Disentangling Canted Phases and Phase Separation Regions with Spin Waves in Doped Manganites

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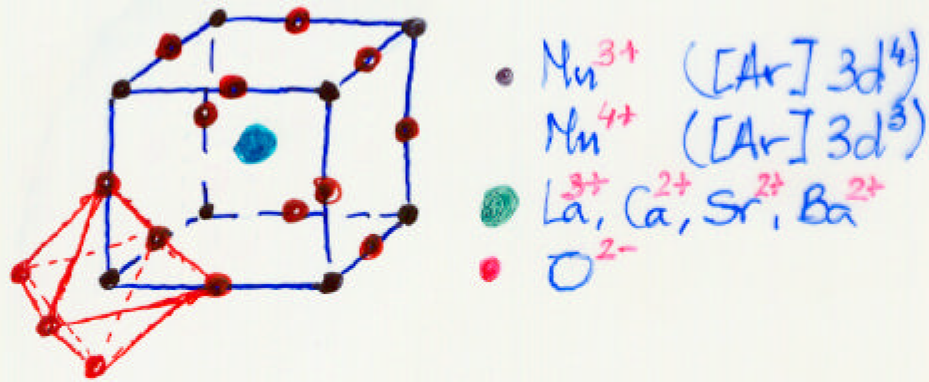
Outline

I. Continuum Double Exchange Model

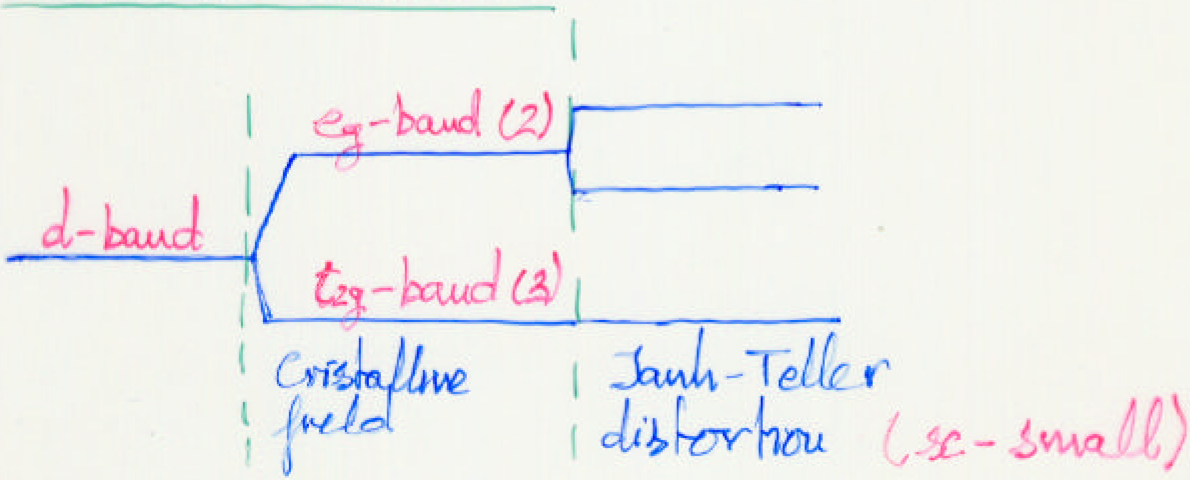
I.1. Manganites characteristics

general formulae $La_{1-x}A_xMnO_3$ (A-divalent)

Perovskite structure



Electronic Structure



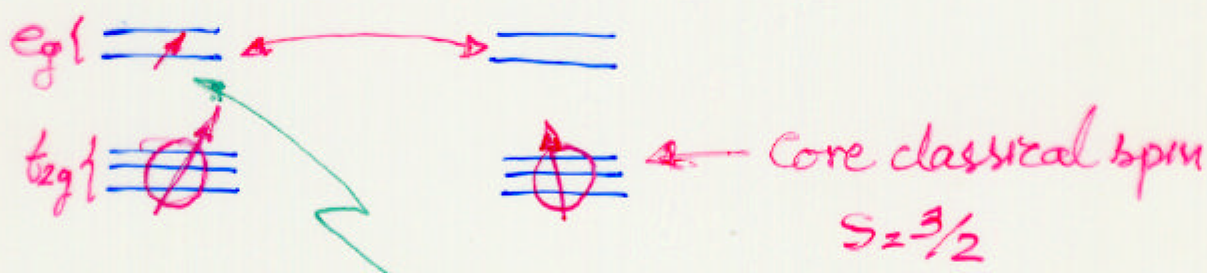
Magnetic Structure

$LaMnO_3$ $(x=0)$, $AMnO_3$ $(x=1)$ → AF, Semiconductor

$La_{1-x}A_xMnO_3$ $(0.2 < x < 0.4)$ → F, Conductor

Double Exchange Model

$Mn^{3+}\ (d^4)$ $\qquad$ $Mn^{4+}\ (d^3)$



The Hund interaction aligns the conduction electrons

Kondo problem with AF interaction

$$H = -t^{\ell} \sum_{\langle ij\rangle\sigma} c^{\dagger}_{i\sigma} c_{j\sigma} - J_H \sum_{i\sigma\sigma'} \vec{S}_i\, c^{\dagger}_{i\sigma} \frac{\vec{\sigma}_{\sigma\sigma'}}{2} c_{i\sigma'} + J^{\ell}_{AF} \sum_{\langle ij\rangle} \vec{S}_i \vec{S}_j$$

The hopping term produce
i) movement of conduction electrons
ii) change the sublattice

I.2. Continuum Double-Exchange Model (CDEM)

- CDEM describes low energy and long distance properties
- CDEM need fields slowly varying over the system

$\vec{M}_1(x), \Psi_1(x) \longrightarrow$ Sublattice 1

$\vec{M}_2(x), \Psi_2(x) \longrightarrow$ Sublattice 2

$$\mathcal{L}(x) = \Psi_1^\dagger(x)\left[(1+i\epsilon)i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H \frac{\vec{\sigma}}{2}\vec{M}_1(x)\right]\Psi_1(x) +$$
$$+ \Psi_2^\dagger(x)\left[(1+i\epsilon)i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H \frac{\vec{\sigma}}{2}\vec{M}_2(x)\right]\Psi_2(x) +$$
$$+ t\left(\Psi_1^\dagger(x)\Psi_2(x) + \Psi_2^\dagger(x)\Psi_1(x)\right) - J_{AF}\vec{M}_1(x)\vec{M}_2(x)$$

$t \sim z t^\ell$ $\qquad J_{AF} \sim \frac{z J_{AF}^\ell}{a^3}$

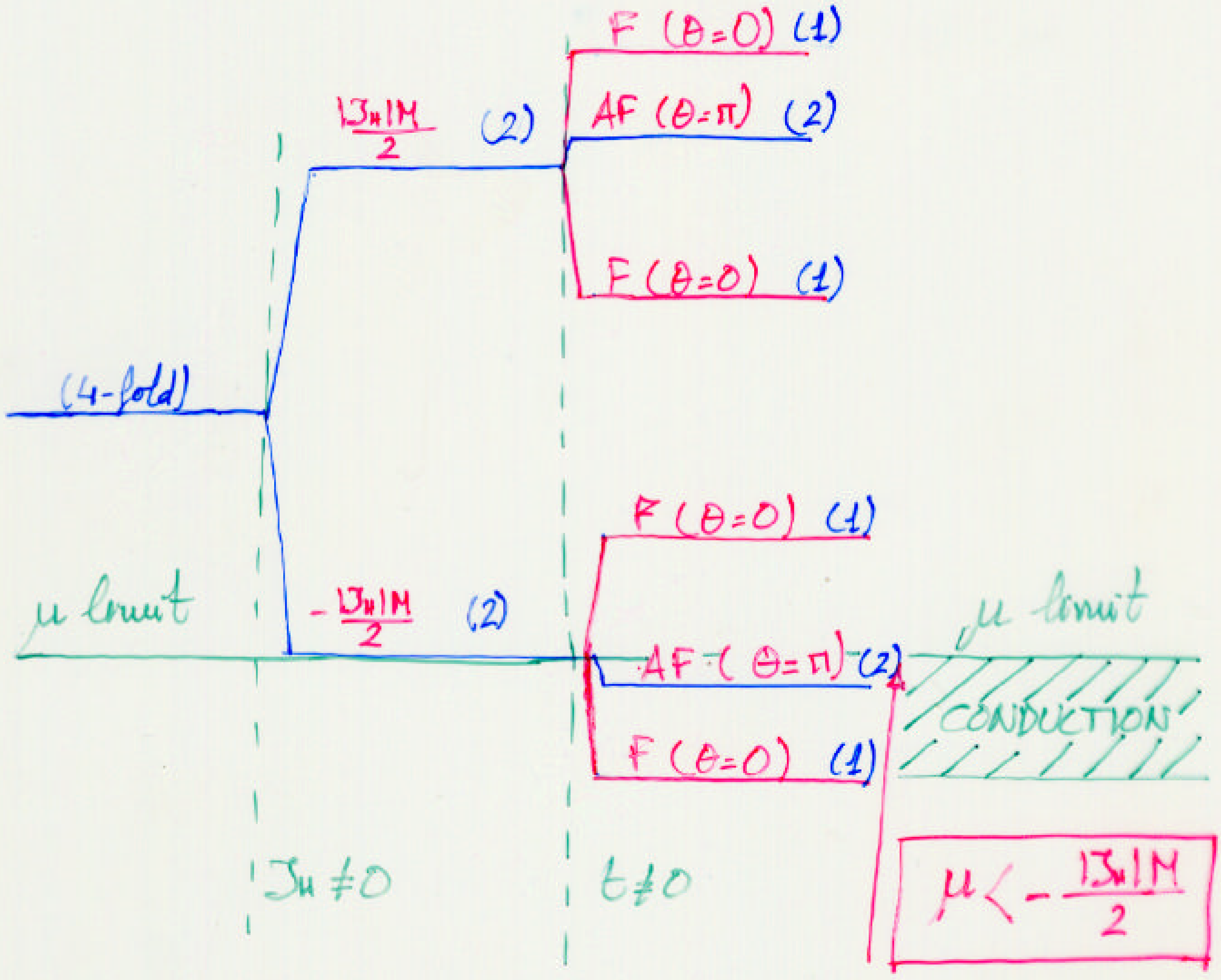
$J_H \sim J_H^\ell$

$$\frac{1}{2m} \sim a^2 t^\ell \sim z a t$$

$z = 6$ coordination

Energy levels of the CDEM

$$E = \frac{k^2}{2m} \pm \frac{|J_H|M}{2}\sqrt{1+\gamma^2 \pm 2\gamma\cos\frac{\theta}{2}} \qquad \gamma = \frac{2t}{J_H M}$$



$$\mu = -\frac{|J_H|M}{2}\sqrt{1+\gamma^2-2\gamma\, y_0} \qquad y_0 < y_0^{max} = \frac{\gamma}{2}$$

I.3. Effective Potential and Phase Diagram

Integration of fermion fields in the path integral for constant configurations $\vec{M}_1, \vec{M}_2$

$$V_{eff} = J_{AF}\,\vec{M}_1\cdot\vec{M}_2 + \frac{i}{VT}\,\mathrm{Tr}\log\hat{O}(\vec{M}_1,\vec{M}_2)$$

$$\hat{O}(\vec{M}_1,\vec{M}_2) = \begin{pmatrix} (1+i\varepsilon)i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H\frac{\vec{\sigma}}{2}\vec{M}_1 & t \\ t & (1+i\varepsilon)i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H\frac{\vec{\sigma}}{2}\vec{M}_2 \end{pmatrix}$$

The effective potential for $\gamma = \frac{2t}{J_H M} \ll 1$

$$V_{eff} = J_{AF}M^2\left[(2y^2-1) - A\left((y_0+y)^{5/2}\Theta(y_0+y) + (y_0-y)^{5/2}\Theta(y_0-y)\right)\right]$$

$$y \equiv \cos\frac{\theta}{2} \qquad A \equiv \frac{z^{3/2}}{15\pi^2}\frac{t}{J_{AF}M^2a^3}$$

Validity $\quad y_0 < y_0^{max} = \frac{\gamma}{2} \longrightarrow A > \frac{2z^{1/2}}{15\pi^2}\frac{z|J_H|M}{2(J_{AF}M^2a^3)}\,y_0^{max}$

Phase diagram (y_0, A): Minimization respect "y"

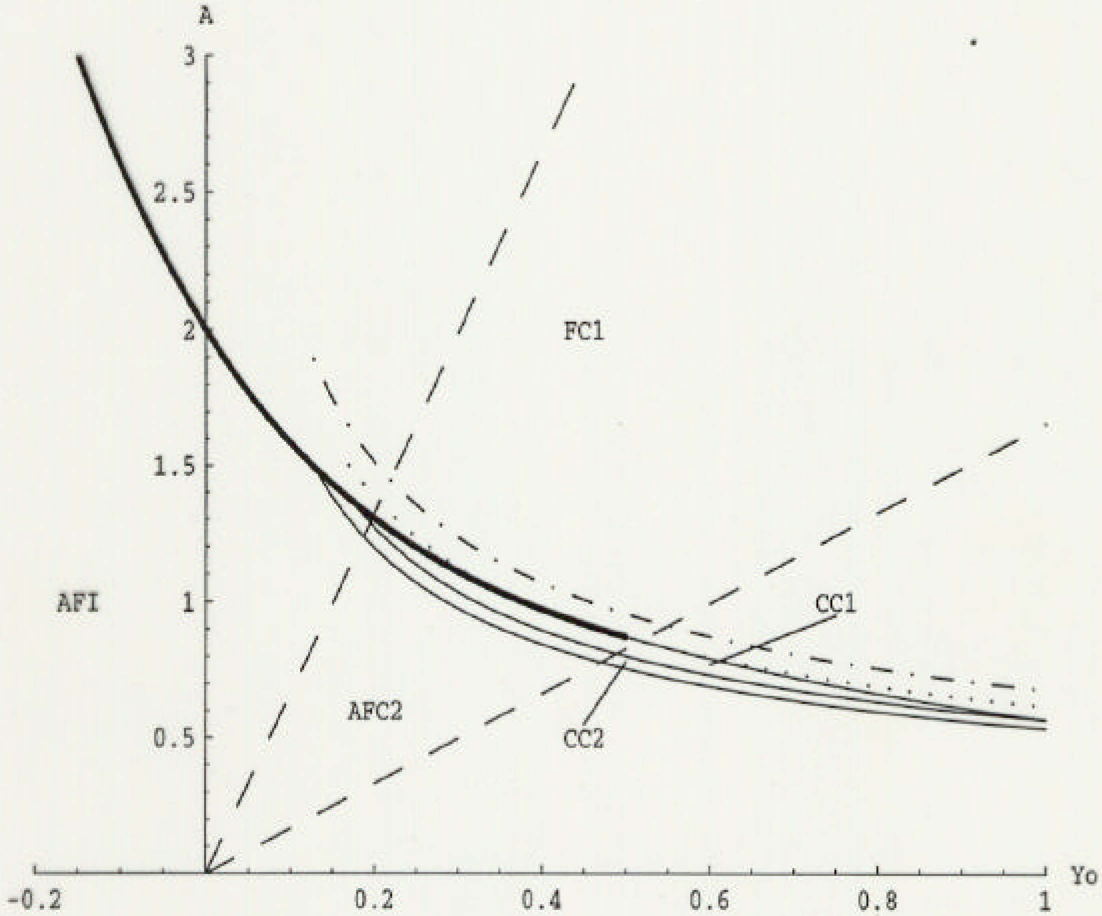


Figure 1: Phase diagram in the (y_0, A) plane. The thick solid line corresponds to first order transitions whereas the remaining solid lines to second order ones. The dotted and dashed dotted lines are the upper stability boundaries for the CC1 and CC2 phases respectively. The two dashed lines are the boundaries for the reliability of our model for $z|J_H|M/2(J_{AF}a^3M^2) \sim 50$ and $z|J_H|M/2(J_{AF}a^3M^2) \sim 200$ respectively. Only the part of the phase diagram to the left of the corresponding dashed line is trustworthy in each case.

$$x = -a^3 \frac{\partial V_{eff}}{\partial \mu} = -\frac{a^3}{t} \frac{\partial V_{eff}}{\partial y_0}.$$

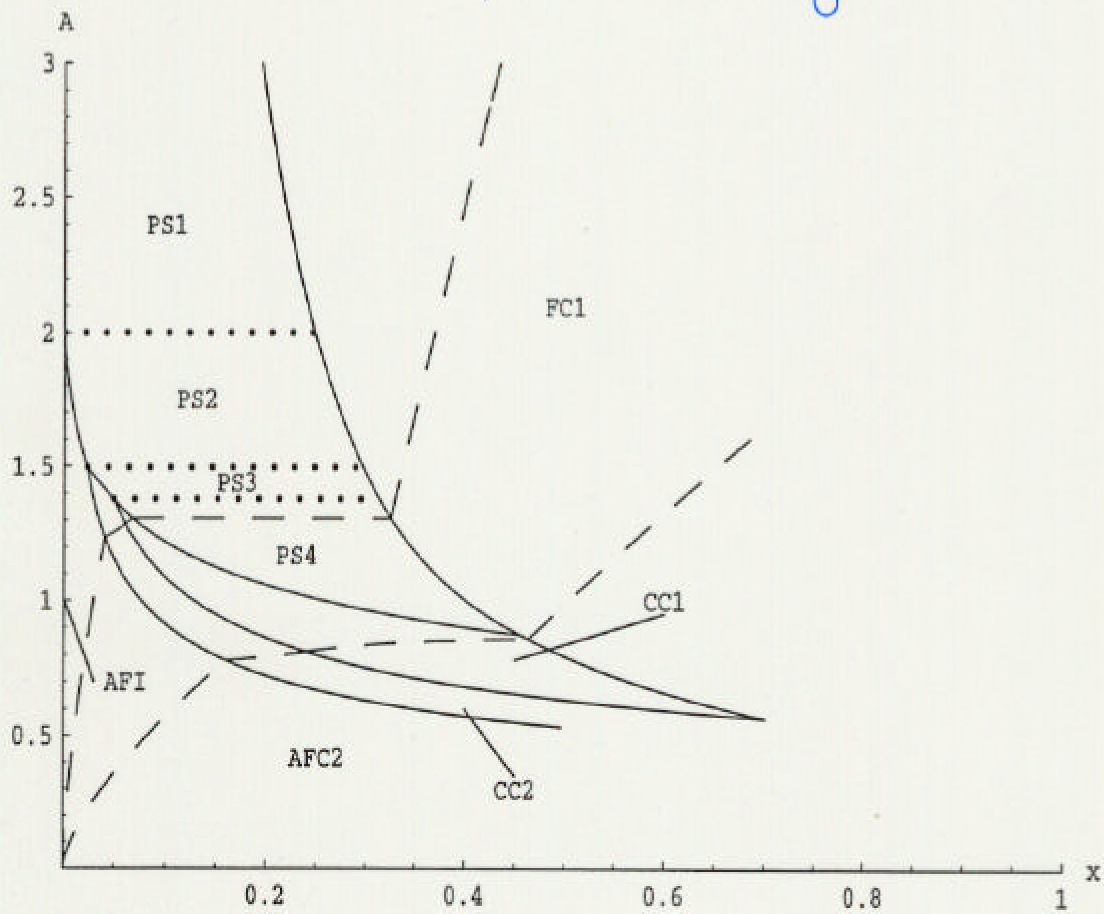


Figure 2: Phase diagram in the (x, A) plane. PSi $(i = 1, 2, 3, 4)$ indicates the new regions where the phases at their boundary may coexist. The $x = 0$ axis corresponds to the AFI phase. The two dashed lines are the boundaries for the reliability of our model for $z|J_H|M/2(J_{AF}a^3M^2) \sim 50$ and $z|J_H|M/2(J_{AF}a^3M^2) \sim 200$ respectively. Only the part of the phase diagram to the left of the corresponding dashed line is trustworthy in each case.

$$\frac{t}{J_{AF}a^3M^2} \sim 10\text{-}20 \Longrightarrow \boxed{A \sim 1\text{-}2}$$

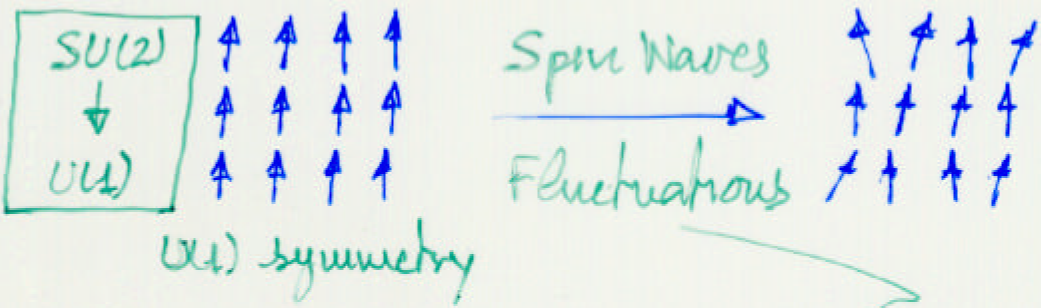
II. Interaction with Spin Waves

II.1. Goldstone Modes and Spin Waves

A magnetically ordered state breaks spontaneously the spin symmetry $(SU(2))$

In a situation of SSB gapless excitation (Goldstone modes) appear

SSB of spin symmetry $\Rightarrow$ Spin Waves



$$\Psi_0(x) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \longrightarrow \Psi(x) = U(x)\,\Psi_0(x)$$

$$U(x) = e^{i(\pi^1(x) S^1 + \pi^2(x) S^2)} \quad \in SU(2)/U(1)$$

$$U(x) \xrightarrow{SU(2)} g\,U(x)\,h^{+}(g,U) \qquad \begin{matrix} g \in SU(2) \\ h^{+} \in U(1) \end{matrix}$$

General SSB: $G \longrightarrow H$

Goldstone modes: $U(x) \in G/H$

$$U(x) \longrightarrow g\,U(x)\,h^{+}(g,U) \qquad \begin{cases} g \in G \\ h^{+} \in H \end{cases}$$

Space-time transformation

F, AF: $SU(2) \to U(1)$; $U(x) \in SU(2)/U(1)$

Canted: $SU(2) \to 1$; $V(x) \in SU(2)$

F: All the points are equivalent

Point group: $U(x) \to U(x)$
Time reversal: $U(x) \to U(x)C$

AF: Two sublattices with opposite magnetization

Point group { Same subl. $U(x) \to U(x)$
{ Change subl. $U(x) \to U(x)C$
Time reversal $U(x) \to U(x)C$

C: Two sublattices with magnetizations forming angle θ

Point group { Same subl. $V(x) \to V(x)$
{ Change subl. $V(x) \to V(x)R$
Time reversal $V(x) \to V(x)C$

II.2. Effective lagrangian

An expansion in derivatives of invariant term under SU(2) and space-time transformations

F, AF: $U^\dagger i\partial_\mu U = a_\mu^-(x) S_+ + a_\mu^+(x) S_- + a_\mu^3(x) S^3$

$$SU(2): \left\{ \begin{array}{l} a_\mu^-(x) \longrightarrow e^{i\theta(x)} a_\mu^-(x) \\ a_\mu^+(x) \longrightarrow e^{-i\theta(x)} a_\mu^+(x) \\ a_\mu^3(x) \longrightarrow a_\mu^3(x) + \partial_\mu\theta(x) \end{array} \right\} \text{U(1) local}$$

$$\text{Space-time}: \left\{ \begin{array}{ll} F: & a_0^3(x) \longrightarrow a_0^3(x) \\ AF: & a_0^3(x) \longrightarrow -a_0^3(x) \end{array} \right.$$

Only term with one time derivative which can appears on the effective lagrangian.

F: $\mathcal{L}(x) \sim \frac{1}{2} a_0^3 - \frac{1}{2m} a_i^- a_i^+ \sim \pi^- i\partial_t \pi^+ - \frac{1}{2m} \partial_i\pi^-\partial_i\pi^+$

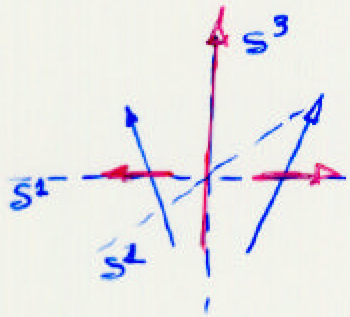
Schrödinger eq.: Quadratic dispersion relation

AF: $\mathcal{L}(x) \sim a_0^- a_0^+ - v^2 a_i^- a_i^+ \sim \partial_t\pi^-\partial_t\pi^+ - v^2 \partial_i\pi^-\partial_i\pi^+$

Wave eq: Linear dispersion relation

C.1 $V^{+} i\partial_\mu V = b^-_\mu(x) S_+ + b^+_\mu(x) S_- + b^3_\mu(x) S^3$

$SU(2): b^a_\mu(x) \longrightarrow b^a_\mu(x)$ Invariance



Ferromagnetic proyection in 3
Antiferromagnetic proyection in 1-2

$V(x) = U(x) H(x)$ $\quad U(x) = e^{i(\pi^- S_+ + \pi^+ S_-)} \in SU(2)/U(1)$
$\quad H(x) = e^{i\pi^3 S^3} \in U(1)$

$$\left.\begin{aligned} b^-_\mu(x) &= e^{-i\pi^3(x)} a^-_\mu(x) \\ b^+_\mu(x) &= e^{i\pi^3(x)} a^+_\mu(x) \\ b^3_\mu(x) &= a^3_\mu(x) - \partial_\mu \pi^3(x) \end{aligned}\right\}$$

$$\mathcal{L}(x) \sim \frac{1}{2} b^3_0 - A b^-_i b^+_i - \frac{B}{2}(b^+_i b^+_i + b^-_i b^-_i) + \frac{1}{2} b^3_0 b^3_0 - \frac{v^2}{2} b^3_i b^3_i$$

$$\mathcal{L}(x) \sim \pi^- i\partial_t \pi^+ - \frac{1}{2m'} \partial_i \pi^- \partial_i \pi^+ + \frac{1}{2}\partial_t \pi^3 \partial_t \pi^3 - \frac{v^2}{2} \partial_i \pi^3 \partial_i \pi^3$$

$\frac{1}{2m'} = \sqrt{A^2 - B^2}$

$\pi^+(x) \rightarrow$ Quadratic dispersion relation
$\pi^3(x) \rightarrow$ Linear dispersion relation

II.3. Interaction Spin Waves - Conduction fermions

$$\mathcal{L}(x) = (\Psi_1^+ \Psi_2^+) \begin{pmatrix} i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H \frac{\vec{\sigma}}{2} \vec{M}_1(x) & \\ t & i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H \frac{\vec{\sigma}}{2} \vec{M}_2(x) \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

Spin Waves are fluctuations in $\boxed{\vec{M}_i(x) = R^a{}_b(x)\, M_i^b}$

vacuum constant configuration

Separation of the interaction with spin waves:

$$\frac{\vec{\sigma}}{2} \vec{M}_i(x) = \frac{\sigma^a}{2} R^a{}_b(x)\, M_i^b = V(x) \frac{\vec{\sigma}}{2} V^+(x)\, \vec{M}_i$$

Representacion adjunta — Representación fundamental

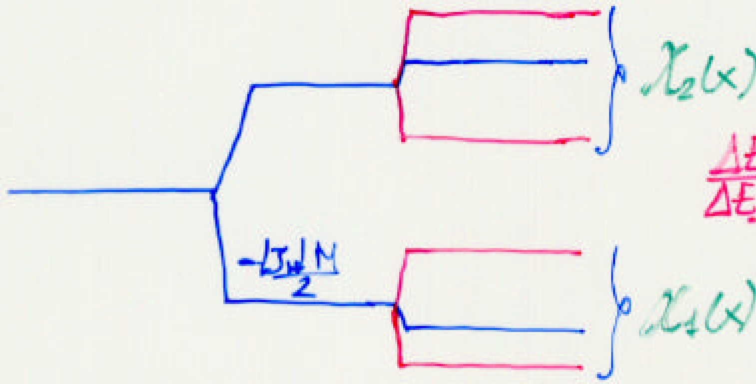
Change of variables: $\Psi_i(x) \longrightarrow V(x)\, \Psi_i(x)$

$$\mathcal{L}(x) = (\Psi_1^+ \Psi_2^+) \begin{pmatrix} i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H \frac{\vec{\sigma}}{2} \vec{M}_1 + \hat{O}^{sw} & t \\ t & i\partial_t + \frac{\partial_i^2}{2m} + \mu + J_H \frac{\vec{\sigma}}{2} \vec{M}_2 + \hat{O}^{sw} \end{pmatrix} \begin{pmatrix} \Psi_1 \\ \Psi_2 \end{pmatrix}$$

$$\boxed{\begin{aligned} \hat{O}^{sw} = {} & \left(b_0^- - \frac{1}{2m}\{i\partial_i, b_i^-\}\right) S_+ + \left(b_0^+ - \frac{1}{2m}\{i\partial_i, b_i^+\}\right) S_- + \\ & + \left(b_0^3 - \frac{1}{2m}\{i\partial_i, b_i^3\}\right) S^3 - \frac{1}{2m}\left(b_i^- b_i^+ + \frac{1}{4} b_i^3 b_i^3\right) \end{aligned}}$$

We can rewrite the lagrangian in terms of the eigenstates which diagonalize the ground state interaction

$$\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix} = P \begin{pmatrix}\Psi_1\\ \Psi_2\end{pmatrix}$$



$$\frac{\Delta E_t}{\Delta E_{J_H}} = \frac{2t}{|J_H|M} = \gamma \ll 1$$

$$\mathcal{L}(x) = (\chi_1^+ \; \chi_2^+)\left[\begin{pmatrix}\hat{L}_1 & \\ & \hat{L}_2\end{pmatrix} + \begin{pmatrix}\hat{W}_{11} & \hat{W}_{12}\\ \hat{W}_{21} & \hat{W}_{22}\end{pmatrix}\right]\begin{pmatrix}\chi_1\\ \chi_2\end{pmatrix}$$

$$\begin{pmatrix}\hat{W}_{11} & \hat{W}_{12}\\ \hat{W}_{21} & \hat{W}_{22}\end{pmatrix} = P\begin{pmatrix}\hat{O}^{sw} & \\ & \hat{O}^{sw}\end{pmatrix}P^+$$

Relevant physics happens around $\chi_1(x)$
We integrate out the states $\chi_2(x)$

For each state F, AF, C

$$\boxed{\begin{aligned}\mathcal{L}(x) &= \chi_1^+(x)\,(\hat{L}_1 + \hat{W}_{11})\,\chi_1(x)\\ \hat{L}_1 &= i\partial_t + \frac{\partial_i^2}{2m} + \mu + \frac{|J_H|M}{2}\sqrt{1+\gamma^2 \pm 2\gamma\cos\frac{\theta}{2}}\end{aligned}}$$

Canted

$$
\begin{aligned}
\hat{W}_{11} \;=\; & -\frac{1}{2mf_\pi^2}\left(\partial_i\pi^-\partial_i\pi^+ + \frac{f_\pi^2}{4f_3^2}\partial_i\pi^3\partial_i\pi^3\right) \\
& + \frac{1}{2f_\pi^2}\cos\frac{\theta}{2}\Big[\pi^-\left(i\partial_0\pi^+ + \frac{1}{2m}\{\partial_i,\partial_i\pi^+\}\right) - \pi^+\left(i\partial_0\pi^- + \frac{1}{2m}\{\partial_i,\partial_i\pi^-\}\right) \\
& \qquad + \frac{if_\pi^2}{f_3}\left(\underline{i\partial_0\pi^3 + \frac{1}{2m}\{\partial_i,\partial_i\pi^3\}}\right)\Big] \\
& + \frac{1}{2f_\pi f_3}\sin\frac{\theta}{2}\Big[if_3\left(\underline{i\partial_0\pi^- + i\partial_0\pi^+ + \frac{1}{2m}\{\partial_i,\partial_i\pi^- + \partial_i\pi^+\}}\right) \\
& \qquad + \pi^3\left(i\partial_0\pi^- - i\partial_0\pi^+ + \frac{1}{2m}\{\partial_i,\partial_i\pi^- - \partial_i\pi^+\}\right) \\
& \qquad + \frac{1}{2m}\partial_i\pi^3(\partial_i\pi^- - \partial_i\pi^+)\Big)\Big]\,(S_+ + S_-)
\end{aligned}
$$

θ angle dependence

Dependence on only one spin wave 

Ferromagnetic

$$
\begin{aligned}
\hat{W}_{11} \;=\; & -\frac{1}{2mf_\pi^2}\partial_i\pi^-\partial_i\pi^+ \\
& + \frac{1}{2f_\pi^2}\left[\pi^-\left(i\partial_0\pi^+ + \frac{1}{2m}\{\partial_i,\partial_i\pi^+\}\right) - \pi^+\left(i\partial_0\pi^- + \frac{1}{2m}\{\partial_i,\partial_i\pi^-\}\right)\right]
\end{aligned}
$$

Two spin waves interaction 

Antiferromagnetic

$$
\begin{aligned}
\hat{W}_{11} \;=\; & -\frac{1}{2mf_\pi^2}\partial_i\pi^-\partial_i\pi^+ \\
& + \frac{1}{f_\pi^2}\left[\pi^-\left(i\partial_0\pi^+ + \frac{1}{2m}\{\partial_i,\partial_i\pi^+\}\right) - \pi^+\left(i\partial_0\pi^- + \frac{1}{2m}\{\partial_i,\partial_i\pi^-\}\right)\right] S^3
\end{aligned}
$$

Two spin waves interaction 

III. Spin Waves in Doped Manganites

III.1. Heisenberg term Contribution

The derivative term to add to the CDEM is given by:

$$\mathcal{H} = -\frac{J_{AF}\,a^2}{2z}\,\partial_i \vec{M}_1(x)\,\partial_i \vec{M}_2(x)$$

In order to introduce the time derivative terms needed for the lagrangian we introduce:

TOTAL: $\vec{\Sigma}(x) = \frac{1}{2}\left(\vec{M}_1(x) + \vec{M}_2(x)\right)$

STAGGERED: $\vec{\Omega}(x) = \vec{M}_1(x) - \vec{M}_2(x)$

In terms of these fields the simplest lag.

$$\mathcal{L}^{(4)}(x) = \frac{1}{a^3\Sigma^2}\int_0^1 d\lambda\,\vec{\Sigma}(\lambda)\left(\partial_0\vec{\Sigma}(\lambda)\times\partial_\lambda\vec{\Sigma}(\lambda)\right) + \frac{J_{AF}\,a^2}{2z}\,\partial_i\vec{\Sigma}\,\partial_i\vec{\Sigma}$$
$$+\frac{z}{12 J_{AF}\,a^6\,\Omega^2}\left[\frac{1}{2}\,\partial_0\vec{\Omega}\,\partial_0\vec{\Omega} - \frac{3J_{AF}^2\,a^8\,\Omega^2}{2z^2}\,\partial_i\vec{\Omega}\,\partial_i\vec{\Omega}\right]$$

$\rightarrow$ Total magnetization: Ferromagnetic

Staggered magnetization: Antiferromagnetic

In terms of the spin waves fields $V(x)$

$$\vec{N}_1(x) = \mathrm{tr}\left(V^+(x)\,\vec{S}\,V(x)\,P_1\right)$$

$$\vec{N}_2(x) = \mathrm{tr}\left(V^+(x)\,\vec{S}\,V(x)\,P_2\right)$$

and after taking derivatives:

$$\mathcal{L}^{(1)} = \frac{2\Sigma}{a^3}\left[\frac{1}{2}b_0^3 + \frac{J_{AF}a^5}{8z\Sigma}\left(8\Sigma^2-\Omega^2\right) b_i^- b_i^+ + \frac{J_{AF}a^5\Omega^2}{8z\Sigma}\left(b_i^+ b_i^+ + b_i^- b_i^-\right)\right] + \frac{z}{6J_{AF}a^6}\left[\frac{1}{4}b_0^3 b_0^3 - \frac{3J_{AF}^2 a^7 \Omega^2}{4z^2} b_i^3 b_i^3\right]$$

III.2. Contribution from Conduction Fermions

The interaction with the fermions:

$$\mathcal{L}(x) = \chi_1^\dagger(x)\,(\hat{L}_1 + \hat{W}_{11})\,\chi_1(x)$$

ground state term Spin Waves term

$$\hat{W}_{11} \begin{cases} \text{F, AF: Two spin waves} \\ \text{C: One spin wave} \end{cases}$$

After integrating out the fermions we obtain an effective action, that up to two spin waves:

$$S_{eff}^{(2)} = -i\,\mathrm{Tr}\log(\hat{L}_1 + \hat{W}_{11}) =$$

$$= -i\,\mathrm{Tr}\log\hat{L}_1 \underbrace{- i\,\mathrm{Tr}(\hat{L}_1^{-1}\hat{W}_{11})}_{S_{eff}^{(2,1)}} + \underbrace{\frac{i}{2}\,\mathrm{Tr}(\hat{L}_1^{-1}\hat{W}_{11}\hat{L}_1^{-1}\hat{W}_{11})}_{S_{eff}^{(2,2)}} + \ldots$$

$$\boxed{\mathcal{L}^{(2,1)} = \frac{\varkappa}{f_\pi^2 a^3}\cos\frac{\theta}{2}\,\pi^- i\partial_0\pi^+ - \frac{\varkappa}{2ma^3}\left(\frac{1}{f_\pi^2}\partial_i\pi^-\partial_i\pi^+ + \frac{1}{2f_3^2}\partial_i\pi^3\partial_i\pi^3\right)}$$

We have considered: $\dfrac{\varkappa}{a^3} = -i\displaystyle\int\frac{dq}{(2\pi)^4}\,\mathrm{tr}\,L_1^{-1}(q)\,e^{iq_0\eta}$

In the calculation of the second term: $S_{eff}^{(2,2)}$
the Vacuum Polarization Tensor appears

$$\Pi_{ab}^{(i,j)}(p) \equiv -i \int \frac{dq}{(2\pi)^4} (p+q)^i \mathcal{L}_{1a}^{-1}(p+q)\, q^j\, \mathcal{L}_{1b}^{-1}(q)$$

$\hat{W}_H$ is a derivative coupling (contains at least one derivative)
$S_{eff}^{(2,2)}$ is second order in derivatives

It is enough to consider the Vacuum Polarization Tensor in the limit: $p \to 0$

$$\Pi_{aa}^{(0,0)}(0) = \frac{m k_a}{4\pi^2}\left[-2 + x_a \log\left|\frac{1+x_a}{1-x_a}\right| - i\pi |x_a|\, \Theta(1-|x_a|)\right]$$

$$\Pi_{aa}^{(0,3)}(0) = \frac{m v}{|\vec{p}|}\, \Pi_{aa}^{(0,0)}(0)$$

$$\sum_a \Pi_{aa}^{(3,3)}(0) = -\frac{m x}{a^3} + \left(\frac{m v}{|\vec{p}|}\right)^2 \sum_a \Pi_{aa}^{(0,0)}(0)$$

$$\Pi_{+-}^{(\kappa,\kappa)}(0) = -\frac{m}{15\pi^2}\, \frac{k_+^5\,\Theta(-\Omega_+) - k_-^5\,\Theta(-\Omega_-)}{k_+^2 - k_-^2} = -\frac{m J_{AF} M^2}{t}\, \Pi_{+-}$$

$$k_\pm \equiv \sqrt{-2m\,\Omega_\pm} \qquad x_a \equiv \frac{m v}{k_a |\vec{p}|}$$

According to the previous relations the time derivatives on the spin wave $\pi^3(x)$ cancels, avoiding the spontaneous decay of $\pi^3(x)$ into fermions

$$S_{eff}^{(2,2)} \sim -\int du\, dw\, e^{-ip(u-w)} \left\{ \frac{1}{4f_3^2} \cos^2\frac{\theta}{2} \sum_a \Pi_{aa}^{(0,0)}(0) \right.$$

$$\left[\partial_0 \pi^3(u)\, \partial_0 \pi^3(w) + 2\, \partial_0 \pi^3(u)\, \frac{\nu}{p^2}\, p^i \partial_i \pi^3(w) + \right.$$

$$\left.\left. + \frac{\nu}{p^2} p^i \partial_i \pi^3(u)\, \frac{\nu}{p^2} p^j \partial_j \pi^3(w) \right] \right\}$$

$$\frac{\nu}{p^2} p^i \partial_i \pi^3 = -\partial_0 \pi^3$$

The contribution of the other terms:

$$\mathcal{L}^{(2,2)} = \frac{J_{AF} M^2}{2mt f_\pi^2}\, \Pi_{+-} \sin^2\frac{\theta}{2} \left[\partial_i \pi^- \partial_i \pi^+ + \frac{1}{2} \left(\partial_i \pi^+ \partial_i \pi^+ + \partial_i \pi^- \partial_i \pi^- \right) \right]$$

$$+ \frac{\varkappa}{4ma^3 f_3^2} \cos^2\frac{\theta}{2}\, \partial_i \pi^3 \partial_i \pi^3$$

$$CC1: \quad \Pi_{+-} = \frac{4}{5} \left(\frac{6\pi^2 x}{z^{3/2}} \right)$$

$$CC2: \quad \Pi_{+-} = \frac{1}{\sqrt{3}} \frac{4}{5} \sqrt{ \frac{4}{5A} \left(\frac{6\pi^2 x}{z^{3/2}} \right) - \gamma^2 } + \frac{A}{2} \left(\frac{6\pi^2 x}{z^{3/2}} \right)$$

III.3. Dispersion Relation Parameters

In the canted case:

$$\mathcal{L}(x) = \pi^- i\partial_0\pi^+ - B\,\partial_i\pi^-\partial_i\pi^+ - \frac{C}{2}\left(\partial_i\pi^+\partial_i\pi^+ + \partial_i\pi^-\partial_i\pi^-\right)$$

$$+ \frac{1}{2}\partial_0\pi^3\partial_0\pi^3 - \frac{v^2}{2}\partial_i\pi^3\partial_i\pi^3$$

$$B = \frac{1}{2m}\frac{z^{3/2}}{15\pi^2 A}\frac{1}{(2N+x)y}\left[(1-3y^2) + \frac{5A}{2}\left(\frac{6\pi^2 x}{z^{3/2}}\right) - \Pi_{+-}(1-y^2)\right]$$

$$C = -\frac{1}{2m}\frac{z^{3/2}}{15\pi^2 A}\frac{1}{(2N+x)y}\left[1 + \Pi_{+-}\right](1-y^2)$$

$$\frac{1}{2m'} = \sqrt{B^2 - C^2} \sim \sqrt{2 - 4y^2 + \frac{5A}{4}\left(\frac{6\pi^2 x}{z^{3/2}}\right)}\cdot$$

$$\cdot\sqrt{\frac{5A}{2}\left(\frac{6\pi^2 x}{z^{3/2}}\right) - 2y^2 - 2\Pi_{+-}(1-y^2)}$$

$$v^2 = \frac{6J_{AF}a^3 N^2}{2mz}\frac{z^{3/2}}{15\pi^2 A}\frac{1}{N^2}\left[2 + \frac{5A}{2}\left(\frac{6\pi^2 x}{z^{3/2}}\right)\right](1-y^2)$$

In the Ferromagnetic limit

$$\mathcal{L} = \pi_i^- \partial_0 \pi^+ - \frac{1}{2m'} \partial_i \pi^- \partial_i \pi^+$$

$$\frac{1}{2m'} = \frac{1}{2m} \frac{z^{3/2}}{15\pi^2 A} \frac{1}{(2N+x)} \left[-2 + \frac{5A}{2} \left(\frac{6\pi^2 x}{z^{3/2}} \right) \right]$$

and the Antiferromagnetic limit

$$\mathcal{L} = \partial_0 \pi^- \partial_0 \pi^+ - v^2 \partial_i \pi^- \partial_i \pi^+$$

$$v^2 = \frac{6 J_{AF} a^2 N^2}{2mz} \frac{z^{3/2}}{15\pi^2 A} \frac{1}{N^2} \left[2 + \frac{5A}{2} \left(\frac{6\pi^2 x}{z^{3/2}} \right) \right]$$

IV. Canted Phases vs. Phase Separation

Phase Separation Region guess:

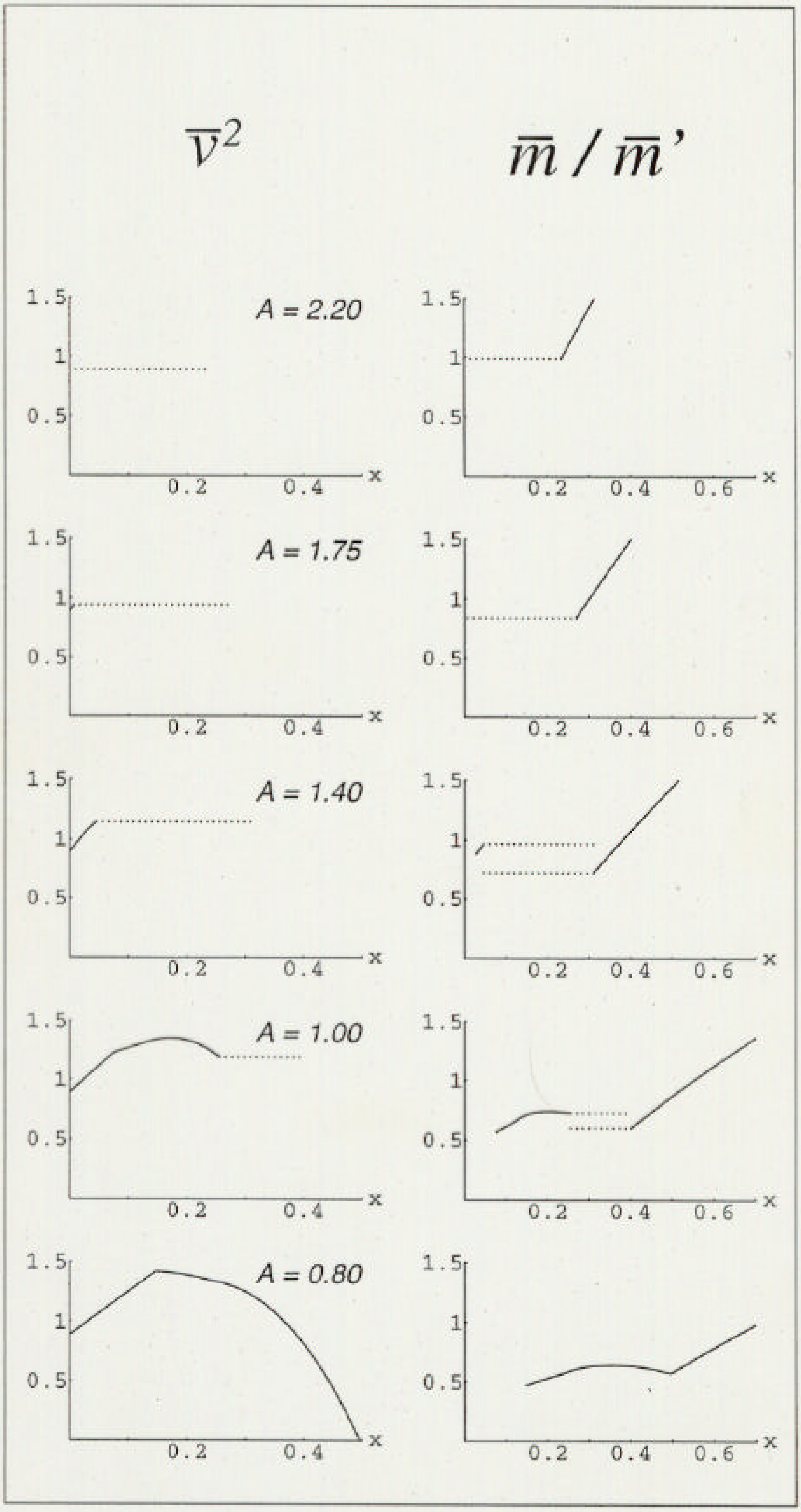
- Two macroscopic F and AF domains
- The interphase does not modifies qualitatively the properties

PS: One F and two AF branches
C: One F and one AF branches

PS: Splitting of the AF branches by a magnetic field
C: No modification of the AF branch by a magnetic field.

Different behaviour of the mass and velocity with the doping

PS: Figure A: 2,20
C: Figure A: 0,80



$\overline{v}^2$
$\overline{m} / \overline{m}'$
A = 2.20
A = 1.75
A = 1.40
A = 1.00
A = 0.80
1.5
1
0.5
0.2
0.4
0.6
x

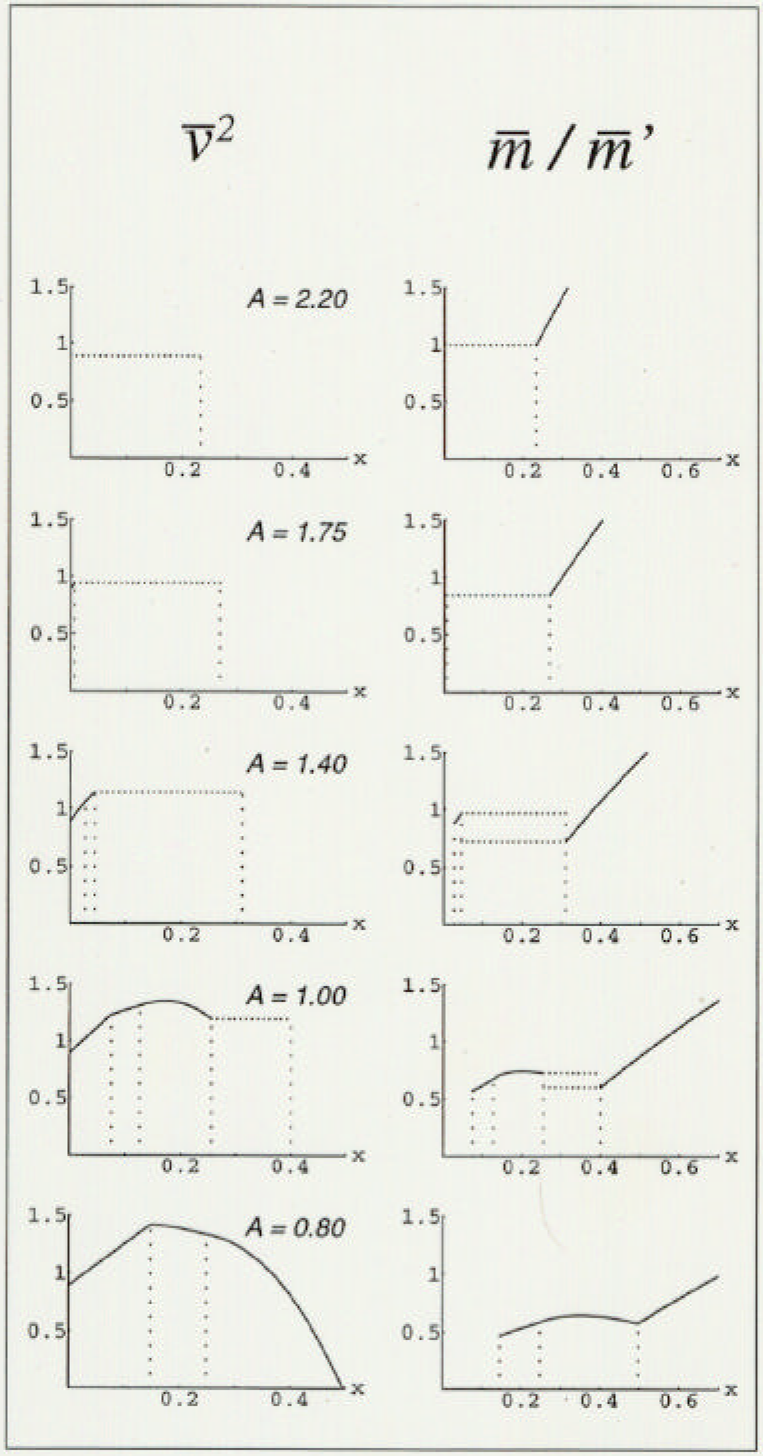


Figure 1: The dependence of the velocities and the masses with the doping for five different values of the parameter A ($\sim t/J_{AF}$). $\bar{v}^2 = (15\pi^2 A)2mzv^2/6z^{3/2}(J_{AF}a^3M^2)$ and $\bar{m}/\bar{m}' = (15\pi^2 A)m/z^{3/2}m'$. The horizontal dotted lines correspond to the phase separation regions, and the vertical dotted lines correspond to the phase transitions.

V. Conclusions

I. Continuum Double Exchange Model

- A simple model in the continuum was presented to describe the rich phase diagram for doped manganites

- Phase diagram presents

  AFI: Antiferromagnetic insulator
  AFC2: " conductor (2-bands)
  CC2: Canted " (2-bands)
  CC1: " " (1-band)
  FC1: Ferromagnetic " (1-band)
  PS: Phase Separation

- The canted phases are thermodynamically stable

II. Interaction with Spin Waves

- The effective lagrangian for the spin waves in the low energy and momentum regime

  F: One spin wave with quadratic D.R.
  AF: Two spin waves with linear D.R.
  C: One F and one AF spin waves

- Interaction lagrangian between the spin waves and the conduction fermions

III. Spin Waves in Doped Manganites

- The contribution to the spin waves from:
  the Heisenberg term, and
  the charge carriers.

- The doping dependence of the mass and velocity of the spin waves.

IV. Canted Phases vs. Phase Separation

- Three distinctive characteristics which allow the experimental differenciation between Phase Separation Regions and Canted Phases.