Effective field theory approach to spin wave mediated non-reciprocal effects in antiferromagnetic crystals

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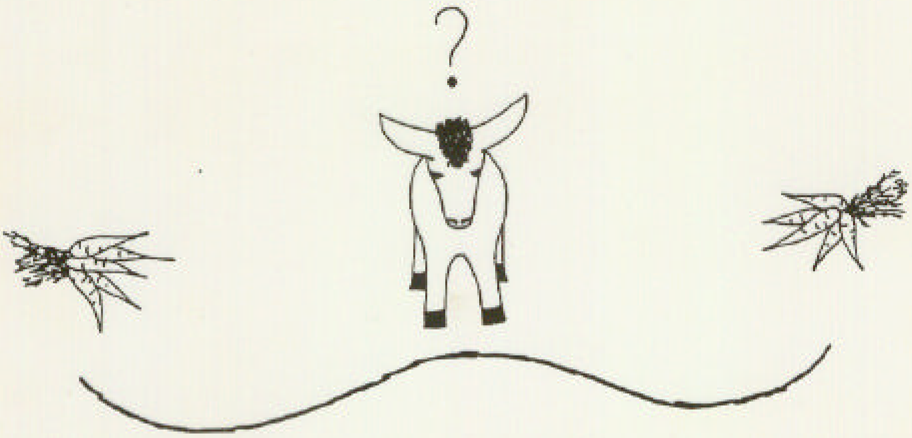
José María Román
Joan Soto

I. Introduction

Spontaneous symmetry breakdown

When the symmetry of the theory (action, hamiltonian, eq. of motion) is not respected by the ground state it is said that symmetry is spontaneously broken.



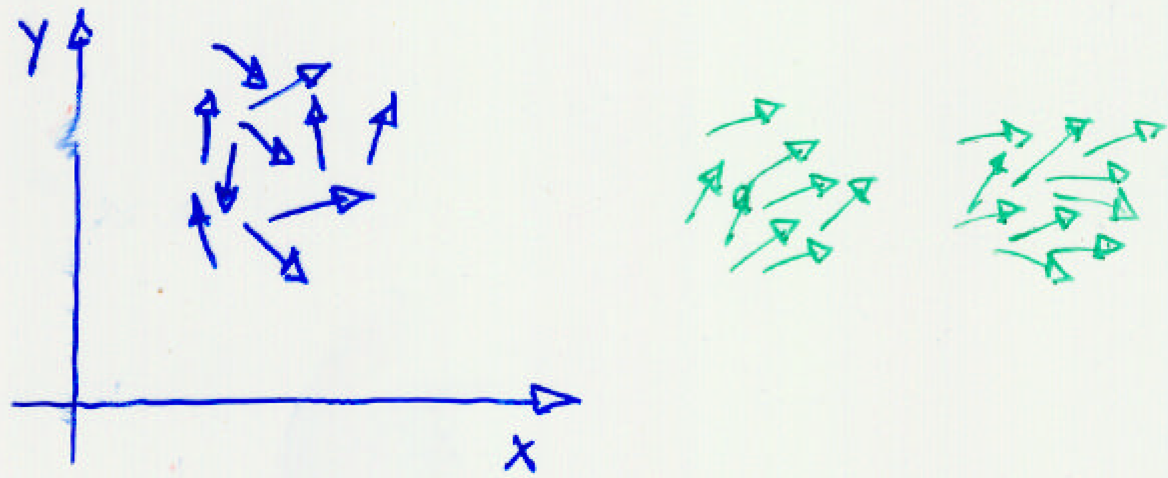
C.M.: Superconductividad ($U(1)_{em} \to 1$)
Phonons ($T_3 \to$ Lattice)
Spin waves (magnons) ($SU(2) \to U(1)$)

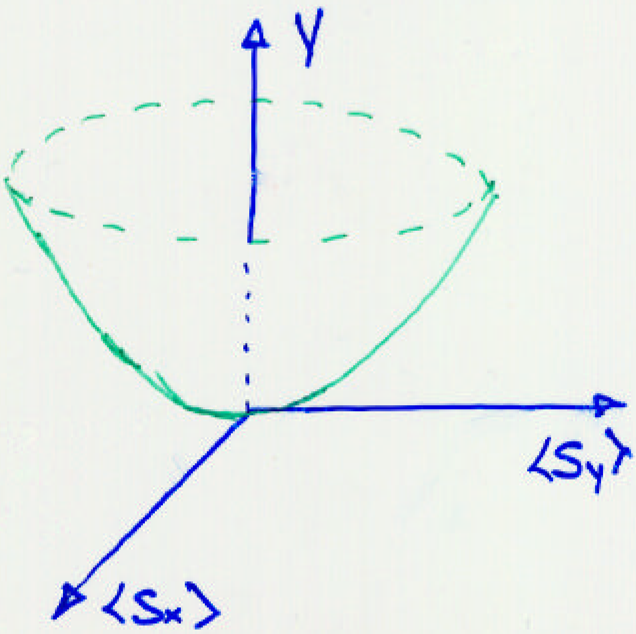
QFT: Standard model symmetry breaking
($SU(2)_L \otimes U(1)_Y \to U(1)_{em}$)
Chiral symmetry breaking (QCD)
($SU(3)_L \otimes SU(3)_R \to SU(3)_V$)

Goldstone bosons

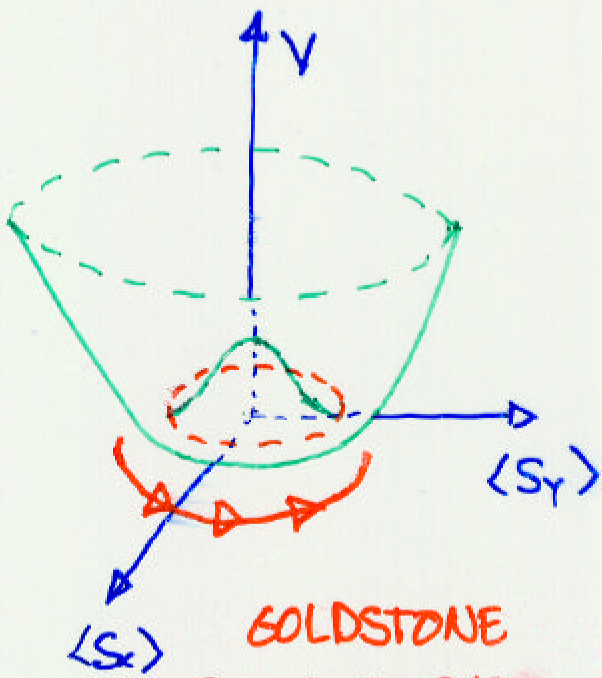
In CM and QFT systems the SSB of internal continuous symmetries plays a dramatic role in the spectrum of the theory

Gapless excitations ≡ Goldstone bosons





NO SYMMETRY BREAKING



GOLDSTONE EXCITATIONS

<u>THE POINT</u>: The Goldstone bosons are the unique modes present in the low energy and momentum regime. Their dynamics is derived from symmetry considerations only (without solving the eq. of motion).

<u>INTENTION</u>: Contruct an effective action describing the Goldstone bosons dynamics in the low energy and momentum regime.

<u>REQUIREMENTS</u>: Invariance under the symmetries (internal and external) of the underlying theory.

F, AF: Heisenberg model: $H = \sum_{\langle i,j \rangle} J_{ij} \vec{S}_i \vec{S}_j$

SU(2) $\otimes$ Space group $\otimes$ Time reversal

$\hookrightarrow$ $\begin{cases} \text{Rotational: Point group} \\ \text{Translational} \end{cases}$

χPT: ~~Three~~ Three flavours massles QCD: $\mathcal{L} = \bar{q}_L i\not{\partial} q_L + \bar{q}_R i\not{\partial} q_R$

$SU(3)_L \otimes SU(3)_R \otimes$ Poincaré $\otimes$ Time reversal

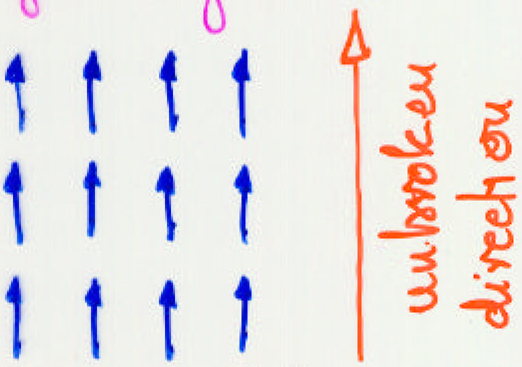
$\hookrightarrow$ $\begin{cases} \text{Rotational: Lorentz} \\ \text{Translational} \end{cases}$

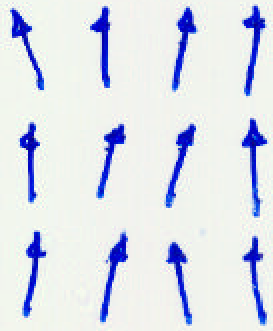
II. Effective fields and its transformations

The Goldstone bosons are represented by elements of the coset G/H
($G \to$ symmetry of the theory, $H \to$ ground state sym.)

$$U(x) \in G/H$$

Goldstone bosons are smooth fluctuations of the ground state





Ground state

Spin wave

$$\begin{pmatrix}1\\0\end{pmatrix} \qquad e^{i\frac{\sqrt{2}}{f_\pi}\left(\pi^1(x)S^1+\pi^2(x)S^2\right)}\begin{pmatrix}1\\0\end{pmatrix}$$

broken generators

F, AF: $\quad U(x) \in SU(2)/U(1)$ (coset)

χPT: $\quad \xi(x) \in SU(3)_L \otimes SU(3)_R / SU(3)_V$ (coset = group)

Transformations under the internal group

$\Psi_0 \to$ ground state

$U(x) \to$ goldstone fluctuations

$\Psi(x) \sim U(x)\Psi_0$

$$g \in G: \Psi(x) \longrightarrow g\Psi(x) \sim g\,U(x)\Psi_0$$

$$\boxed{U(x) \longrightarrow g\,U(x)\,h^{+}(g, U(x))} \qquad \begin{matrix} g \in G \\ h \in H \end{matrix}$$

restores gU to the coset

F, AF: $U(x) \to g\,U(x)\,h^{+}$ $\qquad \begin{matrix} g \in SU(2) \\ h \in U(1) \end{matrix}$

χPT: $\xi(x) \equiv \xi_L(x) \otimes \xi_R(x) \longrightarrow \underbrace{(g_L \otimes g_R)}_{SU(3)_L \otimes SU(3)_R}\,\xi(x)\,\underbrace{g^{+}_{L+R}}_{SU(3)_V}$

$$U(x) \equiv \xi_R(x)\,\xi_L(x) \longrightarrow R\,U(x)\,L^{+}$$

$R \in SU(3)_R$

$L \in SU(3)_L$

Space-time transformations

F, AF: Space group $\otimes$ T $\longrightarrow$ Point group $\otimes$ T

Low energy approach
with translation invariance

χPT: Poincaré $\otimes$ T $\longrightarrow$ Lorentz $\otimes$ T

Rotational group

F:
$$U(x) \longrightarrow U(x)$$
$$x\uparrow \longrightarrow x\uparrow$$

AF:
$$U(x) \longrightarrow U(x)$$
$$x\uparrow \longrightarrow x\uparrow$$
$$U(x) \longrightarrow U(x)\,C\,h^{+}$$
$$x\uparrow \longrightarrow x\downarrow$$

$$\begin{cases} C = e^{-i\pi S^2} \\ C S^3 C^{+} = -S^3 \end{cases}$$

χPT:
$$U(x) \longrightarrow U(x)$$

- F and QCD ground states are invariants
- AF ground state breaks some symmetries

$$\begin{matrix}\uparrow & \downarrow \\ \downarrow & \uparrow\end{matrix} \xrightarrow{C_4} \begin{matrix}\rightarrow & \leftarrow \\ \leftarrow & \rightarrow\end{matrix} \equiv \begin{matrix}\downarrow & \uparrow \\ \uparrow & \downarrow\end{matrix}$$

after SU(2) compensation

Time reversal is an antiunitary transformation

A wave function transformation

$$\Psi(x) \longrightarrow C\Psi^*(x) \qquad C^\dagger S^a C = -S^{aT}$$
$$C^\dagger = -C$$

F, AF : Ground state breaks T symmetry

$$U(x) \longrightarrow C U^*(x) h_t^\dagger \equiv U(x) C h_t^\dagger$$

χPT : QCD ground state respects T symmetry

$$U(x) \longrightarrow U^*(x)$$

- The fact that space-time symmetries are broken by the F and AF ground states in a different form is determinant in the construction of the effective lagrangians

III. Effective action

We want to construct an effective action for $U(x)$ invariant under the previous transformations in the low energy and momentum regime with as few derivatives as possible.

Internal group invariance.

F, AF: $U(x) \longrightarrow g U(x) h^{+}$: $h = e^{i\theta(x) S^{3}} \in U(1)$

$\partial_\mu U(x)$ no good transf. under $SU(2)$

$$U^{+}(x)\, i\partial_\mu U(x) \equiv a_\mu^{-}(x) S_+ + a_\mu^{+}(x) S_- + a_\mu^{3}(x) S^{3}$$

$$a_\mu^{-} = \frac{1}{2S} \operatorname{tr}\left([U^{+} i\partial_\mu U, S_-] P_+\right) \longrightarrow e^{i\theta(x)} a_\mu^{-}$$

$$a_\mu^{+} = -\frac{1}{2S} \operatorname{tr}\left([U^{+} i\partial_\mu U, S_+] P_+\right) \longrightarrow e^{-i\theta(x)} a_\mu^{+}$$

$$a_\mu^{3} = \frac{1}{S} \operatorname{tr}\left(U^{+} i\partial_\mu U P_+\right) \longrightarrow a_\mu^{3} + \partial_\mu \theta(x)$$

{associated to the unbroken symmetry} $\underline{D_\mu \equiv \partial_\mu - i a_\mu^{3}}$ on a_μ^{-}

$$\boxed{\text{Invariants under } SU(2): \quad a_\mu^{+} a_\nu^{-} \,;\; a_\mu^{3}}$$

χPT: $U(x) \longrightarrow R\, U(x) L^{+}$: $R, L \to SU(3)$ global

$\partial_\mu U(x)$ good transf. under $SU(3)_L \otimes SU(3)_R$

$$\boxed{\text{Invariants under } SU(3)_L \otimes SU(3)_R: \quad \operatorname{tr}\left(\partial_\mu U^{+} \partial_\nu U\right)}$$

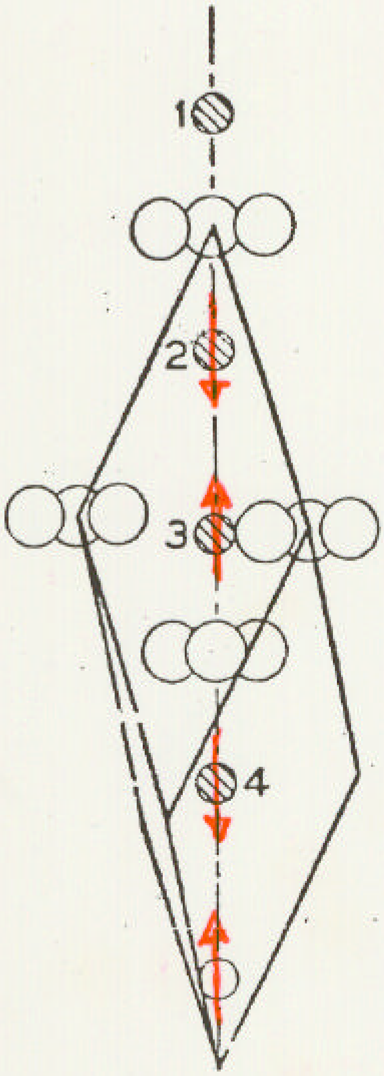
Cristalographic point group

$\bar{3}m = \{ C_{3z}^{+}, I, \sigma_y \}$

$\begin{cases} z = x + iy \\ \bar{z} = x - iy \end{cases}$

$C_{3z}^{+} : \begin{cases} z \to e^{i2\pi/3} z \\ \bar{z} \to e^{-i2\pi/3} \bar{z} \\ x^3 \to x^3 \end{cases}$; $I : \begin{cases} z \to -z \\ \bar{z} \to -\bar{z} \\ x^3 \to -x^3 \end{cases}$; $\sigma_y : \begin{cases} z \to \bar{z} \\ \bar{z} \to z \\ x^3 \to x^3 \end{cases}$

→ It is broken by the AF ground state



Invariance under space-time group

F, AF: The inversion I stablishes the differences

F $\quad I_F: \begin{cases} a_0^- \to a_0^- \\ a_i^- \to a_i^- \\ a_0^3 \to a_0^3 \\ a_i^3 \to a_i^3 \end{cases}$ $\qquad \xi: \{C_{3z}^+, \sigma_y\} = \begin{cases} a_\mu^- \to a_{\xi\mu}^- \\ a_\mu^3 \to a_{\xi\mu}^3 \end{cases}$

AF $\quad I_{AF}: \begin{cases} a_0^- \to -a_0^+ \\ a_i^- \to a_i^+ \\ a_0^3 \to -a_0^3 \\ a_i^3 \to a_i^3 \end{cases}$ $\qquad T: \begin{cases} a_\mu^- \to -a_{t\mu}^+ \\ a_\mu^3 \to -a_{t\mu}^3 \end{cases}$

F: $\boxed{\mathcal{L}(x) = A\, a_0^3 + B(a_z^+ a_{\bar z}^- + a_z^- a_{\bar z}^+) + C\, a_3^+ a_3^-}$

↳ Quadratic dispersion relation

AF: $\boxed{\mathcal{L}(x) = A\, a_0^+ a_0^- + B(a_z^+ a_{\bar z}^- + a_z^- a_{\bar z}^+) + C\, a_3^+ a_3^-}$

↳ Linear dispersion relation

Up to now a_0^3 were eliminated from the AF-effective action only after the calculation of some observables (magnetisation)

XPT: Lorentz invariance: Contraction of indices

$$\boxed{\mathcal{L}(x) = A\, \mathrm{tr}(\partial_\mu U^+ \partial^\mu U)}$$

Topological terms

These terms are naturally included in our formalism.
Advantages over the standard O(3)-sigma model formalism.

Blocks: $n^a(x) \in O(3) \quad n^2 = 1 \Rightarrow SU(2)/U(1)$

Topological terms must be introduced like an interpolation over a new dimension

$$n^a(x) \sim \mathrm{tr}\,(U^\dagger S^a U S^3)$$

$$a_0^3 = \frac{1}{S}\,\mathrm{tr}\,(U^\dagger i\partial_0 U P_+) \sim \int_0^1 d\lambda\, \epsilon^{\alpha\beta}\epsilon_{abc}\, n^a \partial_\alpha n^b \partial_\beta n^c$$

interpolation

In our formalism Chern-Simons are also included:

$$\epsilon^{\mu\nu\rho}\, a_\mu^3 \partial_\nu a_\rho^3$$

IV. Explicit symmetry breaking terms

These terms produce a gap in the spectrum

F, AF: Spin-orbit and magnetic dipole interactions

SO: $$H = \sum_{\langle i,j\rangle} J_{ij}\, \vec{S}_i \cdot \vec{S}_j + \sum_{\langle i,j\rangle} \underbrace{\vec{D}_{ij}}_{\text{antisymmetric}} (\vec{S}_i \times \vec{S}_j) + \sum_{\langle i,j\rangle} \underbrace{M^{ab}_{ij}}_{\text{symmetric}} S^a_i S^b_j$$

$$\left.\begin{array}{l} D \sim (\Delta g/g)\, J \\ M \sim (\Delta g/g)^2 J \end{array}\right\} \Delta g \sim 10^{-2} g$$

g → gyromagnetic factor

magnetic dipole: $$H = \mu^2 \sum_{i \neq j} \left(\frac{\vec{S}_i \cdot \vec{S}_j}{\underbrace{|\vec{x}_i - \vec{x}_j|^3}_{J_{ij}}} - \frac{(\vec{S}_i \hat{x}_i)(\vec{S}_j \hat{x}_j)}{\underbrace{|\vec{x}_i - \vec{x}_j|^3}_{M^{ab}_{ij}}} \right)$$

χPT: Mass of the quarks

$$\mathcal{L}(x) = \bar{q}_R i \not{\partial} q_R + \bar{q}_L i \not{\partial} q_L + \bar{q}_R M q_L + \bar{q}_L M q_R$$

In order to introduce in the effective action terms which break the symmetry in the same way we assume the explicit breaking terms transform such that the underlying theory remains invariant

F, AF: $D^a(\vec{x}_i,\vec{x}_j) \longrightarrow R^a{}_b D^b(\vec{x}_i,\vec{x}_j)$
$M^{ab}(\vec{x}_i,\vec{x}_j) \longrightarrow R^a{}_c R^b{}_d M^{cd}(\vec{x}_i,\vec{x}_j)$ $\Big\}$ $R \in SO(3)$

In the local limit. Compatibility with symmetry

$D^a(\vec{x}_i,\vec{x}_j) \longrightarrow D^a_{pq}$
$M^{ab}(\vec{x}_i,\vec{x}_j) \longrightarrow M^{ab}$ $\Big\}$ $D^-_{z\bar z} = -D^+_{\bar z\bar z}$; $D^-_{3\bar z} = -D^+_{3z}$
M^{-+} ; M^{33}

To have covariant transformations under $SU(2)$

$D_{pq} \equiv D^a_{pq} S^a \longrightarrow g\, D_{pq}\, g^+$
$M \equiv M^{ab}(S^{\prime a} \otimes S^{b\prime}) \longrightarrow (g\otimes g)\, M\, (g^+\otimes g^+)$

Elements transforming under $U(1)$ local like a's

$U^+(x)\, D_{pq}\, U(x) = d^-_{pq}(x)\, S_+ + d^+_{pq}(x)\, S_- + d^3_{pq}(x)\, S^3$

$(U^+(x)\otimes U^+(x))\, M\, (U(x)\otimes U(x)) = m^{ab}(x)\, (S^{\prime a}\otimes S^{b\prime})$

d's and m's have similar representations to a's

$d^-_{pq} \longrightarrow e^{i\theta(x)} d^-_{pq}$
$d^+_{pq} \longrightarrow e^{-i\theta(x)} d^+_{pq}$
$d^3_{pq} \longrightarrow d^3_{pq}$

$m^{--} \longrightarrow e^{i2\theta(x)} m^{--}$
$m^{-3} \longrightarrow e^{i\theta(x)} m^{-3}$
$m^{-+} \longrightarrow m^{-+}$
$m^{33} \longrightarrow m^{33}$

$\boxed{\text{Invariants under } SU(2):\ d^+_{pq} d^-_{rs},\ d^3_{pq} d^3_{rs},\ m^{-+},\ m^{33}}$

χPT: $M \longrightarrow R M L^+$

$\boxed{\text{Invariants under } SU(3)_L \otimes SU(3)_R:\ \mathrm{tr}\,(M(U+U^+))}$

F, AF: The space time transformations are similar to those of a's. Again I transformation is different for F and AF.

F: $I_F: \begin{cases} d^-_{pq} \to d^-_{pq} \\ d^3_{pq} \to d^3_{pq} \\ w^{ab} \to w^{ab} \end{cases}$

$\xi: \{C^+_{3z}, \sigma_v\}: \begin{cases} d^-_{pq} \to d^-_{\xi p\, \xi q} \\ d^3_{pq} \to d^3_{\xi p\, \xi q} \\ w^{ab} \to w^{ab} \end{cases}$

AF: $I_{AF}: \begin{cases} d^-_{pq} \longrightarrow -d^+_{pq} \\ d^3_{pq} \longrightarrow -d^3_{pq} \\ w^{--} \longrightarrow w^{++} \\ w^{-3} \longrightarrow w^{+3} \\ w^{-+} \longrightarrow w^{-+} \\ w^{33} \longrightarrow w^{33} \end{cases}$

$T: \begin{cases} d^-_{pq} \longrightarrow -d^+_{pq} \\ d^3_{pq} \longrightarrow -d^3_{pq} \\ w^{--} \longrightarrow w^{++} \\ w^{-3} \longrightarrow w^{+3} \\ w^{-+} \longrightarrow w^{-3} \\ w^{33} \longrightarrow w^{33} \end{cases}$

Invariants: $(d^+_{zz} d^-_{\bar z\bar z} + d^-_{zz} d^+_{\bar z\bar z})$; $(d^+_{3z} d^-_{3\bar z} + d^-_{3z} d^+_{3\bar z})$;
$(d^+_{3z} d^-_{zz} + d^-_{3z} d^+_{zz}) + (d^-_{3\bar z} d^+_{\bar z\bar z} + d^+_{3\bar z} d^-_{\bar z\bar z})$;
$d^3_{zz} d^3_{\bar z\bar z}$; $d^3_{3z} d^3_{3\bar z}$; $(d^3_{3z} d^3_{zz} + d^3_{3\bar z} d^3_{\bar z\bar z})$
w^{-+}; w^{33}

F: $$\boxed{\mathcal{L} = \pi^+ i\partial_0 \pi^- - \frac{1}{2m} \partial_i \pi^+ \partial_i \pi^- - \frac{1}{2\gamma m} \partial_3 \pi^+ \partial_3 \pi^- - \Delta^2 \pi^+ \pi^-}$$

$\boxed{\Delta \sim D}$

AF: $$\boxed{\mathcal{L} = \partial_0 \pi^+ \partial_0 \pi^- - v^2 \partial_i \pi^+ \partial_i \pi^- - (\gamma v)^2 \partial_3 \pi^+ \partial_3 \pi^- - \Delta^2 \pi^+ \pi^-}$$

χPT: M transforms trivially under Lorentz

$$\boxed{\mathcal{L}(x) = \frac{1}{2} \partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{2} M_\pi^2 \pi^+ \pi^-}$$

$\boxed{M_\pi \sim \sqrt{m_q}}$

V. Electromagnetic coupling

F, AF: Normally e.m. fields are introduced in a theory by means of a minimal coupling

$$\partial_\mu \longrightarrow D_\mu \equiv \partial_\mu - ieQA_\mu$$

Spin waves have not electric charge
Non-minimal coupling: $F_{\mu\nu} \sim \vec{E}, \vec{B}$

$\vec{E}, \vec{B}$ do not break SU(2). Under space-time

$\vec{E} \rightarrow$ transforms like a vector
$\vec{B} \rightarrow$ transforms like a pseudovector

$$T: \begin{cases} \vec{E} \longrightarrow \vec{E} \\ \vec{B} \longrightarrow -\vec{B} \end{cases}$$

χPT: Mesons have electric charge: minimal coupling is available

$$\partial_\mu \longrightarrow D_\mu \equiv \partial_\mu - ieQA_\mu$$

$$\boxed{\mathcal{L}(x) = A \, \mathrm{tr}\left((D_\mu U)^\dagger D^\mu U\right)}$$

F, AF: Pauli coupling: $-\mu\vec{S}\vec{B}$ arises in a non-relativistic theory. It breaks the SU(2) invariance

$$H=-\mu\sum_i \vec{S}_i\vec{B}(x_i) \longrightarrow L(x)=\sum_i \Psi^\dagger(x_i)\, i\,(\partial_0 - i\mu\vec{S}\vec{B}(x_i))\,\Psi(x_i)$$

second quantisation language

covariant derivative form.

We associate a source, $A_0(x)\sim\mu\vec{S}\vec{B}(x)$, which transforms like a connexion under time-dependent SU(2)

$$A_0(x) \longrightarrow g(t)A_0(x)g^\dagger(t) + i\,g(t)\partial_0 g^\dagger(t)$$

Pauli term is introduced in the effective theory by means of the covariant time derivative

$$D_0 \equiv \partial_0 - iA_0(x)$$

New constants are not necessary

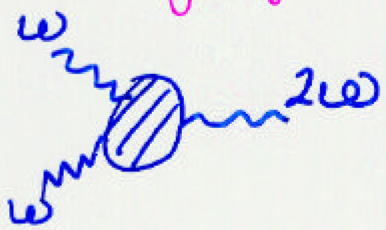
χPT: In QCD left and right currents are coupled to a connexion in order to introduce j_L, j_R in the effective theory by means of covariant derivatives:

VI. Non-reciprocal effects in SHG and GB

1. Introduction

Non-reciprocal effects: Those which are not invariant under the inversion of time

SHG: Second harmonic generation
Non-linear effect which doubles the frecuency of the light.



GB: Gyrotropic Birefringence

Birefringence: Arises from anysotropy of the dielectric tensor

$$\text{Uniaxial crystal} \rightarrow \epsilon^{ab} = \begin{pmatrix} \epsilon_\perp & & \\ & \epsilon_\perp & \\ & & \epsilon_\parallel \end{pmatrix}$$

Gyrotropic: A magnetization contribution introduces a linearly dependence on $\vec{k}$. If the sense of incidence is reversed the optic axis rotates.

2. Orders of magnitude

Blocks : a_0, a_i, d_{pq}, m, $\vec{E}$, $\vec{B}$

Characteristic energy and momentum of the system are given by e.m. field

$$\left.\begin{array}{l} \text{energy} \sim \omega \\ \text{momentum} \sim |\vec{E}| \sim \omega \end{array}\right\} \quad (\hbar = c = 1)$$

These must be small with respect the typical scales of the system to make sense the effective theory

$$\left.\begin{array}{l} \text{energy} \sim J \sim 10\,\text{meV} \\ \text{momentum} \sim 1/a \sim 0{,}1\,\text{Å}^{-1} \end{array}\right\} \Rightarrow \boxed{\begin{array}{l} \partial_0 / J \ll 1 \\ a\partial_i = v\partial_i / J \ll 1 \end{array}}$$

$v = Ja \sim 10^{-4} c$

The suppression of the spin-orbit terms:

$$\left.\begin{array}{l} D \sim (\Delta g/g) J \sim 10^{-2} J \\ M \sim (\Delta g/g)^2 J \sim 10^{-4} J \end{array}\right\} \Rightarrow \text{gap} \sim D$$

Spin waves effects would be enhanced if we take:

$$\boxed{\partial_0 \sim D}$$

Pauli term is associated to the time derivative

$$\mu B / J \ll 1$$

Non-minimal couplings, $\vec{E}$, $\vec{B}$, come from minimal couplings to higher modes

$\vec{E} \to$ scalar potential $\to$ time derivative
$\vec{B} \to$ vector potential $\to$ space derivative

$$\left.\begin{array}{l} eaE / J \ll 1 \\ ea^2B = eavB / J \ll 1 \end{array}\right\}$$

Setting the electric field amplitude to the energy gap, $E \sim D$, the following suppressions hold ($E \sim B$ in e.m. waves)

$$\boxed{\begin{array}{lcl} \partial_0,\ eaE,\ d & \sim & 10^{-2}\,J \\ m & \sim & 10^{-4}\,J \\ \mu B & \sim & 10^{-5}\,J \\ v\partial_i,\ eavB & \sim & 10^{-6}\,J \end{array}}$$

3. Effective action

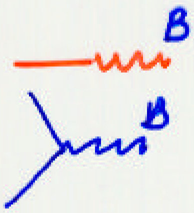
Effective action is constructed up to the first e.m. coupling

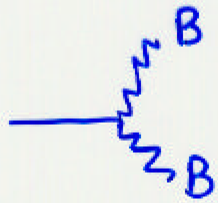
$O(10^{-4})$: $a_0^+ a_0^-$ → Contains Pauli terms $(10^{-7}; 10^{-10})$
d^+d^-, d^3d^3, m → Contribute to the mass gap
$E^z E^{\bar z}; E^3E^3$ → Contact terms: Birefringence

$O(10^{-6})$: $i[(d^+_{\bar z\bar z} a_0^- - d^-_{\bar z\bar z} a_0^+)E^z - (d^-_{zz} a_0^+ - d^+_{zz} a_0^-)E^{\bar z}]$
$i[(d^+_{3z} a_0^- - d^-_{3z} a_0^+)E^z - (d^-_{3\bar z} a_0^+ - d^+_{3\bar z} a_0^-)E^{\bar z}]$

→ First e.m. couplings mediated by spin-orbit
Contains Pauli terms (10^{-9})

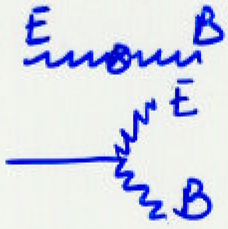
Taking into account the Pauli coupling the following spin waves and e.m. fields arise:

$a_0^+ a_0^-$ → 

$: 10^{-7}$; 

$: 10^{-10}$

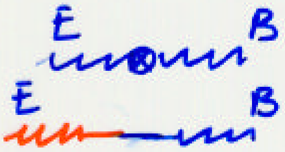
$d^+ a_0^- E$ → 

$: 10^{-6}$; 

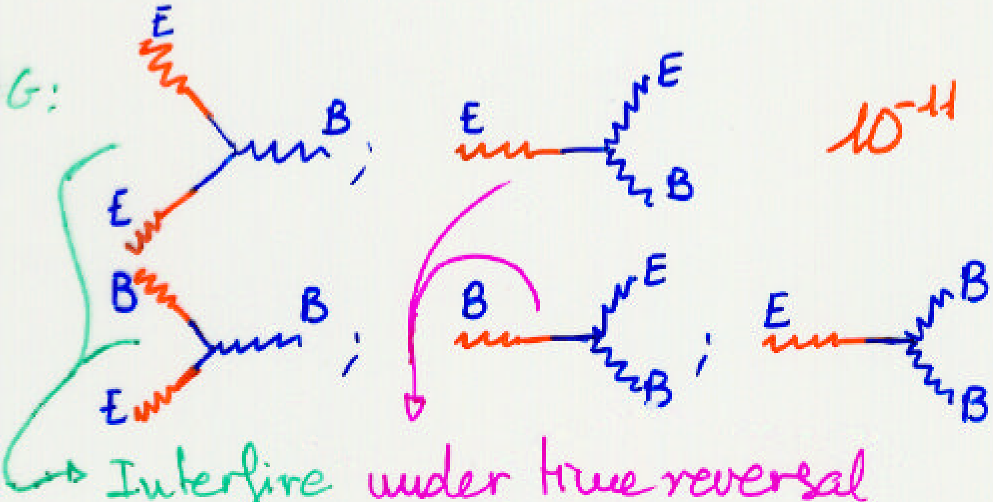
$: 10^{-9}$

We look for contributions to GB and SHG:

Propagator: $P(x-y) = \int \frac{d^4q}{(2\pi)^4} \frac{1}{\omega^2 - \Delta^2} e^{-iq(x-y)}$ $\sim 10^{+4}$

GB: 

10^{-9}

SHG: 

10^{-11}

10^{-12}

Additional contributions to SHG to 10^{-12} could be written if we find vertices of the form:

 : 10^{-8}

$a_0^+ D_0 a_0^- E^3$ (c)

$d^3 d^+ a_0^- E$ (di)

 : 10^{-10}

$d^+ D_0 a_0^- EE;\ d^+ a_0^- E \partial_0 E$ (ei)

$d^3 d^+ d^- EE;\ u^{+3} d^- EE$ (fi)

$d^3 EB$ (gi)

 : 10^{-12}

$d^+ d^- EE \partial_0 E$ (h)

$E \partial_0 E B$ (ji)

$E E \partial_i E$ (ki)

$$
\begin{aligned}
& S[\pi,E,B] = \int dx \Big\{ \partial_0\pi^+\partial_0\pi^- - \Delta^2\pi^+\pi^- \\
\Delta \sim D \qquad & + i\mu(\pi^+\partial_0\pi^- - \pi^-\partial_0\pi^+)B^3 \\
f_\pi^2 \sim \frac{1}{Ja^3} \quad \lambda \sim ea\frac{D}{J} \qquad & - \frac{1}{2}f_\pi[\partial_0\pi^+(\mu B^{\bar z} + \lambda E^{\bar z}) + \partial_0\pi^-(\mu B^z + \lambda E^z)] \\
& - \frac{1}{2}i\mu f_\pi[\pi^+(\mu B^{\bar z} + \lambda E^{\bar z}) - \pi^-(\mu B^z + \lambda E^z)]B^3 \\
& + \frac{1}{4}\mu\lambda f_\pi^2(E^z B^{\bar z} + E^{\bar z} B^z) \\
b_i \sim (ea)^2 \qquad & + f_\pi^2\big(b_1 E^z E^{\bar z} + b_2 E^3 E^3\big) \\
c \sim \frac{ea}{J} \qquad & + ic(\partial_0\pi^+\partial_0^2\pi^- - \partial_0\pi^-\partial_0^2\pi^+)E^3 \\
& + id_1(\pi^+\partial_0\pi^+E^z - \pi^-\partial_0\pi^-E^{\bar z}) \\
d_i \sim ea\left(\frac{D}{J}\right)^2 \qquad & + id_2(\pi^+\partial_0\pi^-E^3 - \pi^-\partial_0\pi^+E^3) \\
& + ie_1 f_\pi(\partial_0\pi^+E^z\partial_0E^z - \partial_0\pi^-E^{\bar z}\partial_0E^{\bar z}) \\
e_i \sim \left(\frac{ea}{J}\right)^2\frac{D}{J} \qquad & + ie_2 f_\pi(\partial_0\pi^+\partial_0E^{\bar z}E^3 - \partial_0\pi^-\partial_0E^zE^3) \\
& + ie_3 f_\pi(\partial_0\pi^+E^{\bar z}\partial_0E^3 - \partial_0\pi^-E^z\partial_0E^3) \\
f_i \sim (ea)^2\left(\frac{D}{J}\right)^3 \qquad & + if_1 f_\pi(\pi^+E^zE^z - \pi^-E^{\bar z}E^{\bar z}) \\
& + if_2 f_\pi(\pi^+E^{\bar z}E^3 - \pi^-E^zE^3) \\
& + ig_1 f_\pi(\pi^+E^zB^z - \pi^-E^{\bar z}B^{\bar z}) \\
g_i \sim (ea)^2 Da \qquad & + ig_2 f_\pi(\pi^+E^{\bar z}B^3 - \pi^-E^zB^3) \\
& + ig_3 f_\pi(\pi^+E^3B^{\bar z} - \pi^-E^3B^z) \\
h \sim (ea)^3 D^2 \qquad & + ihf_\pi^2(E^z\partial_0E^{\bar z} - E^{\bar z}\partial_0E^z)E^3 \\
j_i \sim (ea)^3\frac{a}{J} \qquad & + ij_1 f_\pi^2(E^z\partial_0E^{\bar z} - E^{\bar z}\partial_0E^z)B^3 \\
& + ij_2 f_\pi^2(E^z\partial_0E^3B^{\bar z} - E^3\partial_0E^{\bar z}B^z) \\
& + k_1 f_\pi^2(E^z\partial_{\bar z}E^z + E^{\bar z}\partial_zE^{\bar z})E^3 \\
k_i \sim (ea)^3\frac{a}{J} \qquad & + k_2 f_\pi^2 E^zE^{\bar z}\partial_3E^3 \\
& + k_3 f_\pi^2 E^zE^{\bar z}(\partial_zE^z + \partial_{\bar z}E^{\bar z}) \\
& + k_4 f_\pi^2 E^3E^3(\partial_zE^z + \partial_{\bar z}E^{\bar z})\Big\}
\end{aligned}
$$

$$
\begin{aligned}
S_{eff}[E,B] \; = \; & \int dx f_\pi^2 \left[b_1 E^z E^{\bar z} + b_2 E^3 E^3 + \frac{1}{4}\mu\lambda (E^z B^{\bar z} + E^{\bar z} B^z) \right] \\
& + \int dx dy f_\pi^2 \left[\frac{1}{4}\mu\lambda \left(E^z \partial_0^2 P(x-y) B^{\bar z} + B^z \partial_0^2 P(x-y) E^{\bar z} \right) \right] \\
& + \int dx dy f_\pi^2 \Big\{ -\frac{1}{4} i\mu\lambda^2 E^z \left(B^3 \partial_0 P(x-y) + \partial_0 P(x-y) B^3 \right) E^{\bar z} \\
& \qquad - \frac{1}{4} i\mu^2\lambda \Big[E^z \left(B^3 \partial_0 P(x-y) + \partial_0 P(x-y) B^3 \right) B^{\bar z} \\
& \qquad\qquad + B^z \left(B^3 \partial_0 P(x-y) + \partial_0 P(x-y) B^3 \right) E^{\bar z} \Big] \\
& \qquad - \frac{1}{4} i\lambda e_1 \left[E^z \partial_0^3 P(x-y) E^z E^{\bar z} + E^{\bar z} E^{z} \partial_0^3 P(x-y) E^{\bar z} \right] \\
& \qquad - \frac{1}{2} i\lambda \Big[E^z \partial_0^2 P(x-y) (e_2 E^{\bar z} \partial_0 E^3 + e_3 \partial_0 E^{\bar z} E^3) \\
& \qquad\qquad + (e_2 E^z \partial_0 E^3 + e_3 \partial_0 E^z E^3) \partial_0^2 P(x-y) E^{\bar z} \Big] \\
& \qquad + \frac{1}{2} i\lambda \Big[E^z \partial_0 P(x-y) (f_1 E^z E^{\bar z} + f_2 E^{\bar z} E^3) \\
& \qquad\qquad + (f_1 E^{\bar z} E^{z} + f_2 E^z E^3) \partial_0 P(x-y) E^{\bar z} \Big] \\
& \qquad + \frac{1}{2} i\lambda \Big[E^z \partial_0 P(x-y) (g_1 E^z B^{\bar z} + g_2 E^{\bar z} B^3 + g_3 E^3 B^{\bar z}) \\
& \qquad\qquad + (g_1 E^{\bar z} B^{z} + g_2 E^z B^3 + g_3 E^3 B^z) \partial_0 P(x-y) E^{\bar z} \Big] \Big\} \\
& + \int dx du dy f_\pi^2 \Big\{ -\frac{1}{4} i\lambda^2 E^z \Big(\partial_0^2 P(x-u) (\mu B^3 + d_2 E^3) \partial_0 P(u-y) \\
& \qquad\qquad + \partial_0 P(x-u) (\mu B^3 + d_2 E^3) \partial_0^2 P(u-y) \Big) E^{\bar z} \\
& \qquad - \frac{1}{4} i\mu^2\lambda \Big[E^z \Big(\partial_0^2 P(x-u) B^3 \partial_0 P(u-y) + \partial_0 P(x-u) B^3 \partial_0^2 P(u-y) \Big) B^{\bar z} \\
& \qquad\qquad + B^z \Big(\partial_0^2 P(x-u) B^3 \partial_0 P(u-y) + \partial_0 P(x-u) B^3 \partial_0^2 P(u-y) \Big) E^{\bar z} \Big] \\
& \qquad + \frac{1}{4} i\lambda^2 c E^z \Big(\partial_0^2 P(x-u) E^3 \partial_0^3 P(u-y) + \partial_0^3 P(x-u) E^3 \partial_0^2 P(u-y) \Big) E^{\bar z} \\
& \qquad + \frac{1}{8} i d_1 \lambda^2 \Big[E^z \Big(\partial_0^2 P(x-u) E^z \partial_0 P(u-y) - \partial_0 P(x-u) E^z \partial_0^2 P(u-y) \Big) E^{\bar z} \\
& \qquad\qquad - E^{\bar z} \Big(\partial_0^2 P(x-u) E^{\bar z} \partial_0 P(u-y) - \partial_0 P(x-u) E^{\bar z} \partial_0^2 P(u-y) \Big) E^{z} \Big] \Big\} \\
& + \int dx f_\pi^2 \Big[ih (E^z \partial_0 E^{\bar z} - E^{\bar z} \partial_0 E^z) E^3 \\
& \qquad + i j_1 (E^z \partial_0 E^{\bar z} - E^{\bar z} \partial_0 E^z) B^3 + i j_2 (E^z B^{\bar z} - E^{\bar z} B^z) \partial_0 E^3 \\
& \qquad + k_1 (E^z \partial_{\bar z} E^z + E^{\bar z} \partial_z E^{\bar z}) E^3 + k_2 E^z E^{\bar z} \partial_3 E^3 \\
& \qquad + k_3 E^z E^{\bar z} (\partial_z E^z + \partial_{\bar z} E^{\bar z}) + k_4 E^3 E^3 (\partial_z E^z + \partial_{\bar z} E^{\bar z}) \Big]
\end{aligned}
$$

4. Linear and non-linear susceptibilities

$$L_{int} = - H_{int} \Rightarrow \quad P^a = \frac{\delta S_{eff}}{\delta E^a} \; ; \; M^a = \frac{\delta S_{eff}}{\delta B^a}$$

The symmetry of the ground state constrains the form of the susceptibility tensors

GB and SHG are dynamical effects: $32 : \{ C_{3z}^{+}, C_{2y} \}$

Linear:

$$P^z = \chi_E^{z\bar z} E^z + \chi_B^{z\bar z} B^z$$
$$P^3 = \chi_E^{33} E^3 + \chi_B^{33} B^3$$

$$M \; (\chi \to \gamma)$$

Non-linear:

$$P^z = \chi_{EE}^{zzz} E^{\bar z} E^{\bar z} + 2 \chi_{EE}^{z\bar z 3} E^z E^3 +$$
$$+ \chi_{EB}^{zzz} E^{\bar z} B^{\bar z} + \chi_{EB}^{z\bar z 3} E^z B^3 + \chi_{EB}^{z 3 \bar z} E^3 B^z$$
$$+ \chi_{BB}^{zzz} B^{\bar z} B^{\bar z} + \chi_{BB}^{z \bar z 3} B^z B^3$$

$$P^3 = 2 \chi_{EE}^{3 \bar z z} E^z E^{\bar z} + \chi_{EB}^{3 \bar z z} \left(E^z B^{\bar z} - E^{\bar z} B^z \right) + 2 \chi_{BB}^{3 \bar z z} B^z B^{\bar z}$$

$$M \; (\chi \to \gamma)$$

Linear susceptibilities:

$$
\begin{aligned}
\chi_E^{z\bar{z}}(\omega) &= 2b_1 f_\pi^2 \\
\chi_B^{z\bar{z}}(\omega) &= \frac{1}{2}\mu\lambda f_\pi^2[1-\omega^2 P(\omega)] \\
\chi_E^{33}(\omega) &= 2b_2 f_\pi^2
\end{aligned}
$$

$P(\omega) = \frac{1}{\omega^2 - \Delta^2}$

$$
\gamma_E^{z\bar{z}}(\omega) = \frac{1}{2}\mu\lambda f_\pi^2[1-\omega^2 P(\omega)]
$$

$\chi_B^{z\bar{z}}(\omega)$ and $\gamma_E^{z\bar{z}}(\omega)$ gives the GB effect. It is proportional to the spin wave gap, depending on spin-orbit interaction.

Susceptibilities contributing to SHG:

$$
\begin{aligned}
\chi_{EE}^{zzz}(\omega,\omega) &= \lambda e_1 f_\pi^2\omega^3[P(\omega)-4P(2\omega)] + 2\lambda f_1 f_\pi^2\omega[P(\omega)-P(2\omega)] \\
&\quad + \frac{1}{2}\lambda d_1 f_\pi^2\omega^3 P(\omega)[P(\omega)+2P(2\omega)] \\
\chi_{EE}^{z\bar{z}3}(\omega,\omega) &= \frac{1}{2}\lambda e_2 f_\pi^2\omega^3[P(\omega)+4P(2\omega)] - \lambda e_3 f_\pi^2\omega^3[P(\omega)-2P(2\omega)] \\
&\quad - \frac{1}{2}\lambda f_2 f_\pi^2\omega[P(\omega)+2P(2\omega)] - \frac{3}{2}\lambda^2 d_2 f_\pi^2\omega^3[\underline{P(\omega)P(2\omega)}] \\
&\quad - 3\lambda^2 c f_\pi^2\omega^5 P(\omega)P(2\omega) - 3h f_\pi^2\omega \\
\chi_{EB}^{zzz}(\omega,\omega) &= \lambda g_1 f_\pi^2\omega[P(\omega)-2P(2\omega)] \\
\chi_{EB}^{z\bar{z}3}(\omega,\omega) &= \frac{1}{2}\mu\lambda^2 f_\pi^2\omega[P(\omega)+2P(2\omega)-6\omega^2 P(\omega)P(2\omega)] \\
&\quad - \lambda g_2 f_\pi^2\omega[P(\omega)+2P(2\omega)] - 6j_1 f_\pi^2\omega \\
\chi_{EB}^{z3\bar{z}}(\omega,\omega) &= -2\lambda g_3 f_\pi^2\omega P(2\omega) + 2j_2 f_\pi^2\omega \\
\chi_{BB}^{z\bar{z}3}(\omega,\omega) &= \frac{1}{4}\mu^2\lambda f_\pi^2\omega[P(\omega)+2P(\underline{2\omega})-6\omega^2 \underline{P(\omega)P(2\omega)}] \\
\chi_{EE}^{3\bar{z}z}(\omega,\omega) &= -\lambda e_2 f_\pi^2\omega^3 P(\omega) + \frac{1}{2}\lambda e_3 f_\pi^2\omega^3 P(\omega) \\
\chi_{EB}^{3\bar{z}z}(\omega,\omega) &= -\frac{1}{2}\lambda g_3 f_\pi^2\omega P(\omega) - 2j_2 f_\pi^2\omega
\end{aligned}
$$

$$
\begin{aligned}
\gamma_{EE}^{zzz}(\omega,\omega) &= \lambda g_1 f_\pi^2\omega P(\omega) \\
\gamma_{EE}^{z\bar{z}3}(\omega,\omega) &= -\frac{1}{2}\lambda g_3 f_\pi^2\omega P(\omega) + j_2 f_\pi^2\omega \\
\gamma_{EB}^{z\bar{z}3}(\omega,\omega) &= \frac{1}{2}\mu^2\lambda f_\pi^2\omega[P(\omega)+2P(2\omega)-6\omega^2 P(\omega)P(2\omega)]
\end{aligned}
$$

They have a non-trivial dependence in ω given by the spin wave propagator $P(\omega)$.

VII. Summary

- We give a systematic way to construct effective lagrangians for SW in F and AF materials. exploiting the internal symmetry breaking pattern, $SU(2) \rightarrow U(1)$, as well as crystal point group and time reversal symmetries

- Explicit symmetry breaking SO terms and coupling to the e.m. field are introduced.

- Important points of these formalism
  - Very easy introduction of topological terms
  - Differences between F and AF arise from symmetries
  - Pauli term does not introduce new parameters
  - It is valid for any material with the same symmetry (spin arbitrary)

- Following the above procedure we construct an effective lagrangian for SW couple to e.m. field in an AF material: Cr_2O_3

- By integrating out the SW we obtain an interaction lagrangian for the e.m. field which encodes the response of the material in the microwave region.

- Susceptibilities for GB and SHG have been computed for the interaction lagrangian.