Options on volatility index

Statistical programming languages

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Outline

Options

The buyer of the **call option** has the right to buy an agreed quantity of a particular commodity(**the underlying**) from the seller of the option at a certain time (**the expiration date**) for a certain price (**the strike price**).
Hence **the pay-off**, i.e. the value of the call option at expiry, is

$$\max\{S_T - K, 0\}$$

The Black–Scholes formula

Black–Scholes formula gives a theoretical estimate of the price of European-style options

$$C(S,t) = N(d_1)S - N(d_2)Ke^{-r(T-t)},$$

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left[\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)\right]$$

$$d_2 = d_1 - \sigma\sqrt{T-t}$$

- N is the cumulative distribution function of the standard normal distribution
- $T - t$ is the time to maturity
- S is the spot price of the underlying asset
- K is the strike price
- r is the risk free rate
- σ is the volatility of returns of the underlying asset

Implied volatility

Implied volatility of an option contract is that value of the volatility of the underlying instrument which, when input in an option pricing model will return a theoretical value equal to the current market price of the option. In general, the value of an option depends on an estimate of the future realized price volatility, σ, of the underlying. Or, mathematically:

$$C = f(\sigma, \cdot)$$

where C is the theoretical value of an option, and f is a pricing model that depends on σ, along with other inputs.

Implied volatility

The function f is monotonically increasing in σ, so a unique solution exists. In general, a pricing model function, f, does not have a closed-form solution for its inverse.Instead, a root finding technique is used to solve the equation:

$$f(\sigma_{\bar{C}}, \cdot) - \bar{C} = 0$$

Newton-Raphson method was used for such a purpose in this paper.

VSTOXX index

Eurex Exchange offers futures and options on the VSTOXX, the European benchmark for equity volatility. The VSTOXX is derived from market prices of implied volatility and is calculated every five seconds by using real time EURO STOXX 50 Index Options bid/ask prices.

Several academic studies demonstrate that volatility is negatively correlated to equity returns, making it an attractive asset class for equity und managers. It can be used to increase portfolio diversification or improve risk return profiles

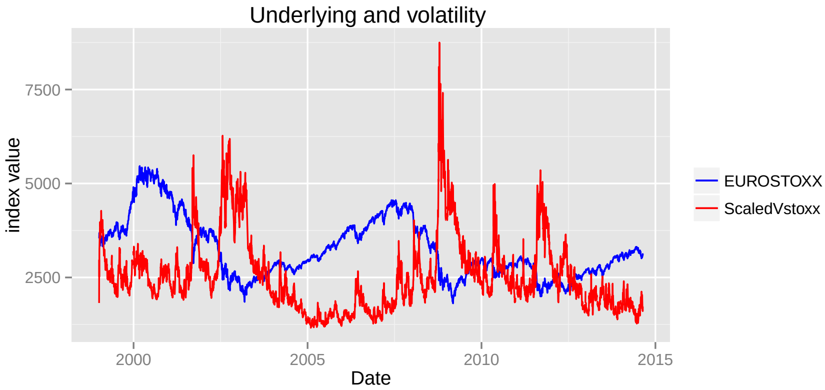

Figure 1: VSTOXX and ESTOXX 50

Why trade volatility derivatives?

- Hedge your portfolio exposure, especially as a disaster hedge
- Diversify your portfolio by adding a new asset class

We need a model to price volatility options !

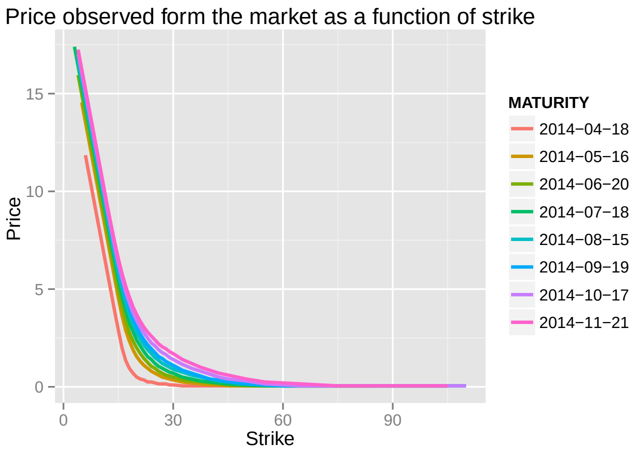

Figure 2: VSTOXX call options traded on 2014-03-31

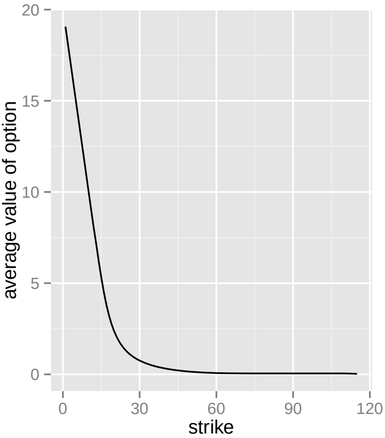

Figure 3: VSTOXX call options traded on 2014-03-31

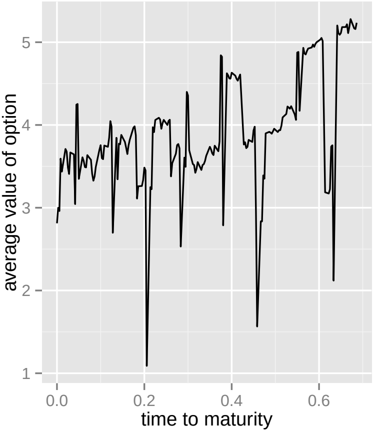

Figure 4: VSTOXX call options traded on 2014-03-31

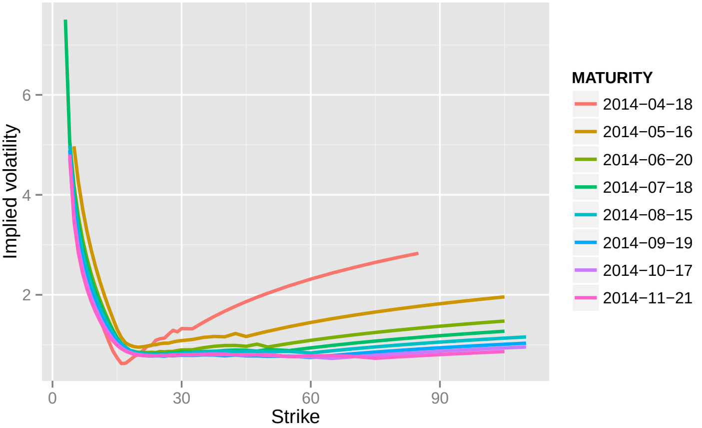

Figure 5: implied volatility of VSTOXX call options traded on 2014-03-31

Gruenbichler and Longstaff(1996) model

In this model European volatility call option price is obtained by assuming that the volatility follows a square–root diffusion process, which is known to exhibit convenient features for volatility modeling, like positivity and mean reversion

$$dV_t = \kappa_V(\theta_V - V_t)dt + \sigma_V\sqrt{V_t}dZ_t$$

Gruenbichler and Longstaff(1996) model

- V_t is the time t value of the volatility index, for example the VSTOXX
- θ_V is a mean of the volatility index, assumed positive
- κ_V is the rate at which V_t reverts, assumed positive
- σ_V is the volatility of the volatility, assumed positive
- Z_t is a standard Brownian motion

Gruenbichler and Longstaff(1996) model

Having made assumation about dynamics of the underlying asset, value of an option may be computed as discounted expectation of the terminal payoff with respect to some martingale measure

$$H_0 = P(0, T)\,\mathsf{E}_Q(H_T).$$

Gruenbichler and Longstaff(1996) model

formula for a call option:

$$C(V_0, K, T) = D(T) \cdot e^{-\beta T} \cdot V_0 \cdot Q(\gamma \cdot K|\nu + 4, \lambda)$$

$$+ \ \mathrm{D(T)} \cdot \left(\frac{\alpha}{\beta}\right) \cdot \left(1 - e^{-\beta T}\right) \cdot Q(\gamma \cdot K|\nu + 2, \lambda)$$

$$-\mathrm{D(T)} \cdot K \cdot Q(\gamma \cdot K|\nu, \lambda)$$

Gruenbichler and Longstaff(1996) model

- $\alpha = \kappa\theta$
- $\beta = \kappa + \zeta$
- $\gamma = \frac{4\beta}{\sigma^2\left(1-e^{-\beta T}\right)}$
- $\nu = \frac{4\alpha}{\sigma^2}$
- $\lambda = \gamma \cdot e^{-\beta T} \cdot V$
- $D(T)$ is the appropriate discount factor
- Q is the complementary non-central χ^2 distribution

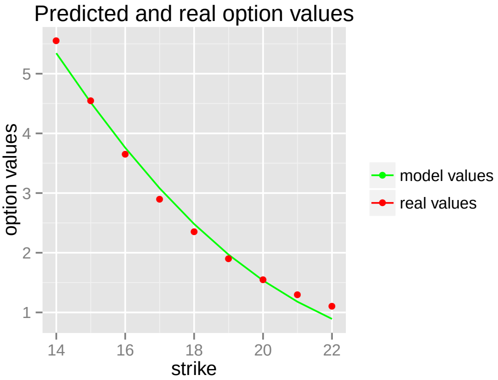

Figure 6: call option prices from model compared to observed from the market on 2014-03-31 (maturity 2014-05-16, MSE 0.056)

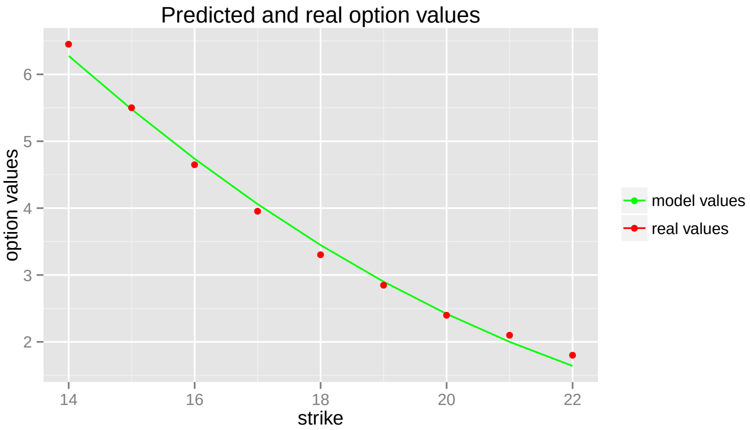

Figure 7: call option prices from model compared to observed from the market on 2014-03-31 (maturity 2014-07-18, MSE 0.037)

Summary

Options on volatility index (VSTOXX)

- **descriptive characteristics**
  - Average values
  - Implied volatility
- **Valuation of volatility Options**
  - Gruenbichler and Longstaff(1996) model

References

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