Содержание

1   final/template/template.cpp

```cpp
// team : SPb ITMO University Komanda
#include <bits/stdc++.h>
#ifdef SIR
   #define err(...) fprintf(stderr, __VA_ARGS__)
#else
   #define err(...) 42
#endif

#define db(x) cerr << #x << " = " << x << endl
#define db2(x, y) cerr << "(" << #x << ", " << #y <<
    ") = (" << x << ", " << y << ")\n";
#define db3(x, y, z) cerr << "(" << #x << ", " << #y
    << ", " << #z << ") = (" << x << ", " << y <<
    ", " << z << ")\n"
#define dbv(a) cerr << #a << " = "; for (auto xxxx:
    a) cerr << xxxx << " "; cerr << endl

using namespace std;

typedef long long ll;

void solve() {

}

int main() {
#ifdef SIR
   freopen("input.txt", "r", stdin), freopen("output.
    txt", "w", stdout);
#endif
   ios_base::sync_with_stdio(0);
   cin.tie(0);
   solve();
   return 0;
}
```

2   Practice round

- Посабмитить задачи каждому человеку.

- Распечатать решение.

- IDE для джавы.

- Сравнить скорость локального компьютера и сервера.

- Проверить int128.

- Проверить прагмы. Например, на bitset.

3   final/stuff/debug.cpp

```cpp
#include <bits/stdc++.h>
#define _GLIBCXX_DEBUG

using namespace std;

template <class T>
struct MyVector : vector<T> {
  MyVector() : vector<T>() { }
  MyVector( int n ) : vector<T>(n) { }
  T &operator [] ( int i )  { return vector<T>::at(i
    ); }
  T operator [] ( int i ) const { return vector<T>::
    at(i); }
};
```

```
14  /** Есливвашемкодевместовсех           int [] и vector<int> ←
        использовать MyVector<int >,
15    вывувидитевсе    range check errorы- */
16  MyVector<int> b(10), a;
17
18  int main() {
19    MyVector<int> a(50);
20    for (int i = 1; i <= 600; i++) a[i] = i;
21    cout << a[500] << "\n";
22  }
```

4   final/template/fastIO.cpp

```
1   #include <cstdio>
2   #include <algorithm>
3
4   /** Interface */
5
6   inline int readInt();
7   inline int readUInt();
8   inline bool isEof();
9
10  /** Read */
11
12  static const int buf_size = 100000;
13  static char buf[buf_size];
14  static int buf_len = 0, pos = 0;
15
16  inline bool isEof() {
17    if (pos == buf_len) {
18      pos = 0, buf_len = fread(buf, 1, buf_size, stdin←
          );
19      if (pos == buf_len) return 1;
20    }
21    return 0;
22  }
23
24  inline int getChar() { return isEof() ? -1 : buf[pos←
        ++]; }
25
26  inline int readChar() {
27    int c = getChar();
28    while (c != -1 && c <= 32) c = getChar();
29    return c;
30  }
31
32  inline int readUInt() {
33    int c = readChar(), x = 0;
34    while ('0' <= c && c <= '9') x = x * 10 + c - '0',←
        c = getChar();
35    return x;
36  }
37
38  inline int readInt() {
39    int s = 1, c = readChar();
40    int x = 0;
41    if (c == '-') s = -1, c = getChar();
42    while ('0' <= c && c <= '9') x = x * 10 + c - '0',←
        c = getChar();
43    return s == 1 ? x : -x;
44  }
45
46
47  // 10M int [0..1e9]
48  // cin 3.02
49  // scanf 1.2
50  // cin sync_with_stdio(false) 0.71
51  // fastRead getchar 0.53
52  // fastRead fread 0.15
```

5   final/template/optimizations.cpp

```
1   inline void fasterLLDivMod(unsigned long long x, ←
        unsigned y, unsigned &out_d, unsigned &out_m) {
2     unsigned xh = (unsigned)(x >> 32), xl = (unsigned)←
        x, d, m;
3   #ifdef __GNUC__
4     asm(
5       "divl %4; \n\t"
6       : "=a" (d), "=d" (m)
7       : "d" (xh), "a" (xl), "r" (y)
```

```
8       );
9   #else
10      __asm {
11        mov edx, dword ptr[xh];
12        mov eax, dword ptr[xl];
13        div dword ptr[y];
14        mov dword ptr[d], eax;
15        mov dword ptr[m], edx;
16      };
17  #endif
18      out_d = d; out_m = m;
19  }
20
21  // have no idea what sse flags are really cool; list←
        of some of them
22  // — very good with bitsets
23  #pragma GCC optimize("O3")
24  #pragma GCC target("sse,sse2,sse3,ssse3,sse4,popcnt,←
        abm,mmx")
```

6   final/template/useful.cpp

```
1   #include "ext/pb_ds/assoc_container.hpp"
2   using namespace __gnu_pbds;
3
4   template <typename T> using ordered_set = tree<T, ←
        null_type, less<T>, rb_tree_tag, ←
        tree_order_statistics_node_update>;
5   template <typename K, typename V> using ordered_map ←
        = tree<K, V, less<K>, rb_tree_tag, ←
        tree_order_statistics_node_update>;
6
7   // HOW TO USE ::
8   // — order_of_key(10) returns the number of ←
        elements in set/map strictly less than 10
9   // — *find_by_order(10) returns 10-th smallest ←
        element in set/map (0-based)
10
11  bitset<N> a;
12  for (int i = a._Find_first(); i != a.size(); i = a.←
        _Find_next(i)) {
13    cout << i << endl;
14  }
```

7   final/template/Template.java

```
1   import java.util.*;
2   import java.io.*;
3
4   public class Template {
5     FastScanner in;
6     PrintWriter out;
7
8     public void solve() throws IOException {
9       int n = in.nextInt();
10      out.println(n);
11    }
12
13    public void run() {
14      try {
15        in = new FastScanner();
16        out = new PrintWriter(System.out);
17
18        solve();
19
20        out.close();
21      } catch (IOException e) {
22        e.printStackTrace();
23      }
24    }
25
26    class FastScanner {
27      BufferedReader br;
28      StringTokenizer st;
29
30      FastScanner() {
31        br = new BufferedReader(new InputStreamReader(←
        System.in));
32      }
33
34      String next() {
```

```
35          while (st == null || !st.hasMoreTokens()) {
36            try {
37              st = new StringTokenizer(br.readLine());
38            } catch (IOException e) {
39              e.printStackTrace();
40            }
41          }
42          return st.nextToken();
43        }
44
45        int nextInt() {
46          return Integer.parseInt(next());
47        }
48    }
49
50    public static void main(String[] arg) {
51      new Template().run();
52    }
53  }
```

8 final/template/bitset.cpp

```
1
2   const int SZ = 6;
3   const int BASE = pw(SZ);
4   const int MOD = BASE - 1;
5
6   struct Bitset {
7     typedef unsigned long long T;
8     vector<T> data;
9     int n;
10    void resize(int nn) {
11      n = nn;
12      data.resize((n + BASE - 1) / BASE);
13    }
14    void set(int pos, int val) {
15      int id = pos >> SZ;
16      int rem = pos & MOD;
17      data[id] ^= data[id] & pw(rem);
18      data[id] |= val * pw(rem);
19    }
20    int get(int pos) {
21      return (data[pos >> SZ] >> (pos & MOD)) & 1;
22    }
23    // k > 0 -> (*this) << k
24    // k < 0 -> (*this) >> (-k)
25    Bitset shift(int k) {
26      Bitset res;
27      res.resize(n);
28      int s = k / BASE;
29      int rem = k % BASE;
30      if (rem < 0) {
31        rem += BASE;
32        s--;
33      }
34      int p1 = BASE - rem;
35      T mask = (p1 == 64)? -1: pw(p1) - 1;
36      for (int i = max(0, -s); i < sz(data) - max(s, ↩
      0); i++) {
37        res.data[i + s] |= (data[i] & mask) << rem;
38      }
39      if (rem != 0) {
40        for (int i = max(0, -s - 1); i < sz(data) - ↩
      max(s + 1, 0); i++) {
41          res.data[i + s + 1] |= (data[i] >> p1) & (pw↩
      (rem) - 1);
42        }
43      }
44      int cc = data.size() * BASE - n;
45      res.data.back() <<= cc;
46      res.data.back() >>= cc;
47      return res;
48    }
49  };
```

9 final/numeric/fft.cpp

```
1   namespace fft
2   {
3     const int maxBase = 21;
4     const int maxN = 1 << maxBase;
5
6     struct num
7     {
8       dbl x, y;
9       num(){}
10      num(dbl xx, dbl yy): x(xx), y(yy) {}
11      num(dbl alp): x(cos(alp)), y(sin(alp)) {}
12    };
13
14    inline num operator + (num a, num b) { return num(↩
    a.x + b.x, a.y + b.y); }
15    inline num operator - (num a, num b) { return num(↩
    a.x - b.x, a.y - b.y); }
16    inline num operator * (num a, num b) { return num(↩
    a.x * b.x - a.y * b.y, a.x * b.y + a.y * b.x); ↩
    }
17    inline num conj(num a) { return num(a.x, -a.y); }
18
19    const dbl PI = acos(-1);
20
21    num root[maxN];
22    int rev[maxN];
23    bool rootsPrepared = false;
24
25    void prepRoots()
26    {
27      if (rootsPrepared) return;
28      rootsPrepared = true;
29      root[1] = num(1, 0);
30      for (int k = 1; k < maxBase; ++k)
31      {
32        num x(2 * PI / pw(k + 1));
33        for (int i = pw(k - 1); i < pw(k); ++i)
34        {
35          root[2 * i] = root[i];
36          root[2 * i + 1] = root[i] * x;
37        }
38      }
39    }
40
41    int base, N;
42
43    int lastRevN = -1;
44    void prepRev()
45    {
46      if (lastRevN == N) return;
47      lastRevN = N;
48      forn(i, N) rev[i] = (rev[i >> 1] >> 1) + ((i & ↩
    1) << (base - 1));
49    }
50
51    void fft(num *a, num *f)
52    {
53      forn(i, N) f[i] = a[rev[i]];
54      for (int k = 1; k < N; k <<= 1) for (int i = 0; ↩
    i < N; i += 2 * k) forn(j, k)
55      {
56        num z = f[i + j + k] * root[j + k];
57        f[i + j + k] = f[i + j] - z;
58        f[i + j] = f[i + j] + z;
59      }
60    }
61
62    num a[maxN], b[maxN], f[maxN], g[maxN];
63    ll A[maxN], B[maxN], C[maxN];
64
65    void _multMod(int mod)
66    {
67      forn(i, N)
68      {
69        int x = A[i] % mod;
70        a[i] = num(x & (pw(15) - 1), x >> 15);
71      }
72      forn(i, N)
73      {
74        int x = B[i] % mod;
75        b[i] = num(x & (pw(15) - 1), x >> 15);
76      }
77      fft(a, f);
78      fft(b, g);
79
80      forn(i, N)
81      {
82        int j = (N - i) & (N - 1);
```

3

```
 83        num a1 = (f[i] + conj(f[j])) * num(0.5, 0);
 84        num a2 = (f[i] - conj(f[j])) * num(0, -0.5);
 85        num b1 = (g[i] + conj(g[j])) * num(0.5 / N, 0);
 86        num b2 = (g[i] - conj(g[j])) * num(0, -0.5 / N);
 87        a[j] = a1 * b1 + a2 * b2 * num(0, 1);
 88        b[j] = a1 * b2 + a2 * b1;
 89      }
 90
 91      fft(a, f);
 92      fft(b, g);
 93
 94      forn(i, N)
 95      {
 96        ll aa = f[i].x + 0.5;
 97        ll bb = g[i].x + 0.5;
 98        ll cc = f[i].y + 0.5;
 99        C[i] = (aa + bb % mod * pw(15) + cc % mod * pw(30)) % mod;
100      }
101    }
102
103    void prepAB(int n1, int n2)
104    {
105      base = 1;
106      N = 2;
107      while (N < n1 + n2) base++, N <<= 1;
108
109      for (int i = n1; i < N; ++i) A[i] = 0;
110      for (int i = n2; i < N; ++i) B[i] = 0;
111
112      prepRoots();
113      prepRev();
114    }
115
116    void mult(int n1, int n2)
117    {
118      prepAB(n1, n2);
119      forn(i, N) a[i] = num(A[i], B[i]);
120      fft(a, f);
121      forn(i, N)
122      {
123        int j = (N - i) & (N - 1);
124        a[i] = (f[j] * f[j] - conj(f[i] * f[i])) * num(0, -0.25 / N);
125      }
126      fft(a, f);
127      forn(i, N) C[i] = (ll)round(f[i].x);
128    }
129
130
131    void multMod(int n1, int n2, int mod)
132    {
133      prepAB(n1, n2);
134      _multMod(mod);
135    }
136
137    int D[maxN];
138
139    void multLL(int n1, int n2)
140    {
141      prepAB(n1, n2);
142
143      int mod1 = 1.5e9;
144      int mod2 = mod1 + 1;
145
146      _multMod(mod1);
147
148      forn(i, N) D[i] = C[i];
149
150      _multMod(mod2);
151
152      forn(i, N)
153      {
154        C[i] = D[i] + (C[i] - D[i] + (ll)mod2) * (ll)mod1 % mod2 * mod1;
155      }
156    }
157    // HOW TO USE ::
158    // -- set correct maxBase
159    // -- use mult(n1, n2), multMod(n1, n2, mod) and multLL(n1, n2)
160    // -- input : A[], B[]
161    // -- output : C[]
162 }
```

10   final/numeric/fftint.cpp

```
 1  namespace fft {
 2    const int MOD = 998244353;
 3    const int maxB = 20;
 4    const int maxN = 1 << maxB;
 5    const int initROOT = 646;
 6
 7    int root[maxN];
 8    int rev[maxN];
 9    int N;
10
11    ll inv(ll a, ll m = MOD) {
12      if (a == 0) return 0;
13      return ((1 - inv(m % a, a) * m) / a + m) % m;
14    }
15
16    void init(int cur_base) {
17      N = 1 << cur_base;
18      for (int i = 0; i < N; i++) rev[i] = (rev[i >> 1] >> 1) + ((i & 1) << (cur_base - 1));
19
20
21      int ROOT = initROOT;
22      for (int i = cur_base; i < 20; i++) ROOT = mul(ROOT, ROOT);
23
24      int NN = N >> 1;
25      int z = 1;
26      for (int i = 0; i < NN; i++) {
27        root[i + NN] = z;
28        z = z * (ll)ROOT % MOD;
29      }
30      for (int i = NN - 1; i > 0; --i) root[i] = root[2 * i];
31    }
32
33    void fft(int *a, int *f) {
34      for (int i = 0; i < N; i++) f[i] = a[rev[i]];
35      for (int k = 1; k < N; k <<= 1) {
36        for (int i = 0; i < N; i += 2 * k) {
37          for (int j = 0; j < k; j++) {
38            int z = f[i + j + k] * (ll)root[j + k] % MOD;
39            f[i + j + k] = (f[i + j] - z + MOD) % MOD;
40            f[i + j] = (f[i + j] + z) % MOD;
41          }
42        }
43      }
44    }
45
46    int A[maxN], B[maxN], C[maxN];
47    int F[maxN], G[maxN];
48
49    void _mult(int eq) {
50      fft(A, F);
51      if (eq)
52        for (int i = 0; i < N; i++)
53          G[i] = F[i];
54      else fft(B, G);
55      int invN = inv(N);
56      for (int i = 0; i < N; i++) A[i] = F[i] * (ll)G[i] % MOD * invN % MOD;
57      reverse(A + 1, A + N);
58      fft(A, C);
59    }
60
61    void mult(int n1, int n2, int eq = 0) {
62      int n = n1 + n2, cur_base = 0;
63      while ((1 << cur_base) < n) cur_base++;
64      init(cur_base + 1);
65
66      for (int i = n1; i < N; ++i) A[i] = 0;
67      for (int i = n2; i < N; ++i) B[i] = 0;
68
69      _mult(eq);
70
71      //forn(i, n1 + n2) C[i] = 0;
72      //forn(i, n1) forn(j, n2) C[i + j] = (C[i + j] + A[i] * (ll)B[j]) % mod;
73    }
74 }
```

11   final/numeric/berlekamp.cpp

```
 1  vector<int> berlekamp(vector<int> s) {
 2    int l = 0;
 3    vector<int> la(1, 1);
 4    vector<int> b(1, 1);
```

```cpp
5      for (int r = 1; r <= (int)s.size(); r++) {
6        int delta = 0;
7        for (int j = 0; j <= l; j++) {
8          delta = (delta + 1LL * s[r - 1 - j] * la[j]) %↩
    MOD;
9        }
10       b.insert(b.begin(), 0);
11       if (delta != 0) {
12         vector<int> t(max(la.size(), b.size()));
13         for (int i = 0; i < (int)t.size(); i++) {
14           if (i < (int)la.size()) t[i] = (t[i] + la[i↩
    ]) % MOD;
15           if (i < (int)b.size()) t[i] = (t[i] - 1LL * ↩
    delta * b[i] % MOD + MOD) % MOD;
16         }
17         if (2 * l <= r - 1) {
18           b = la;
19           int od = inv(delta);
20           for (int &x : b) x = 1LL * x * od % MOD;
21           l = r - l;
22         }
23         la = t;
24       }
25     }
26     assert((int)la.size() == l + 1);
27     assert(l * 2 + 30 < (int)s.size());
28     reverse(la.begin(), la.end());
29     return la;
30 }
31
32 vector<int> mul(vector<int> a, vector<int> b) {
33     vector<int> c(a.size() + b.size() - 1);
34     for (int i = 0; i < (int)a.size(); i++) {
35         for (int j = 0; j < (int)b.size(); j++) {
36             c[i + j] = (c[i + j] + 1LL * a[i] * b[j]) % ↩
    MOD;
37         }
38     }
39     vector<int> res(c.size());
40     for (int i = 0; i < (int)res.size(); i++) res[i] =↩
    c[i] % MOD;
41     return res;
42 }
43
44 vector<int> mod(vector<int> a, vector<int> b) {
45     if (a.size() < b.size()) a.resize(b.size() - 1);
46
47     int o = inv(b.back());
48     for (int i = (int)a.size() - 1; i >= (int)b.size()↩
    - 1; i--) {
49         if (a[i] == 0) continue;
50         int coef = 1LL * o * (MOD - a[i]) % MOD;
51         for (int j = 0; j < (int)b.size(); j++) {
52             a[i - (int)b.size() + 1 + j] = (a[i - (int)b.↩
    size() + 1 + j] + 1LL * coef * b[j]) % MOD;
53         }
54     }
55     while (a.size() >= b.size()) {
56         assert(a.back() == 0);
57         a.pop_back();
58     }
59     return a;
60 }
61
62 vector<int> bin(int n, vector<int> p) {
63     vector<int> res(1, 1);
64     vector<int> a(2); a[1] = 1;
65     while (n) {
66         if (n & 1) res = mod(mul(res, a), p);
67         a = mod(mul(a, a), p);
68         n >>= 1;
69     }
70     return res;
71 }
72
73 int f(vector<int> t, int m) {
74     vector<int> v = berlekamp(t);
75     vector<int> o = bin(m - 1, v);
76     int res = 0;
77     for (int i = 0; i < (int)o.size(); i++) res = (res↩
    + 1LL * o[i] * t[i]) % MOD;
78     return res;
79 }
```

12   final/numeric/blackbox.cpp

```cpp
1 namespace blackbox
```

```cpp
2  {
3      int A[N];
4      int B[N];
5      int C[N];
6
7      int magic(int k, int x)
8      {
9          B[k] = x;
10         C[k] = (C[k] + A[0] * (ll)B[k]) % mod;
11         int z = 1;
12         if (k == N - 1) return C[k];
13         while ((k & (z - 1)) == (z - 1))
14         {
15             //mult B[k - z + 1 ... k] x A[z .. 2 * z - 1]
16             forn(i, z) fft::A[i] = A[z + i];
17             forn(i, z) fft::B[i] = B[k - z + 1 + i];
18             fft::multMod(z, z, mod);
19             forn(i, 2 * z - 1) C[k + 1 + i] = (C[k + 1 + i↩
    ] + fft::C[i]) % mod;
20             z <<= 1;
21         }
22         return C[k];
23     }
24     // A — constant array
25     // magic(k, x):: B[k] = x, returns C[k]
26     // !! WARNING !! better to set N twice the size ↩
    needed
27 }
```

13   final/numeric/crt.cpp

```cpp
1 int CRT(int a1, int m1, int a2, int m2) {
2     return (a1 - a2 % m1 + m1) * (ll)rev(m2, m1) % m1 ↩
    * m2 + a2;
3 }
```

14   final/numeric/extendedgcd.cpp

```cpp
1 int gcd(int a, int b, int &x, int &y) {
2     if (a == 0) {
3         x = 0, y = 1;
4         return b;
5     }
6     int x1, y1;
7     int d = gcd(b % a, a, x1, y1);
8     x = y1 - (b / a) * x1;
9     y = x1;
10    return d;
11 }
```

15   final/numeric/mulMod.cpp

```cpp
1 ll mul( ll a, ll b, ll m ) { // works for MOD 8e18
2     ll k = (ll)((long double)a * b / m);
3     ll r = a * b - m * k;
4     if (r < 0) r += m;
5     if (r >= m) r -= m;
6     return r;
7 }
```

16   final/numeric/modReverse.cpp

```cpp
1 int rev(int x, int m) {
2     if (x == 1) return 1;
3     return (1 - rev(m % x, x) * (ll)m) / x + m;
4 }
```

17   final/numeric/pollard.cpp

```cpp
namespace pollard
{
  using math::p;

  vector<pair<ll, int>> getFactors(ll N)  {
    vector<ll> primes;

    const int MX = 1e5;
    const ll MX2 = MX * (ll)MX;

    assert(MX <= math::maxP && math::pc > 0);

    function<void(ll)> go = [&go, &primes](ll n) {
      for (ll x : primes) while (n % x == 0) n /= x;
      if (n == 1) return;
      if (n > MX2) {
        auto F = [&](ll x) {
          ll k = ((long double)x * x) / n;
          ll r = (x * x - k * n + 3) % n;
          return r < 0 ? r + n : r;
        };
        ll x = mt19937_64()() % n, y = x;
        const int C = 3 * pow(n, 0.25);

        ll val = 1;
        forn(it, C) {
          x = F(x), y = F(F(y));
          if (x == y) continue;
          ll delta = abs(x - y);
          ll k = ((long double)val * delta) / n;
          val = (val * delta - k * n) % n;
          if (val < 0) val += n;
          if (val == 0)   {
            ll g = __gcd(delta, n);
            go(g), go(n / g);
            return;
          }
          if ((it & 255) == 0) {
            ll g = __gcd(val, n);
            if (g != 1) {
              go(g), go(n / g);
              return;
            }
          }
        }
      }
      primes.pb(n);
    };

    ll n = N;

    for (int i = 0; i < math::pc && p[i] < MX; ++i) ↵
    if (n % p[i] == 0) {
      primes.pb(p[i]);
      while (n % p[i] == 0) n /= p[i];
    }
    go(n);
    sort(primes.begin(), primes.end());

    vector<pair<ll, int>> res;
    for (ll x : primes) {
      int cnt = 0;
      while (N % x == 0) {
        cnt++;
        N /= x;
      }
      res.push_back({x, cnt});
    }
    return res;
  }
}
```

18   final/numeric/poly.cpp

```cpp
struct poly
{
  vi v;
  poly() {}
  poly(vi vv)
  {
    v = vv;
  }
  int size()
  {
    return (int)v.size();
  }
  poly cut(int maxLen)
  {
    if (maxLen < sz(v)) v.resize(maxLen);
    return *this;
  }
  poly norm()
  {
    while (sz(v) > 1 && v.back() == 0) v.pop_back();
    return *this;
  }
  inline int& operator [] (int i)
  {
    return v[i];
  }
  void out(string name="")
  {
    stringstream ss;
    if (sz(name)) ss << name << "=";
    int fst = 1;
    forn(i, sz(v)) if (v[i])
    {
      int x = v[i];
      int sgn = 1;
      if (x > mod / 2) x = mod-x, sgn = -1;
      if (sgn == -1) ss << "-";
      else if (!fst) ss << "+";
      fst = 0;
      if (!i || x != 1)
      {
        ss << x;
        if (i > 0) ss << "*x";
        if (i > 1) ss << "^" << i;
      }
      else
      {
        ss << "x";
        if (i > 1) ss << "^" << i;
      }
    }
    if (fst) ss <<"0";
    string s;
    ss >> s;
    eprintf("%s\n", s.data());
  }
};

poly operator + (poly A, poly B)
{
  poly C;
  C.v = vi(max(sz(A), sz(B)));
  forn(i, sz(C))
  {
    if (i < sz(A)) C[i] = (C[i] + A[i]) % mod;
    if (i < sz(B)) C[i] = (C[i] + B[i]) % mod;
  }
  return C.norm();
}

poly operator - (poly A, poly B)
{
  poly C;
  C.v = vi(max(sz(A), sz(B)));
  forn(i, sz(C))
  {
    if (i < sz(A)) C[i] = (C[i] + A[i]) % mod;
    if (i < sz(B)) C[i] = (C[i] + mod - B[i]) % mod;
  }
  return C.norm();
}

poly operator * (poly A, poly B)
{
  poly C;
  C.v = vi(sz(A) + sz(B) - 1);

  forn(i, sz(A)) fft::A[i] = A[i];
  forn(i, sz(B)) fft::B[i] = B[i];
  fft::multMod(sz(A), sz(B), mod);
  forn(i, sz(C)) C[i] = fft::C[i];
  return C.norm();
}

poly inv(poly A, int n) // returns A^-1 mod x^n
{
  assert(sz(A) && A[0] != 0);
  A.cut(n);

  auto cutPoly = [](poly &from, int l, int r)
  {
```

```
102      poly R;
103      R.v.resize(r - 1);
104      for (int i = l; i < r; ++i)
105      {
106         if (i < sz(from)) R[i - 1] = from[i];
107      }
108      return R;
109    };
110
111    function<int(int, int)> rev = [&rev](int x, int m)↩
         ->int
112    {
113       if (x == 1) return 1;
114       return (1 - rev(m % x, x) * (ll)m) / x + m;
115    };
116
117    poly R({rev(A[0], mod)});
118    for (int k = 1; k < n; k <<= 1)
119    {
120       poly A0 = cutPoly(A, 0, k);
121       poly A1 = cutPoly(A, k, 2 * k);
122       poly H = A0 * R;
123       H = cutPoly(H, k, 2 * k);
124       poly R1 = (((A1 * R).cut(k) + H) * (poly({0}) - ↩
         R)).cut(k);
125       R.v.resize(2 * k);
126       forn(i, k) R[i + k] = R1[i];
127    }
128    return R.cut(n).norm();
129 }
130
131 pair<poly, poly> divide(poly A, poly B)
132 {
133    if (sz(A) < sz(B)) return {poly({0}), A};
134
135    auto rev = [](poly f)
136    {
137       reverse(all(f.v));
138       return f;
139    };
140
141    poly q = rev((inv(rev(B), sz(A) - sz(B) + 1) * rev↩
         (A)).cut(sz(A) - sz(B) + 1));
142    poly r = A - B * q;
143
144    return {q, r};
145 }
```

```
27     }
28
29    void pivot(int r, int s) {
30       T *a = D[r].data(), inv = 1 / a[s];
31       rep(i,0,m+2) if (i != r && abs(D[i][s]) > eps) {
32          T *b = D[i].data(), inv2 = b[s] * inv;
33          rep(j,0,n+2) b[j] -= a[j] * inv2;
34          b[s] = a[s] * inv2;
35       }
36       rep(j,0,n+2) if (j != s) D[r][j] *= inv;
37       rep(i,0,m+2) if (i != r) D[i][s] *= -inv;
38       D[r][s] = inv;
39       swap(B[r], N[s]);
40    }
41
42    bool simplex(int phase) {
43       int x = m + phase - 1;
44       for (int it = 0; it < 100; it++) {
45          int s = -1;
46          rep(j,0,n+1) if (N[j] != -phase) ltj(D[x]);
47          if (D[x][s] >= -eps) return true;
48          int r = -1;
49          rep(i,0,m) {
50             if (D[i][s] <= eps) continue;
51             if (r == -1 || MP(D[i][n+1] / D[i][s], B[i])
52                 < MP(D[r][n+1] / D[r][s], B[r])) r =↩
         i;
53          }
54          if (r == -1) return false;
55          pivot(r, s);
56       }
57    }
58
59    T solve(vd &x) {
60       int r = 0;
61       rep(i,1,m) if (D[i][n+1] < D[r][n+1]) r = i;
62       if (D[r][n+1] < -eps) {
63          pivot(r, n);
64          if (!simplex(2) || D[m+1][n+1] < -eps) return ↩
         -inf;
65          rep(i,0,m) if (B[i] == -1) {
66             int s = 0;
67             rep(j,1,n+1) ltj(D[i]);
68             pivot(i, s);
69          }
70       }
71       bool ok = simplex(1); x = vd(n);
72       rep(i,0,m) if (B[i] < n) x[B[i]] = D[i][n+1];
73       return ok ? D[m][n+1] : inf;
74    }
75 };
```

19　final/numeric/simplex.cpp

```
1  typedef double T; // long double, Rational, double +↩
         mod<P>...
2  typedef vector<T> vd;
3  typedef vector<vd> vvd;
4
5  const T eps = 1e-8, inf = 1/.0;
6  #define MP make_pair
7  #define ltj(X) if(s == -1 || MP(X[j],N[j]) < MP(X[s↩
         ],N[s])) s=j
8  #define sz(X) ((X).size())
9  #define rep(i,l,r) for (int i = (l); i < (r); i++)
10
11 struct LPSolver {
12    // Description: Solves a general linear ↩
         maximization problem: maximize $c^T x$ subject ↩
         to $Ax \le b$, $x \ge 0$.
13    // A is a matrix with shape (number of ↩
         inequalities, number of variables)
14    // Returns -inf if there is no solution, inf if ↩
         there are arbitrarily good solutions, or the ↩
         maximum value of $c^T x$ otherwise.
15    // The input vector is set to an optimal $x$ (or ↩
         in the unbounded case, an arbitrary solution ↩
         fulfilling the constraints).
16
17    int m, n;
18    vector<int> N, B;
19    vvd D;
20
21    LPSolver(const vvd& A, const vd& b, const vd& c) :
22       m(sz(b)), n(sz(c)), N(n+1), B(m), D(m+2, vd(n↩
         +2)) {
23       rep(i,0,m) rep(j,0,n) D[i][j] = A[i][j];
24       rep(i,0,m) { B[i] = n+i; D[i][n] = -1; D[i][n+1]↩
         = b[i];}
25       rep(j,0,n) { N[j] = j; D[m][j] = -c[j]; }
26       N[n] = -1; D[m+1][n] = 1;
```

20　final/numeric/sumLine.cpp

```
1  // sum(i=0..n-1) (a+b*i) div m
2  ll solve(ll n, ll a, ll b, ll m) {
3     if (b == 0) return n * (a / m);
4     if (a >= m) return n * (a / m) + solve(n, a % m, b↩
         , m);
5     if (b >= m) return n * (n - 1) / 2 * (b / m) + ↩
         solve(n, a, b % m, m);
6     return solve((a + b * n) / m, (a + b * n) % m, m, ↩
         b);
7  }
```

21　final/numeric/integrate.cpp

```
1  function<dbl(dbl, dbl, function<dbl(dbl)>)> f = [&](↩
         dbl L, dbl R, function<dbl(dbl)> g) {
2     const int ITERS = 1000000;
3     dbl ans = 0;
4     dbl step = (R - L) * 1.0 / ITERS;
5     for (int it = 0; it < ITERS; it++) {
6        double xl = L + step * it;
7        double xr = L + step * (it + 1);
8        dbl x1 = (xl + xr) / 2;
9        dbl x0 = x1 - (x1 - xl) * sqrt(3.0 / 5);
10       dbl x2 = x1 + (x1 - xl) * sqrt(3.0 / 5);
11       ans += (5 * g(x0) + 8 * g(x1) + 5 * g(x2)) / 18 ↩
         * step;
12    }
```

```
13    return ans;
14  };
```

22 final/geom/commonTangents.cpp

```
1
2
3  vector<Line> commonTangents(pt A, dbl rA, pt B, dbl ↩
       rB) {
4    vector<Line> res;
5    pt C = B - A;
6    dbl z = C.len2();
7    for (int i = -1; i <= 1; i += 2) {
8      for (int j = -1; j <= 1; j += 2) {
9        dbl r = rB * j - rA * i;
10       dbl d = z - r * r;
11       if (ls(d, 0)) continue;
12       d = sqrt(max(0.01, d));
13       pt magic = pt(r, d) / z;
14       pt v(magic % C, magic * C);
15       dbl CC = (rA * i - v % A) / v.len2();
16       pt O = v * -CC;
17       res.pb(Line(O, O + v.rotate()));
18     }
19   }
20   return res;
21  }
22
23  // HOW TO USE ::
24  // --    *D*---------------*F*
25  // --    *...* -      -*...*
26  // --   *.....* -    - *.....*
27  // --   *.......* -  - *.......*
28  // --  *...A...*  --  *...B...*
29  // --   *.......* -  - *.......*
30  // --   *.....* -    - *.....*
31  // --    *...* -      -*...*
32  // --    *C*---------------*E*
33  // -- res = {CE, CF, DE, DF}
```

23 final/geom/halfplaneIntersection.cpp

```
1  int getPart(pt v) {
2    return ls(v.y, 0) || (eq(0, v.y) && ls(v.x, 0));
3  }
4
5  int cmpV(pt a, pt b) {
6    int partA = getPart(a);
7    int partB = getPart(b);
8    if (partA < partB) return 1;
9    if (partA > partB) return -1;
10   if (eq(0, a * b)) return 0;
11   if (0 < a * b) return -1;
12   return 1;
13  }
14
15  double planeInt(vector<Line> l) {
16    sort(all(l), [](Line a, Line b) {
17        int r = cmpV(a.v, b.v);
18        if (r != 0) return r < 0;
19        return a.O % a.v.rotate() > b.O % a.v.rotate()↩
       ;
20      });
21
22    l.resize(unique(all(l), [](Line A, Line B) { ↩
       return cmpV(A.v, B.v) == 0; }) - l.begin());
23    for (int i = 0; i < sz(l); i++)
24      l[i].id = i;
25
26    // if an infinite answer is possible
27    int flagUp = 0;
28    int flagDown = 0;
29    for (int i = 0; i < sz(l); i++) {
30      int part = getPart(l[i].v);
31      if (part == 1) flagUp = 1;
32      if (part == 0) flagDown = 1;
33    }
34    if (!flagUp || !flagDown) return -1;
35
36    for (int i = 0; i < sz(l); i++) {
37      pt v = l[i].v;
38      pt u = l[(i + 1) % sz(l)].v;
39      if (eq(0, v * u) && ls(v % u, 0)) {
40        pt dir = l[i].v.rotate();
41        if (le(l[(i + 1) % sz(l)].O % dir, l[i].O % ↩
       dir)) return 0;
42        return -1;
```

8

```
43        }
44        if (ls(v * u, 0))
45          return -1;
46      }
47      // main part
48      vector<Line> st;
49      for (int tt = 0; tt < 2; tt++) {
50        for (auto L: l) {
51          for (; sz(st) >= 2 && le(st[sz(st) - 2].v * (←
          st.back() * L - st[sz(st) - 2].O), 0); st.←
          pop_back());
52          st.pb(L);
53          if (sz(st) >= 2 && le(st[sz(st) - 2].v * st.←
          back().v, 0)) return 0; // useless line
54        }
55      }
56      vector<int> use(sz(l), -1);
57      int left = -1, right = -1;
58      for (int i = 0; i < sz(st); i++) {
59        if (use[st[i].id] == -1) {
60          use[st[i].id] = i;
61        }
62        else {
63          left = use[st[i].id];
64          right = i;
65          break;
66        }
67      }
68      vector<Line> tmp;
69      for (int i = left; i < right; i++)
70        tmp.pb(st[i]);
71      vector<pt> res;
72      for (int i = 0; i < (int)tmp.size(); i++)
73        res.pb(tmp[i] * tmp[(i + 1) % tmp.size()]);
74      double area = 0;
75      for (int i = 0; i < (int)res.size(); i++)
76        area += res[i] * res[(i + 1) % res.size()];
77      return area / 2;
78    }
```

24 final/geom/minDisc.cpp

```
1
2   pair<pt, dbl> minDisc(vector<pt> p) {
3     int n = p.size();
4     pt O = pt(0, 0);
5     dbl R = 0;
6     random_shuffle(all(p));
7     for (int i = 0; i < n; i++) {
8       if (ls(R, (O - p[i]).len())) {
9         O = p[i];
10        R = 0;
11        for (int j = 0; j < i; j++) {
12          if (ls(R, (O - p[j]).len())) {
13            O = (p[i] + p[j]) / 2;
14            R = (p[i] - p[j]).len() / 2;
15            for (int k = 0; k < j; k++) {
16              if (ls(R, (O - p[k]).len())) {
17                Line l1((p[i] + p[j]) / 2, (p[i] + p[j←
              ]) / 2 + (p[i] - p[j]).rotate());
18                Line l2((p[k] + p[j]) / 2, (p[k] + p[j←
              ]) / 2 + (p[k] - p[j]).rotate());
19                O = l1 * l2;
20                R = (p[i] - O).len();
21              }
22            }
23          }
24        }
25      }
26    }
27    return {O, R};
28  }
```

25 final/geom/convexHull3D-N2.cpp

```
1
2   struct Plane {
3     pt O, v;
4     vector<int> id;
```

```
5   };
6
7   vector<Plane> convexHull3(vector<pt> p) {
8     vector<Plane> res;
9     int n = p.size();
10    for (int i = 0; i < n; i++)
11      p[i].id = i;
12    for (int i = 0; i < 4; i++) {
13      vector<pt> tmp;
14      for (int j = 0; j < 4; j++)
15        if (i != j)
16          tmp.pb(p[j]);
17      res.pb({tmp[0], (tmp[1] - tmp[0]) * (tmp[2] - ←
      tmp[0]), {tmp[0].id, tmp[1].id, tmp[2].id}});
18      if ((p[i] - res.back().O) % res.back().v > 0) {
19        res.back().v = res.back().v * -1;
20        swap(res.back().id[0], res.back().id[1]);
21      }
22    }
23    vector<vector<int>> use(n, vector<int>(n, 0));
24    int tmr = 0;
25    for (int i = 4; i < n; i++) {
26      int cur = 0;
27      tmr++;
28      vector<pair<int,int>> curEdge;
29      for (int j = 0; j < sz(res); j++) {
30        if ((p[i] - res[j].O) % res[j].v > 0) {
31          for (int t = 0; t < 3; t++) {
32            int v = res[j].id[t];
33            int u = res[j].id[(t + 1) % 3];
34            use[v][u] = tmr;
35            curEdge.pb({v, u});
36          }
37        }
38        else {
39          res[cur++] = res[j];
40        }
41      }
42      res.resize(cur);
43      for (auto x: curEdge) {
44        if (use[x.S][x.F] == tmr) continue;
45        res.pb({p[i], (p[x.F] - p[i]) * (p[x.S] - p[i←
        ]), {x.F, x.S, i}});
46      }
47    }
48    return res;
49  }
50
51  // plane in 3d
52  //(A, v) * (B, u) -> (O, n)
53
54  pt n = v * u;
55  pt m = v * n;
56  double t = (B - A) % u / (u % m);
57  pt O = A - m * t;
```

26 final/geom/convexDynamic.cpp

```
1   struct convex {
2     map<ll, ll> M;
3     bool get(int x, int y) {
4       if (M.size() == 0)
5         return false;
6       if (M.count(x))
7         return M[x] >= y;
8       if (x < M.begin()->first || x > M.rbegin()->←
      first)
9         return false;
10
11      auto it1 = M.lower_bound(x), it2 = it1;
12      it1--;
13
14      return pt(pt(*it1), pt(x, y)) % pt(pt(*it1), pt←
      (*it2)) >= 0;
15    }
16    void add(int x, int y) {
17      if (get(x, y)) return;
18
19      pt P(x, y);
20      M[x] = y;
21
22      auto it = M.lower_bound(x), it1 = it;
23      it1--;
24      auto it2 = it1;
25      it2--;
26
27      if (it != M.begin() && it1 != M.begin()) {
```

```
28          while (it1 != M.begin() && (pt(pt(*it2), pt(*↵
         it1)) % pt(pt(*it1), P)) >= 0) {
29          M.erase(it1);
30          it1 = it2;
31          it2--;
32        }
33      }
34      it1 = it, it1++;
35      if (it1 == M.end()) return;
36      it2 = it1, it2++;
37
38      if (it1 != M.end() && it2 != M.end()) {
39        while (it2 != M.end() && (pt(P, pt(*it1)) % pt↵
         (pt(*it1), pt(*it2))) >= 0) {
40          M.erase(it1);
41          it1 = it2;
42          it2++;
43        }
44      }
45    }
46  } H, J;
47
48  int solve() {
49    int q;
50    cin >> q;
51    while (q--) {
52      int t, x, y;
53      cin >> t >> x >> y;
54      if (t == 1) {
55        H.add(x, y);
56        J.add(x, -y);
57      }
58      else {
59        if (H.get(x, y) && J.get(x, -y))
60          puts("YES");
61        else
62          puts("NO");
63      }
64    }
65    return 0;
66  }
```

27   final/geom/polygonArcCut.cpp

```
1
2  struct Meta {
3    int type; // 0 - seg, 1 - circle
4    pt O;
5    dbl R;
6  };
7
8  const Meta SEG = {0, pt(0, 0), 0};
9
10 vector<pair<pt, Meta>> cut(vector<pair<pt, Meta>> p,↵
        Line l) {
11   vector<pair<pt,Meta>> res;
12   int n = p.size();
13   for (int i = 0; i < n; i++) {
14     pt A = p[i].F;
15     pt B = p[(i + 1) % n].F;
16     if (le(0, l.v * (A - l.O))) {
17       if (eq(0, l.v * (A - l.O)) && p[i].S.type == 1↵
        && ls(0, l.v % (p[i].S.O - A)))
18         res.pb({A, SEG});
19       else
20         res.pb(p[i]);
21     }
22     if (p[i].S.type == 0) {
23       if (sign(l.v * (A - l.O)) * sign(l.v * (B - l.↵
        O)) == -1) {
24         pt FF = Line(A, B) * l;
25         res.pb(make_pair(FF, SEG));
26       }
27     }
28     else {
29       pt E, F;
30       if (intCL(p[i].S.O, p[i].S.R, l, E, F)) {
31         if (onArc(p[i].S.O, A, E, B))
32           res.pb({E, SEG});
33         if (onArc(p[i].S.O, A, F, B))
34           res.pb({F, p[i].S});
35       }
36     }
37   }
38   return res;
39 }
```

28   final/geom/polygonTangent.cpp

```
1  pt tangent(vector<pt>& p, pt O, int cof) {
2    int step = 1;
3    for (; step < (int)p.size(); step *= 2);
4    int pos = 0;
5    int n = p.size();
6    for (; step > 0; step /= 2) {
7      int best = pos;
8      for (int dx = -1; dx <= 1; dx += 2) {
9        int id = ((pos + step * dx) % n + n) % n;
10       if ((p[id] - O) * (p[best] - O) * cof > 0)
11         best = id;
12     }
13     pos = best;
14   }
15   return p[pos];
16 }
```

29   final/geom/checkPlaneInt.cpp

```
1
2  bool checkPoint(vector<Line> l, pt& ret) {
3    random_shuffle(all(l));
4    pt A = l[0].O;
5    for (int i = 1; i < sz(l); i++) {
6      if (!le(0, l[i].v * (A - l[i].O))) {
7        dbl mn = -INF;
8        dbl mx = INF;
9        for (int j = 0; j < i; j++) {
10         if (eq(l[j].v * l[i].v, 0)) {
11           if (l[j].v % l[i].v < 0 && (l[j].O - l[i].↵
        O) % l[i].v.rotate() <= 0) {
12             return false;
13           }
14         }
15         else {
16           pt u = l[j].v.rotate();
17           dbl proj = (l[j].O - l[i].O) % u / (l[i].v↵
        % u);
18           if (l[i].v * l[j].v > 0) {
19             mx = min(mx, proj);
20           }
21           else {
22             mn = max(mn, proj);
23           }
24         }
25       }
26       if (mn <= mx) {
27         A = l[i].O + l[i].v * mn;
28       }
29       else {
30         return false;
31       }
32     }
33   }
34   ret = A;
35   return true;
36 }
```

30   final/geom/furthestPoints.cpp

```
1  ll furthestPoints(vector<pt> p) {
2    int n = p.size();
3    int cur = 1;
4    ll answer = 0;
5    for (int i = 0; i < n; i++) {
6      for (; (p[(i + 1) % n] - p[i]) * (p[(cur + 1) % ↵
        n] - p[cur]) > 0; cur = (cur + 1) % n);
7      answer = max(answer, (p[i] - p[cur]).len2());
8    }
9    return answer;
10 }
```

31   final/geom/chtDynamic.cpp

```cpp
const ll is_query = -(1LL << 62);

struct Line {
  ll m, b;
  mutable function<const Line *()> succ;

  bool operator<(const Line &rhs) const {
    if (rhs.b != is_query) return m < rhs.m;
    const Line *s = succ();
    if (!s) return 0;
    ll x = rhs.m;
    return b - s->b < (s->m - m) * x;
  }
};

struct HullDynamic : public multiset<Line> {
  bool bad(iterator y) {
    auto z = next(y);
    if (y == begin()) {
      if (z == end()) return 0;
      return y->m == z->m && y->b <= z->b;
    }
    auto x = prev(y);
    if (z == end()) return y->m == x->m && y->b <= x->b;
    return (x->b - y->b) * (z->m - y->m) >= (y->b - z->b) * (y->m - x->m);
  }

  void insert_line(ll m, ll b) {
    auto y = insert({m, b});
    y->succ = [=] { return next(y) == end() ? 0 : &*next(y); };
    if (bad(y)) {
      erase(y);
      return;
    }
    while (next(y) != end() && bad(next(y))) erase(next(y));
    while (y != begin() && bad(prev(y))) erase(prev(y));
  }

  ll eval(ll x) {
    auto l = *lower_bound((Line) {x, is_query});
    return l.m * x + l.b;
  }
};
```

32   final/strings/eertree.cpp

```cpp
namespace eertree {
  const int INF = 1e9;
  const int N = 5e6 + 10;
  char _s[N];
  char *s = _s + 1;
  int to[N][2];
  int suf[N], len[N];
  int sz, last;

  const int odd = 1, even = 2, blank = 3;

  void go(int &u, int pos) {
    while (u != blank && s[pos - len[u] - 1] != s[pos]) {
      u = suf[u];
    }
  }

  int add(int pos) {
    go(last, pos);
    int u = suf[last];
    go(u, pos);
    int c = s[pos] - 'a';
    int res = 0;
    if (!to[last][c]) {
      res = 1;
      to[last][c] = sz;
      len[sz] = len[last] + 2;
      suf[sz] = to[u][c];
      sz++;
    }
    last = to[last][c];
    return res;
  }

  void init() {
    to[blank][0] = to[blank][1] = even;
    len[blank] = suf[blank] = INF;
    len[even] = 0, suf[even] = odd;
    len[odd] = -1, suf[odd] = blank;
    last = even;
    sz = 4;
  }
}
```

33   final/strings/manacher.cpp

```cpp
vector<int> Pal1(string s) {
  int n = (int)s.size();
  vector<int> d1(n);
  int l = 0, r = -1;
  for (int i = 0, k; i < n; i++) {
    if (i > r) k = 1;
    else k = min(d1[l + r - i], r - i);
    while (0 <= i - k && i + k < n && s[i - k] == s[i + k]) k++;
    d1[i] = k;
    if (i + k - 1 > r) r = i + k - 1, l = i - k + 1;
  }
  return d1;
}

vector<int> Pal2(string s) {
  int n = (int)s.size();
  vector<int> d2(n);
  int l = 0, r = -1;
  for (int i = 0, k; i < n; i++) {
    if (i > r) k = 0;
    else k = min(d2[l + r - i + 1], r - i + 1);
    while (i + k < n && i - k - 1 >= 0 && s[i + k] == s[i - k - 1]) k++;
    d2[i] = k;
    if (i + k - 1 > r) l = i - k, r = i + k - 1;
  }
  return d2;
}
```

34 final/strings/sufAutomaton.cpp

```
namespace SA {
  const int MAXN = 1 << 18;
  const int SIGMA = 26;

  int sz, last;
  int nxt[MAXN][SIGMA];
  int link[MAXN], len[MAXN], pos[MAXN];

  void init() {
    memset(nxt, -1, sizeof(nxt));
    memset(link, -1, sizeof(link));
    memset(len, 0, sizeof(len));
    last = 0;
    sz = 1;
  }

  void add(int c) {
    int cur = sz++;
    len[cur] = len[last] + 1;
    pos[cur] = len[cur];
    int p = last;
    last = cur;
    for (; p != -1 && nxt[p][c] == -1; p = link[p])
    nxt[p][c] = cur;
    if (p == -1) {
      link[cur] = 0;
      return;
    }
    int q = nxt[p][c];
    if (len[p] + 1 == len[q]) {
      link[cur] = q;
      return;
    }
    int clone = sz++;
    memcpy(nxt[clone], nxt[q], sizeof(nxt[q]));
    len[clone] = len[p] + 1;
    pos[clone] = pos[q];
    link[clone] = link[q];
    link[q] = link[cur] = clone;
    for (; p != -1 && nxt[p][c] == q; p = link[p])
    nxt[p][c] = clone;
  }

  int n;
  string s;
  int l[MAXN], r[MAXN];
  int e[MAXN][SIGMA];

  void getSufTree(string _s) {
    memset(e, -1, sizeof(e));
    s = _s;
    n = s.length();
    reverse(s.begin(), s.end());
    init();
    for (int i = 0; i < n; i++) add(s[i] - 'a');
    reverse(s.begin(), s.end());
    for (int i = 1; i < sz; i++) {
      int j = link[i];
      l[i] = n - pos[i] + len[j];
      r[i] = n - pos[i] + len[i];
      e[j][s[l[i]] - 'a'] = i;
    }
  }
}
```

35 final/strings/sufArray.cpp

```
int n;
char s[N];
int p[N], pn[N], c[N], cn[N], cnt[N];
int o[N];
int lcp[N];

void build() {
  for (int i = 0; i < 256; i++) cnt[i] = 0;
  for (int i = 0; i < n; i++) cnt[(int)s[i]]++;
  for (int i = 1; i < 256; i++) cnt[i] += cnt[i -
      1];
  for (int i = n - 1; i >= 0; i--) p[--cnt[(int)s[i
      ]]] = i;
  int cl = 1;
  c[p[0]] = 0;
```

```
  for (int i = 1; i < n; i++) {
    cl += s[p[i]] != s[p[i - 1]];
    c[p[i]] = cl - 1;
  }

  for (int len = 1; len < n; len <<= 1) {
    for (int i = 0; i < cl; i++) cnt[i] = 0;
    for (int i = 0; i < n; i++) cnt[c[i]]++;
    for (int i = 1; i < cl; i++) cnt[i] += cnt[i -
        1];
    for (int i = 0; i < n; i++) pn[i] = (p[i] - len
        + n) % n;
    for (int i = n - 1; i >= 0; i--) p[--cnt[c[pn[i
        ]]]] = pn[i];
    cl = 1;
    cn[p[0]] = 0;
    for (int i = 1; i < n; i++) {
      cl += c[p[i]] != c[p[i - 1]] || c[(p[i] + len)
      % n] != c[(p[i - 1] + len) % n];
      cn[p[i]] = cl - 1;
    }
    for (int i = 0; i < n; i++) c[i] = cn[i];
  }

  for (int i = 0; i < n; i++) o[p[i]] = i;

  int z = 0;
  for (int i = 0; i < n; i++) {
    int j = o[i];
    if (j == n - 1) {
      z = 0;
    } else {
      while (s[i + z] == s[p[j + 1] + z]) z++;
    }
    lcp[j] = z;
    z -= !!z;
  }
}
```

36 final/strings/sufArrayLinear.cpp

```
const int dd = (int)2e6 + 3;

ll cnt2[dd];
int AN;
int A[3 * dd + 100];
int cnt[dd + 1];  // Should be >= 256
int SA[dd + 1];

/* Used by suffix_array. */
void radix_pass(int* A, int AN, int* R, int RN, int*
    D) {
  memset(cnt, 0, sizeof(int) * (AN + 1));
  int* C = cnt + 1;
  for(int i = 0; i < RN; i++) ++C[A[R[i]]];
  for(int i = -1, v = 0; i <= AN && v < RN; v += C[i
      ++]) swap(v, C[i]);
  for(int i = 0; i < RN; i++) D[C[A[R[i]]]++] = R[i];
}

/* DC3 in O(N) using 20N bytes of memory. Stores the
    suffix array of the string
 * [A,A+AN) into SA where SA[i] (0<=i<=AN) gives the
    starting position of the
 * i-th least suffix of A (including the empty
    suffix).
 */
void suffix_array(int* A, int AN) {
  // Base case... length 1 string.
  if (!AN) {
    SA[0] = 0;
  } else if (AN == 1) {
    SA[0] = 1; SA[1] = 0;
    return;
  }

  // Sort all strings of length 3 starting at non-
      multiples of 3 into R.
  int RN = 0;
  int* SUBA = A + AN + 2;
  int* R = SUBA + AN + 2;
  for(int i = 1; i < AN; i += 3) SUBA[RN++] = i;
  for(int i = 2; i < AN; i += 3) SUBA[RN++] = i;
  A[AN + 1] = A[AN] = -1;
  radix_pass(A + 2, AN - 2, SUBA, RN, R);
  radix_pass(A + 1, AN - 1, R, RN, SUBA);
  radix_pass(A + 0, AN - 0, SUBA, RN, R);
```

```
41
42   // Compute the relabel array if we need to ←
          recursively solve for the
43   // non-multiples.
44   int resfix, resmul, v;
45   if(AN % 3 == 1) {
46     resfix = 1; resmul = RN >> 1;
47   } else {
48     resfix = 2; resmul = RN + 1 >> 1;
49   }
50   for(int i = v = 0; i < RN; i++) {
51     v += i && (A[R[i - 1] + 0] != A[R[i] + 0] ||
52         A[R[i - 1] + 1] != A[R[i] + 1] ||
53         A[R[i - 1] + 2] != A[R[i] + 2]);
54     SUBA[R[i] / 3 + (R[i] % 3 == resfix) * resmul] = v←
          ;
55   }
56
57   // Recursively solve if needed to compute relative ←
          ranks in the final suffix
58   // array of all non-multiples.
59   if(v + 1 != RN) {
60     suffix_array(SUBA, RN);
61     SA[0] = AN;
62     for(int i = 1; i <= RN; i++) {
63       SA[i] = SA[i] < resmul ? 3 * SA[i] + (resfix←
            ==1?2:1) :
64           3 * (SA[i] - resmul) + resfix;
65     }
66   } else {
67     SA[0] = AN;
68     memcpy(SA + 1, R, sizeof(int) * RN);
69   }
70
71   // Compute the relative ordering of the multiples.
72   int NMN = RN;
73   for(int i = RN = 0; i <= NMN; i++) {
74     if(SA[i] % 3 == 1) {
75       SUBA[RN++] = SA[i] - 1;
76     }
77   }
78   radix_pass(A, AN, SUBA, RN, R);
79
80   // Compute the reverse SA for what we know so far.
81   for(int i = 0; i <= NMN; i++) {
82     SUBA[SA[i]] = i;
83   }
84
85   // Merge the orderings.
86   int ii = RN - 1;
87   int jj = NMN;
88   int pos;
89   for(pos = AN; ii >= 0; pos--) {
90     int i = R[ii];
91     int j = SA[jj];
92     int v = A[i] - A[j];
93     if(!v) {
94       if(j % 3 == 1) {
95         v = SUBA[i + 1] - SUBA[j + 1];
96       } else {
97         v = A[i + 1] - A[j + 1];
98         if(!v) v = SUBA[i + 2] - SUBA[j + 2];
99       }
100    }
101    SA[pos] = v < 0 ? SA[jj--] : R[ii--];
102  }
103 }
104
105 char s[dd + 1];
106
107 /* Copies the string in s into A and reduces the ←
          characters as needed. */
108 void prep_string() {
109   int v = AN = 0;
110   memset(cnt, 0, 256 * sizeof(int));
111   for(char* ss = s; *ss; ++ss, ++AN) cnt[*ss]++;
112   for(int i = 0; i < AN; i++) cnt[s[i]]++;
113   for(int i = 0; i < 256; i++) cnt[i] = cnt[i] ? v++ ←
          : -1;
114   for(int i = 0; i < AN; i++) A[i] = cnt[s[i]];
115 }
116
117 /* Computes the reverse SA index. REVSA[i] gives the←
          index of the suffix
118  * starting a i in the SA array. In other words, ←
          REVSA[i] gives the number of
119  * suffixes before the suffix starting at i. This ←
          can be useful in itself but
120  * is also used for compute_lcp().
121  */
122 int REVSA[dd + 1];
123 void compute_reverse_sa() {
124   for(int i = 0; i <= AN; i++) {
```

```
125     REVSA[SA[i]] = i;
126   }
127 }
128
129 /* Computes the longest common prefix between ←
          adjacent suffixes. LCP[i] gives
130  * the longest common suffix between the suffix ←
          starting at i and the next
131  * smallest suffix. Runs in O(N) time.
132  */
133 int LCP[dd + 1];
134 void compute_lcp() {
135   int len = 0;
136   for(int i = 0; i < AN; i++, len = max(0, len - 1)) ←
          {
137     int s = REVSA[i];
138     int j = SA[s - 1];
139     for(; i + len < AN && j + len < AN && A[i + len] ←
          == A[j + len]; len++);
140     LCP[s] = len;
141   }
142 }
```

37   final/strings/duval.cpp

```cpp
1  void duval(string s) {
2    int n = (int) s.length();
3    int i=0;
4    while (i < n) {
5      int j=i+1, k=i;
6      while (j < n && s[k] <= s[j]) {
7        if (s[k] < s[j])
8          k = i;
9        else
10         ++k;
11       ++j;
12     }
13     while (i <= k) {
14       cout << s.substr(i, j-k) << ' ';
15       i += j - k;
16     }
17   }
18 }
```

38   final/graphs/centroid.cpp

```cpp
// original author: burunduk1, rewritten by me (←
    enot110)
// !!! warning !!! this code is not tested well
const int N = 1e5, K = 17;

int pivot, level[N], parent[N];
vector<int> v[N];

int get_pivot( int x, int xx, int n ) {
  int size = 1;
  for (int y : v[x])
  {
    if (y != xx && level[y] == -1) size += get_pivot←
    (y, x, n);
  }
  if (pivot == -1 && (size * 2 >= n || xx == -1)) ←
    pivot = x;
  return size;
}

void build( int x, int xx, int dep, int size ) {
  assert(dep < K);
  pivot = -1;
  get_pivot(x, -1, size);
  x = pivot;
  level[x] = dep, parent[x] = xx;
  for (int y : v[x]) if (level[y] == -1)
  {
    build(y, x, dep + 1, size / 2);
  }
}
```

39   final/graphs/dominatorTree.cpp

```cpp
namespace domtree {
  const int K = 18;
  const int N = 1 << K;

  int n, root;
  vector<int> e[N], g[N];
  int sdom[N], dom[N];
  int p[N][K], h[N], pr[N];
  int in[N], out[N], tmr, rev[N];

  void init(int _n, int _root) {
    n = _n;
    root = _root;
    tmr = 0;
    for (int i = 0; i < n; i++) {
      e[i].clear();
      g[i].clear();
      in[i] = -1;
    }
  }

  void addEdge(int u, int v) {
    e[u].push_back(v);
    g[v].push_back(u);
  }

  void dfs(int v) {
    in[v] = tmr++;
    for (int to : e[v]) {
      if (in[to] != -1) continue;
      pr[to] = v;
      dfs(to);
    }
    out[v] = tmr - 1;
  }

  int lca(int u, int v) {
    if (h[u] < h[v]) swap(u, v);
    for (int i = 0; i < K; i++) if ((h[u] - h[v]) & ←
    (1 << i)) u = p[u][i];
    if (u == v) return u;
    for (int i = K - 1; i >= 0; i--) {
      if (p[u][i] != p[v][i]) {
        u = p[u][i];
        v = p[v][i];
      }
    }
    return p[u][0];
```

```cpp
  }

  void solve(int _n, int _root, vector<pair<int, int←
  > > _edges) {
    init(_n, _root);
    for (auto ed : _edges) addEdge(ed.first, ed.←
    second);

    dfs(root);
    for (int i = 0; i < n; i++) if (in[i] != -1) rev←
    [in[i]] = i;
    segtree tr(tmr); // a[i]:=min(a[i],x) and return←
     a[i]
    for (int i = tmr - 1; i >= 0; i--) {
      int v = rev[i];
      int cur = i;
      for (int to : g[v]) {
        if (in[to] == -1) continue;
        if (in[to] < in[v]) cur = min(cur, in[to]);
        else cur = min(cur, tr.get(in[to]));
      }
      sdom[v] = rev[cur];
      tr.upd(in[v], out[v], in[sdom[v]]);
    }
    for (int i = 0; i < tmr; i++) {
      int v = rev[i];
      if (i == 0) {
        dom[v] = v;
        h[v] = 0;
      } else {
        dom[v] = lca(sdom[v], pr[v]);
        h[v] = h[dom[v]] + 1;
      }
      p[v][0] = dom[v];
      for (int j = 1; j < K; j++) p[v][j] = p[p[v][j←
    - 1]][j - 1];
    }
    for (int i = 0; i < n; i++) if (in[i] == -1) dom←
    [i] = -1;
  }
}
```

40   final/graphs/generalMatching.cpp

```cpp
//COPYPASTED FROM E-MAXX
namespace GeneralMatching {
  const int MAXN = 256;
  int n;
  vector<int> g[MAXN];
  int match[MAXN], p[MAXN], base[MAXN], q[MAXN];
  bool used[MAXN], blossom[MAXN];

  int lca (int a, int b) {
    bool used[MAXN] = { 0 };
    for (;;) {
      a = base[a];
      used[a] = true;
      if (match[a] == -1) break;
      a = p[match[a]];
    }
    for (;;) {
      b = base[b];
      if (used[b]) return b;
      b = p[match[b]];
    }
  }

  void mark_path (int v, int b, int children) {
    while (base[v] != b) {
      blossom[base[v]] = blossom[base[match[v]]] = ←
        true;
      p[v] = children;
      children = match[v];
      v = p[match[v]];
    }
  }

  int find_path (int root) {
    memset (used, 0, sizeof used);
    memset (p, -1, sizeof p);
    for (int i=0; i<n; ++i)
      base[i] = i;

    used[root] = true;
    int qh=0, qt=0;
    q[qt++] = root;
    while (qh < qt) {
```

```
43        int v = q[qh++];
44        for (size_t i=0; i<g[v].size(); ++i) {
45          int to = g[v][i];
46          if (base[v] == base[to] || match[v] == to) ←
              continue;
47          if (to == root || (match[to] != -1 && p[←
              match[to]] != -1)) {
48            int curbase = lca(v, to);
49            memset(blossom, 0, sizeof blossom);
50            mark_path(v, curbase, to);
51            mark_path(to, curbase, v);
52            for (int i=0; i<n; ++i)
53              if (blossom[base[i]]) {
54                base[i] = curbase;
55                if (!used[i]) {
56                  used[i] = true;
57                  q[qt++] = i;
58                }
59              }
60          }
61          else if (p[to] == -1) {
62            p[to] = v;
63            if (match[to] == -1)
64              return to;
65            to = match[to];
66            used[to] = true;
67            q[qt++] = to;
68          }
69        }
70      }
71      return -1;
72    }
73
74    vector<pair<int, int> > solve(int _n, vector<pair<←
        int, int> > edges) {
75      n = _n;
76      for (int i = 0; i < n; i++) g[i].clear();
77      for (auto o : edges) {
78        g[o.first].push_back(o.second);
79        g[o.second].push_back(o.first);
80      }
81      memset(match, -1, sizeof match);
82      for (int i=0; i<n; ++i) {
83        if (match[i] == -1) {
84          int v = find_path(i);
85          while (v != -1) {
86            int pv = p[v], ppv = match[pv];
87            match[v] = pv, match[pv] = v;
88            v = ppv;
89          }
90        }
91      }
92      vector<pair<int, int> > ans;
93      for (int i = 0; i < n; i++) {
94        if (match[i] > i) {
95          ans.push_back(make_pair(i, match[i]));
96        }
97      }
98      return ans;
99    }
100 }
```

41   final/graphs/heavyLight.cpp

```
1  namespace hld {
2    const int N = 1 << 17;
3    int par[N], heavy[N], h[N];
4    int root[N], pos[N];
5    int n;
6    vector<vector<int> > e;
7    segtree tree;
8
9    int dfs(int v) {
10     int sz = 1, mx = 0;
11     for (int to : e[v]) {
12       if (to == par[v]) continue;
13       par[to] = v;
14       h[to] = h[v] + 1;
15       int cur = dfs(to);
16       if (cur > mx) heavy[v] = to, mx = cur;
17       sz += cur;
18     }
19     return sz;
20   }
21
22   template <typename T>
23   void path(int u, int v, T op) {
```

```
24       for (; root[u] != root[v]; v = par[root[v]]) {
25         if (h[root[u]] > h[root[v]]) swap(u, v);
26         op(pos[root[v]], pos[v] + 1);
27       }
28       if (h[u] > h[v]) swap(u, v);
29       op(pos[u], pos[v] + 1);
30     }
31
32     void init(vector<vector<int> > _e) {
33       e = _e;
34       n = e.size();
35       tree = segtree(n);
36       memset(heavy, -1, sizeof(heavy[0]) * n);
37       par[0] = -1;
38       h[0] = 0;
39       dfs(0);
40       for (int i = 0, cpos = 0; i < n; i++) {
41         if (par[i] == -1 || heavy[par[i]] != i) {
42           for (int j = i; j != -1; j = heavy[j]) {
43             root[j] = i;
44             pos[j] = cpos++;
45           }
46         }
47       }
48     }
49
50     void add(int v, int x) {
51       tree.add(pos[v], x);
52     }
53
54     int get(int u, int v) {
55       int res = 0;
56       path(u, v, [&](int l, int r) {
57         res = max(res, tree.get(l, r));
58       });
59       return res;
60     }
61 }
```

42   final/graphs/hungary.cpp

```
1  namespace hungary
2  {
3    const int N = 210;
4
5    int a[N][N];
6    int ans[N];
7
8    int calc(int n, int m)
9    {
10     ++n, ++m;
11     vi u(n), v(m), p(m), prev(m);
12     for (int i = 1; i < n; ++i)
13     {
14       p[0] = i;
15       int x = 0;
16       vi mn(m, inf);
17       vi was(m, 0);
18       while (p[x])
19       {
20         was[x] = 1;
21         int ii = p[x], dd = inf, y = 0;
22         for (int j = 1; j < m; ++j) if (!was[j])
23         {
24           int cur = a[ii][j] - u[ii] - v[j];
25           if (cur < mn[j]) mn[j] = cur, prev[j] = x;
26           if (mn[j] < dd) dd = mn[j], y = j;
27         }
28         forn(j, m)
29         {
30           if (was[j]) u[p[j]] += dd, v[j] -= dd;
31           else mn[j] -= dd;
32         }
33         x = y;
34       }
35       while (x)
36       {
37         int y = prev[x];
38         p[x] = p[y];
39         x = y;
40       }
41     }
42     for (int j = 1; j < m; ++j)
43     {
44       ans[p[j]] = j;
45     }
46     return -v[0];
```

```
47    }
48    // HOW TO USE ::
49    // -- set values to a[1..n][1..m] (n <= m)
50    // -- run calc(n, m) to find MINIMUM
51    // -- to restore permutation use ans[]
52    // -- everything works on negative numbers
53    //
54    // !! i don't understand this code, it's ←
         copypasted from e-maxx (and rewrited by enot110←
         )
55 }
```

43   final/graphs/minCost.cpp

```
1  ll findflow(int s, int t) {
2    ll cost = 0;
3    ll flow = 0;
4
5    forn(i, N) G[i] = inf;
6
7    queue<int> q;
8
9    q.push(s);
10   used[s] = true;
11   G[s] = 0;
12
13   while (q.size()) {
14     int v = q.front();
15     used[v] = false;
16     q.pop();
17
18     forn(i, E[v].size()) {
19       auto &e = E[v][i];
20       if (e.f < e.c && G[e.to] > G[v] + e.w) {
21         G[e.to] = G[v] + e.w;
22         if (!used[e.to]) {
23           q.push(e.to);
24           used[e.to] = true;
25         }
26       }
27     }
28   }
29
30   while (1) {
31     forn(i, N)
32       d[i] = inf, p[i] = { -1, -1 }, used[i] = 0;
33
34     d[s] = 0;
35     while (1) {
36       int v = -1;
37       forn(i, N) {
38         if (!used[i] && d[i] != inf && (v == -1 || d←
              [i] < d[v]))
39           v = i;
40       }
41       if (v == -1)
42         break;
43       used[v] = 1;
44
45       forn(i, E[v].size()) {
46
47         auto &e = E[v][i];
48         if (e.f < e.c && d[e.to] > d[v] + e.w + G[v]←
                - G[e.to]) {
49           p[e.to] = mp(v, i);
50           d[e.to] = d[v] + e.w + G[v] - G[e.to];
51         }
52       }
53
54     }
55     if (p[t].first == -1) {
56       break;
57     }
58     int add = inf;
59     for (int i = t; p[i].first != -1; i = p[i].first←
          ) {
60       add = min(add, E[p[i].first][p[i].second].c - ←
            E[p[i].first][p[i].second].f);
61     }
62     for (int i = t; p[i].first != -1; i = p[i].first←
          ) {
63       auto &e = E[p[i].first][p[i].second];
64       cost += 1ll * add * e.w;
65       e.f += add;
66       E[e.to][e.back].f -= add;
67     }
68     flow += add;
```

```
69     if (add == 0)
70       break;
71     forn(i, N)
72       G[i] += d[i];
73   }
74   return cost;
75 }
```

44   final/graphs/minCostNegCycle.cpp

```
1  struct Edge {
2    int from, to, cap, flow;
3    double cost;
4
5  };
6
7  struct Graph {
8    int n;
9    vector<Edge> edges;
10   vector<vector<int> > e;
11
12   Graph(int _n) {
13     n = _n;
14     e.resize(n);
15   }
16
17   void addEdge(int from, int to, int cap, double ←
        cost) {
18     e[from].push_back(edges.size());
19     edges.push_back({ from, to, cap, 0, cost });
20     e[to].push_back(edges.size());
21     edges.push_back({ to, from, 0, 0, -cost });
22   }
23
24   void maxflow() {
25     while (1) {
26       queue<int> q;
27       vector<int> d(n, INF);
28       vector<int> pr(n, -1);
29       q.push(0);
30       d[0] = 0;
31       while (!q.empty()) {
32         int v = q.front();
33         q.pop();
34         for (int i = 0; i < (int)e[v].size(); i++) {
35           Edge cur = edges[e[v][i]];
36           if (d[cur.to] > d[v] + 1 && cur.flow < cur←
                .cap) {
37             d[cur.to] = d[v] + 1;
38             pr[cur.to] = e[v][i];
39             q.push(cur.to);
40           }
41         }
42       }
43       if (d[n - 1] == INF) break;
44       int v = n - 1;
45       while (v) {
46         edges[pr[v]].flow++;
47         edges[pr[v] ^ 1].flow--;
48         v = edges[pr[v]].from;
49       }
50     }
51   }
52
53   bool findcycle() {
54     int iters = n;
55     vector<int> changed;
56     for (int i = 0; i < n; i++) changed.push_back(i)←
          ;
57
58     vector<vector<double> > d(iters + 1, vector<←
          double>(n, INF));
59     vector<vector<int> > p(iters + 1, vector<int>(n,←
          -1));
60     d[0].assign(n, 0);
61     for (int it = 0; it < iters; it++) {
62       d[it + 1] = d[it];
63       vector<int> nchanged(n, 0);
64       for (int v : changed) {
65         for (int id : e[v]) {
66           Edge cur = edges[id];
67           if (d[it + 1][cur.to] > d[it][v] + cur.←
                cost && cur.flow < cur.cap) {
68             d[it + 1][cur.to] = d[it][v] + cur.cost;
69             p[it + 1][cur.to] = id;
70             nchanged[cur.to] = 1;
71           }
```

```
72          }
73        }
74        changed.clear();
75        for (int i = 0; i < n; i++) if (nchanged[i]) ←
              changed.push_back(i);
76      }
77      if (changed.empty()) return 0;
78
79      int bestU = 0, bestK = 1;
80      double bestAns = INF;
81      for (int u = 0; u < n; u++) {
82        double curMax = -INF;
83        for (int k = 0; k < iters; k++) {
84          double curVal = (d[iters][u] - d[k][u]) / (←
                iters - k);
85          curMax = max(curMax, curVal);
86        }
87        if (bestAns > curMax) {
88          bestAns = curMax;
89          bestU = u;
90        }
91      }
92
93      int v = bestU;
94      int it = iters;
95      vector<int> was(n, -1);
96      while (was[v] == -1) {
97        was[v] = it;
98        v = edges[p[it][v]].from;
99        it--;
100     }
101     int vv = v;
102     it = was[v];
103     double sum = 0;
104     do {
105       edges[p[it][v]].flow++;
106       sum += edges[p[it][v]].cost;
107       edges[p[it][v] ^ 1].flow--;
108       v = edges[p[it][v]].from;
109       it--;
110     } while (v != vv);
111     return 1;
112   }
113 };
```

45　final/graphs/retro.cpp

```
1  namespace retro
2  {
3    const int N = 4e5 + 10;
4
5    vi v[N];
6    vi vrev[N];
7
8    void add(int x, int y)
9    {
10     v[x].pb(y);
11     vrev[y].pb(x);
12   }
13
14   const int UD = 0;
15   const int WIN = 1;
16   const int LOSE = 2;
17
18   int res[N];
19   int moves[N];
20   int deg[N];
21   int q[N], st, en;
22
23   void calc(int n)
24   {
25     forn(i, n) deg[i] = sz(v[i]);
26     st = en = 0;
27     forn(i, n) if (!deg[i])
28     {
29       q[en++] = i;
30       res[i] = LOSE;
31     }
32     while (st < en)
33     {
34       int x = q[st++];
35       for (int y : vrev[x])
36       {
37         if (res[y] == UD && (res[x] == LOSE || (--←
deg[y] == 0 && res[x] == WIN)))
38         {
39           res[y] = 3 - res[x];
```

```
40           moves[y] = moves[x] + 1;
41           q[en++] = y;
42         }
43       }
44     }
45   }
46 }
```

46　final/graphs/mincut.cpp

```
1  const int MAXN = 500;
2  int n, g[MAXN][MAXN];
3  int best_cost = 1000000000;
4  vector<int> best_cut;
5
6  void mincut() {
7    vector<int> v[MAXN];
8    for (int i=0; i<n; ++i)
9      v[i].assign(1, i);
10   int w[MAXN];
11   bool exist[MAXN], in_a[MAXN];
12   memset(exist, true, sizeof exist);
13   for (int ph=0; ph<n-1; ++ph) {
14     memset(in_a, false, sizeof in_a);
15     memset(w, 0, sizeof w);
16     for (int it=0, prev; it<n-ph; ++it) {
17       int sel = -1;
18       for (int i=0; i<n; ++i)
19         if (exist[i] && !in_a[i] && (sel == -1 || w[←
i] > w[sel]))
20           sel = i;
21       if (it == n-ph-1) {
22         if (w[sel] < best_cost)
23           best_cost = w[sel], best_cut = v[sel];
24         v[prev].insert(v[prev].end(), v[sel].begin←
(), v[sel].end());
25         for (int i=0; i<n; ++i)
26           g[prev][i] = g[i][prev] += g[sel][i];
27         exist[sel] = false;
28       }
29       else {
30         in_a[sel] = true;
31         for (int i=0; i<n; ++i)
32           w[i] += g[sel][i];
33         prev = sel;
34       }
35     }
36   }
37 }
```

47　final/graphs/twoChineseFast.cpp

```
1  namespace twoc {
2    struct Heap {
3      static Heap* null;
4      ll x, xadd;
5      int ver, h;
6      /* ANS */ int ei;
7      Heap *l, *r;
8      Heap(ll xx, int vv) : x(xx), xadd(0), ver(vv), h←
(1), l(null), r(null) {}
9      Heap(const char*) : x(0), xadd(0), ver(0), h(0),←
 l(this), r(this) {}
10     void add(ll a) { x += a; xadd += a; }
11     void push() {
12       if (l != null) l->add(xadd);
13       if (r != null) r->add(xadd);
14       xadd = 0;
15     }
16   };
17   Heap *Heap::null = new Heap("wqeqw");
18   Heap* merge(Heap *l, Heap *r) {
19     if (l == Heap::null) return r;
20     if (r == Heap::null) return l;
21     l->push(); r->push();
22     if (l->x > r->x)
23       swap(l, r);
24     l->r = merge(l->r, r);
25     if (l->l->h < l->r->h)
26       swap(l->l, l->r);
27     l->h = l->r->h + 1;
```

```
28      return l;
29    }
30    Heap *pop(Heap *h) {
31      h->push();
32      return merge(h->l, h->r);
33    }
34    const int N = 666666;
35    struct DSU {
36      int p[N];
37      void init(int nn) { iota(p, p + nn, 0); }
38      int get(int x) { return p[x] == x ? x : p[x] = ←
        get(p[x]); }
39      void merge(int x, int y) { p[get(y)] = get(x); }
40    } dsu;
41    Heap *eb[N];
42    int n;
43    /* ANS */ struct Edge {
44    /* ANS */   int x, y;
45    /* ANS */   ll c;
46    /* ANS */ };
47    /* ANS */ vector<Edge> edges;
48    /* ANS */ int answer[N];
49    void init(int nn) {
50      n = nn;
51      dsu.init(n);
52      fill(eb, eb + n, Heap::null);
53      edges.clear();
54    }
55    void addEdge(int x, int y, ll c) {
56      Heap *h = new Heap(c, x);
57      /* ANS */ h->ei = sz(edges);
58      /* ANS */ edges.push_back({x, y, c});
59      eb[y] = merge(eb[y], h);
60    }
61    ll solve(int root = 0) {
62      ll ans = 0;
63      static int done[N], pv[N];
64      memset(done, 0, sizeof(int) * n);
65      done[root] = 1;
66      int tt = 1;
67      /* ANS */ int cnum = 0;
68      /* ANS */ static vector<ipair> eout[N];
69      /* ANS */ for (int i = 0; i < n; ++i) eout[i].←
        clear();
70      for (int i = 0; i < n; ++i) {
71        int v = dsu.get(i);
72        if (done[v])
73          continue;
74        ++tt;
75        while (true) {
76          done[v] = tt;
77          int nv = -1;
78          while (eb[v] != Heap::null) {
79            nv = dsu.get(eb[v]->ver);
80            if (nv == v) {
81              eb[v] = pop(eb[v]);
82              continue;
83            }
84            break;
85          }
86          if (nv == -1)
87            return LINF;
88          ans += eb[v]->x;
89          eb[v]->add(-eb[v]->x);
90          /* ANS */ int ei = eb[v]->ei;
91          /* ANS */ eout[edges[ei].x].push_back({++←
        cnum, ei});
92          if (!done[nv]) {
93            pv[v] = nv;
94            v = nv;
95            continue;
96          }
97          if (done[nv] != tt)
98            break;
99          int v1 = nv;
100         while (v1 != v) {
101           eb[v] = merge(eb[v], eb[v1]);
102           dsu.merge(v, v1);
103           v1 = dsu.get(pv[v1]);
104         }
105       }
106     }
107     /* ANS */ memset(answer, -1, sizeof(int) * n);
108     /* ANS */ answer[root] = 0;
109     /* ANS */ set<ipair> es(all(eout[root]));
110     /* ANS */ while (!es.empty()) {
111     /* ANS */   auto it = es.begin();
112     /* ANS */   int ei = it->second;
113     /* ANS */   es.erase(it);
114     /* ANS */   int nv = edges[ei].y;
115     /* ANS */   if (answer[nv] != -1)
116     /* ANS */     continue;
117     /* ANS */   answer[nv] = ei;
118     /* ANS */   es.insert(all(eout[nv]));
119     /* ANS */ }
120     /* ANS */ answer[root] = -1;
121     return ans;
122   }
123   /* Usage: twoc::init(vertex_count);
124    *   twoc::addEdge(v1, v2, cost);
125    *   twoc::solve(root); - returns cost or LINF
126    * twoc::answer contains index of ingoing edge for←
        each vertex
127    */
128 }
```

48   final/graphs/linkcut.cpp

```
1  #include <iostream>
2  #include <cstdio>
3  #include <cassert>
4
5  using namespace std;
6
7  // BEGIN ALGO
8
9  const int MAXN = 110000;
10
11 typedef struct _node{
12   _node *l, *r, *p, *pp;
13   int size; bool rev;
14   _node();
15   explicit _node(nullptr_t){
16     l = r = p = pp = this;
17     size = rev = 0;
18   }
19   void push(){
20     if (rev){
21       l->rev ^= 1; r->rev ^= 1;
22       rev = 0; swap(l,r);
23     }
24   }
25   void update();
26 }* node;
27 node None = new _node(nullptr);
28 node v2n[MAXN];
29 _node::_node(){
30   l = r = p = pp = None;
31   size = 1; rev = false;
32 }
33 void _node::update(){
34   size = (this != None) + l->size + r->size;
35   l->p = r->p = this;
36 }
37 void rotate(node v){
38   assert(v != None && v->p != None);
39   assert(!v->rev); assert(!v->p->rev);
40   node u = v->p;
41   if (v == u->l)
42     u->l = v->r, v->r = u;
43   else
44     u->r = v->l, v->l = u;
45   swap(u->p,v->p); swap(v->pp,u->pp);
46   if (v->p != None){
47     assert(v->p->l == u || v->p->r == u);
48     if (v->p->r == u) v->p->r = v;
49     else v->p->l = v;
50   }
51   u->update(); v->update();
52 }
53 void bigRotate(node v){
54   assert(v->p != None);
55   v->p->p->push();
56   v->p->push();
57   v->push();
58   if (v->p->p != None){
59     if ((v->p->l == v) ^ (v->p->p->r == v->p))
60       rotate(v->p);
61     else
62       rotate(v);
63   }
64   rotate(v);
65 }
66 inline void Splay(node v){
67   while (v->p != None) bigRotate(v);
68 }
69 inline void splitAfter(node v){
70   v->push();
71   Splay(v);
72   v->r->p = None;
```

18

```
73   v->r->pp = v;
74   v->r = None;
75   v->update();
76  }
77  void expose(int x){
78   node v = v2n[x];
79   splitAfter(v);
80   while (v->pp != None){
81    assert(v->p == None);
82    splitAfter(v->pp);
83    assert(v->pp->r == None);
84    assert(v->pp->p == None);
85    assert(!v->pp->rev);
86    v->pp->r = v;
87    v->pp->update();
88    v = v->pp;
89    v->r->pp = None;
90   }
91   assert(v->p == None);
92   Splay(v2n[x]);
93  }
94  inline void makeRoot(int x){
95   expose(x);
96   assert(v2n[x]->p == None);
97   assert(v2n[x]->pp == None);
98   assert(v2n[x]->r == None);
99   v2n[x]->rev ^= 1;
100 }
101 inline void link(int x,int y){
102  makeRoot(x); v2n[x]->pp = v2n[y];
103 }
104 inline void cut(int x,int y){
105  expose(x);
106  Splay(v2n[y]);
107  if (v2n[y]->pp != v2n[x]){
108   swap(x,y);
109   expose(x);
110   Splay(v2n[y]);
111   assert(v2n[y]->pp == v2n[x]);
112  }
113  v2n[y]->pp = None;
114 }
115 inline int get(int x,int y){
116  if (x == y) return 0;
117  makeRoot(x);
118  expose(y); expose(x);
119  Splay(v2n[y]);
120  if (v2n[y]->pp != v2n[x]) return -1;
121  return v2n[y]->size;
122 }
123 // END ALGO
124
125 _node mem[MAXN];
126
127
128 int main(){
129  freopen("linkcut.in","r",stdin);
130  freopen("linkcut.out","w",stdout);
131
132  int n,m;
133  scanf("%d %d",&n,&m);
134
135  for (int i = 0; i < n; i++)
136   v2n[i] = &mem[i];
137
138  for (int i = 0; i < m; i++){
139   int a,b;
140   if (scanf(" link %d %d",&a,&b) == 2)
141    link(a-1,b-1);
142   else if (scanf(" cut %d %d",&a,&b) == 2)
143    cut(a-1,b-1);
144   else if (scanf(" get %d %d",&a,&b) == 2)
145    printf("%d\n",get(a-1,b-1));
146   else
147    assert(false);
148  }
149  return 0;
150 }
```

49 final/graphs/chordaltree.cpp

```
1  void chordaltree(vector<vector<int>> e) {
2    int n = e.size();
3
4    vector<int> mark(n);
5    set<pair<int, int> > st;
```

```
6     for (int i = 0; i < n; i++) st.insert({-mark[i], i
      });
7
8     vector<int> vct(n);
9     vector<pair<int, int> > ted;
10    vector<vector<int> > who(n);
11    vector<vector<int> > verts(1);
12    vector<int> cliq(n, -1);
13    cliq.push_back(0);
14    vector<int> last(n + 1, n);
15    int prev = n + 1;
16    for (int i = n - 1; i >= 0; i--) {
17      int x = st.begin()->second;
18      st.erase(st.begin());
19      if (mark[x] <= prev) {
20        vector<int> cur = who[x];
21        cur.push_back(x);
22        verts.push_back(cur);
23        ted.push_back({cliq[last[x]], (int)verts.size
        () - 1});
24      } else {
25        verts.back().push_back(x);
26      }
27      for (int y : e[x]) {
28        if (cliq[y] != -1) continue;
29        who[y].push_back(x);
30        st.erase({-mark[y], y});
31        mark[y]++;
32        st.insert({-mark[y], y});
33        last[y] = x;
34      }
35      prev = mark[x];
36      vct[i] = x;
37      cliq[x] = (int)verts.size() - 1;
38    }
39
40    int k = verts.size();
41    vector<int> pr(k);
42    vector<vector<int> > g(k);
43    for (auto o : ted) {
44      pr[o.second] = o.first;
45      g[o.first].push_back(o.second);
46    }
47 }
```

50 final/graphs/minimization.cpp

```
1  namespace mimimi /* ^_^ */ {
2    const int N = 100555;
3    const int S = 3;
4    int e[N][S];
5    int label[N];
6    vector<int> eb[N][S];
7    int ans[N];
8    void solve(int n) {
9      for (int i = 0; i < n; ++i)
10       for (int j = 0; j < S; ++j)
11         eb[i][j].clear();
12     for (int i = 0; i < n; ++i)
13       for (int j = 0; j < S; ++j)
14         eb[e[i][j]][j].push_back(i);
15     vector<unordered_set<int>> classes(*max_element(
       label, label + n) + 1);
16     for (int i = 0; i < n; ++i)
17       classes[label[i]].insert(i);
18     for (int i = 0; i < sz(classes); ++i)
19       if (classes[i].empty()) {
20         classes[i].swap(classes.back());
21         classes.pop_back();
22         --i;
23       }
24     for (int i = 0; i < sz(classes); ++i)
25       for (int v : classes[i])
26         ans[v] = i;
27     for (int i = 0; i < sz(classes); ++i)
28       for (int c = 0; c < S; ++c) {
29         unordered_map<int, unordered_set<int>>
        involved;
30         for (int v : classes[i])
31           for (int nv : eb[v][c])
32             involved[ans[nv]].insert(nv);
33         for (auto &pp : involved) {
34           int cl = pp.X;
35           auto &cls = classes[cl];
36           if (sz(pp.Y) == sz(cls))
37             continue;
38           for (int x : pp.Y)
```

```
39            cls.erase(x);
40            if (sz(cls) < sz(pp.Y))
41              cls.swap(pp.Y);
42            for (int x : pp.Y)
43              ans[x] = sz(classes);
44            classes.push_back(move(pp.Y));
45          }
46        }
47      }
48      /* Usage: initialize edges: e[vertex][character]
49                labels: label[vertex]
50          solve(n)
51          ans[] - classes
52      */
53  }
```

```
66        color[c[i]] = 1;
67      }
68      vector<int> x1, x2;
69      for (int i = 0; i < m; ++i) if (!used[i]) {
70        if (gauss.check(a[i])) {
71          x1.push_back(i);
72        }
73        if (!color[c[i]]) {
74          x2.push_back(i);
75        }
76      }
77      vector<int> path = G.get_path(x1, x2);
78      if (!path.size()) return;
79      for (int i : path) used[i] ^= 1;
80      get_ans(used, m);
81  }
```

51　final/graphs/matroidIntersection.cpp

```cpp
1   struct Graph {
2     vector<vector<int>> G;
3
4     Graph(int n = 0) {
5       G.resize(n);
6     }
7
8     void add_edge(int v, int u) {
9       G[v].push_back(u);
10    }
11
12    vector<int> get_path(vector<int> &s, vector<int> &
        t) {
13      int n = G.size();
14      vector<int> dist(n, inf), pr(n, -1);
15      queue<int> Q;
16      for (int i : s) {
17        dist[i] = 0;
18        Q.push(i);
19      }
20      while (!Q.empty()) {
21        int v = Q.front();
22        Q.pop();
23        for (int to : G[v]) if (dist[to] > dist[v] +
        1) {
24          dist[to] = dist[v] + 1;
25          pr[to] = v;
26          Q.push(to);
27        }
28      }
29      int V = -1;
30      for (int i : t) if (V == -1 || dist[i] < dist[V
        ]) {
31        V = i;
32      }
33      if (V == -1 || dist[V] == inf) return {};
34      vector<int> path;
35      while (V != -1) {
36        path.push_back(V);
37        V = pr[V];
38      }
39      return path;
40    }
41  };
42
43  void get_ans(vector<int> &used, int m) {
44    Graph G(m);
45    for (int i = 0; i < m; ++i) if (used[i]) {
46      Gauss gauss;
47      vector<int> color(130, 0);
48      for (int j = 0; j < m; ++j) if (used[j] && j !=
        i) {
49        gauss.add(a[j]);
50        color[c[j]] = 1;
51      }
52      for (int j = 0; j < m; ++j) if (!used[j]) {
53        if (gauss.check(a[j])) {
54          G.add_edge(i, j);
55        }
56        if (!color[c[j]]) {
57          G.add_edge(j, i);
58        }
59      }
60    }
61
62    Gauss gauss;
63    vector<int> color(130, 0);
64    for (int i = 0; i < m; ++i) if (used[i]) {
65      gauss.add(a[i]);
```

dbl Simpson() { return (F(-1) + 4 * F(0) + F(1)) / 6; } dbl Runge2() { return (F(-sqrtl(1.0 / 3)) + F(sqrtl(1.0 / 3))) / 2; } dbl Runge3() { return (F(-sqrtl(3.0 / 5)) * 5 + F(0) * 8 + F(sqrtl(3.0 / 5)) * 5) / 18; }

Simpson и Runge2 – точны для полиномов степени $<=$ 3 Runge3 – точен для полиномов степени $<=$ 5

—

Явный Рунге-Кутт четвертого порядка, ошибка $O(h^4)$

$y' = f(x, y)$ $y\_(n+1) = y\_n + (k1 + 2 * k2 + 2 * k3 + k4) * h / 6$

$k1 = f(xn, yn)$ $k2 = f(xn + h/2, yn + h/2 * k1)$ $k3 = f(xn + h/2, yn + h/2 * k2)$ $k4 = f(xn + h, yn + h * k3)$

Методы Адамса-Башфорта

$y\_n+3 = y\_n+2 + h * (23/12 * f(x\_n+2,y\_n+2)$ $- 4/3 * f(x\_n+1,y\_n+1) + 5/12 * f(x\_n,y\_n))$ $y\_n+4 = y\_n+3 + h * (55/24 * f(x\_n+3,y\_n+3) - 59/24 * f(x\_n+2,y\_n+2) + 37/24 * f(x\_n+1,y\_n+1) - 3/8 * f(x\_n,y\_n))$ $y\_n+5 = y\_n+4 + h * (1901/720 * f(x\_n+4,y\_n+4) - 1387/360 * f(x\_n+3,y\_n+3) + 109/30 * f(x\_n+2,y\_n+2) - 637/360 * f(x\_n+1,y\_n+1) + 251/720 * f(x\_n,y\_n))$

—

Извлечение корня по простому модулю (от Сережи) $3 <= p$, $1 <= a < p$, найти $x^2 = a$

1) Если $a^{((p - 1)/2)} != 1$, return -1 2) Выбрать случайный $1 <= i < p$ 3) $T(x) = (x + i)^{((p - 1)/2)} \mod (x^2 - a) = bx + c$ 4) Если $b != 0$ то вернуть c/b, иначе к шагу 2)

—

Иногда вместо того чтобы считать первообразный у простого числа, можно написать чекер ответа и перебирать случайный первообразный.

Иногда можно представить ответ в виде многочлена и вместо подсчета самих к-тов посчитать значения и проинтерполировать

—

Лемма Бернсайда:

Группа G действует на множество X Тогда число классов эквивалентности $=$ (sum $|f(g)|$ for g in G) / $|G|$ где $f(g)$ = число x (из X) : $g(x) == x$

—

Число простых быстрее $O(n)$:

$dp(n, k)$ – число чисел от 1 до n в которых все простые $>= p[k]$ $dp(n, 1) = n$ $dp(n, j) = dp(n, j + 1) + dp(n / p[j], j)$, т. е. $dp(n, j + 1) = dp(n, j) - dp(n / p[j], j)$

Если $p[j], p[k] > \mathrm{sqrt}(n)$ то $dp(n, j) + j == dp(n, k) + k$

Делаешь все оптимайзы сверху, но не считаешь глубже $dp(n, k)$, $n < K$ Потом фенвиком+сортировкой подсчитываешь за $(K+Q)\log$ все эти запросы Делаешь во второй раз, но на этот раз берешь прекальканные значения

Если $\mathrm{sqrt}(n) < p[k] < n$ то (число простых до n)=$dp(n, k) + k - 1$

—

$\mathrm{sum}(k=1..n)\ k^2 = n(n+1)(2n+1)/6$

$\mathrm{sum}(k=1..n)\ k^3 = n^2(n+1)^2/4$

Чиселки:

Фибоначчи 45: 1134903170 46: 1836311903 47: 2971215073 91: 4660046610375530309 92: 7540113804746346429 93: 12200160415121876738

Числа с кучей делителей 20: d(12)=6 50: d(48)=10 100: d(60)=12 1000: d(840)=32 10^4: d(9240)=64 10^5: d(83160)=128 10^6: d(720720)=240 10^7: d(8648640)=448 10^8: d(91891800)=768 10^9: d(931170240)=1344 10^{11}: d(97772875200)=4032 10^{12}: d(963761198400)=6720 10^{15}: d(866421317361600)=26880 10^{18}: d(897612484786617600)=103680

Bell numbers: 0:1, 1:1, 2:2, 3:5, 4:15, 5:52, 6:203, 7:877, 8:4140, 9:21147, 10:115975, 11:678570, 12:4213597, 13:27644437, 14:190899322, 15:1382958545, 16:10480142147, 17:82864869804, 18:682076806159, 19:5832742205057, 20:51724158235372, 21:474869816156751, 22:4506715738447323, 23:44152005855084346

Catalan numbers: 0:1, 1:1, 2:2, 3:5, 4:14, 5:42, 6:132, 7:429, 8:1430, 9:4862, 10:16796, 11:58786, 12:208012, 13:742900, 14:2674440, 15:9694845, 16:35357670, 17:129644790, 18:477638700, 19:1767263190, 20:6564120420, 21:24466267020, 22:91482563640, 23:343059613650, 24:1289904147324, 25:4861946401452

Partitions numbers: 0:1, 1:1, 2:2, 3:3, 4:5, 5:7, 6:11, 7:15, 8:22, 9:30, 10:42, 20:627, 30:5604, 40:37338, 50:204226, 60:966467, 70:4087968, 80:15796476, 90:56634173, 100:190569292

prod (k=1..+inf) $(1-x^k)$ = sum(q=-inf..+inf) $(-1)^q x^{((3q^2-q)/2)}$

Table of Integrals*

Basic Forms

$$\int x^n dx = \frac{1}{n+1}x^{n+1} \tag{1}$$

$$\int \frac{1}{x}dx = \ln|x| \tag{2}$$

$$\int u\,dv = uv - \int v\,du \tag{3}$$

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b| \tag{4}$$

Integrals of Rational Functions

$$\int \frac{1}{(x+a)^2}dx = -\frac{1}{x+a} \tag{5}$$

$$\int (x+a)^n dx = \frac{(x+a)^{n+1}}{n+1}, n \neq -1 \tag{6}$$

$$\int x(x+a)^n dx = \frac{(x+a)^{n+1}((n+1)x-a)}{(n+1)(n+2)} \tag{7}$$

$$\int \frac{1}{1+x^2}dx = \tan^{-1}x \tag{8}$$

$$\int \frac{1}{a^2+x^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a} \tag{9}$$

$$\int \frac{x}{a^2+x^2}dx = \frac{1}{2}\ln|a^2+x^2| \tag{10}$$

$$\int \frac{x^2}{a^2+x^2}dx = x - a\tan^{-1}\frac{x}{a} \tag{11}$$

$$\int \frac{x^3}{a^2+x^2}dx = \frac{1}{2}x^2 - \frac{1}{2}a^2\ln|a^2+x^2| \tag{12}$$

$$\int \frac{1}{ax^2+bx+c}dx = \frac{2}{\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{13}$$

$$\int \frac{1}{(x+a)(x+b)}dx = \frac{1}{b-a}\ln\frac{a+x}{b+x}, \ a \neq b \tag{14}$$

$$\int \frac{x}{(x+a)^2}dx = \frac{a}{a+x} + \ln|a+x| \tag{15}$$

$$\int \frac{x}{ax^2+bx+c}dx = \frac{1}{2a}\ln|ax^2+bx+c| - \frac{b}{a\sqrt{4ac-b^2}}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} \tag{16}$$

Integrals with Roots

$$\int \sqrt{x-a}\,dx = \frac{2}{3}(x-a)^{3/2} \tag{17}$$

$$\int \frac{1}{\sqrt{x\pm a}}dx = 2\sqrt{x\pm a} \tag{18}$$

$$\int \frac{1}{\sqrt{a-x}}dx = -2\sqrt{a-x} \tag{19}$$

$$\int x\sqrt{x-a}\,dx = \frac{2}{3}a(x-a)^{3/2} + \frac{2}{5}(x-a)^{5/2} \tag{20}$$

$$\int \sqrt{ax+b}\,dx = \left(\frac{2b}{3a} + \frac{2x}{3}\right)\sqrt{ax+b} \tag{21}$$

$$\int (ax+b)^{3/2}dx = \frac{2}{5a}(ax+b)^{5/2} \tag{22}$$

$$\int \frac{x}{\sqrt{x\pm a}}dx = \frac{2}{3}(x\mp 2a)\sqrt{x\pm a} \tag{23}$$

$$\int \sqrt{\frac{x}{a-x}}dx = -\sqrt{x(a-x)} - a\tan^{-1}\frac{\sqrt{x(a-x)}}{x-a} \tag{24}$$

$$\int \sqrt{\frac{x}{a+x}}dx = \sqrt{x(a+x)} - a\ln\left[\sqrt{x} + \sqrt{x+a}\right] \tag{25}$$

$$\int x\sqrt{ax+b}\,dx = \frac{2}{15a^2}(-2b^2 + abx + 3a^2x^2)\sqrt{ax+b} \tag{26}$$

$$\int \sqrt{x(ax+b)}\,dx = \frac{1}{4a^{3/2}}\left[(2ax+b)\sqrt{ax(ax+b)} - b^2\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right|\right] \tag{27}$$

$$\int \sqrt{x^3(ax+b)}\,dx = \left[\frac{b}{12a} - \frac{b^2}{8a^2x} + \frac{x}{3}\right]\sqrt{x^3(ax+b)} + \frac{b^3}{8a^{5/2}}\ln\left|a\sqrt{x} + \sqrt{a(ax+b)}\right| \tag{28}$$

$$\int \sqrt{x^2\pm a^2}\,dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \pm \frac{1}{2}a^2\ln\left|x + \sqrt{x^2\pm a^2}\right| \tag{29}$$

$$\int \sqrt{a^2-x^2}\,dx = \frac{1}{2}x\sqrt{a^2-x^2} + \frac{1}{2}a^2\tan^{-1}\frac{x}{\sqrt{a^2-x^2}} \tag{30}$$

$$\int x\sqrt{x^2\pm a^2}\,dx = \frac{1}{3}\left(x^2\pm a^2\right)^{3/2} \tag{31}$$

$$\int \frac{1}{\sqrt{x^2\pm a^2}}dx = \ln\left|x + \sqrt{x^2\pm a^2}\right| \tag{32}$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\frac{x}{a} \tag{33}$$

$$\int \frac{x}{\sqrt{x^2\pm a^2}}dx = \sqrt{x^2\pm a^2} \tag{34}$$

$$\int \frac{x}{\sqrt{a^2-x^2}}dx = -\sqrt{a^2-x^2} \tag{35}$$

$$\int \frac{x^2}{\sqrt{x^2\pm a^2}}dx = \frac{1}{2}x\sqrt{x^2\pm a^2} \mp \frac{1}{2}a^2\ln\left|x + \sqrt{x^2\pm a^2}\right| \tag{36}$$

$$\int \sqrt{ax^2+bx+c}\,dx = \frac{b+2ax}{4a}\sqrt{ax^2+bx+c} + \frac{4ac-b^2}{8a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx^+c)}\right| \tag{37}$$

$$\int x\sqrt{ax^2+bx+c} = \frac{1}{48a^{5/2}}\left(2\sqrt{a}\sqrt{ax^2+bx+c} \times (-3b^2+2abx+8a(c+ax^2)) + 3(b^3-4abc)\ln\left|b+2ax+2\sqrt{a}\sqrt{ax^2+bx+c}\right|\right) \tag{38}$$

$$\int \frac{1}{\sqrt{ax^2+bx+c}}dx = \frac{1}{\sqrt{a}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{39}$$

$$\int \frac{x}{\sqrt{ax^2+bx+c}}dx = \frac{1}{a}\sqrt{ax^2+bx+c} - \frac{b}{2a^{3/2}}\ln\left|2ax+b+2\sqrt{a(ax^2+bx+c)}\right| \tag{40}$$

$$\int \frac{dx}{(a^2+x^2)^{3/2}} = \frac{x}{a^2\sqrt{a^2+x^2}} \tag{41}$$

Integrals with Logarithms

$$\int \ln ax\,dx = x\ln ax - x \tag{42}$$

$$\int \frac{\ln ax}{x}dx = \frac{1}{2}(\ln ax)^2 \tag{43}$$

$$\int \ln(ax+b)dx = \left(x + \frac{b}{a}\right)\ln(ax+b) - x, a \neq 0 \tag{44}$$

$$\int \ln(x^2+a^2)\,\mathrm{d}x = x\ln(x^2+a^2) + 2a\tan^{-1}\frac{x}{a} - 2x \tag{45}$$

$$\int \ln(x^2-a^2)\,\mathrm{d}x = x\ln(x^2-a^2) + a\ln\frac{x+a}{x-a} - 2x \tag{46}$$

$$\int \ln\left(ax^2+bx+c\right)\mathrm{d}x = \frac{1}{a}\sqrt{4ac-b^2}\tan^{-1}\frac{2ax+b}{\sqrt{4ac-b^2}} - 2x + \left(\frac{b}{2a} + x\right)\ln\left(ax^2+bx+c\right) \tag{47}$$

$$\int x\ln(ax+b)dx = \frac{bx}{2a} - \frac{1}{4}x^2 + \frac{1}{2}\left(x^2 - \frac{b^2}{a^2}\right)\ln(ax+b) \tag{48}$$

$$\int x\ln\left(a^2-b^2x^2\right)\mathrm{d}x = -\frac{1}{2}x^2 + \frac{1}{2}\left(x^2 - \frac{a^2}{b^2}\right)\ln\left(a^2-b^2x^2\right) \tag{49}$$

Integrals with Exponentials

$$\int e^{ax}dx = \frac{1}{a}e^{ax} \tag{50}$$

$$\int \sqrt{x}e^{ax}dx = \frac{1}{a}\sqrt{x}e^{ax} + \frac{i\sqrt{\pi}}{2a^{3/2}}\mathrm{erf}\left(i\sqrt{ax}\right), \quad \text{where } \mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt \tag{51}$$

$$\int xe^x dx = (x-1)e^x \tag{52}$$

$$\int xe^{ax}dx = \left(\frac{x}{a} - \frac{1}{a^2}\right)e^{ax} \tag{53}$$

$$\int x^2 e^x dx = \left(x^2 - 2x + 2\right)e^x \tag{54}$$

$$\int x^2 e^{ax}dx = \left(\frac{x^2}{a} - \frac{2x}{a^2} + \frac{2}{a^3}\right)e^{ax} \tag{55}$$

$$\int x^3 e^x dx = \left(x^3 - 3x^2 + 6x - 6\right)e^x \tag{56}$$

$$\int x^n e^{ax}\,\mathrm{d}x = \frac{x^n e^{ax}}{a} - \frac{n}{a}\int x^{n-1}e^{ax}\,\mathrm{d}x \tag{57}$$

$$\int x^n e^{ax}\,\mathrm{d}x = \frac{(-1)^n}{a^{n+1}}\Gamma[1+n, -ax], \quad \text{where } \Gamma(a,x) = \int_x^\infty t^{a-1}e^{-t}\,\mathrm{d}t \tag{58}$$

$$\int e^{ax^2}\,\mathrm{d}x = -\frac{i\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(ix\sqrt{a}\right) \tag{59}$$

$$\int e^{-ax^2}\,\mathrm{d}x = \frac{\sqrt{\pi}}{2\sqrt{a}}\mathrm{erf}\left(x\sqrt{a}\right) \tag{60}$$

$$\int xe^{-ax^2}\,\mathrm{d}x = -\frac{1}{2a}e^{-ax^2} \tag{61}$$

$$\int x^2 e^{-ax^2}\,\mathrm{d}x = \frac{1}{4}\sqrt{\frac{\pi}{a^3}}\mathrm{erf}(x\sqrt{a}) - \frac{x}{2a}e^{-ax^2} \tag{62}$$

Integrals with Trigonometric Functions

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax \qquad (63)$$

$$\int \sin^2 ax \, dx = \frac{x}{2} - \frac{\sin 2ax}{4a} \qquad (64)$$

$$\int \sin^n ax \, dx =$$
$$-\frac{1}{a}\cos ax \; {}_2F_1\left[\frac{1}{2}, \frac{1-n}{2}, \frac{3}{2}, \cos^2 ax\right] \qquad (65)$$

$$\int \sin^3 ax \, dx = -\frac{3\cos ax}{4a} + \frac{\cos 3ax}{12a} \qquad (66)$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax \qquad (67)$$

$$\int \cos^2 ax \, dx = \frac{x}{2} + \frac{\sin 2ax}{4a} \qquad (68)$$

$$\int \cos^p ax \, dx = -\frac{1}{a(1+p)}\cos^{1+p} ax \times$$
$$\quad {}_2F_1\left[\frac{1+p}{2}, \frac{1}{2}, \frac{3+p}{2}, \cos^2 ax\right] \qquad (69)$$

$$\int \cos^3 ax \, dx = \frac{3\sin ax}{4a} + \frac{\sin 3ax}{12a} \qquad (70)$$

$$\int \cos ax \sin bx \, dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)}, a \neq b \qquad (71)$$

$$\int \sin^2 ax \cos bx \, dx = -\frac{\sin[(2a-b)x]}{4(2a-b)}$$
$$\quad + \frac{\sin bx}{2b} - \frac{\sin[(2a+b)x]}{4(2a+b)} \qquad (72)$$

$$\int \sin^2 x \cos x \, dx = \frac{1}{3}\sin^3 x \qquad (73)$$

$$\int \cos^2 ax \sin bx \, dx = \frac{\cos[(2a-b)x]}{4(2a-b)} - \frac{\cos bx}{2b}$$
$$\quad - \frac{\cos[(2a+b)x]}{4(2a+b)} \qquad (74)$$

$$\int \cos^2 ax \sin ax \, dx = -\frac{1}{3a}\cos^3 ax \qquad (75)$$

$$\int \sin^2 ax \cos^2 bx \, dx = \frac{x}{4} - \frac{\sin 2ax}{8a} - \frac{\sin[2(a-b)x]}{16(a-b)}$$
$$\quad + \frac{\sin 2bx}{8b} - \frac{\sin[2(a+b)x]}{16(a+b)} \qquad (76)$$

$$\int \sin^2 ax \cos^2 ax \, dx = \frac{x}{8} - \frac{\sin 4ax}{32a} \qquad (77)$$

$$\int \tan ax \, dx = -\frac{1}{a}\ln\cos ax \qquad (78)$$

$$\int \tan^2 ax \, dx = -x + \frac{1}{a}\tan ax \qquad (79)$$

$$\int \tan^n ax \, dx = \frac{\tan^{n+1} ax}{a(1+n)} \times$$
$$\quad {}_2F_1\left(\frac{n+1}{2}, 1, \frac{n+3}{2}, -\tan^2 ax\right) \qquad (80)$$

$$\int \tan^3 ax \, dx = \frac{1}{a}\ln\cos ax + \frac{1}{2a}\sec^2 ax \qquad (81)$$

$$\int \sec x \, dx = \ln|\sec x + \tan x| = 2\tanh^{-1}\left(\tan\frac{x}{2}\right) \qquad (82)$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax \qquad (83)$$

$$\int \sec^3 x \, dx = \frac{1}{2}\sec x \tan x + \frac{1}{2}\ln|\sec x + \tan x| \qquad (84)$$

$$\int \sec x \tan x \, dx = \sec x \qquad (85)$$

$$\int \sec^2 x \tan x \, dx = \frac{1}{2}\sec^2 x \qquad (86)$$

$$\int \sec^n x \tan x \, dx = \frac{1}{n}\sec^n x, n \neq 0 \qquad (87)$$

$$\int \csc x \, dx = \ln\left|\tan\frac{x}{2}\right| = \ln|\csc x - \cot x| + C \qquad (88)$$

$$\int \csc^2 ax \, dx = -\frac{1}{a}\cot ax \qquad (89)$$

$$\int \csc^3 x \, dx = -\frac{1}{2}\cot x \csc x + \frac{1}{2}\ln|\csc x - \cot x| \qquad (90)$$

$$\int \csc^n x \cot x \, dx = -\frac{1}{n}\csc^n x, n \neq 0 \qquad (91)$$

$$\int \sec x \csc x \, dx = \ln|\tan x| \qquad (92)$$

Products of Trigonometric Functions and Monomials

$$\int x\cos x \, dx = \cos x + x\sin x \qquad (93)$$

$$\int x\cos ax \, dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax \qquad (94)$$

$$\int x^2 \cos x \, dx = 2x\cos x + \left(x^2 - 2\right)\sin x \qquad (95)$$

$$\int x^2 \cos ax \, dx = \frac{2x\cos ax}{a^2} + \frac{a^2 x^2 - 2}{a^3}\sin ax \qquad (96)$$

$$\int x^n \cos x \, dx = -\frac{1}{2}(i)^{n+1}\left[\Gamma(n+1, -ix)\right.$$
$$\quad \left. +(-1)^n \Gamma(n+1, ix)\right] \qquad (97)$$

$$\int x^n \cos ax \, dx = \frac{1}{2}(ia)^{1-n}\left[(-1)^n \Gamma(n+1, -iax)\right.$$
$$\quad \left. -\Gamma(n+1, ixa)\right] \qquad (98)$$

$$\int x\sin x \, dx = -x\cos x + \sin x \qquad (99)$$

$$\int x\sin ax \, dx = -\frac{x\cos ax}{a} + \frac{\sin ax}{a^2} \qquad (100)$$

$$\int x^2 \sin x \, dx = \left(2 - x^2\right)\cos x + 2x\sin x \qquad (101)$$

$$\int x^2 \sin ax \, dx = \frac{2 - a^2 x^2}{a^3}\cos ax + \frac{2x\sin ax}{a^2} \qquad (102)$$

$$\int x^n \sin x \, dx = -\frac{1}{2}(i)^n\left[\Gamma(n+1, -ix) - (-1)^n\Gamma(n+1, -ix)\right] \qquad (103)$$

Products of Trigonometric Functions and Exponentials

$$\int e^x \sin x \, dx = \frac{1}{2}e^x(\sin x - \cos x) \qquad (104)$$

$$\int e^{bx}\sin ax \, dx = \frac{1}{a^2 + b^2}e^{bx}(b\sin ax - a\cos ax) \qquad (105)$$

$$\int e^x \cos x \, dx = \frac{1}{2}e^x(\sin x + \cos x) \qquad (106)$$

$$\int e^{bx}\cos ax \, dx = \frac{1}{a^2 + b^2}e^{bx}(a\sin ax + b\cos ax) \qquad (107)$$

$$\int xe^x \sin x \, dx = \frac{1}{2}e^x(\cos x - x\cos x + x\sin x) \qquad (108)$$

$$\int xe^x \cos x \, dx = \frac{1}{2}e^x(x\cos x - \sin x + x\sin x) \qquad (109)$$

Integrals of Hyperbolic Functions

$$\int \cosh ax \, dx = \frac{1}{a}\sinh ax \qquad (110)$$

$$\int e^{ax}\cosh bx \, dx =$$
$$\begin{cases} \dfrac{e^{ax}}{a^2 - b^2}[a\cosh bx - b\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} + \dfrac{x}{2} & a = b \end{cases} \qquad (111)$$

$$\int \sinh ax \, dx = \frac{1}{a}\cosh ax \qquad (112)$$

$$\int e^{ax}\sinh bx \, dx =$$
$$\begin{cases} \dfrac{e^{ax}}{a^2 - b^2}[-b\cosh bx + a\sinh bx] & a \neq b \\ \dfrac{e^{2ax}}{4a} - \dfrac{x}{2} & a = b \end{cases} \qquad (113)$$

$$\int e^{ax}\tanh bx \, dx =$$
$$\begin{cases} \dfrac{e^{(a+2b)x}}{(a+2b)}{}_2F_1\left[1 + \dfrac{a}{2b}, 1, 2 + \dfrac{a}{2b}, -e^{2bx}\right] \\ \quad -\dfrac{1}{a}e^{ax}{}_2F_1\left[\dfrac{a}{2b}, 1, 1E, -e^{2bx}\right] & a \neq b \\ \dfrac{e^{ax} - 2\tan^{-1}[e^{ax}]}{a} & a = b \end{cases} \qquad (114)$$

$$\int \tanh ax \, dx = \frac{1}{a}\ln\cosh ax \qquad (115)$$

$$\int \cos ax \cosh bx \, dx = \frac{1}{a^2 + b^2}[a\sin ax \cosh bx$$
$$\quad + b\cos ax \sinh bx] \qquad (116)$$

$$\int \cos ax \sinh bx \, dx = \frac{1}{a^2 + b^2}[b\cos ax \cosh bx +$$
$$\quad a\sin ax \sinh bx] \qquad (117)$$

$$\int \sin ax \cosh bx \, dx = \frac{1}{a^2 + b^2}[-a\cos ax \cosh bx +$$
$$\quad b\sin ax \sinh bx] \qquad (118)$$

$$\int \sin ax \sinh bx \, dx = \frac{1}{a^2 + b^2}[b\cosh bx \sin ax -$$
$$\quad a\cos ax \sinh bx] \qquad (119)$$

$$\int \sinh ax \cosh ax \, dx = \frac{1}{4a}[-2ax + \sinh 2ax] \qquad (120)$$

$$\int \sinh ax \cosh bx \, dx = \frac{1}{b^2 - a^2}[b\cosh bx \sinh ax$$
$$\quad - a\cosh ax \sinh bx] \qquad (121)$$