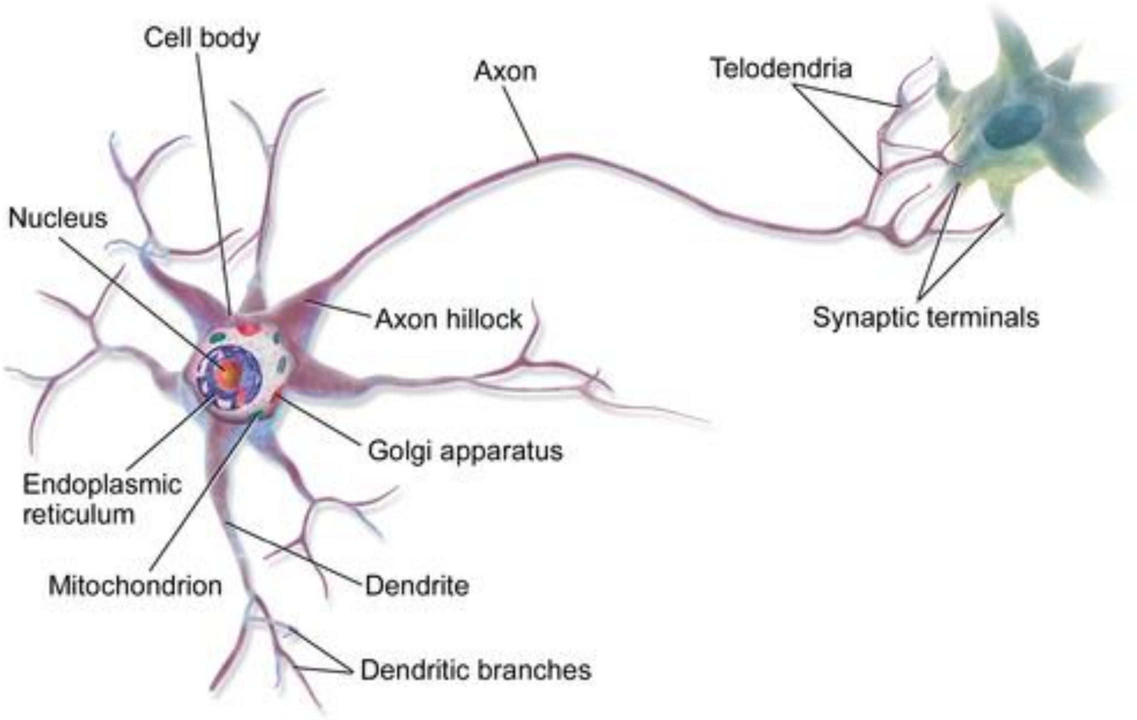
layer name	output size	18-layer	34-layer	50-layer	101-layer	152-layer
conv1	112×112	7×7, 64, stride 2				
conv2_x	56×56	3×3 max pool, stride 2				
		$\begin{bmatrix}3\times3, 64\\3\times3, 64\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 64\\3\times3, 64\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 64\\3\times3, 64\\1\times1, 256\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 64\\3\times3, 64\\1\times1, 256\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 64\\3\times3, 64\\1\times1, 256\end{bmatrix}\times3$
conv3_x	28×28	$\begin{bmatrix}3\times3, 128\\3\times3, 128\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 128\\3\times3, 128\end{bmatrix}\times4$	$\begin{bmatrix}1\times1, 128\\3\times3, 128\\1\times1, 512\end{bmatrix}\times4$	$\begin{bmatrix}1\times1, 128\\3\times3, 128\\1\times1, 512\end{bmatrix}\times4$	$\begin{bmatrix}1\times1, 128\\3\times3, 128\\1\times1, 512\end{bmatrix}\times8$
conv4_x	14×14	$\begin{bmatrix}3\times3, 256\\3\times3, 256\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 256\\3\times3, 256\end{bmatrix}\times6$	$\begin{bmatrix}1\times1, 256\\3\times3, 256\\1\times1, 1024\end{bmatrix}\times6$	$\begin{bmatrix}1\times1, 256\\3\times3, 256\\1\times1, 1024\end{bmatrix}\times23$	$\begin{bmatrix}1\times1, 256\\3\times3, 256\\1\times1, 1024\end{bmatrix}\times36$
conv5_x	7×7	$\begin{bmatrix}3\times3, 512\\3\times3, 512\end{bmatrix}\times2$	$\begin{bmatrix}3\times3, 512\\3\times3, 512\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 512\\3\times3, 512\\1\times1, 2048\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 512\\3\times3, 512\\1\times1, 2048\end{bmatrix}\times3$	$\begin{bmatrix}1\times1, 512\\3\times3, 512\\1\times1, 2048\end{bmatrix}\times3$
	1×1	average pool, 1000-d fc, softmax				
FLOPs		1.8×10^9	3.6×10^9	3.8×10^9	7.6×10^9	11.3×10^9

Summary

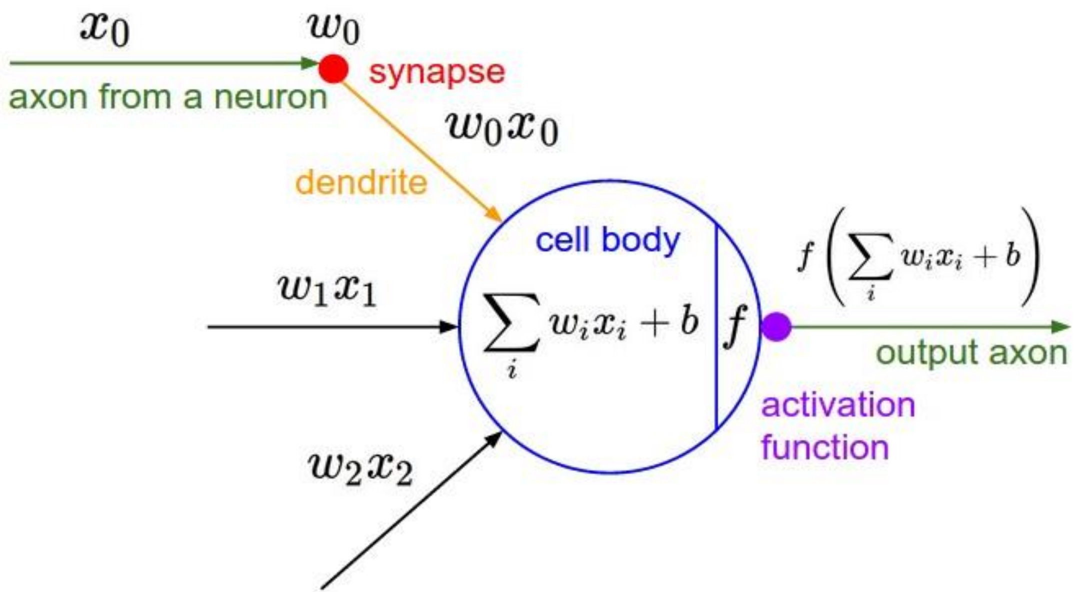
- ConvNets stack CONV,ReLU,POOL,FC layers
- Trend towards smaller filters and deeper architectures
- Trend towards getting rid of POOL/FC layers (just CONV)
- Early architectures look like

  [(CONV-RELU)*N-POOL?]*M-(FC-RELU)*K,SOFTMAX
- but recent advances such as ResNet/GoogLeNet use only Conv-ReLU, 1x1 convolutions, global max pooling and Softmax

Activation Functions: Biological Inspiration



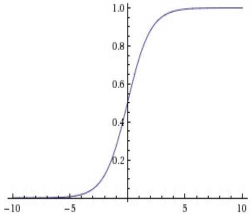
Activation Functions



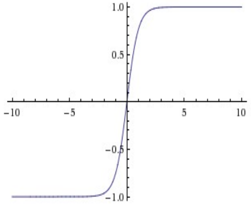
Activation Functions

Sigmoid

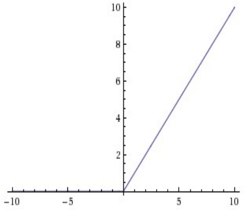
$$\sigma(x) = 1/(1+e^{-x})$$



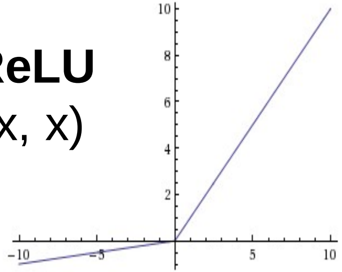
tanh tanh(x)



ReLU max(0,x)

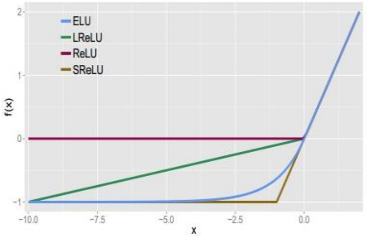


Leaky ReLU
max(0.1x, x)



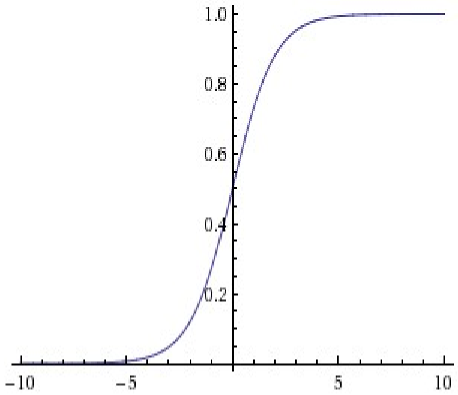
Maxout $\max(w_1^T x + b_1, w_2^T x + b_2)$

ELU $$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$



Slide based on cs231n by Fei-Fei Li & Andrej Karpathy & Justin Johnson

Activation Functions

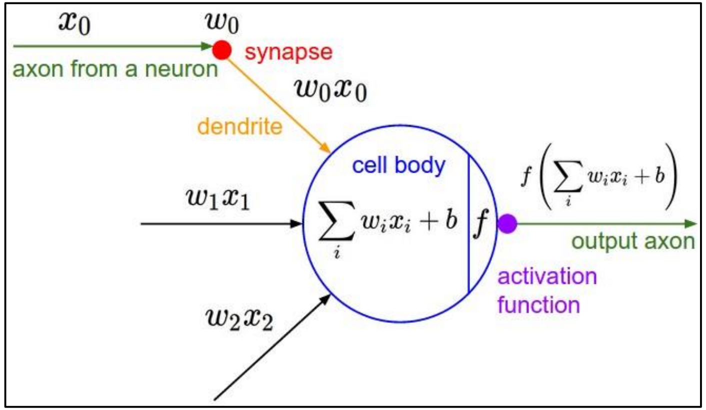


Sigmoid (logistic function)

$$\sigma(x) = 1/(1+e^{-x})$$

- Squashes numbers to range [0,1] – can kill gradients.
- A key element in LSTM networks – “control signals”
- Good for learning “logical” functions, – good for non-linear control (robots)
- Not as good for image networks (replaced by RELU)
- Not zero-centered

Consider what happens when the input to a neuron (x) is always positive:

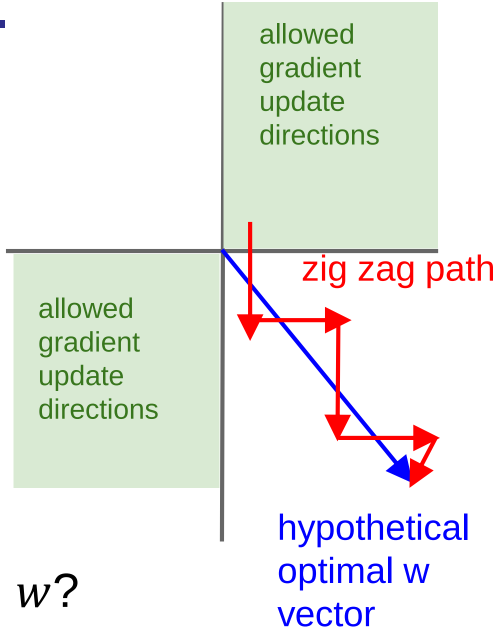


$$f\left(\sum_i w_i x_i + b\right)$$

What can we say about the gradients on w?

Consider what happens when the input to a neuron is always positive...

$$f\left(\sum_i w_i x_i + b\right)$$

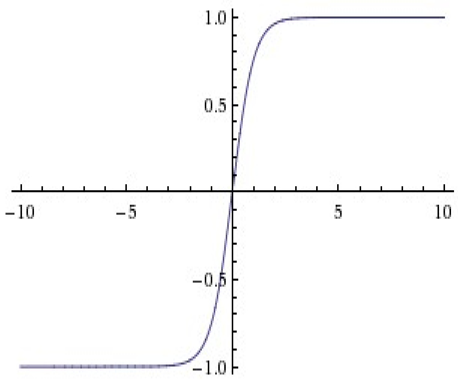


What can we say about the gradients on w?

Always all positive or all negative :(

(this is also why you want zero-mean data!)

Activation Functions

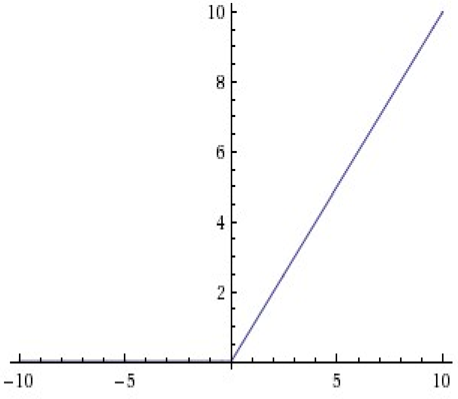


tanh(x)

- Squashes numbers to range [-1,1]
- Zero centered (nice)
- Still kills gradients when saturated :(
- Also used in LSTMs for bounded, signed values.
- Not as good for binary functions

[LeCun et al., 1991]

Activation Functions

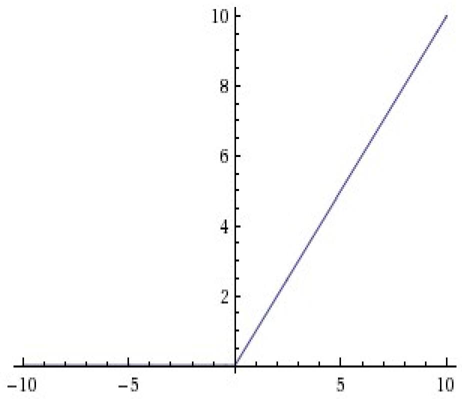


ReLU
(Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Converges faster than sigmoid/tanh on image data (e.g. 6x)
- Not suitable for logical functions
- Not for control in recurrent nets

[Krizhevsky et al., 2012]

Activation Functions

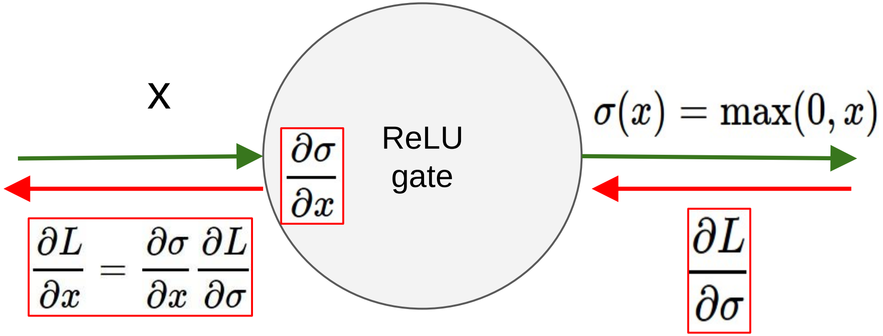


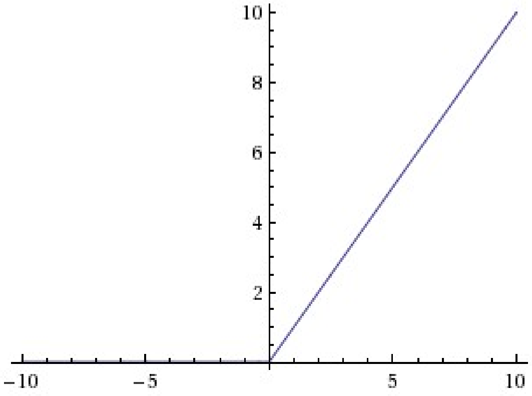
ReLU
(Rectified Linear Unit)

- Computes $f(x) = \max(0, x)$
- Does not saturate (in +region)
- Very computationally efficient
- Converges much faster than sigmoid/tanh in practice (e.g. 6x)

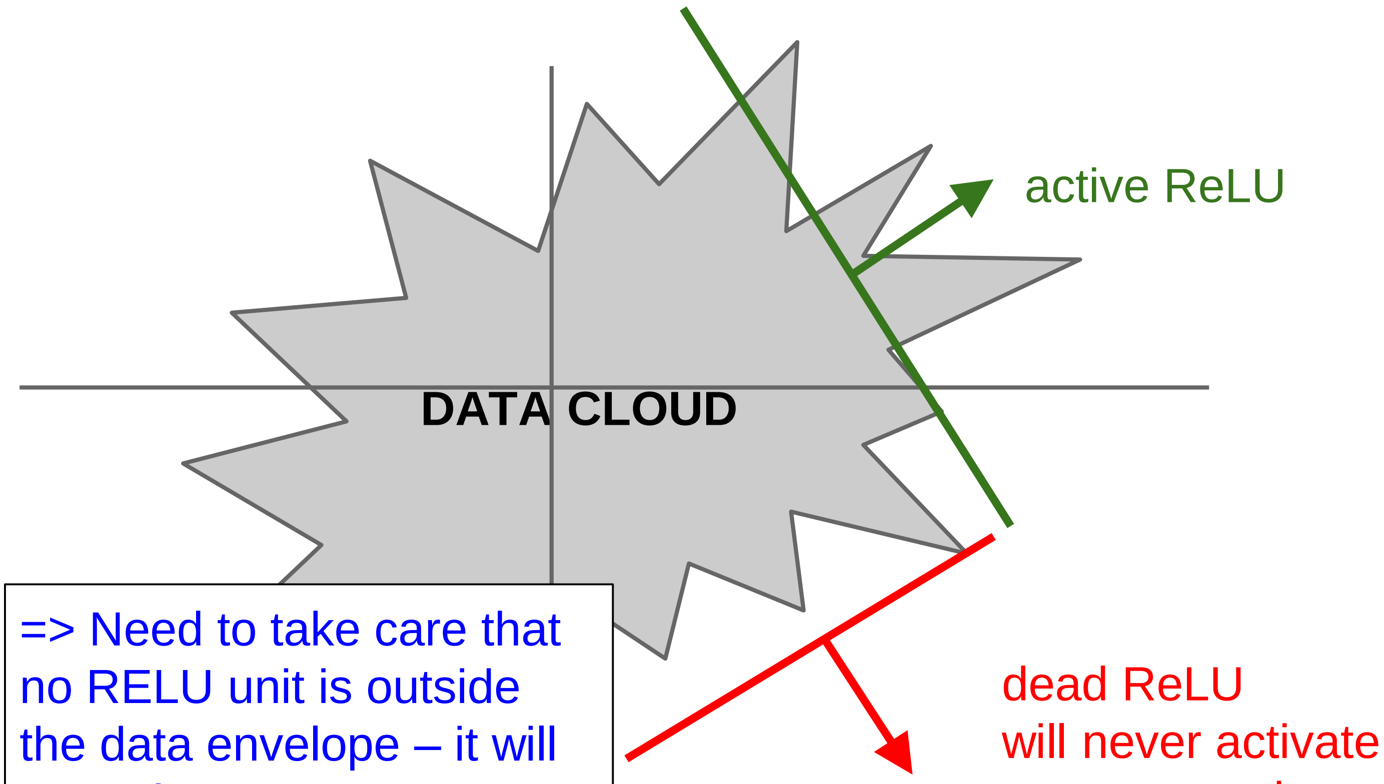
- Not zero-centered output
- An annoyance:

hint: what is the gradient when x < 0?





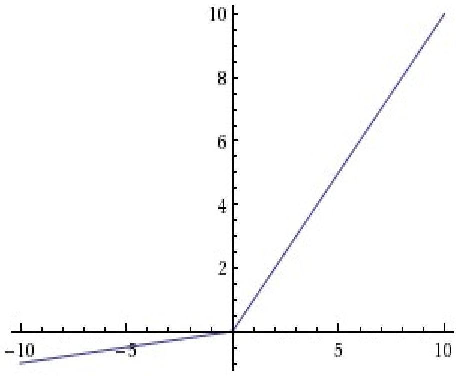
What happens when x = -10?
What happens when x = 0?
What happens when x = 10?



=> Need to take care that no RELU unit is outside the data envelope – it will never learn.

Activation Functions

[Mass et al., 2013]
[He et al., 2015]



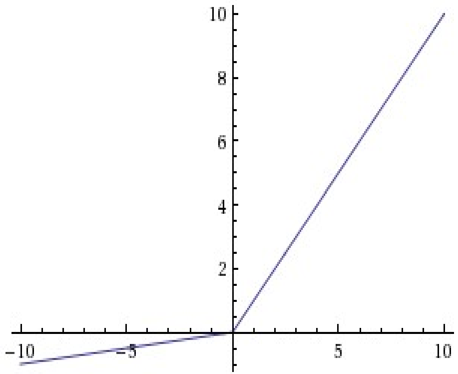
- Does not saturate
- Converges faster than sigmoid/tanh on image data(e.g. 6x)
- **will not “die”.**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Activation Functions

[Mass et al., 2013]
[He et al., 2015]



- Does not saturate
- Converges faster than sigmoid/tanh on image data (e.g. 6x)
- **will not "die".**

Leaky ReLU

$$f(x) = \max(0.01x, x)$$

Parametric Rectifier (PReLU)

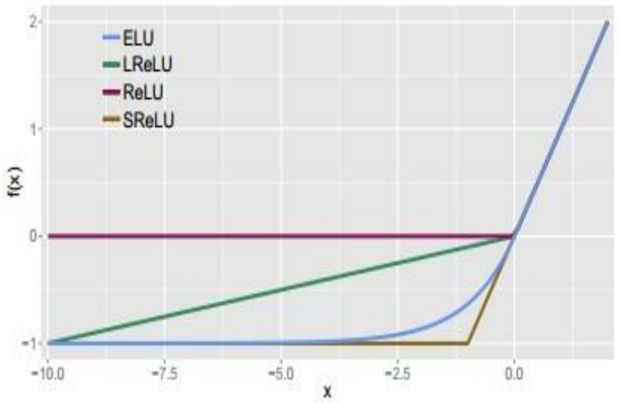
$$f(x) = \max(\alpha x, x)$$

backprop into α (parameter)

Activation Functions

[Clevert et al., 2015]

Exponential Linear Units (ELU)



$$f(x) = \begin{cases} x & \text{if } x > 0 \\ \alpha\,(\exp(x) - 1) & \text{if } x \le 0 \end{cases}$$

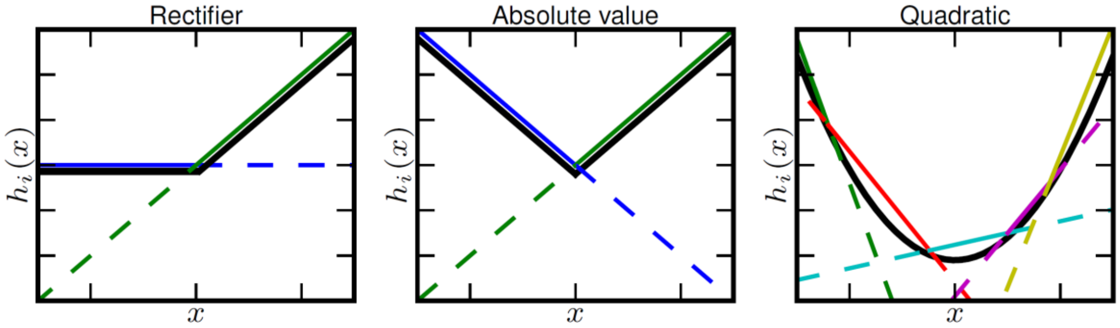
- All benefits of ReLU
- Does not die
- Closer to zero mean outputs

Maxout "Neuron"

[Goodfellow et al., 2013]

- Does not have the basic form of dot product -> nonlinearity
- Generalizes ReLU and Leaky ReLU
- Linear Regime! Does not saturate! Does not die!

$\max(w_1^T x + b_1, w_2^T x + b_2)$ can learn:



Problem: doubles the number of parameters/neuron :(

TLDR: In practice:

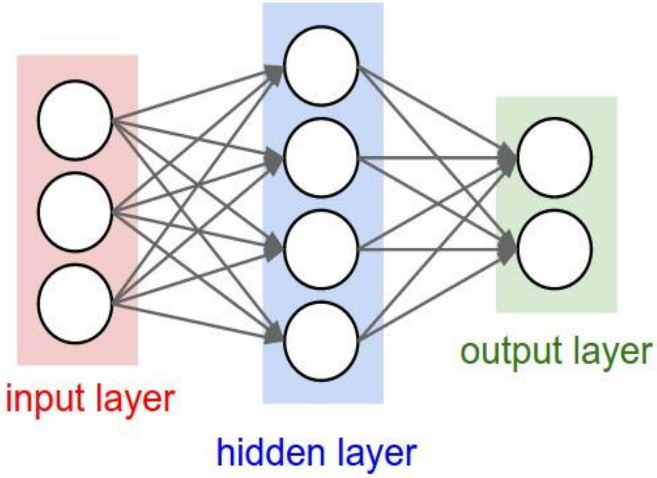
Try everything. Usually:

- Use ReLU on early image layers.
- Try out Leaky ReLU / Maxout / ELU.
- Use sigmoids for smooth functions, e.g. robot control.
- Sigmoids are also good for logical functions AND/OR.

Weight Initialization

Weight Initialization

- Q: what happens when W=0 init is used?



Weight Initialization

First idea: **Fixed random initialization**

e.g. gaussian with zero mean and fixed variance

$$W_{ij} \sim \mathcal{N}(0, 0.0001)$$

Works OK for small networks. Used in Alexnet.

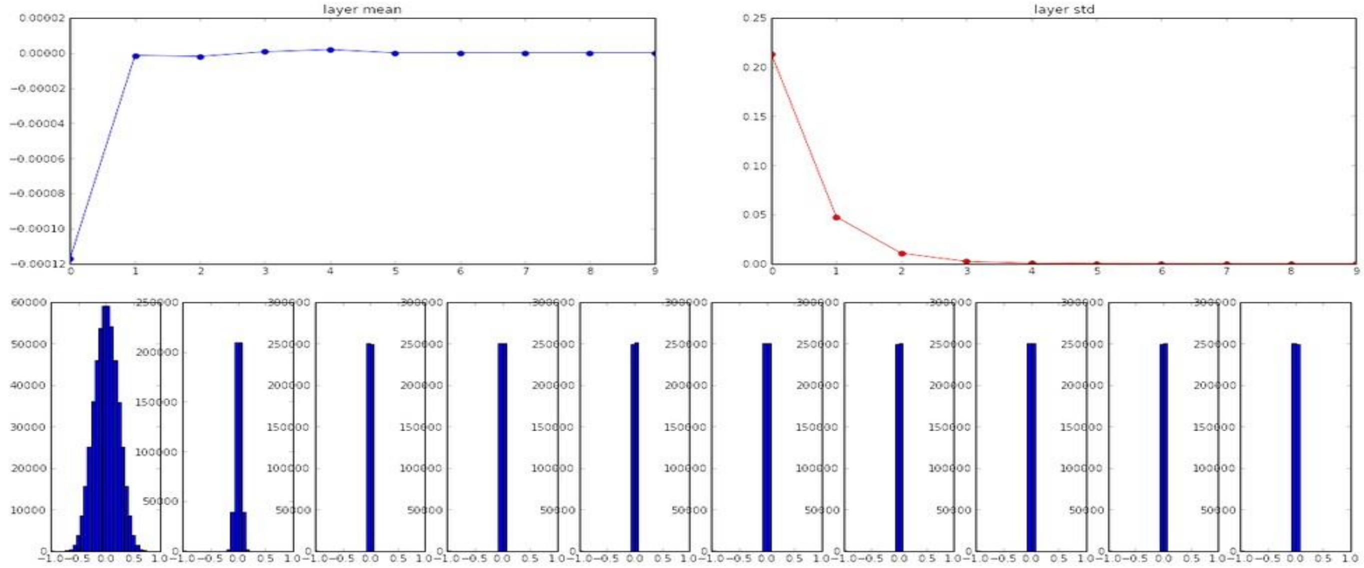
Problem is that activations are scaled by weights in the forward pass.

If the initial weights are too small/large, the activations will vanish or explode during the forward pass.

The gradients will do the same in the backward pass…

Activations with Random Weight init

```
input layer had mean 0.000927 and std 0.998388
hidden layer 1 had mean -0.000117 and std 0.213081
hidden layer 2 had mean -0.000001 and std 0.047551
hidden layer 3 had mean -0.000002 and std 0.010630
hidden layer 4 had mean 0.000001 and std 0.002378
hidden layer 5 had mean 0.000002 and std 0.000532
hidden layer 6 had mean -0.000000 and std 0.000119
hidden layer 7 had mean 0.000000 and std 0.000026
hidden layer 8 had mean -0.000000 and std 0.000006
hidden layer 9 had mean 0.000000 and std 0.000001
hidden layer 10 had mean -0.000000 and std 0.000000
```



Analysis of activation growth

Consider a linear layer (affine or convolutional), and define the **fan-in F** as the number of input neurons that contribute to each output.

- For an affine layer, the fan-in is just the number of neurons in the input layer.
- For a convolutional layer, the fan-in is $F_H \times F_W \times C$ where C is the depth of the input layer.

Then each output activation

$$a_{out} = \sum_{i=1}^{F} W_i a\_i$$

and assuming the activations and weights are independent, same variance:

$$\mathrm{Var}(a_{out}) = F\,\mathrm{Var}(W)\,\mathrm{Var}(a_{in})$$

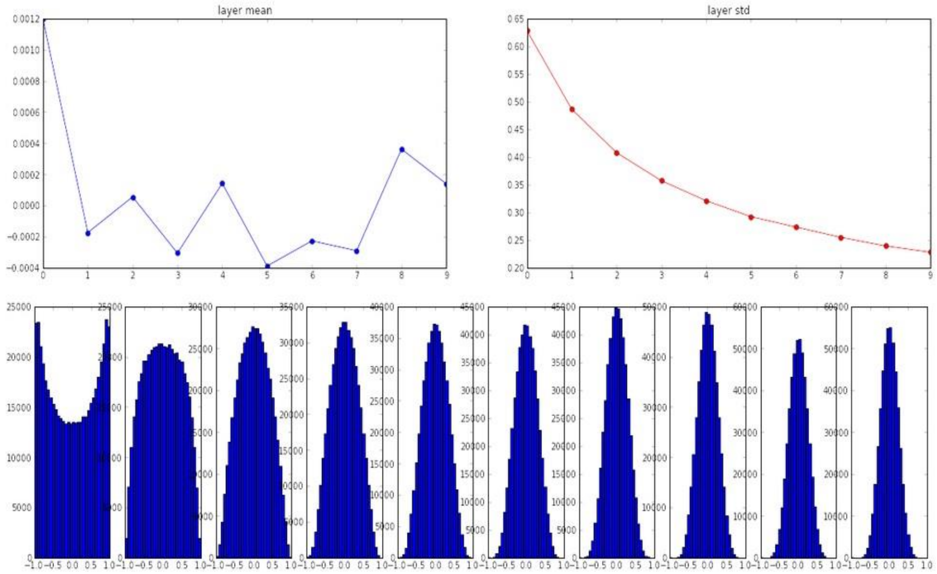
To get the same variance on input and output, we want $\mathbf{Var(\boldsymbol{W}) = 1/\boldsymbol{F}}$

Xavier Initialization

```
input layer had mean 0.001800 and std 1.001311
hidden layer 1 had mean 0.001198 and std 0.627953
hidden layer 2 had mean -0.000175 and std 0.486051
hidden layer 3 had mean 0.000055 and std 0.407723
hidden layer 4 had mean -0.000306 and std 0.357108
hidden layer 5 had mean 0.000142 and std 0.320917
hidden layer 6 had mean -0.000389 and std 0.292116
hidden layer 7 had mean -0.000228 and std 0.273387
hidden layer 8 had mean -0.000291 and std 0.254935
hidden layer 9 had mean 0.000361 and std 0.239266
hidden layer 10 had mean 0.000139 and std 0.228008
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

“Xavier initialization” [Glorot et al., 2010]

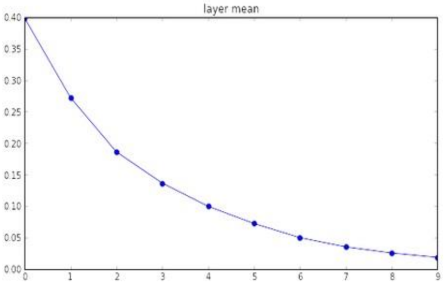


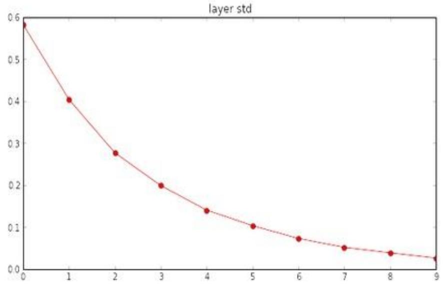
Accounting for ReLUs

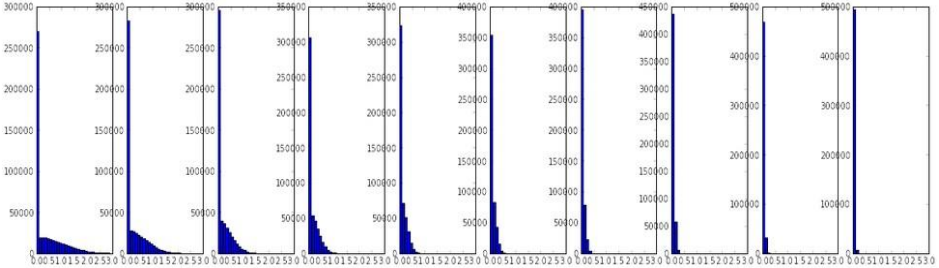
```
input layer had mean 0.000501 and std 0.999444
hidden layer 1 had mean 0.398623 and std 0.582273
hidden layer 2 had mean 0.272352 and std 0.403795
hidden layer 3 had mean 0.186076 and std 0.276912
hidden layer 4 had mean 0.136442 and std 0.198685
hidden layer 5 had mean 0.099568 and std 0.140299
hidden layer 6 had mean 0.072234 and std 0.103280
hidden layer 7 had mean 0.049775 and std 0.072748
hidden layer 8 had mean 0.035138 and std 0.051572
hidden layer 9 had mean 0.025404 and std 0.038583
hidden layer 10 had mean 0.018408 and std 0.026076
```

```python
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in) # layer initialization
```

Our analysis assumed that layers were linear.
Not accurate with ReLU layers.





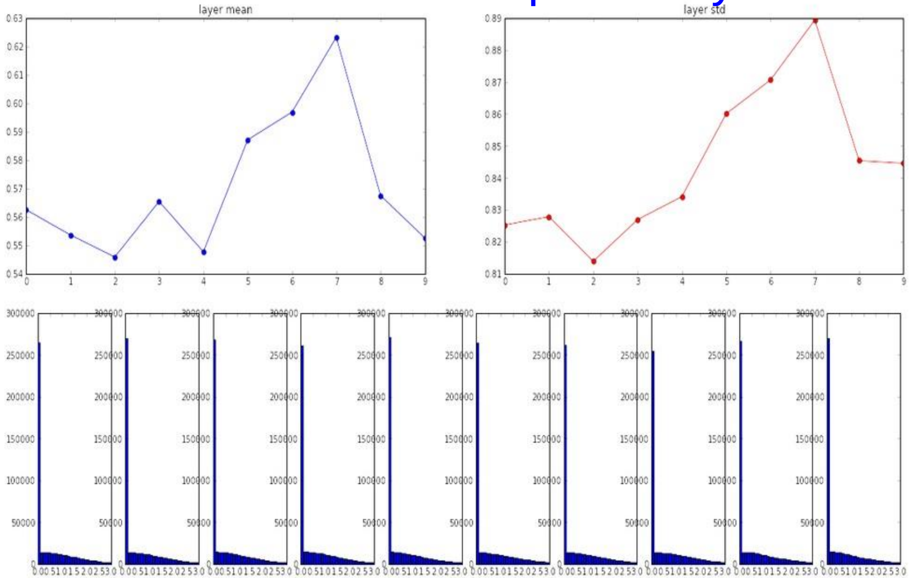


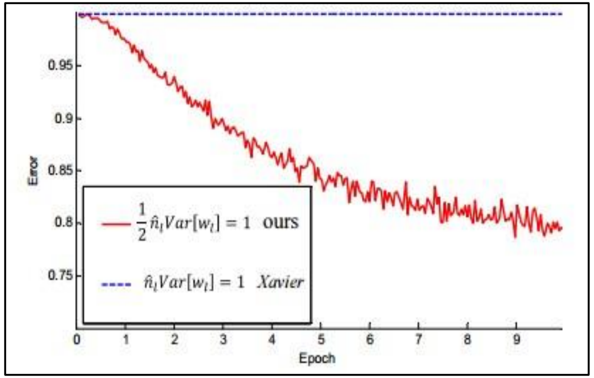
ReLU adjustment to Xavier

```
input layer had mean 0.000501 and std 0.999444
hidden layer 1 had mean 0.562488 and std 0.825232
hidden layer 2 had mean 0.553614 and std 0.827835
hidden layer 3 had mean 0.545867 and std 0.813855
hidden layer 4 had mean 0.565396 and std 0.826902
hidden layer 5 had mean 0.547678 and std 0.834092
hidden layer 6 had mean 0.587103 and std 0.860035
hidden layer 7 had mean 0.596867 and std 0.870610
hidden layer 8 had mean 0.623214 and std 0.889348
hidden layer 9 had mean 0.567498 and std 0.845357
hidden layer 10 had mean 0.552531 and std 0.844523
```

```
W = np.random.randn(fan_in, fan_out) / np.sqrt(fan_in/2) # layer initialization
```

He et al., 2015 (note additional /2)
factor of 2 doesn't seem like much, but it applies multiplicatively 150 times in a large ResNet.





Batch Normalization

[Ioffe and Szegedy, 2015]

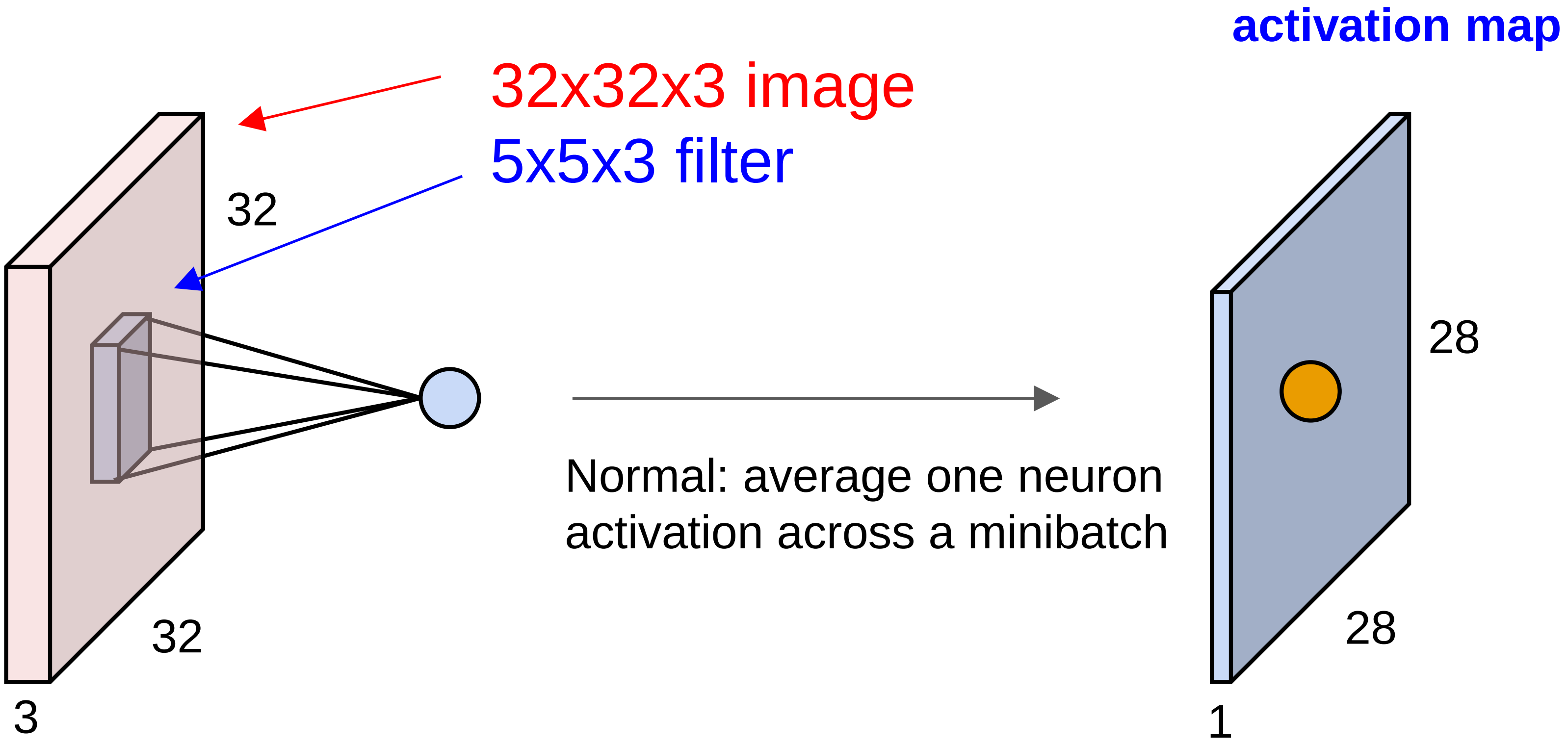
"you want unit gaussian activations? just make them so."

Consider a batch of activations at some layer.
To make each dimension unit gaussian, apply:

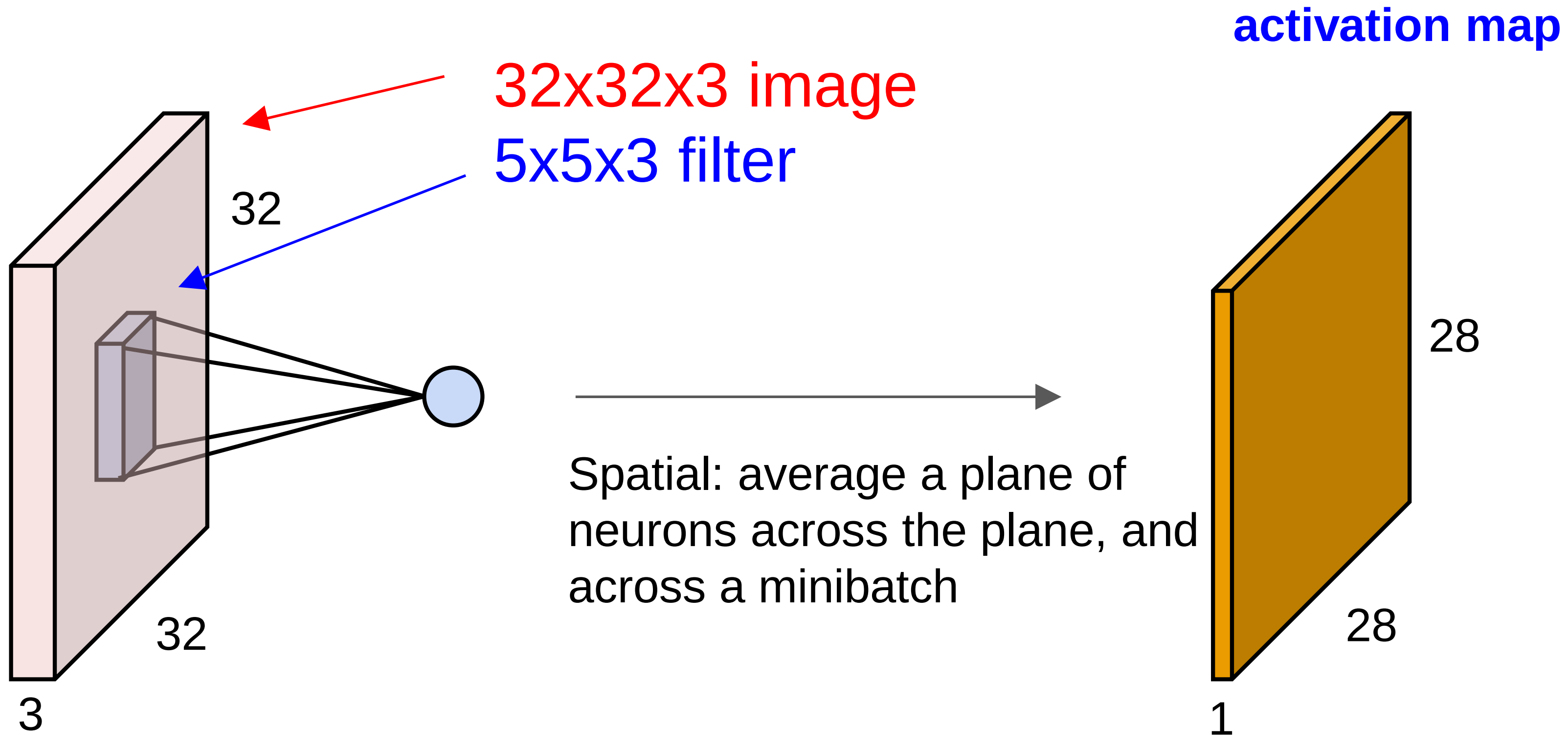
$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

this is a vanilla differentiable function...

Batch Norm: Per-Activation (default) or Spatial



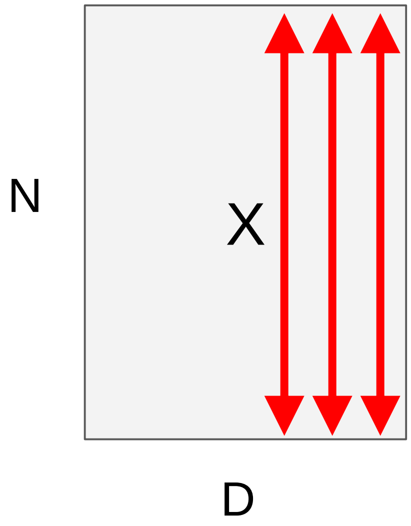
Batch Norm: Spatial



Batch Normalization

[Ioffe and Szegedy, 2015]

"you want unit gaussian activations?
just make them so."



1. compute the empirical mean and variance independently for each dimension.

2. Normalize

$$\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}$$

Batch Normalization

[Ioffe and Szegedy, 2015]

Normalize:

$$\boxed{\widehat{x}^{(k)} = \frac{x^{(k)} - \mathrm{E}[x^{(k)}]}{\sqrt{\mathrm{Var}[x^{(k)}]}}}$$

And then allow the network to squash the range if it wants to:

$$\boxed{y^{(k)} = \gamma^{(k)} \widehat{x}^{(k)} + \beta^{(k)}}$$

Note, the network can learn:

$$\gamma^{(k)} = \sqrt{\mathrm{Var}[x^{(k)}]}$$

$$\beta^{(k)} = \mathrm{E}[x^{(k)}]$$

to recover the identity mapping.

Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1...m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \text{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_{\mathcal{B}} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_{\mathcal{B}}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_{\mathcal{B}})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_{\mathcal{B}}}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \qquad \text{// normalize}$$

$$y_i \leftarrow \gamma\widehat{x}_i + \beta \equiv \text{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Acts as a form of regularization in a funny way, and slightly reduces the need for dropout, maybe

This Time: Batch Normalization

[Ioffe and Szegedy, 2015]

Input: Values of x over a mini-batch: $\mathcal{B} = \{x_{1\ldots m}\}$;
Parameters to be learned: γ, β

Output: $\{y_i = \mathrm{BN}_{\gamma,\beta}(x_i)\}$

$$\mu_\mathcal{B} \leftarrow \frac{1}{m}\sum_{i=1}^{m} x_i \qquad \text{// mini-batch mean}$$

$$\sigma_\mathcal{B}^2 \leftarrow \frac{1}{m}\sum_{i=1}^{m}(x_i - \mu_\mathcal{B})^2 \qquad \text{// mini-batch variance}$$

$$\widehat{x}_i \leftarrow \frac{x_i - \mu_\mathcal{B}}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \qquad \text{// normalize}$$

$$\boxed{y_i \leftarrow \gamma \widehat{x}_i + \beta \equiv \mathrm{BN}_{\gamma,\beta}(x_i) \qquad \text{// scale and shift}}$$

- Improves gradient flow through the network
- Allows higher learning rates
- Reduces the strong dependence on initialization
- Reduces need for dropout

Un-normalization!! Re-compute and apply the optimal scaling and bias for each neuron!
Learn γ and β (same dims as μ and σ^2).
It can (should?) learn the identity mapping!

Batch Normalization Gradients

[Ioffe and Szegedy, 2015]

$$\boxed{\begin{aligned}
\frac{\partial \ell}{\partial \widehat{x}_i} &= \frac{\partial \ell}{\partial y_i} \cdot \gamma \\
\frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} &= \sum_{i=1}^m \frac{\partial \ell}{\partial \widehat{x}_i} \cdot (x_i - \mu_\mathcal{B}) \cdot \frac{-1}{2}(\sigma_\mathcal{B}^2 + \epsilon)^{-3/2} \\
\frac{\partial \ell}{\partial \mu_\mathcal{B}} &= \left(\sum_{i=1}^m \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \right) + \frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} \cdot \frac{\sum_{i=1}^m -2(x_i - \mu_\mathcal{B})}{m}
\end{aligned}}$$

$$\frac{\partial \ell}{\partial x_i} = \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{1}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} + \frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} \cdot \frac{2(x_i - \mu_\mathcal{B})}{m} + \frac{\partial \ell}{\partial \mu_\mathcal{B}} \cdot \frac{1}{m}$$

$$\frac{\partial \ell}{\partial \gamma} = \sum_{i=1}^m \frac{\partial \ell}{\partial y_i} \cdot \widehat{x}_i$$

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^m \frac{\partial \ell}{\partial y_i}$$

Don’t need these directly, they are subexpressions for the other gradients.

Think of this as backprop for the nodes $\hat{x}$, $\sigma_\mathcal{B}^2$, $\mu_\mathcal{B}$, which are all internal to the minibatch update.

Batch Normalization Gradients

[Ioffe and Szegedy, 2015]

$$\frac{\partial \ell}{\partial \widehat{x}_i} = \frac{\partial \ell}{\partial y_i} \cdot \gamma$$

$$\frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot (x_i - \mu_{\mathcal{B}}) \cdot \frac{-1}{2}(\sigma_{\mathcal{B}}^2 + \epsilon)^{-3/2}$$

$$\frac{\partial \ell}{\partial \mu_{\mathcal{B}}} = \left(\sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} \right) + \frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} \cdot \frac{\sum_{i=1}^{m} -2(x_i - \mu_{\mathcal{B}})}{m}$$

$$\boxed{\frac{\partial \ell}{\partial x_i} = \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{1}{\sqrt{\sigma_{\mathcal{B}}^2 + \epsilon}} + \frac{\partial \ell}{\partial \sigma_{\mathcal{B}}^2} \cdot \frac{2(x_i - \mu_{\mathcal{B}})}{m} + \frac{\partial \ell}{\partial \mu_{\mathcal{B}}} \cdot \frac{1}{m}}$$

Gradient to propagate to the input layer

$$\frac{\partial \ell}{\partial \gamma} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i} \cdot \widehat{x}_i$$

$$\frac{\partial \ell}{\partial \beta} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i}$$

Batch Normalization Gradients

[Ioffe and Szegedy, 2015]

$$\frac{\partial \ell}{\partial \widehat{x}_i} = \frac{\partial \ell}{\partial y_i} \cdot \gamma$$

$$\frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} = \sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot (x_i - \mu_\mathcal{B}) \cdot \frac{-1}{2}(\sigma_\mathcal{B}^2 + \epsilon)^{-3/2}$$

$$\frac{\partial \ell}{\partial \mu_\mathcal{B}} = \left(\sum_{i=1}^{m} \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{-1}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} \right) + \frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} \cdot \frac{\sum_{i=1}^{m} -2(x_i - \mu_\mathcal{B})}{m}$$

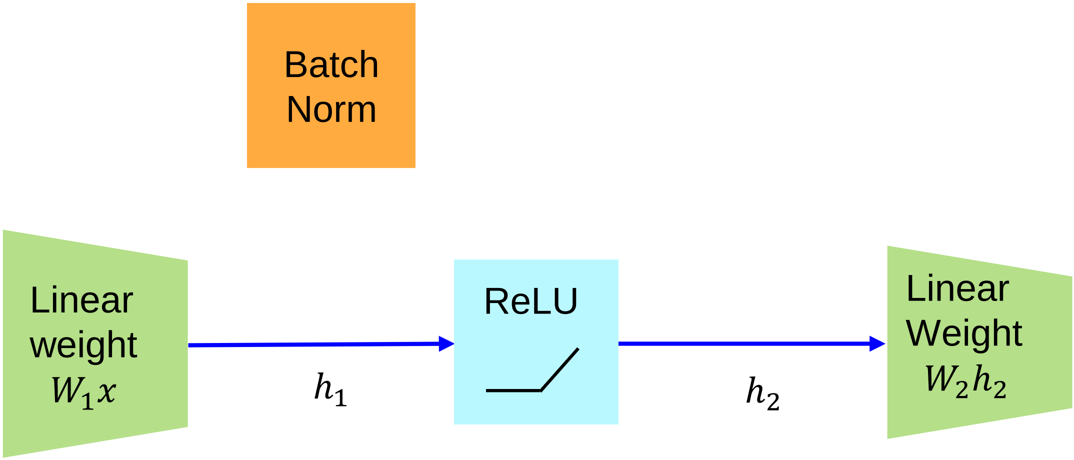
$$\frac{\partial \ell}{\partial x_i} = \frac{\partial \ell}{\partial \widehat{x}_i} \cdot \frac{1}{\sqrt{\sigma_\mathcal{B}^2 + \epsilon}} + \frac{\partial \ell}{\partial \sigma_\mathcal{B}^2} \cdot \frac{2(x_i - \mu_\mathcal{B})}{m} + \frac{\partial \ell}{\partial \mu_\mathcal{B}} \cdot \frac{1}{m}$$

$$\boxed{\begin{aligned} \frac{\partial \ell}{\partial \gamma} &= \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i} \cdot \widehat{x}_i \\ \frac{\partial \ell}{\partial \beta} &= \sum_{i=1}^{m} \frac{\partial \ell}{\partial y_i} \end{aligned}}$$

Gradients for the learnable parameters γ and β.

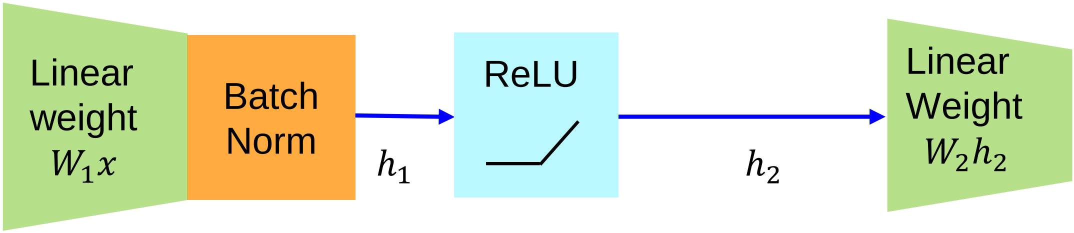
Where to do BatchNorm ?

BatchNorm is just a linear scale/bias layer in the limit of large batch sizes (μ, σ^2).



Where to do BatchNorm ?

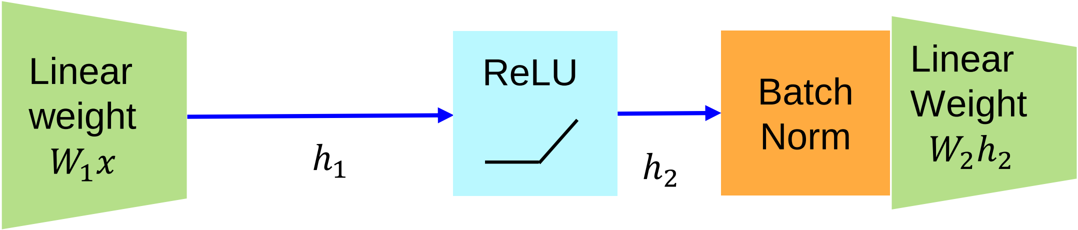
BatchNorm is just a linear scale/bias layer in the limit of large batch sizes (μ, σ^2).



?? Linear weight layer should
already have optimal scale/bias
in its output

Where to do BatchNorm ?

BatchNorm is just a linear scale/bias layer in the limit of large batch sizes (μ, σ^2).



?? Any bias/scaling by the batch norm layer should be over-ruled by the second linear weight layer.

Where to do BatchNorm ?

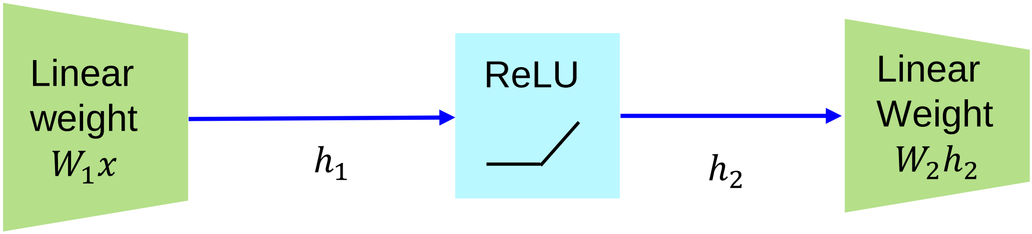
Hard to argue that it is doing normalization, since it will often learn the identity…

Is it really a pseudo-random regularizer (via $1/\sqrt{\sigma^2}$), like dropout?

- Seems to reduce need for dropout

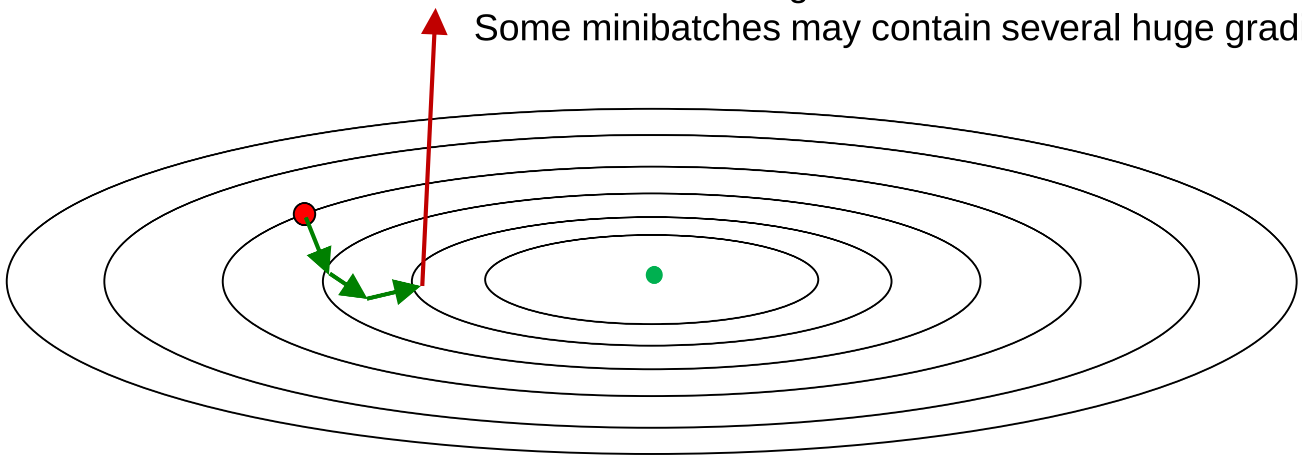
Does it enact a kind of activation/gradient clipping?

- Allows higher learning rates

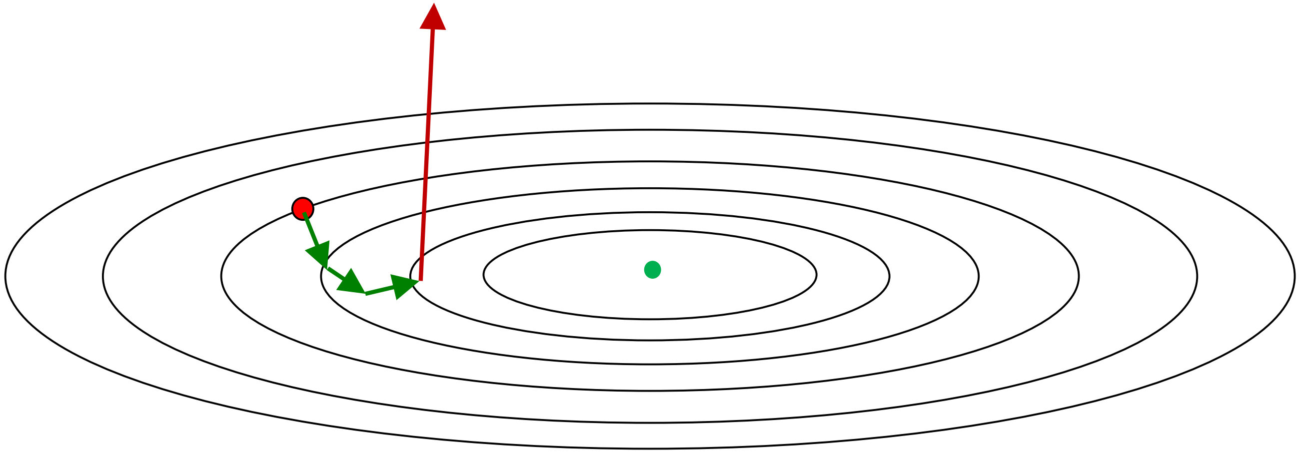


Gradient Magnitudes:

Occasional “monster” gradients
Gradient magnitudes far from gaussian, long-tailed distributions
Gradients much larger for mis-classified instances
Some minibatches may contain several huge gradients



Gradient Clipping by Value:

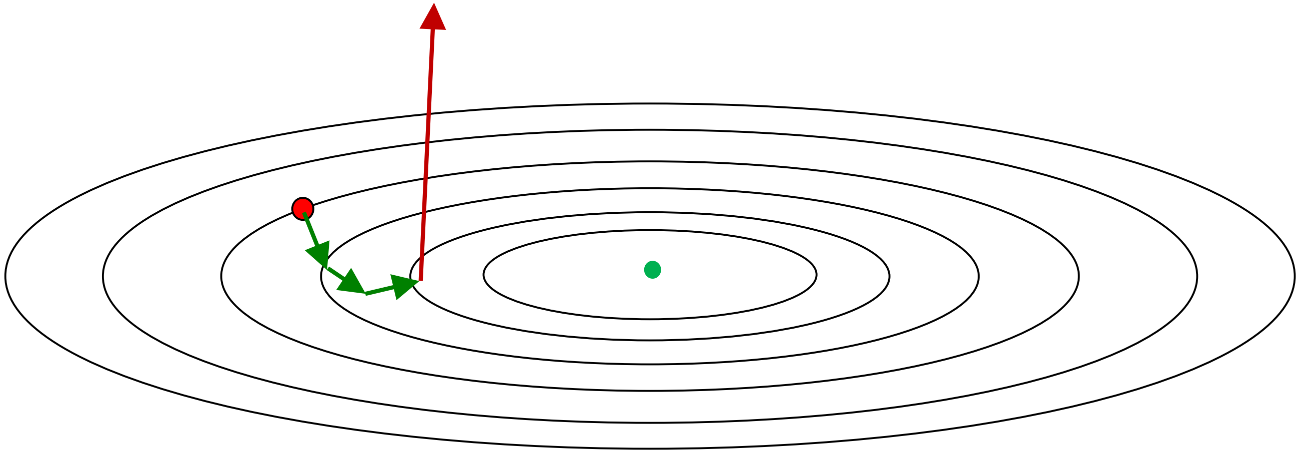


Simply limit the magnitude of each gradient:

$$\bar{g}_i = \min\big(g_{\max}, \max(-g_{\max}, g_i)\big)$$

so $|\bar{g}_i| \le g_{\max}$. But how to set $g_{\max}$? Use minibatch stats?

Gradient Clipping by Norm:

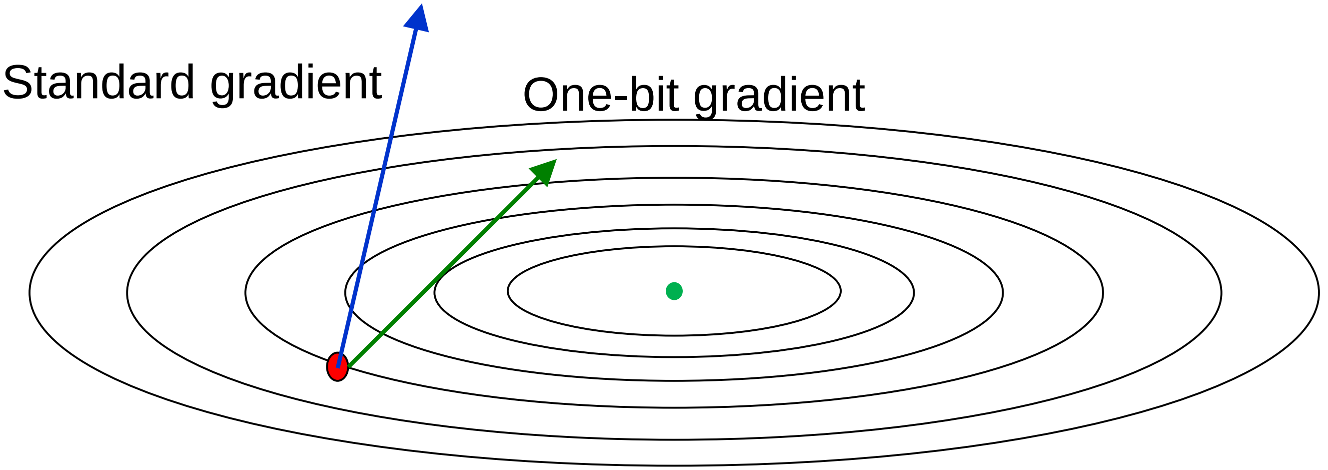


Clip to limit the norm of the gradient:

clipped_grad[i] = grad[i] * clip_norm / max(norm(grad), clip_norm)

Still need to set clip_norm, use a multiple of median norm(grad) ?

One-bit Gradients!



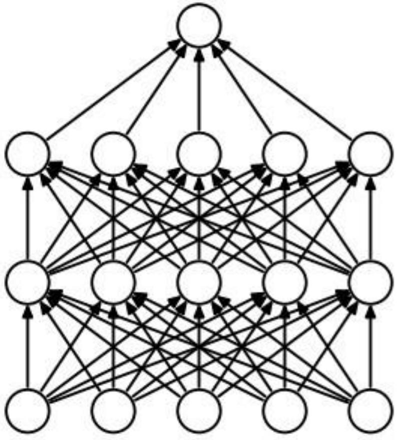
If we clip all gradient dimensions, we are left only with their sign: $\bar{g}_i = g_{\max}(-1,1,1,-1,1,\ldots)$

This actually works on some problems with little or no loss of accuracy!:
(see "1-Bit Stochastic Gradient Descent and Application to Data-Parallel Distributed Training of Speech DNNs" by Seide et al. 2014)

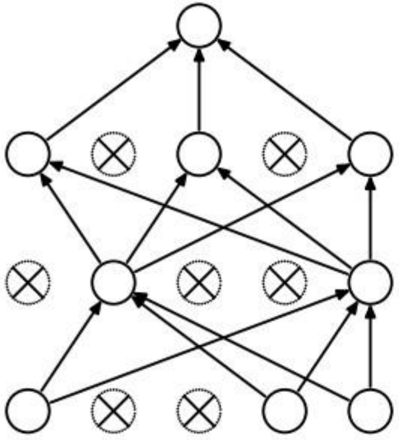
Dropout

"randomly set some neurons to zero in the forward pass"

i.e. multiply by random bernoulli variables with parameter p.



(a) Standard Neural Net



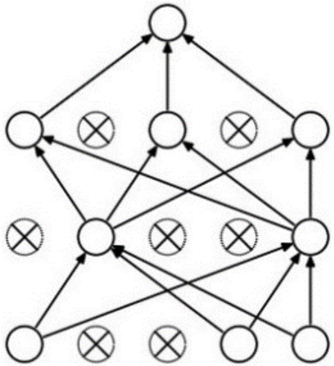
(b) After applying dropout.

Note, p is the probability of keeping a neuron (note, incorrect in assignment 1)

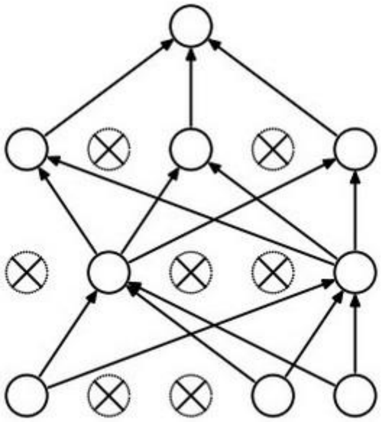
[Srivastava et al., 2014]

```python
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  """ X contains the data """

  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = np.random.rand(*H1.shape) < p # first dropout mask
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = np.random.rand(*H2.shape) < p # second dropout mask
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)
```

Example forward pass with a 3-layer network using dropout



Waaaait a second…
How could this possibly be a good idea?



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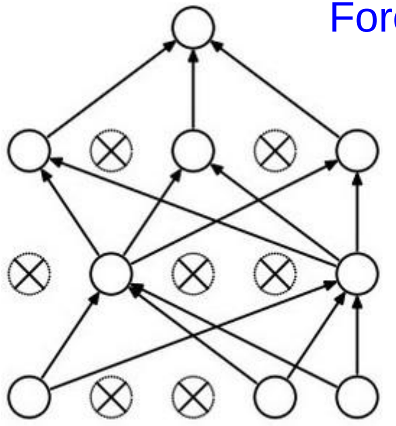
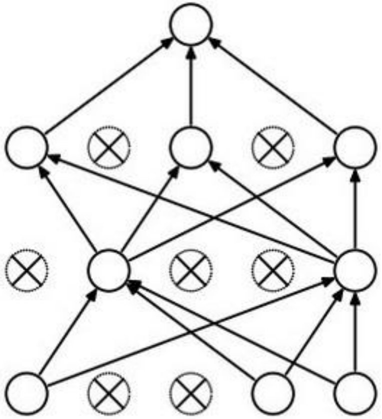


Forces the network to have a redundant representation.

Waaaait a second…
How could this possibly be a good idea?



Another interpretation:

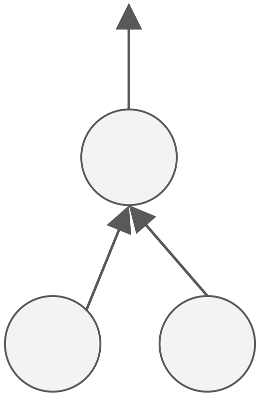
Dropout is training a large ensemble of models (that share parameters).

Each binary mask is one model, gets trained on only ~one datapoint.

At test time….

Can in fact do this with a single forward pass! (approximately)

Leave all input neurons turned on (no dropout).

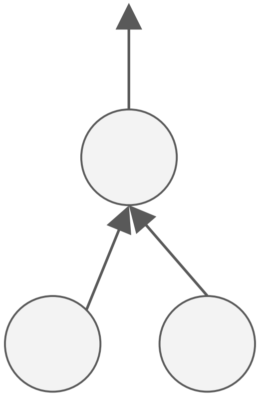


(this can be shown to be an approximation to evaluating the whole ensemble)

At test time….

Can in fact do this with a single forward pass! (approximately)

Leave all input neurons turned on (no dropout).



Q: Suppose that with all inputs present at test time the output of this neuron is x.

What would its output be during training time, in expectation? (e.g. if p = 0.5)

We can do something approximate analytically

```
def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
  H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
  out = np.dot(W3, H2) + b3
```

At test time all neurons are active always
=> We must scale the activations so that for each neuron:
<u>output at test time</u> = <u>expected output at training time</u>

Dropout Summary

```
""" Vanilla Dropout: Not recommended implementation (see notes below) """

p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  """ X contains the data """

  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = np.random.rand(*H1.shape) < p # first dropout mask
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = np.random.rand(*H2.shape) < p # second dropout mask
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)

def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) * p # NOTE: scale the activations
  H2 = np.maximum(0, np.dot(W2, H1) + b2) * p # NOTE: scale the activations
  out = np.dot(W3, H2) + b3
```

drop in forward pass

scale at test time

More common: “Inverted dropout”

```
p = 0.5 # probability of keeping a unit active. higher = less dropout

def train_step(X):
  # forward pass for example 3-layer neural network
  H1 = np.maximum(0, np.dot(W1, X) + b1)
  U1 = (np.random.rand(*H1.shape) < p) / p # first dropout mask. Notice /p!
  H1 *= U1 # drop!
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  U2 = (np.random.rand(*H2.shape) < p) / p # second dropout mask. Notice /p!
  H2 *= U2 # drop!
  out = np.dot(W3, H2) + b3

  # backward pass: compute gradients... (not shown)
  # perform parameter update... (not shown)

def predict(X):
  # ensembled forward pass
  H1 = np.maximum(0, np.dot(W1, X) + b1) # no scaling necessary
  H2 = np.maximum(0, np.dot(W2, H1) + b2)
  out = np.dot(W3, H2) + b3
```

test time is unchanged!

Summary

- **ConvNet Case Studies:**
  - LeNet
  - AlexNet
  - VGG
  - GoogLeNet
  - ResNet
- Activation Functions and typical uses
- Weight Initialization principles
- Batch Normalization
- Dropout