Nonlinear Iteration Procedure

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A modification of the nonlinear iteration procedure proposed by (Moon et al., 1999) is used as a global solution technique. Let us consider the interface between the nodes k and $k+1$. The face-averaged neutron current J^k_{x+} at the right face of the node k is expressed as

$$J^k_{x+} = -\widehat{D}^k \, (\Phi^k_{x+} - \bar{\Phi}^k) - \widehat{D}^k_{x+} \, (\Phi^k_{x+} + \bar{\Phi}^k), \tag{1}$$

where

$\widehat{D}^k = 2\, D^k / h^k$ is the nondimensional diffusion coefficient of the node k;

Φ^k_{x+} is face-averaged neutron current J^k_{x+} at the right face of the node;

$\widehat{D}^k_{x+}$ is the unknown nodal coupling diffusion coefficient at the right face of the node.

Respectively, the face-averaged neutron current J^{k+1}_{x-} at the left face of the node $k+1$ is expressed as

$$J^{k+1}_{x-} = \widehat{D}^{k+1} \, (\Phi^{k+1}_{x-} - \bar{\Phi}^{k+1}) - \widehat{D}^{k+1}_{x-} \, (\Phi^{k+1}_{x-} + \bar{\Phi}^{k+1}). \tag{2}$$

Using the flux and current continuity conditions, the face-averaged neutron flux is eliminated and the face-averaged neutron current is expressed as

$$J^k_{x+} = -\frac{(\widehat{D}^{k+1}\widehat{D}^k - \widehat{D}^{k+1}_{x-}\widehat{D}^k_{x+})}{\widehat{D}^{k+1} + \widehat{D}^{k+1}_{x-} + \widehat{D}^k + \widehat{D}^k_{x+}} \, (\bar{\Phi}^{k+1} - \bar{\Phi}^k) - \frac{(\widehat{D}^{k+1}\widehat{D}^k_{x+} - \widehat{D}^k\widehat{D}^{k+1}_{x-})}{\widehat{D}^{k+1} + \widehat{D}^{k+1}_{x-} + \widehat{D}^k + \widehat{D}^k_{x+}} \, (\bar{\Phi}^{k+1} + \bar{\Phi}^k). \tag{3}$$

Substituting this expression for the neutron current into a neutron balance equation we obtain a coarse-mesh finite-difference (CMFD) form of the neutron diffusion equations. Nonlinear iterations start with the nodal coupling coefficients $\widehat{D}^k_{x+}$ and $\widehat{D}^{k+1}_{x-}$ set to zero. After several iterations solving CMFD equations, a new approximation for the node-averaged neutron fluxes and the eigenvalue is obtained. Using equations of the nodal method values of the face-averaged neutron current are computed. Requiring that the CMFD method reproduce these values of the neutron current the nodal coupling diffusion coefficients are determined as

$$(\widehat{D}^k_{x+})_{gg} = \frac{-J^k_{gx+} + \widehat{D}^k_g\,(\bar{\Phi}^k_g - \bar{\Phi}^k_{gx+})}{\bar{\Phi}^k_{gx+} + \bar{\Phi}^k_g}.$$

$$(\widehat{D}^{k+1}_{x-})_{gg} = \frac{J^{k+1}_{gx-} + \widehat{D}^{k+1}_g\,(\bar{\Phi}^{k+1}_g - \bar{\Phi}^{k+1}_{gx-})}{\bar{\Phi}^{k+1}_{gx-} + \bar{\Phi}^{k+1}_g}.$$

The nonlinear iterations are performed till convergence, when the nodal coupling coefficients do not change anymore. As a result a global solution procedure is decoupled into the two processes: an iterative solution of the CMFD equations, where the node-averaged neutron fluxes and the eigenvalue are computed and a solution of the nodal equations to compute the face-averaged neutron currents and the nodal coupling coefficients.

Using Eqs. (1),(2) and the flux and current continuity we can also express the face-average neutron flux as

$$\Phi^k_{gx+} = \frac{(\widehat{D}^k - \widehat{D}^k_{x+})\,\bar{\Phi}^k + (\widehat{D}^{k+1} - \widehat{D}^{k+1}_{x-})\,\bar{\Phi}^{k+1}}{\widehat{D}^{k+1} + \widehat{D}^{k+1}_{x-} + \widehat{D}^k + \widehat{D}^k_{x+}}.$$

In the case of the node at the boundary, boundary conditions are used instead of the flux and current continuity. Considering, for example, a boundary at the right face of the node k. The boundary condition is written as

$$f\,\bar{\Phi}^k_{x-} - c\,J^k_{x-} = 0.$$

Using Eq. (1) the face-averaged flux is written as

$$J^k_{x+} = \frac{f\,(\widehat{D}^k - \widehat{D}^k_{x+})\,\bar{\Phi}^k}{f + c\,(\widehat{D}^k + \widehat{D}^k_{x+})},$$

and the face-averaged flux is given by

$$\Phi^k_{x+} = \frac{c\,(\widehat{D}^k - \widehat{D}^k_{x+})\,\bar{\Phi}^k}{f + c\,(\widehat{D}^k + \widehat{D}^k_{x+})}.$$

For the node k with the boundary at the left face of the node the boundary condition is written as

$$f\,\bar{\Phi}^k_{x+} + c\,J^k_{x+} = 0.$$

Using Eq. (2) the face-averaged flux is written as

$$J^k_{x-} = -\frac{f\,(\widehat{D}^k - \widehat{D}^k_{x-})\,\bar{\Phi}^k}{f + c\,(\widehat{D}^k + \widehat{D}^k_{x-})},$$

and the face-averaged flux is given by

$$\Phi_{x-}^{k} = \frac{c\,(\widehat{D}^{k} - \widehat{D}_{x-}^{k})\,\bar{\Phi}^{k}}{f + c\,(\widehat{D}^{k} + \widehat{D}_{x-}^{k})}.$$

References

Moon, K. S., Noh, J. M., Cho, N. Z., and Hong, S. G. (1999). Acceleration of the analytuic function expansion method by two-factor two-node nonlinear iteration. *Nucl. Sci. Eng.*, 132:194–202.