# ECN3019 Games Theory – Assessment 1

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## Contents



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## I. Game Theory and p-Beauty Contest Game

Game theory offers a means for analysing strategic interactions. The strategies are the result of a reasoning process in which the players incorporate the behaviour of the other players in their decisions.

The 'p-Beauty Contest' game is an example conducted in the first place by Nagel in 1995 that can be analysed by the game theory. Its principle is to ask each participant to choose a number in the closed interval [0,100]. A mean is computed from all the chosen numbers and multiplied by a known positive parameter *p*: this is the target of all players. The winner would be the person who has picked the closest number to this target**.** This kind of game is categorized by the game theorists as a simultaneous-move game where all players must decide at the same time. The 'p-Beauty Contest' game is the result of a decision-making process where the players must make their decisions by guessing about the other players' choices.

The game is played for a certain number of rounds. After each round, the results, i.e. all chosen numbers, the mean *m* and the winning number are exposed to the players as a feedback. Nagel conducted four sessions of the game repeated four times with p = 2/3 and noticed a convergence. When $0 \leq p < 1$, the game theory predicts a unique Nash equilibrium, i.e. an equilibrium where each player chooses his "best response" to the other's choice. This equilibrium occurs when all the players announce zero.

Nevertheless, all players might not use the Nash Equilibrium strategy initially. Out-of-equilibrium answers may be explained by another reasoning strategy involving a finite depth of reasoning on players' beliefs about one another. This kind of strategy can be thought of as a degree of belief strategy. An irrational and naive degree of strategy would be for the player to choose a number randomly. In this case, the player forms a zero-order belief. If everyone follows this kind of beliefs, the average of all chosen numbers should be 50. This can be extended to the first-order beliefs, where the player forms his beliefs by considering that all the other players choose a number randomly. This means that the best answer for someone following this form of beliefs is 50 (i.e. the average obtained by choosing random numbers between 0 and 100) times *p*. If we extend it order by order, this reasoning can be continued up to k-order beliefs where the number chosen would be 50 times $p^k$ and the logical conclusion of this would be the Nash equilibrium zero.

## II. The results of the experimentation

The data studied concern the guessing of 30 individuals playing the 'p-Beauty Contest' game. Three rounds have been played and the behaviours of the players have been saved in an Excel spreadsheet. Hence, the aim of this section is to compare these results with the theory behind it. Do the players' answers correspond to the statement of the game theory's economists? We will present each round one by one and show how reality and theory are related.

The three following tables show the means and medians recorded after each round, that we would analyse more precisely in the next sub-sections. It is firstly important to stress that the means and medians decline significantly between periods, so is the winning number.

**Mean**

| 1st round | 2nd round | 3rd round |
|---|---|---|
| 20.80 | 10.93 | 5.47 |

**Median**

| 1st round | 2nd round | 3rd round |
|---|---|---|
| 15 | 7,5 | 4 |

**Mean times p = ⅔ and winning number**

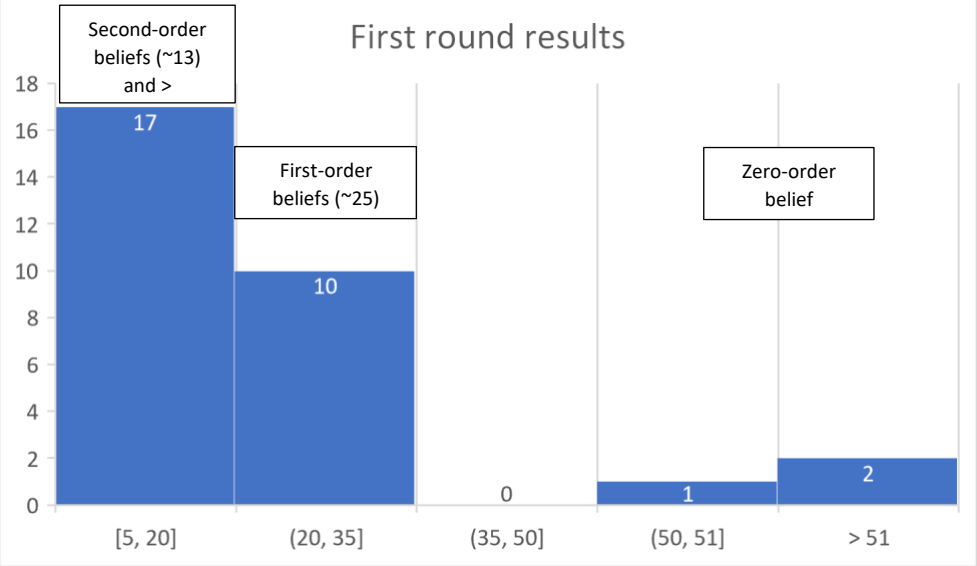
| | 1st round | 2nd round | 3rd round |
|---|---|---|---|
| | 10.40 | 5.465 ~ 5.5 | 2.735 ~ 2.7 |
| **Winning number** | 10 (two winners) | 6 (two winners) | 3 (three winners) |

## 1. First round

In the first round the players have no information about the behaviour of the others. This put them in a unique situation that they will not face in the second and third rounds. Hence, they must think by themselves and form their own expectations about the others' answers in a different way than in the later periods. Afterwards, the players would be able to gather information about the effective behaviour of the others and plan their choices according to this gathered information.

Therefore, the data collected in the first period need to be analysed in a different way than the subsequent periods.

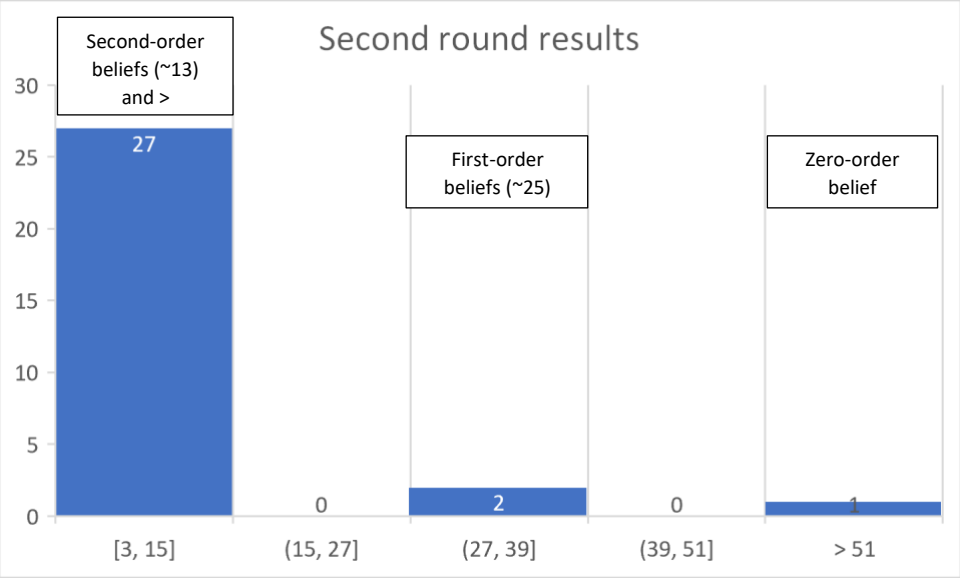
The following histogram (*page 3*) produced with Excel presents the data and the frequency of the numbers chosen in the first round. We can notice that most of the players chose to apply the two-order beliefs strategy, and deeper (interval [5,20]). It is a proof of overall rationality despite of the lack of information available in this special round. Nevertheless, three of them chose to adopt the zero-order belief strategy (numbers greater than 51), probably because they did not understand the game. It might change in the next round(s).




As a result, the mean of all the players' choices is 20.8 which means that the winners of this round are the ones that chose the closest number to 20.8*1/2 = 10.4. Two players chose the number 10 and win this round.

## 2. Second round

In this round, people have now the knowledge of the players overall behaviour and a better understanding of the game. They can adapt their choice according to the previous results.



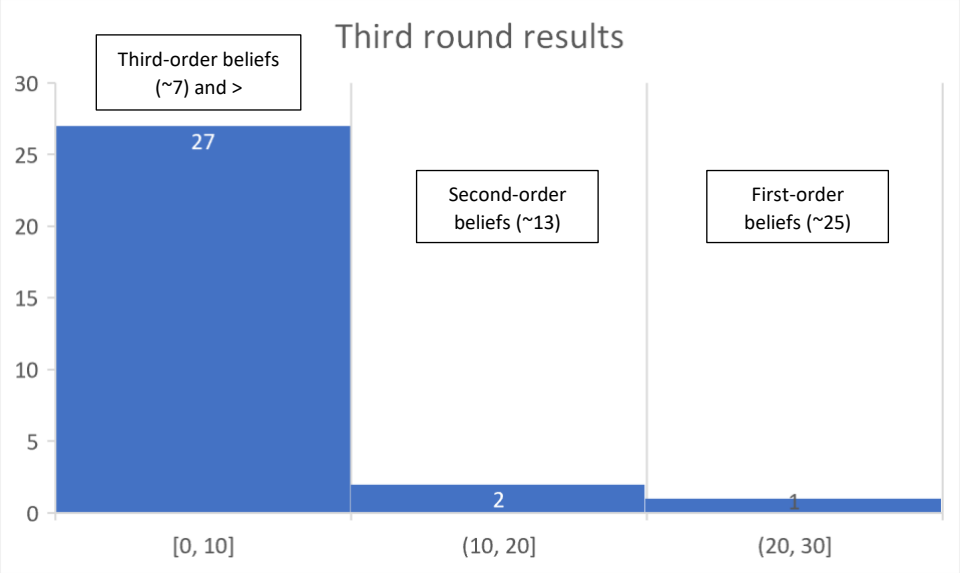

Again, nobody chose the Nash Equilibrium, as shown in the histogram above. But, the great majority of players chose a small number in the closed interval [3, 15]. Only 3 players still deviated. Hence, people managed to adopt a deeper order of belief closer to the previous winning number 10, the new mean being 10.93. Furthermore, we computed a t-test on each round's two means using the SPSS software and wrote down the results (*Table 1*). These results confirm that the means are statistically different, i.e. that the difference between the two means are unlikely to be due to chance.

| ***Rounds:*** | 1 | 2 |
|---|---|---|
| Mean | 20.80 | 10.93 |
| Std. Deviation | 15.226 | 10.399 |
| Std. Error of the Mean | 2.780 | 1.899 |
| ***T-test result*** | ***2.931 **** | |
| ***** significant at the 1% level | | |
| *The standard deviation gives us an idea of the spread of the data (choices in round 2 are hence less spread out than choices in round 1)* | | |

*Table 1: t-statistics results between rounds 1 and 2*

As a result, the winners of the second round are the ones that chose the closest number to 5.5, which is 6. Two players indeed picked this number and win this round.

## 3. Third round




This third and last round shows the beginning of a convergence towards the Nash Equilibrium. Players took account of the precedent winning number and concentrated their choice around it. Hence, they chose even smaller numbers and two of them chose the Nash Equilibrium 0. This can be better seen in *Table 2* that highlights again the statistical significance of the means' differences, and the decreasing standard deviation, between the two last rounds.

| ***Rounds:*** | **2** | **3** |
|---|---|---|
| Mean | 10.93 | 5.47 |
| Std. Deviation | 10.399 | 5.631 |
| Std. Error of the Mean | 1.899 | 1.028 |
| ***T-test result*** | ***2.532 **** | |
| ***** significant at the 10% level | | |

*Table 2: t-statistics results between round 2 and 3*

Perhaps, by repeating a few more rounds all the players would make the Nash Equilibrium their choices. Three rounds are not enough to make any conclusion on the players' final behaviour, but the results for now are consistent with the theorists' expectations.

# III. Critical evaluation of Kocher & Sutter (2005) and Weber (2003)


## Abstract

Weber along with Kocher and Sutter managed to extend Nagel's work in respectively 2003 and 2005. They experimented the p-Beauty contest game by introducing new studied characteristics in their analyses. Their papers study the convergence towards the Nash Equilibrium in different circumstances and lead to several conclusions. In one side, Kocher and Sutter concluded that groups are quicker learner than individuals in understanding the rationality of the beauty-contest game. In another side, Weber showed that exposing no feedback to the players after each round does not affect the existence of a learning process throughout the periods.


## 1. Introduction

Whilst Weber studied the learning behaviour of the players when they play several times the same game, Kocher and Sutter got interested in the process reasoning differences between groups and individuals. This section aims to analyse and compare the findings of those two studies.

Weber's objective is to explore the assumption that there exists a learning process in games resulting from the repeated experience, which we will consider as the hypothesis tested. Subjects in beauty-contest games learn from their past experiences and adapt their choices according to their understanding of the game. The information available could be crucial to determine their decisions. Thus, in his experiment, Weber chose to vary the quantity of information available and analyse the learning capacity of the players throughout the sessions.

As for Kocher and Sutter interests, they focus on the speed's differences of the game's understanding between group players and individual players. Hence, they are also interested in the learning process, but the target here is the type of decision maker. Kocher and Sutter have reasons to believe that groups will reason more deeply than individuals within a p beauty-contest game framework and will win more often when playing against individuals. These two facts correspond to the hypotheses that they want to test. They chose to run two experiments: the first one studies the competition among homogeneous decision makers and in the second one the competition becomes heterogeneous.

## 2. Experimental Design

Both studies chose a $p = 2/3$ and conduct their experiments using students having no knowledge in game theory. They both promised earnings for the winner of each period in order to motivate them.

Weber chose to divide each of his three sessions, composed of ten rounds, into four different treatments. One treatment is a replicate of Nagel's experiment and is called control condition (C). Another one gives no feedback to the subjects at all and is called 'no-feedback no-priming condition' (NP). The two other treatments receive some hints, called 'priming' by Weber, that could eventually help them understanding the game and choosing the best response. In fact, one of them informs the players that an average, a target number and a winner number has been calculated (no-feedback low-priming condition or LP). The other one suggests furthermore to guess and write down the other players choices (no-feedback high-priming condition or HP).

Kocher and Sutter performed two experiments with respectively four and three sessions constituted of four rounds. One experiment regroups two individual-only treatments and two group-only treatments and aims to compare the learning speed differences between the two types of decision makers. Groups were composed of three subjects per group. The other experiment confronts individuals and groups altogether and aims to compare the performances of groups and individuals when confronting. The conditions of guessing were made such that each group or individual could decide far away from the others, without being disturbed.

## 3. Results

The studies both gave a specific analysis on the first period which is different from the others in the sense that all the subjects without exception start the game without any information on the players' behaviour. Hence, no strongly significant differences between treatments are noticed in the first round. Indeed, Weber found no significant choices' differences using a two-tailed Kolmogorov-Smirnov test in all four treatments, even though the initial choices differ lightly. As for Kocher and Sutter, in both experiments, they found no significance in the differences between groups and individuals. Groups even chose higher numbers on average in the first round **(*)**.

The two studies also similarly decided to make use of different types of graphs and tables in order to illustrate their results. The two studies recorded for example the means and medians' choices, decreasing between the periods. The larger decrease was in the treatment C **(**)** in Weber's experiment and in the group treatment in Kocher and Sutter's experiment, which corresponds to their expectations.

**(*)** experiment 1: group mean = 34.9 / individual mean = 30.8; experiment 2: group mean = 40.7 / individual mean = 32.1

**(**)** decrease in mean between periods = 18.1 (C) while decrease = 17.1 (NP), decrease = 8.8 (LP), decrease = 17.2 (HP)

Weber noticed that all his subjects learned from their choices and modified them towards the Nash equilibrium, even when they have no access to a feedback. This learning behaviour is however faster for the treatment C which knows a frequency of lower choices significantly higher (one-tailed Kolmogorov-Smirnov test significant at the 1% level) than in the three no-feedback conditions. He also recorded extreme changes in choices between periods 1 and 10. Indeed, the big majority of players in all treatments decided to lower their choices rather than increasing them. In the condition LP, 8 subjects chose to increase their choice while 19 chose to decrease it. Similarly, in the conditions NP and HP, 2 and none chose respectively to increase their choice while 25 and 24 chose to decrease it. This shows that the assumption tested is true: players can learn from experience even though they are lacking information.

In their first experiment, Kocher and Sutter's comparison of chosen number's modifications (increased, decreased or unchanged) between rounds 2 and 1 shows a significant difference between individuals and groups. Indeed, 11 out of 35 individuals chose a higher number in round 2 than in round 1 while only 1 out of 35 groups did. Subsequently, groups chose systematically lower numbers in the next rounds and employed a significantly deeper order of reasoning than individuals. This confirms the hypothesis that groups are more rational at responding to obvious fact and superior in integrating feedback to their choices, despite of the results recorded in the first round. The second experiment regroups 24 individuals and 12 groups. Each group was paired with 2 individuals. The choices' evolution between the periods highlighted that the group choices were lower, and their win percentage was higher. Hence, the hypothesis that groups are on average better players than individuals by learning faster cannot be rejected. This is probably due to a better processing of information thanks to interactions.

## 4. Conclusion

To conclude, these two extensions of Nagel's experiment are important studies that add value and knowledge to the economics theories. In their studies, Weber, Kocher and Sutter tested innovative hypotheses and concluded with the help of a unique game. They proved that learning from repeated experience in a game such as p-beauty contest is feasible while interactive reasoning is a more efficient and quicker way to understand an intellective task. Other authors managed to extend Nagel's experiment, such as Agranov, Caplin and Tergiman that introduce a choice process protocol or Nagel himself that replaced the mean with either the maximum or the median. They all add a different point of view to the construction of a decision.

# References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