Real Exchange Rate Uncertainty Matters

Trade, Shipping Lags and Default

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Abstract Using a new time-varying measure of real exchange rate uncertainty, I show that there is a negative relation between real exchange rate uncertainty and international trade at the aggregate level. A one standard deviation increase in the real exchange rate uncertainty is associated with a 5% drop in total trade over GDP. Then, using Colombian firm-level data, I document 3 firm-level facts consistent with the existence of a precautionary margin in international trade. When real exchange rate uncertainty increases exporters, 1) reduce their export intensity; 2) are more likely to stop exporting and 3) less likely to start exporting to new markets. Additionally, I find that this behavior is mostly explained by those exporters paying higher interest rates and facing higher shipping lags. These results contrast with the predictions from standard sunk cost models of international trade. As a consequence, these models will under-estimate the effects that real exchange uncertainty has on international trade flows. To overcome this issue, I incorporate firm-level debt default and international shipping lags into a standard dynamic model of trade. In the new model, an increase in the real exchange rate uncertainty increases the probability for exporters to end up in a financially vulnerable situation. To hedge against this risk, exporters respond by increasing mark-ups or quitting the export market, generating a drop in aggregate exports through both the intensive and the extensive margin of trade. Once this extension is calibrated to match firm-level Colombian data, it predicts that a one standard deviation increase in the real exchange rate uncertainty generates a drop in total exports of around 6%.

JEL Classifications: F10, F31, F40

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1   Introduction

It has been more than 40 years since the fall of Bretton Woods and there is no clear consensus about the consequences that foreign exchange rate uncertainty has on international trade flows. After this episode, it has been widely documented that foreign exchange rate became particularly difficult to predict and highly volatile, creating what is known as currency risk. While the problems associated with currency risk are generally well recognized by firms engaging in international activities and by policy makers, the literature was not able to find any meaning-full relation between exchange rate uncertainty and international trade. Having a clear understanding of the relevance and channels through which foreign exchange rate uncertainty affects international trade is imperative in a world that is increasingly connected. This need is not only justified by the importance of improving our understanding in the determinants of international trade among different nations, but also by the implication that this relation can have on the design of optimal exchange rate policies.

To understand the relations between real exchange rate uncertainty and international trade I use a new measure of real exchange rate uncertainty (henceforth RERU) based on regime switching estimation method. Using this measure of RERU, I then document 3 firm-level facts on how firms respond to these shocks consistent with the existence of a "precautionary margin of international trade". In particular, when real exchange rate uncertainty increases exporters, 1) reduce their export intensity; 2) are more likely to stop exporting and 3) less likely to start exporting to new markets. I show that these results are mainly explained by those exporters facing higher shipping lags and paying higher interest rates, suggesting that financial conditions, and the time-intensive nature of international trade are key dimensions to understand these facts. I then extend an otherwise standard dynamic model of international trade to incorporate these two frictions. In particular, I model financial imperfections as a friction on financial contracts where firms are able to default over their debt. I then show that the existence of debt default and shipping lags are essential to generate firms to behave consistently with the empirical results. Finally, I calibrate this model to firm-level data from Colombian and show that RERU has large quantitative effects over international trade.

To develop the new measure of RERU, I relied on regime switching estimation method to estimate the RERU that agents on an economy face. This new measure allows me to clearly distinguish between shocks to the level of the real exchange rate and to its expected volatility. The use of this measure allows me to pin down a time-varying expected uncertainty over the real exchange rate conditional on information available at a particular time. Then, using a panel data of 58 countries I estimate a standard gravity equation expanded with the proposed measure of RERU. I find a negative relationship between total international trade and RERU, in particular, a one standard deviation increase in the RERU is associated with a drop in total trade over GDP of 5%. To give some context, this drop represents near

a third of the drop in trade openness during the trade collapse that occurred during 2008. But what is behind this negative relationship in the data?

According to standard sunk cost models of international trade, as in Dixit (1989a), Dixit (1989b), and Alessandria et al. (2007), when real exchange rate uncertainty increases, all the adjustment in the exports market are due to changes in the extensive margin of trade. In these types of model, firms have to pay a sunk cost to start exporting, after which they face smaller continuation costs to keep exporting. As in any sunk cost investment model, when profits become more uncertain firms find optimal to delay their investment decision. This delay in the investment takes a particular form in these framework - fewer firms are willing to enter to new export markets, and fewer exporters are willing to stop exporting. When uncertainty increases, the only margin of adjustment is given by the extensive margin. These changes in the extensive margin decision have offsetting effects. When RERU increases, on the one hand there will be fewer firms entering to new markets reducing aggregate exports, but on the other hand, exporters will less willing to stop exporting, increasing the aggregate exports.

I test these predictions using Colombia firm-level data. Contrary to the predictions of the standard sunk cost model, the results show that changes in RERU generate trade responses trough both the intensive and extensive margins. With respect to the intensive margin, when real exchange rate uncertainty increases exporters reduce their exports share. This reaction is mostly explained by those exporters shipping to more time-intensive destinations, and paying higher interest rates. The adjustment trough the extensive margin shows that higher RERU reduce the willingness firms to export. In particular, fewer firms start exporting to new markets, and also more exporters stop exporting. These empirical results are robust to a different range of specifications, and to the addition of different types of controls. I argue that this results are consistent with what I called the "precautionary margins of international trade".

The existence of this precautionary margins of international trade, implies that standard sunk cost type of models will under-estimate the impact that real exchange uncertainty has on international trade. This under-estimation of the effect that RERU has on international trade is due mainly to two reasons. The first one is the lack of response in the intensive margin predicted by this model. The second one is due to the prediction that there will be fewer stoppers when RERU is high. These two prediction are contrary to the empirical patterns documented at the firm level.

What can generate the existence of this precautionary margin of international trade? I document that those firms paying higher interest rates and/or facing higher shipping lags are the ones reducing their exports intensity or quitting the export market. This suggests that financial conditions and the time-intensive nature of trade are relevant to understand these firm level results. These empirical findings shows the relevance of these two mechanisms to understand and quantitatively estimate the effects that RERU has on international trade.

Motivated by these empirical facts, I develop a new dynamic model of international trade based on Dixit (1989a) that incorporates the existence of shipping lags and financial risk. I follow Arellano et al. (2018) in modeling financial imperfections as a friction on financial contracts where firms are able to default over their debt and they borrow from a risk neutral lender. I show that to be consistent with empirical facts standard models are missing two key ingredients that shapes the firm-level responses to changes in the RERU. The firm's financial situation, and the shipping lags of the destination market.

The existence of debt default and shipping lags are essential to motivate firms to behave consistently with the empirical pattern. The intuition is as follows, while increases in the real exchange rate uncertainty leave the expected value of exchange rate intact, the probability that firms assign to end up in financially vulnerable situation increases. This leads exporters to engage in precautionary practices, increasing mark-ups or quitting the export market to reduce the risk they face. These precautionary practices lead to drop in aggregate exports through both the extensive and the intensive margin of trade, consistent with the empirical patters in the data. To estimate the quantitative relevance of theses mechanisms, I estimate the extended model to match the key moments of the exporter behaviour and I find that a one changes in the RERU can reduce total exports by 6%.

Literature This paper is contained in the literature that studies how uncertainty affects real allocations. For example Bloom (2009), and Arellano et al. (2018) argue that uncertainty played a significant role in explaining the 2008 crisis due to the capital adjustment costs in the former work, or due to the financial frictions in the latter. Also recent papers have proposed new measures for aggregate volatility, to analyse the effect of uncertainty shocks on aggregate different economic variables as in Jurado et al. (2015), Fernandez-Villaverde et al. (2010), Fernández-Villaverde et al. (2015). Furthermore, there is an increasing interest in understanding how uncertainty can affect international trade, for example, Novy et al. (2014) document that incorporating aggregate volatility shocks into a model with inventories helps to explain the high volatility of international trade flows, also Alessandria et al. (2015) discussed the direction of the causality between idiosyncratic uncertainty and aggregate shocks, and found that in a sunk cost model of exports idiosyncratic uncertainty shocks generate a counterfactual increase in exports, and Fernández-Villaverde et al. (2011) studies how volatility shocks to the interest rate can affect the macroeconomic performance of small open economies.

But in particular, this paper is more closely related with two branches of literature. With the literature that studies how real exchange rate volatility affects international trade, and also with the literature that studies how financial frictions can affect international trade. The former is a literature that started in the early seventies and is summarized by McKenzie (1999), Clark et al. (2004) and Bahmani-Oskooee et al. (2007). The main conclusion is that the literature did not find any meaningful relationship between exchange rate volatility and trade variables at aggregate level. Recently, some papers have studied

the sectoral effects that real exchange uncertainty has over trade as in Lin et al. (2018), Héricourt et al. (2015),Héricourt et al. (2016), they find that firms that belong to sectors that are financially vulnerable tend to respond negatively to exchange rate volatility.

With respect to the second literature, there are several papers suggesting that financial imperfections can affect trade in different ways, Manova (2013) shows that financially developed countries have a comparative advantages in sectors that are more financially vulnerable, Kohn et al. (2014) show that financial imperfections can help to understands exports dynamics during large devaluations, and Kohn et al. (2016) show that financial imperfections can help to explain new exporter dynamics ,more recently, Brooks et al. (2019) found that the relevance of financial imperfections to understand the gains from a trade reform episode depends on the way financial imperfections are modeled, and that for Colombia, the data seems to point out that standard collateral constraint can be misleading to interpret how trade response to trade reforms.

This paper makes four main contributions to these three strands of the literature. First, I propose a new method to measure real exchange rate uncertainty, that can be easily apply to a wide range of countries, and that can distinguish first moment shocks to the real exchange from second moments shocks, which is not possible to achieve using rolling standard deviations over the changes in the real exchange rate or GARCH/ARCH type of methods. I find that using this measure the aggregate relation between real exchange uncertainty and international trade, is meaningful not only statistically but also economically, something that the former literature was not able to find at aggregate level. Second, I show that both the intensive and the extensive margin of trade response respond to changes in the RERU, and that this changes are related to firm's financial situation and the shipping lags its face. Third, I propose a new model building on Alessandria et al. (2007) and Arellano et al. (2018), that can replicate the negative relationship between real exchange uncertainty and trade, and the firm level responses at both the intensive and the extensive margin by incorporating debt default and shipping lags as a novel mechanism. This model is also in line with empirical works showing that exporters may be more likely to go bankrupt as in Antunes et al. (2015). Fourth, I test the relevance of the new measure using simulated data from the model and compare it with other measures used by the literature. I find that the lack of empirical relevance at aggregate level that was found in the literature between real exchange volatility and trade is likely due to the measure of real exchange uncertainty that had been used in the past.

The structure of the paper is as follows. Section 2 briefly describe the data. Section 3 discusses how I construct the measure of real exchange uncertainty and its relationship with others aggregate variables. Section 4 documents the facts relating trade and real exchange uncertainty. Section 5 develops the model. Section 6 presents the quantitative exercises using the model, and section 7 concludes.

2   Data

In this section I present the data used in the empirical analysis. The empirical analysis is divided in two, fist I analyse the relation between time varying real exchange rate uncertainty and trade openness at aggregate level for several countries. Then I focus on how exporters reacts to changes in real exchange rate uncertainty. For the aggregate analysis I use a penal data composed by 58 countries listed in table A.1 of the appendix. The panel goes from 1995-2015, the time is mainly restricted by the availability of the real exchange rate series for some countries. I use this panel the aggregate relevance of real exchange rate uncertainty shocks over total trade, exports and imports. The sources for the aggregate data correspond to:

- Bank of international Settlement: Monthly effective real exchange rate (RER) indices.
- Penn world tables and World development indicators: Aggregate variables like exports, imports, GDP, term of trade, price indices.
- CEPII: Gravity equations variables like distance, common language, trade agreements, colonial relationships, entry cost (in monetary term and in time).

Firm level data. The firm level database is a panel that goes from 2006 to 2015 constructed using two sources. The main source is from the Colombian customs data, which reports all international exports at firm-destination-product level at a monthly frequency. The second source comes from "Superintendencia de Sociedades", this data is reported at firm level in an annual frequency, covers around 20 thousand firms, that represents more than 85% of the Colombian GDP, accodrging to the organism. I use this data to construct the financial variables at firm level. The data can be merge for the period 2006-2015, which allows me to construct panel at annual frequency for the mentioned period.

In the appendix A.4, I present some results using firm level data from Chile. The data comes from the manufacturing survey, which is panel that goes 1997-2006. I use this dataset as a robustness check, since allows me to control form variables at firm level that I cannot observe using the Colombian firm level data, like employment, and estimated total factor productivity (TFP) at firm level, and to show that this results are no unique to Colombia.

3   Measuring Real exchange rate uncertainty

This sections develops a new way to measure time varying real exchange uncertainty, motivated by the recent works of Jurado et al. (2015) and Fernandez-Villaverde et al. (2010). The proposed measure of real exchange rate uncertainty relies on a two step procedure, the computation of the forecast error of real exchange rate, and the estimation of the expected volatility for every period of time.

I follow Jurado et al. (2015) in thinking uncertainty as the inability of agents to make accurate predictions of the variables. This imply that the absolute value of the error forecast should be consistently low during low uncertainty periods, and consistently high during high uncertainty periods.

To measure the error forecast of the agents I follow Meese et al. (1983) and Kilian et al. (2003), and I assume that agents forecast the real exchange rate as if it behaves as a random walk[1].

Once I obtain a series of the forecast error, I estimate the expected real exchange rate uncertainty by assuming that the process for the forecast error is characterized by a Markov regime-switching in variances using the method developed by Hamilton (1989) [2]. The use of the Markov regime-switching in variances allows to me distinguished between periods characterized by low and high uncertainty, and to compute an the future expected volatility by agents, conditional on the information available at a particular moment.

Construction of the measure

To construct the time varying measure of real exchange rate uncertainty for each country, I proceed in two steps. In the first step, I compute the forecast error. In the second one, I estimate the Markov-Switching process of the error forecast, to compute the expected volatility at moment in time.

1. **Error forecast computation**: Compute forecast Error of h months ahead, μ_t^h:

$$\mu_{t+h}^h = y_{t+h} - E\left[y_{t+h}|I_t\right]$$

   Where I_t is available information at time t. And y_t represents the the natural logarithm of the real exchange rate index at period t. As mentioned, I assumed that agents predict the RER as random walk. Which implies that:

$$E\left[y_t|I_t\right] = y_t$$

2. **Uncertainty computation**: In this step we need to compute the expected variance of error forecast $\tilde{\sigma}_t^{2h,j} = E_t\left[\sigma_{\mu^h}^2|I_t\right]$. To do it I proceed in several sub-steps:

   (a) *Choose number of states*: Let τ be the amount of states of the underlying process. Fix $\tau = j$ $for j \in \{0; 1; 2; 3\}$

   (b) *Estimation of the process given τ*: Estimate the process for $\mu_t^{h,\tau}$.

$$\mu_t^{h,\tau} = \theta_t^{s,\tau} + \epsilon_t^{s,\tau}$$

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[1] Meese et al. (1983), Engel (1994), and Kilian et al. (2003) have shown that different models used to predict real exchange rate cannot improve the out of sample prediction of assuming that the real exchange rate behaves as a random walk

[2] Fernandez-Villaverde et al. (2010) presents a discussion of different method to estimate time varying variances. I assume the Markov switching behaviour, to be consistent with the model specification

With:

$$\epsilon_t^{s,\tau} \sim N(0, \sigma_{s,\tau}^2)$$

And the estimation of the matrix Π, that represents the transition probability matrix, with dimensions $\tau \times \tau$. Were s denotes the possible states of the economy, $s \in \{s_1; ..; s_\tau\}$

(c) *Computation of probabilities*: Compute the j step ahead forecast probability of $s_{t+j} = s_i$ given information I_t:

$$P_{i,t}^{j,\tau} = prob(s_{t+j}^\tau = s_i^\tau | I_t) \text{for i} \in \{1, 2, ..\tau\} \tag{1}$$

(d) *Optimal τ*: Using the likelihood test, for each country, compute the optimal τ. Let the optimal amount of states be defined as τ^*.

(e) *Computation of uncertainty*: Compute $\tilde{\sigma}^{2h,j}_t = E\left[\sigma^2_{s,\tau^*,t+h}|I_t\right]$ using $P_{i,t}^{j,\tau^*}$ estimated in sub step (c).

There several advantages of using this method, first it only relies on real exchange rate series to be constructed, which is available for several countries after 1995,second it has a clear quantitative mapping to economic models,and third it is easy to compute. But these advantages come at the cost that it needs the real exchange rate to move to capture the uncertainty measure. For example, if the real exchange rate is constant over 5 year, but some agents change their perception about the real exchange rate uncertainty during this period, the measure will not be able to capture this change. A problem share with all the measures used by the literature.

Additionally the measure is completely agnostic of what are the sources generating the changes in the regime, or what determines the probability of change for one regime to another, by assumption these changes are exogenously given [3]. This implies that the empirical results should not be interpreted as causal.

Finally, as stated before, this measure of uncertainty has the advantage that does not directly depends on first moment shocks to the real exchange rate as the common measures used in the literature, such as GARCH types of estimations, or moving average of the standard deviations of the exchange rate that assumes that changes in the real exchange rate generates changes in the volatility [4]. Form what is follow I will use $h = 6$ and $j = 1$, to construct the monthly measure of real exchange rate uncertainty. To get annual estimates or the real exchange rate uncertainty I will take the average of the monthly value over the corresponding year.

Cyclical features of real exchange rate uncertainty. To understand how real exchange rate uncertainty is related wit other aggregate variables, table 1 presents the correlation of the real exchange

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[3] This is why controlling not only for standard gravity equation determinants but for changes in foreign demand, real exchange rate, and GDP is important.

[4] See Fernandez-Villaverde et al. (2010) for a discussion about this point.

uncertainty measure with the cyclical component of other aggregate variables. Panel A shows the relative volatility of each variable with respect to output, while Panel B present the correlation of each variable with the RERU. The table shows that RERU is almost half as volatile as GDP for developed economies, and 80% as volatile as the GDP for emerging economies. Similar to literature in of macroeconomic uncertainty I find that the uncertainty in the real exchange rate is counter-cyclical. The measure of uncertainty is negatively correlated with GDP, consumption , investment, exports and imports, while the correlation with net exports does not seems to be significantly different from zero. Finally, the real exchange rate uncertainty is positively correlated with the real exchange rate, and the nominal exchange rate between the domestic country and the USD. This implies that not controlling for changes in the real exchange rate, could biased the results downward, since movements in the real exchange tend to increase the exports and reduce imports.

To have a general understanding of some events related with changes in real exchange uncertainty, figure 1 presents the time series of the estimated real exchange uncertainty for the United States, figure 1 shows that most of the periods of high uncertainty are associated with some extreme events, like wars, financial and economic crisis arround the world.

4   Real exchange rate uncertainty and international trade

In this section explore the the relation real exchange rate uncertainty and international trade. First I focus on the aggregate level relation between international trade and real exchange rate uncertainty. Then I use firm level Colombian data to explore how real exchange rate uncertainty affects exports behavior, at both the extensive and the intensive margin. To the best of my knowledge, I present 3 new firm-level facts about the relation between exporters and real exchange rate uncertainty. These firm level facts are key to understand why, standard dynamic model of trade would under predict the effects of real exchange rate uncertainty in international trade, and we need theory being able to generate the precautionary behavior at the intensive and extensive margins of trade. [5]

Fact 1: Negative relationship between Real exchange rate uncertainty and trade

Aggregate Trade. To estimate the relationship between trade and real exchange uncertainty, I estimate a gravity equation expand it with the real exchange rate uncertainty measure. For country i in time t I estimate the following equation:

$$log(y_{i,t}) = \beta_0 + \beta_1 log(\tilde{\sigma}_{i,t}^2) + X_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t} \tag{2}$$

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[5] In the appendix I estimate an SVAR in the tradition of Sims (1980) and Bloom (2009). Similar results using Chilean data are presented in the appendix A.4.

Where $y_{i,t}$ represents exports, imports or total trade over GDP, depending on the case. $X_{i,t}$, α_i,γ_t represents aggregate controls for each country, fixed effects by country, and time fixed effects. In particular I control for changes in the GDP, real exchange rate, term of trade, its past changes and lag values, and episodes associated with large devaluations.

Table 2 presents the results, only the parameter of interest β_1 is presented. The estimation, implies that a change of an standard deviation in the real exchange uncertainty is associated with[6]:

1. A drop in exports over GDP between a 4% and 5.5%.
2. A drop in imports over GDP between a 3% and 4% .
3. A drop in total trade over GDP between a 3% and a 4%.

To put the results in perspective, according to the World bank during the 2008 crisis, the drop of exports over GDP for the whole world was on average about 13%. Table 2 shows that controlling for the change of the aggregate variable (column 2), or for past change of them (column 3) do not change the results for exports and total trade over GDP. Once I control for year fixed effects (column 4), the results remain the same. One possible objection for this result, is that exchange rate elasticity could change over time as responds to changes in real exchange rate. If the elasticity to changes in the real exchange change over time, as found in Alessandria et al. (2014), and since the real exchange uncertainty is correlated with changes in the real exchange, it could be that the observed negative relationship is just due to a miss specification. To over come this problem, I estimate an error correction model, to capture this possible different reactions between the long and short run effects of the real exchange rate in exports. The estimation results are presented in table A.2 of the appendix A.1. In this case the negative relationship still holds.[7].

Bilateral Trade. I also estimate a similar equation as before but for the bilateral relationship across countries. This allows me to control by variables that could affect total and bilateral trade, and that are not included in the first case, like bilateral changes in the real exchange rate. For country i and j in time t the estimated equation is given by:

$$log(y_{i,j,t}) = \beta_0 + \beta_1 log(\tilde{\sigma}_{i,t}^2) + \tilde{X}_{i,t} + X_{i,t} + X_{i,j,t} + \alpha_i + \gamma_t + \epsilon_{i,t} \tag{3}$$

Where $y_{i,j,t}$ represents bilateral exports, imports, or total trade over GDP from country i to country j. $X_{i,j,t}$ represents controls at bilateral relationship level and the standard gravity controls. $X_{i,t}$ and $X_{j,t}$ represents aggregate controls at country level. $\alpha_{i,j}$ and γ_t represents fixed effects by bilateral relationship and year fixed effects, respectively.

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[6] A change of an standard deviation with respect to the mean

[7] The estimated equation for the error correction model, (25) is presented in the appendix A.1

The estimation results of the equation (3) are presented in table 3. These results are consistent with the ones found in the previous estimation, controlling for bilateral variables does not seem to change the results. In this case, an standard deviation change in the real exchange rate uncertainty is associated with an average drop of 5% in bilateral exports over GDP, 2.5% in bilateral imports over GDP and 3.1% in total bilateral trade over GDP.

As before, it can be several concerns about some omitted variable problem, or reverse causality problem with estimation (3). It could be that the real exchange rate uncertainty is capturing changes in the uncertainty or volatility of the GDP of a country, or that the changes in real exchange uncertainty are reflecting changes in the co-movement of the domestic economy with foreign countries. To control for this, in table A.5 in appendix A.2 I present the same estimation as in (3), but controlling for these variables. In column 1 I include as a control a moving average correlation over the last three year in the industrial production between the domestic country and the industrial production of the countries that belongs to the G7 , and in column 2-4 I include as a control the rolling standard deviation (over 1,2, and 3 years) of the log changes in the industrial production for domestic and foreign economy. In all these cases, the results holds.

Finally, another possible objection is that the real exchange uncertainty shocks could reflect the fact that an economy is more closed in term of trade, implying the existence of reverse causality [8]. To control for this, I estimate the same equation as in (3) including lags of the trade openness of the domestic economy. Results are presents in table A.5 of appendix A.2. The estimation using these controls, shows that main results still hold, but for exports and total trade, the estimated coefficient reduces up to a 33% in some cases.

Firm level facts

This section focus on how exporters respond to changes in the real exchange uncertainty. I present the estimations at firm level data, to identify possible mechanisms that help explain the aggregate patterns. I divide the analyses between the intensive margin of trade and the extensive margin trade. I document that when real exchange rate uncertainty increases: 1) firms that paid higher interest rates and/or faced higher shipping lags reduce their exports shares by more; 2) firms are more likely to stop exporting; and 3) firms are less likely to start exporting to new markets. These firm-level results contradicts the predictions of the standard dynamic models of trade. More importantly these results shows that using these type of models to understand how real exchange rate uncertainty affects international trade, would lead to sub

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[8] In an standard two country model, the the real exchange rate would be more volatile if there is less trade among the two countries. Even though, the scope of the paper is about changes in real exchange uncertainty, and not in the average volatility of real exchange rate, it can be think, that changes in the total trade over GDP could be generating the movements in the real exchange rate volatility.

estimation of its effects. The sub-estimation is due to inability of these type of model to generate facts 1) and 2).

Fact 2: Intensive margin of trade and RERU

Motivated by a wide variety of works showing that international trade is intensive in time and financial requirements as documented by Manova (2013), Kohn et al. (2014), Kohn et al. (2016), Fillat et al. (2015), Leibovici et al. (2019) among others, and works as Arellano et al. (2018), Khan et al. (2016), showing that the existence of default risk at firm level can induce further adjustment in firm production. I focus on two main mechanism that are important to understand how exporters reacts to uncertainty, financial vulnerability and shipping lags.

To construct a measures that denotes the financial vulnerabilities that firms face, I follow the theoretically predictions in Arellano et al. (2018), and use the interest rate that firms pay as a measure of financial vulnerability. Using the Colombian data described in section 2, I construct the interest rate that firms pays as the interest that the firm paid divided by the total liabilities the firm had over a year[9]. Once I have a interest rate measure for each firm, I group each firm in different groups according to were they are in percentile of the distribution. I group firms in different percentiles each year,I construct a dummy for each percentile denoting if in the year the firm belonged or not to that particular group. I use that dummy, as a measure of default risk that firms face in the main regression. I use other measures as robustness as leverage, or the ratio of interest payment over total profits that firm has, and group them in a several ways. I finally use lagged corresponding dummy interacted with the measure of RERU, to see how different firms reacts to RERU depending on their financial vulnerability.

To construct a measure of shipping lags, I proceed in a similar way. First using the "Doing business" survey of World Bank, I obtain the reported time of each country to process an import, and I use it as a proxy for the total shipping lags that Colombian exporters face. I group each destination according to the this measures in different percentiles, and then I use the corresponding dummy interacted with the RERU measure, to see how different firms reacts to RERU depending on the shipping lags they face. For a firm i, that exports product l to country j, I estimate the following equations:

$$log(es_{i,l,j,t}) = \beta_0 + \beta_1 \log(\tilde{\sigma}_t^2) + \sum_{h=0}^{3} \beta_h^0 \log(\tilde{\sigma}_t^2) \times I_{i,t-1}^h + \sum_{h=0}^{3} \beta_h^1 \times I_{i,t-1}^h + \alpha_{i,l,j} + X_{i,l,j,t} + \hat{X}_{i,t} + \epsilon_{i,j,h,t} \quad (4)$$

Where the dependent variables $es_{i,l,j,t}$ is the export share of the product destination exports, and is constructed as the exports value in pesos of the product-destination divided by total income of the firm in pesos. $\tilde{\sigma}_t^2$ represents the measure of real exchange rate uncertainty, and $I_{i,t-1}^h$ represents a dummy variable

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[9] Ideally I will use the marginal interest rates, since this data is not available I use the average interest rate as a proxy for the marginal interest rate firms pay

that is 1 if the firm belonged the percentile h in the previous period, and zero otherwise, depending on the case, it will represent the financial vulnerability of each firm or the shipping lags that firm face. $\alpha_{i,l,j}$ represents fixed effects by firm, product, destination. $X_{i,j,t}$ represents standard gravity controls, bilateral exchange rates, multilateral real exchange rate, domestic and foreign absorption, term of trade, aggregate productivity , the change of this variables (lag and log difference). $\hat{X}_{i,t}$ represents firm level controls: Number of destination by firm and by firm-product pair, Age of a firm exporting to a market, actual and lag profits, imports share (total imports divide by operational costs)

Financial vulnerable firms and RERU: Table 4 presents the estimations of equation (4) using the interest rate to construct the financial vulnerability dummy. Two main results are striking, exporters reduce their exports shares when real exchange rate uncertainty is high (first row), and those exporter facing financial vulnerable situation reduce their export shares between a 9% and a 6% more (second and third row). The first column presents the results when controlling for the standard gravity considerations fixed effects at firm, product, destination level, and size. The third to the fifth column sequentially aggregates further controls as described by the table. Results remain mostly unchanged as we add these additional controls at firm, home and destination level.

Table A.3 in the appendix presents the estimations of equation (4) using in column one the leverage of the firms (measures as total liabilities over total asset) and in column two using interest payments over total profits as two different measures of financial vulnerability. Results remain unchanged

Shipping lags and RERU: Table 5 presents the results for shipping lags, the estimated equation is the same as in (4) but using the shipping lag dummy. [10] Rows one shows the average reaction of export share to RERU, and rows 2 and 3 the differential reactions for those firms facing higher shipping lags (In the second and third tercile for the distribution). In particular, firms facing higher shipping lags drops their exports share between a 10% and 25% more. This results are consistent with other papers,as Leibovici et al. (2019) that find that shipping lags are relevant to explain trade flows.

Fact 3 and 4: The precautionary extensive margin

The above estimation reflects how the intensive margin at firm level is related with exchange rate uncertainty, now I turn to analyze how the real exchange rate uncertainty affects the extensive margin. Standard sunk cost models predicts that when uncertainty increases, firms are less likely to enter the export markets, and exporter are less likely to stop exporting. While the former predictions holds in the data, the second one is contrary to my results. I create a dummy variable $I_{i,t}^{Stop}$, that equals one if a firm

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[10] This table is not available for the Chilean economy, for the lack of data to construct this variable. The data for Chile is presented as total exports for exporter, and do not discriminate over other destinations.

exported at period t-1 and did not export at t, and is equal to zero if a firm exported at t-1 and at t [11]. Similarly, I create a dummy $I_{i,t}^{Entrant}$ that equals one if the firm export to a given market at t, but did not export to that market at t-1.[12] I estimate the following linear probability model:

$$I_{i,t}^h = \beta_0 + \beta_1 log(\tilde{\sigma}_{t-1}^2) + \xi_{i,l,d} + X_{i,d,l,t} + X_{i,t} + \epsilon_{i,l,d,t} \tag{5}$$

The results are presented in table 6. In this case, an standard deviation increase with respect to the mean, generates between a 5% and a 8% percentage points decrease in the probability of firms entering to new exports markets. And an increase in the probability of exporters to stop exporting between 10% and 20% percentage points.

In conclusion, I have presented 4 facts about real exchange uncertainty and exporter behavior. The first fact, shows that there is a negative association between international trade and real exchange rate uncertainty. When looking at firm level data, fact 2 shows that exporters tend to reduce their exports share when real exchange rate uncertainty is high, and also that those facing higher shipping lags or that are more financially vulnerable tend to reduce their export share by more. Fact 3 and 4, shows that when RERU is high, exporter are more likely to stop exporting and less likely to enter new markets. While the later fact (less likely entry to new markets) is predicted by standard dynamic models of international trade, facts 2 and 4 are not. These model does not predict any change in the intensive margin, and contrary to the empirical results, predicts that firms are less likely to exit. These results implies that using an standard dynamic model of international trade would sub-estimate the effects that RERU has on international trade flows, and that to proper captures this relation we need a model that can properly capture firm level facts. The next section develop a model consistent with all the documented firm level facts.

5   Model

Motivated by facts 2, 3 and 4, I extend the standard sunk cost model of Alessandria et al. (2007) to incorporate two additionally frictions, the existences of shipping lags, and financial friction modeled as in Arellano et al. (2018). [13][14]

To focus on the relevance of each mechanism I develop a partial equilibrium model, in an small open economy, with two types of shocks, a nominal exchange rate shock, and a shock to its volatility [15]. In the model, the economy is populated by two type of agents, producers of varieties and lenders. The producers

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[11] A period t means a year

[12] Note that in this case, the extensive margin is referring to each destination to which the firm is engaging with.

[13] This way of modeling financial frictions is also in lines with Arellano et al. (2012), and Khan et al. (2016), and is motivated by the fact number 2

[14] The motivation for shipping lags is related and trying to capture in a simple way fact number 3.

[15]   Itskhoki et al. (2017) shows that incorporating shocks to the demand of foreign asset, that generate changes in the nominal exchange rate, can help to explain several of the puzzle related with the real exchange rate

of varieties sell their products in competitive monopolistic markets, and can issue debt to a risk neutral lender. The lender is modeled as a representative agents that is risk neutral and lends and borrows money to firms.

Production of varieties. There is a measure μ_{fi} of firms that produce goods. A firm in the model, is a producer of one variety that can holds debt, and can sell to the domestic market, denoted as d, and the foreign market, denoted as f. At the moment each firm is born, the firm draw a productivity z_i from a log normal random variable Z, characterized by the mean μ_z and its variance σ_Z^2, the firm's productivity is fixed over all the life time of the firm. These firms are monopolistically competitive and use labour l in a constant return production function to produce output $y = l z_i$.

There are two main differences between the domestic and foreign market, the timing of production and the currency at which prices are set. In the domestic market, at time t, a firm decides how much to produce and sell in the domestic currency. The firm pays the labour and receives the profits at time t. When selling to the foreign market, the firm face shipping lags, it has to decide production at period t, to sell it at t+1 when it gets the revenues in foreign currency and pays its workers. [16]

As in Alessandria et al. (2007), to enter the export market, each firm has to pay a sunk cost f_s, and once they are in the export market they have to pay a continuation fix cost f_e. The sunk cost is paid at period t by firms (if did not export at t-1) to export at t+1. The fixed cost, f_e is paid once the export is done at t+1. This generates the option value of exporting, making the decision of entry and exit a dynamic one, as in Dixit (1989b) and Alessandria et al. (2007). Furthermore, as noted by Fillat et al. (2015), this will generate risk of exporting, since firms will be willing to stay in the export market even if they expect negative profits, just not to forgo the export option value.

Finally, on the financial side, each firm can issue debt denominated in national currency, b_t, to a risk neutral lender at a price q_t. The main difference with standard exports model, is that firms are now allowed to default over debt, implying they will not issue negative dividends as in Arellano et al. (2018). If firm has to issue negative equity, it will be forced to leave the market and default over debt. This assumption, together with shipping lags, generate firms to be averse to risk. As the volatility of exchange rate increases, the probability of facing lower exchange rate realization will increase, and firms will decide to reduce the risk to which they are exposed to. This reduction of risk can be done in two ways, reducing the labour they hire and hence generating a decrease in export through the intensive margin, or leaving the export market.

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[16] See **gopinath˙international˙2015.** for more details about invoicing. According to Dian data, at least 2/3 of the firms invoice their exports prices in U.S. dollars, for the period between 20011 and 2018.

Domestic and foreign Demand. The demand that a firm i face, is exogenously given by:

$$y_i^{D,j} = A^j (p_i^j)^{-\sigma} \tag{6}$$

For $j \in \{d, f\}$. Since the demand does not fluctuate, the sales to the domestic market does not vary over time. In this set up, the model is capturing the additional riskiness that the exchange rate generate over the nominal demand in the foreign market. The domestic market, is a completely risk-free market to which firms will always sell the same quantities.[17]

Financial imperfection. As stated before, firms cannot issue negative dividends. Each firm pays their equity holder their revenues net of production costs and net payments of debt. Equity payments are not allowed to be negative, by the non-negative equity payout condition:

$$d_t = p_t^d y_t^d - w l_t^d + m_t \{ e^{\xi_t} p_t^f y_t^f - w l_t^f - f_e \} - m_{t+1}(1 - m_t) f_s + q_t b_{t+1} - b_t \geq 0 \tag{7}$$

The first two terms denote the domestic profits, the third term is the profits of exporting if the firm had decided to export ($m_t = 1$) in the previous period. $m_{t+1}(1 - m_t) f_s$ is the sunk cost payment the firm has to do if the firm decides to export the following period but did not export on period t . Finally, $q_t b_{t+1} - b_t$ is the net payments of debt.

The price of the bond $q_t = q_t(z_i, b_{t+1}, l_{t+1}^f, \xi_t, \sigma_{\xi,t})$ reflects the compensation that a risk neutral lender will receive for the loss it will incur in case the firm decides to default. It depends on the aggregate state $S_t = \{\xi_t, \sigma_\xi\}$, the productivity of the firm, and the firm decision $l_{t+1}^f, m_{t+1}, b_{t+1}$.

To characterized the default decision of a firm, lets first define the maximal borrowing that a firm can do as:

$$M_b(z_i, S_t) = \max_{l_{t+1}^f, b_{t+1}, m_{t+1}} q(z_i, b_{t+1}, l_{t+1}^f, S_t) b_{t+1} \tag{8}$$

And let $\bar{l}_{t+1}^f, \bar{b}_{t+1}, \bar{m}_{t+1}$, be the decision that maximized the issuance of new debt. Now lets define the exporter liquidity needs as follows:

$$LN(l_{t+1}^f, z_i, b_{t+1}) = p_{t+1}^d y_{t+1}^d - w l_{t+1}^d - w l_{t+1}^f - f_e + M_b(z_i, S_{t+2}) - b_{t+1} \tag{9}$$

Which denotes how much liquidity from the export market a firm that exports at t+1 will need to cover all the expenses net of the domestic profits it will get. If $LN(.)$ is positive, then domestic profits are enough to cover all the expenses net of debt payments.

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[17] In a general equilibrium environment, this will not hold. The aggregate prices and wages would fluctuate with the exchange rate, making domestic demand to vary over time

For an exporter, define the ξ_{t+1}^* as the minimum exchange rate level at which a firm that is exporting will not default:

$$e^{\xi_{t+1}^*} = \begin{cases} -\frac{LN(l_{t+1}^f, z_i, b_{t+1})}{p_{t+1}^f y_{t+1}^f} & \text{if } LN(l_{t+1}^f, z_i, b_{t+1}) < 0 \text{ and } m_{t+1} = 1 \\ 0 & \text{if } LN(l_{t+1}^f, z_i, b_{t+1}) > 0 \text{ and } m_{t+1} = 1 \end{cases} \tag{10}$$

Equation (10) characterised the threshold level of the exchange rate at which an exporter will default. If the LN(.) is positive, there is no value of $e^{\xi_{t+1}^*}$ at which the firm will default. In this case, domestic profits at t+1 are big enough to pay all the costs net debt payments.

The default decision for a firm that does not export, does not depends on the level of exchange rate, but on the amount of debt it has and its productivity.

Define d_{net} as the indicator variable that indicate when a firm will default.

$$d_{ne,t+1} = \begin{cases} 1 & \text{if } b_{t+1} > p_{t+1}^d y_{t+1}^d - \bar{m}_{t+2} f_s - w l_{t+1}^d + M_b(z_i, S_{t+2}) \text{ and } m_{t+1} = 0 \\ 0 & \text{if } b_{t+1} < p_{t+1}^d y_{t+1}^d - \bar{m}_{t+2} f_s - w l_{t+1}^d + M_b(z_i, S_{t+2}) \text{ and } m_{t+1} = 0 \end{cases} \tag{11}$$

Lender's problem. The lender is assumed to be risk neutral. Given a free risk interest rate, r, and the associated discount rate β, the lender will lend to the firm at prices $q_t(z_i, b_{t+1}, l_{t+1}^f, m_{t+1}, \xi_t, \sigma_{\xi,t})$. For a firm that has decided to export the following period, the bond price will be given by:

$$q_t(z_i, b_{t+1}, l_{t+1}^f, 1, \xi_t, \sigma_{\xi,t}) = \frac{1}{1+r}(1 - F(\xi_t^*|\xi_t, \sigma_{\xi,t})) \tag{12}$$

Where $F(.|\xi, \sigma_\xi)$ is the cumulative distribution function of the exchange rate conditional on the values ξ, σ_ξ. And for a firm that decided not to export will be given by:

$$q_t(z_i, b_{t+1}, l_{t+1}^f, 0, \xi_t, \sigma_{\xi,t}) = \frac{1}{1+r} d_{ne,t+1} \tag{13}$$

Cash on hand. Similar to Arellano et al. (2018), the problem of the firm can be expressed using cash on hand, denoted by x, as a state variable. This simplifies the problem reducing the number of states variables to 5, making the solution of the model easier in computational terms. The cash on hand of each firm in this case is given by:

$$x_t = p_t^d y_t^d - w l_t^d + m_t \{e^{\xi_t} p_t^f y_t^f - w l_t^f - f_e\} - b_t \tag{14}$$

This allows me to simplify the state space to $\{z_i, x_t, m_t, \xi, \sigma_\xi\}$. Given an exchange rate realization, the dividends of firm will be given by:

$$d_t = x_t - (1 - m_t)m_{t+1}f_s + q_t b_{t+1} \tag{15}$$

Now the decision rule for labour, debt, entry and exit of the export market, can be expressed as function of the cash on hand, export status, firm's productivity and the aggregate state of the economy.

Exchange rate process The exchange rate process is assume to follow an AR(1) process with time varying standard deviations. The time varying standard deviation evolves according a Markov chain with only two states denoted by σ_H and σ_L. The process of nominal exchange rate, ξ_t is then given by:

$$\log(\xi_t) = \mu_s^\xi + \rho \log(\xi_{t-1}) + \sigma_s \epsilon_t \tag{16}$$

Where μ_s^ξ has the standard convex correction, such that:

$$\mu_s^\xi = \left(\mu^\xi - \frac{\sigma^2}{2(1 - \rho^2)} \right) (1 - \rho) \tag{17}$$

An the matrix of transition probabilities between states is given by:

$$\Pi = \begin{bmatrix} \pi_{L,L}^s & \pi_{L,H}^s \\ \pi_{H,L}^s & \pi_{H,H}^s \end{bmatrix} \tag{18}$$

Firm's Recursive problem. Each firm is characterized by the productivity z_i, the cash on hand x_t, and the export status m_t. Following Khan et al. (2016), I assume that firms face a probability of dying given by π_d, this is relevant to make firms willing to issue debt, and to be financially constrained. Given an amount of cash on hand, the firms decides how much new debt to issue, its export status tomorrow, and the production for the foreign market, provided that did not default. To simplify notation, I will drop the time subscript.

At the beginning of each period firms decide if default or not. Formally the problem that the firm solves is as follows:

$$V(z, x, b, \xi, \sigma_\xi) = max\{V^c(z, x, m, \xi, \sigma_\xi), V^d\} \tag{19}$$

Were V^c denotes the continuation value, and V^d is a constant value that the firm will get if decides to default, normalized to zero[18]. If the firm decides to continue, it has to solve the following problem:

$$V^c(z,x,m,\xi,\sigma_\xi) = \max_{l^d,l^{f\prime},m^\prime,b^\prime} d(z,x,m,\xi,\sigma_\xi) + \mathbb{E}\{QV(z,x^\prime,m^\prime,\xi^\prime,\sigma_\xi^\prime)|\sigma_\xi,\xi\} \tag{20}$$

s.t.

$$d = x - m^\prime(1-m)f_s + q(z,l^{f\prime},m^\prime,b^\prime)b^\prime \geq 0 \tag{21}$$

$$x^\prime = p^{d\prime}y^{d\prime} - wl^{d\prime}_t + m^\prime\{e^{\xi^\prime}p^{f\prime}y^{f\prime} - wl^{f\prime}_t - f_e\} - b^\prime \tag{22}$$

$$y^d + y^f m = z_i(l^d + l^f m) \tag{23}$$

and (6), (10),(11), (12),(13)

Similar to Dixit (1989b) and Alessandria et al. (2007), on top of the default threshold, there will be two thresholds that characterized the export decision, the entry and the exit threshold. The main difference with their set up, is that the threshold will not only depend on the aggregate variable (the exchange rate level and the volatility in this case) and the firm productivity, but also on the cash on hand level that each firm has. This generates, the possibility of the extensive margin of trade reacting to the aggregate and the individual risk.

Equilibrium Lets $S_t = (\xi,\sigma_\xi)$ and $s_i = (z_i,x_i,m_i)$ denote the aggregate and the idiosyncratic state variables. The equilibrium for this economy, is a set of policies functions $\{l^d(s_i,S), l^f(s_i,S), m^\prime(s_i,S), b^\prime((s_i,S))\}$ a value function $V(s_i,S)$, and a set of prices $\{q(s_i,S), p^d(s_i,S), p^f(s_i,S)\}$ such that given the parameter values, the aggregate state S, and the individual states s_i:

1. The optimal solution for each firm coincides with the policies functions and the value function.

2. The bond prices are given by (10) and (11).

3. Individual markets for each variety clear.

The partial equilibrium nature of the model, implies that firms do not need to keep track of the distribution of cash on hand and export status over the whole economy. This assumption simplify the computational burden of the problem, but at the cost that it is likely that the results of the model can change once the same exercises is done in a general equilibrium. Most likely, the partial equilibrium nature of the exercise, will imply that the model will over estimate the impacts of real exchange uncertainty shocks, this is due to the fact that once prices and wages adjust to real exchange rate shocks, firms will

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[18] This implies will default only when dividends are negative.

not decrease labour as under the partial equilibrium set up. In section B, I present the algorithm I use to solve the firm's and lender problem.

Optimal decisions. Before I present the main quantitative results of the model, I discuss the optimal policies functions related with the amount of trade in the economy, i.e., the labour use for production of exports, the extensive margins of firms, and the amount of debt issue by firms that will decide to export in the following period. The results presented in this sections uses the parameters values presented in table 7.

Figure 3 plots the the minimum exchange rate level at which each firm is willing to enter or exit the export market. From the top graphs, present the entry and exit decision for firms with high levels on cash on hand, while the bottom graphs the entry and exit decision for firms with low level of cash on hand. The red dotted lines, represents the entry and exit decision for states with high volatility of exchange rate, and the blue line the decision under low volatility states. Similarly, figure A.2, presents the entry and exit decision for an standard sunk model, without neither shipping lags, nor default.

Figure 3 displays two results. First, as it is standard in the sunk models, the threshold to enter the export market is higher than the one to exit it, this is due to the option value of exports originated by the existence of the sunk cost to export as shown in Dixit (1989b), and Alessandria et al. (2007). Second, firms require an smaller exchange rate level to enter or exit the export market, when they hold high on cash.

Figure 3, also displays how adding shipping lags and default originates a new force that counteracts the standard standard effects generated by the existence of the option value of exports. When firms face more risk, due to the increase in the exchange rate uncertainty or the low cash on hand, firms are more willing to exit. I call this the precautionary quitting motive. The bottom panel shows that when firms have low cash on hand the existence of additional risk reduce the differences between the entry and the exit threshold.

In the top right panel of 3, it can be seen that when uncertainty increases the precautionary quitting motive and the option value of exports works in opposite directions. When the exchange rate volatility is higher, the least productive firms are more willing to leave the market, and the most productive firms more are willing to stay. In this case the precautionary quitting motive is stronger that the option value of exports for low productive firms. Firms that are more productive face smaller risk in the model because they receive higher profits from the domestic market, and the fixed cost they pay as a share of expected profits is smaller. When uncertainty increases, the increase in the probability of default is not big enough to compensate the effect that uncertainty has over the option value of exporting.

When firms have high cash on hand, high productive firms face small risk, making the quitting decision behave as in the standard model. But as the risk starts increasing, the precautionary quitting motive kicks in reducing the willingness of firms to stay in the export market. [19]

In conclusion, the quitting decision, is driving by two opposing forces in the model, on the one hand, the option value of exports that make firms willing to delay the quitting decision of the export market, and on the other hand, the risk of being in the export market makes firms more willing to quit. In the model, the risk a firm faces increases with exchange rate uncertainty and decreases with the amount of cash on hand. For periods of low uncertainty and high cash on hand, the value of exporting is higher, but it decreases as we increase the uncertainty of real exchange.

To further understand the entry and exit decision, figure 4 presents the debt issuance for firms that had decided to export in the following period. The top panel presents the amount of debt each firm issue conditional on the level of cash on hand and the export status a firm has. The bottom panel, shows that when exporter hold high levels of cash on hand, they decide to reduce their dividends today to increase the saving. This results could help us to understand why firms with high cash on hand are more willing to enter the export market or to stay on it. When firms have high cash on hand, firms decide to save more reducing their exposure to risk in the next period. The reduction on risk made through savings is particularly important for low productivity firms since the cash flows originated in the domestic market is smaller. As it can be appreciated in the third graph of figure 4, firms with low cash on hand will not be able to save as much due to the liability constraint, which does not allow the firm to reduce the risk as expected. This inability to save, increase the exposure to risk, reducing entry, increasing exit, and reducing the intensive margin of labour.

Figure 5, presents the labour decision for firms with different productivity and for different level of uncertainty at the mean level of exchange rate. The top panel shows the labour decision for firms that have exported in the current period, while the bottom panel show the labor decision for firms that have not export in this period. It can be seen, that besides the effects that higher volatility has on the extensive margin, it also affects the intensive margin, on average firms reduce the amount of labour they hire in 3% when uncertainty about real exchange rate is high. The intuition here, is similar as in Khan et al. (2016), and Arellano et al. (2018), the existence of financial imperfection, make firms willing to reduce the labor they hired during high volatile environments to reduce the exposure to bad shocks.

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[19] In figure A.2 of the appendix C, I present the policy function for entry and exit for an standard sunk model in partial equilibrium. In this case, the uncertainty generates a delay and entry and exit as in Dixit (1989b) and Caballero (1992). The increase in uncertainty delay entry, making firm require a higher exchange rate to enter the export market, but on the other hand it decreases the exchange rate level at which are indifferent to keep exporting or quitting the export market.

6   Quantitative results

Now I use the model to perform two quantitative exercise intended to respond the following two answers:

1. Can the proposed mechanism help us to explain the negative relationship between real exchange rate uncertainty shocks and aggregate exports?

2. How different is the proposed method to measure real exchange uncertainty with respect the other methods used in the literature?

To answer these questions, I calibrate the model to match relevant moments of international trade. Once the calibration is done, to answer the first question I simulate a model with and without shocks to real exchange rate, and regress the simulated volatility of the real exchange rate over the aggregate export.

To answer the second question, I will present a second simulation, in this case I estimate the proposed measure of real exchange uncertainty and the rolling standard deviation to test if this method can properly captured the real exchange uncertainty and in what magnitude.

6.1   Calibration

I calibrate three different models. One is the standard sunk cost model without neither shipping lags nor default, I call this the sunk cost model. Then I calibrate the extended version, that I call the shipping lags, and finally, I calibrate the same version presented in the previous section but reducing the death probability of firm by half, I call this version Low π_d. Table 7, presents the calibrated parameters for the three versions, the second column presents the value of each parameter, and the third column the rationale behind it. The model is calibrated at quarterly frequency, and the only parameters estimated within the model are the ratio $\frac{A^{cl}}{A}$, the dispersion and the mean of the firm productivity μ_z, σ_z, and the level of the fixed cost f_e. These parameters are estimated to match the exports share, the export-sale ratio, and the exporter premia measure as the ratio of total shipment between exporter and non exporters. The parameters associated with real exchange rate are estimated outside the model for the Colombian case.

In table 8, I present the values of the targeted moments and the prediction of the model. I decide to target this moments, because are important for the mechanism stated in the model. The exporter premia, and the share of exporter help to discipline the model in term of the relevance of the extensive margin, against the intensive margin, since the magnitudes of the impact of the real exchange uncertainty could be different depending on which effects is more important. While, the ratio of domestic sales over foreign sales is important in the model, since firms face no uncertainty in the domestic market, if a firm's total income depends heavily on the exports income it can over react to uncertainty shocks, since the overall risk would be artificially large.

6.2   Real exchange Uncertainty shocks: The relevance of the mechanism.

I use the model to test how big is the response of aggregate exports to uncertainty shocks. I do this in two ways, first I simulate the model with uncertainty shocks and real exchange rate shocks, and then I simulate the model under pure uncertainty shock, and compare the results between the standard sunk cost model, the proposed extension, that I call Shipping lags, and the proposed extension using a lower value for death probability, called low π_d.

In the standard sunk cost model, the timing for entry and exit, and the timing for payment of f_e and f_s are the same as in the Shipping lags model. The differences relays in that firms will never default neither face shipping lags. Firms are allowed to produce after they observed the exchange rate shocks and dividends will be equal to current profits. The third case, call low π_d, is exactly the same as the proposed model, but with $\pi_d = 0.005$. The comparison between the proposed extension and the standard sunk cost model, will point out the relevance of the two proposed mechanism, while lowering π_d, allows me to test for the relevance of the financial constraint, since a lower π_d will reduce the probability of firms facing financial problems.

I simulate the model over 2800 periods. During the first 300 periods I assume the volatility shocks is always in the low state and that the real exchange is at the mean value. From period 300 to 2800, I simulate the exchange rate following the process described in the model section. I replicate each simulation over 200 time. In each replication I regress the aggregate exports over the real exchange and the volatility value (remember volatility only takes two values representing high or low volatility states).

Foe each replication I estimate the following regression over the generated data:

$$y_t = \beta_0 + \beta_1 \tilde{\sigma}_{\xi,t} + \beta_2 rer_t + \epsilon_t \tag{24}$$

In table 9 I presents the average coefficient for the estimation of equation (24). Row 1 of each panel table 9 runs the specification (24) in levels. While row 2 of each panel presents the results of the same specification but in logs. Column 1 correspond to the standard sunk cost model, column 2 presents the coefficient for the proposed extension of it and column three presents the estimation for the Low π_d version. The results shows that in both cases the prediction of the model implies a negative relationship between aggregate exports and volatility. While the standard sunk cost model predicts near zero response to pure exchange rate shocks, the proposed model, predicts that an standard deviation change (with respect to the mean) in the real exchange volatility generates a drop in aggregate exports of 4%, while for the version with low π_d implies a drop of a 0.6%. When I simulate the uncertainty shocks with shocks to the level of the exchange rate, I find that the standard sunk cost model predicts a drop in aggregate exports of 3.3%, while the estimation for the proposed model is of 10% and 6% for the case with the low probability of dead.

6.3   Real exchange Uncertainty shocks: The relevance of the estimation method

The last exercise for which I will use the model is to test the different measures used to measure the real exchange uncertainty . I will simulate the model exactly in the same way as in the first case, but I will make 150 replications.

Since the model is calibrated at quarterly frequency, I will simulate the real exchange data and the exports at quarterly frequency. For each generated series of the real exchange rate I will generate five different measure of the uncertainty of the exchange rate shocks. The first series is going to be the original (the same as the one use to feed the model), the second measure of uncertainty is going to be the proposed measure of real exchange uncertainty using Markov switching regimes, and the other three measures of uncertainty are going to be rolling standard deviation of the log differences of the real exchange rate over the last 4, 8 and 12 quarters, as used in the literature. The idea of this exercise is to use the model to test how well are these measure of uncertainty doing in order to capture the underlying relationships. Ideally we should expect the coefficient using the first series (the original) to be the same as the one using the other measures.

I estimate equation (24) in logs. The results are presented in table 10, as before I present the average values of the estimated coefficient and the average standard error. The results, shows that when using the proposed measure of real exchange uncertainty, the estimated coefficient is near the true coefficient, since the latter lies on the 95% percent confidence interval of the former. While, using the rolling standard deviation measure, the coefficient are about a half of the true effect.

This results can help to understand why when using the proposed measure of real exchange uncertainty, I find significant responses at aggregate level, while previous papers in the literature did not find a meaningfully economic relationship.

7   Conclusion

In this paper I study how real exchange rate uncertainty affects international trade. In order to answer this question, I propose a new method based on regime switching estimation to estimate a measure of real exchange rate uncertainty. Using this measure I document 4 facts. First, at aggregate level I find that real exchange uncertainty is negatively related with international trade. Then using firm level data, I document three new firm level facts relating exporter behavior and real exchange rate uncertainty.I find that when real exchange rate uncertainty increases 1) firms reduce their export share, and those that pay higher interest rates and/or faced higher shipping lags reduce it by more; 2) firms are more likely to stop exporting; and 3) firms are less likely to start exporting to new markets. This results show the mechanism through which real exchange rate uncertainty affects aggregate international trade, and also show that standard dynamic model of trade are ill-suited to understand how real exchange rate uncertainty affects

international trade. These models do not predict movements in the intensive margin of trade, and have opposite predictions to fact 2. This generates this model to under predict the effects that real exchange uncertainty has on international trade.

Based on this result, I develop a partial equilibrium sunk cost model extended with shipping lags and endogenous default. I find that this extension can replicate all the firm level the empirical findings. I estimate the model to match Colombian exporter data, and I find that a one standard deviation increase in real exchange rate uncertainty can generate a drop in aggregate exports between a 5% and a 10%. Then I use the model to test the ability of different empirical measure to capture the effects that real exchange rate uncertainty has on trade, I show that the standard measure used by the literature can capture at most half of the effects predicted by the model.

The results I find empirically, can also be used to revisit some long lasting questions in international macroeconomics. These results can help to explain why exchange rate volatility seems to be unrelated or "disconnect" with other fundamentals as discussed in Itskhoki et al. (2017).To the extent that real exchange uncertainty is high, this can imply that international trade flows react less to exchange rate movements, implying that exchange rate movements need to be larger to pushed back the economy to its equilibrium levels.

Finally, this model can be easily extended to incorporate second moments shocks to aggregate productivity, or foreign demand. For example, an interesting avenue for future research could be to use this model to understand how foreign uncertainty shocks propagates to different countries, and the relevance of international trade as a propagation mechanism.

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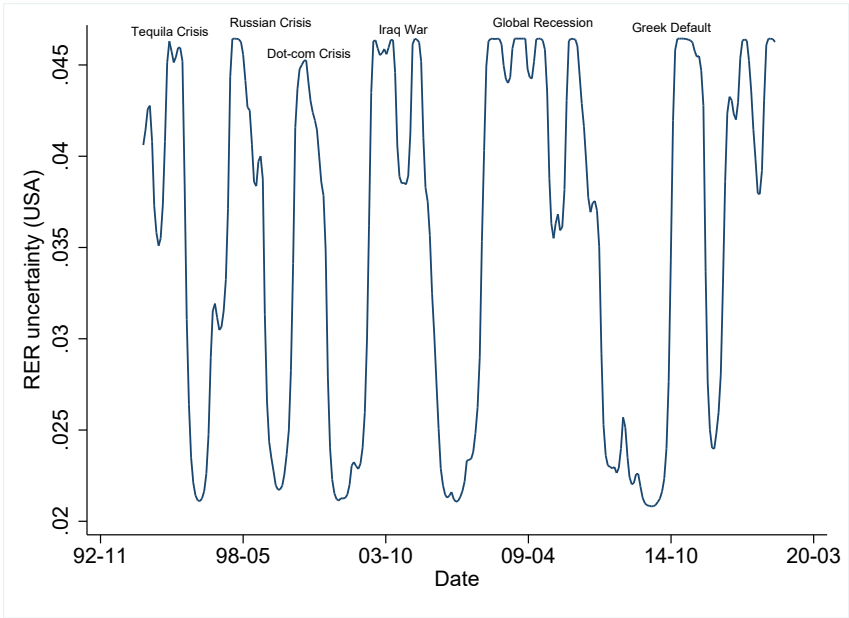
8   Figures

Figure 1: Real Exchange rate uncertainty USA
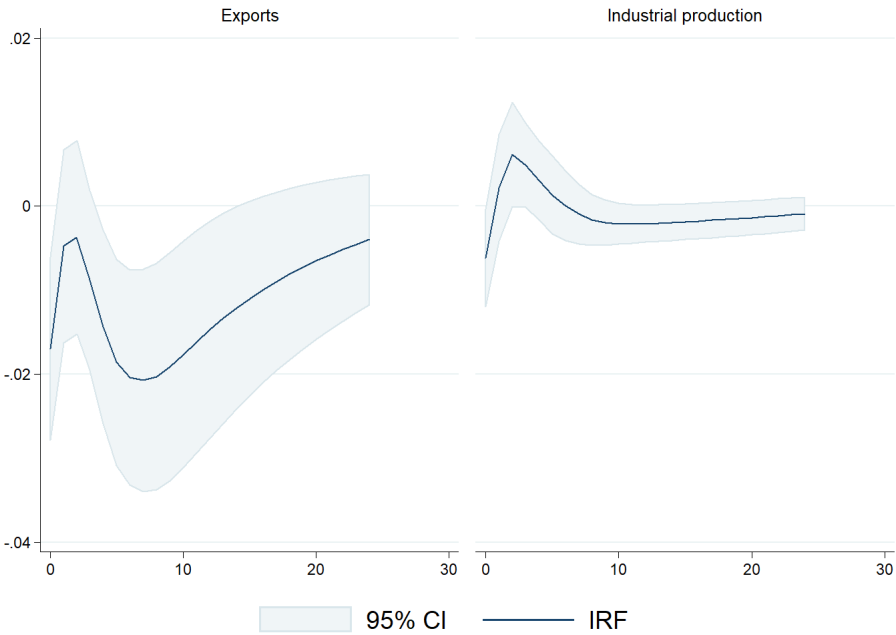
Figure 2: IRF to real exchange uncertainty shock
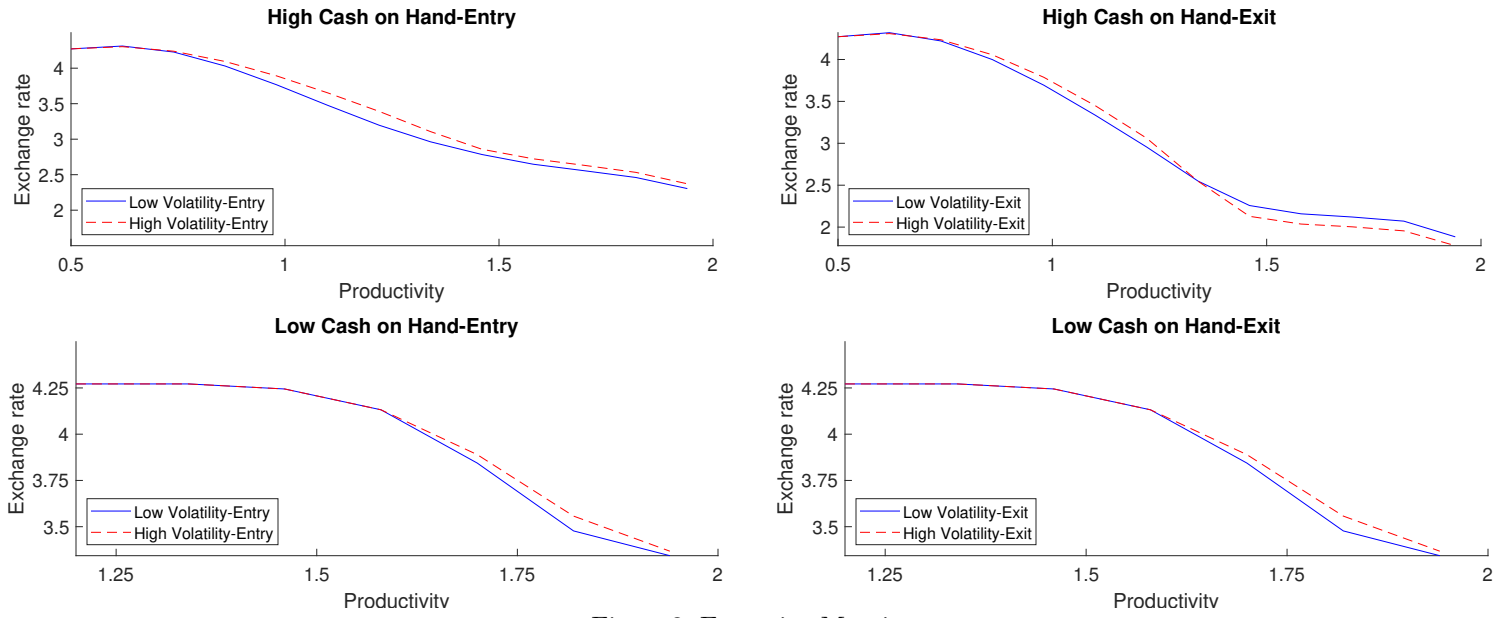
Figure 3: Extensive Margin
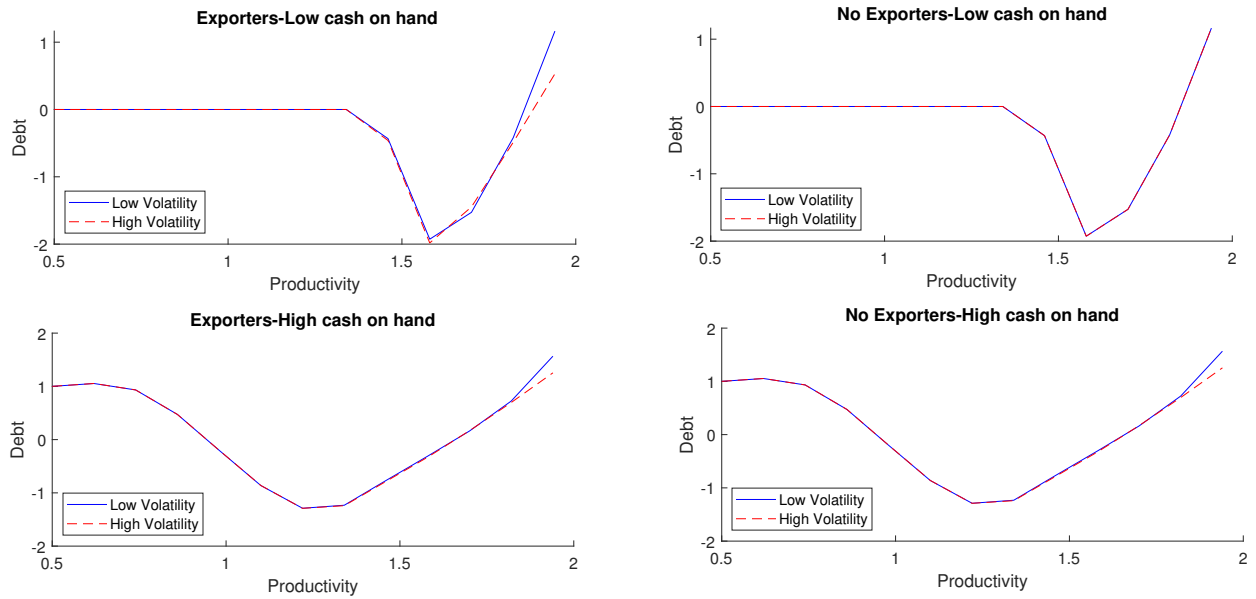
Figure 4: Debt decision
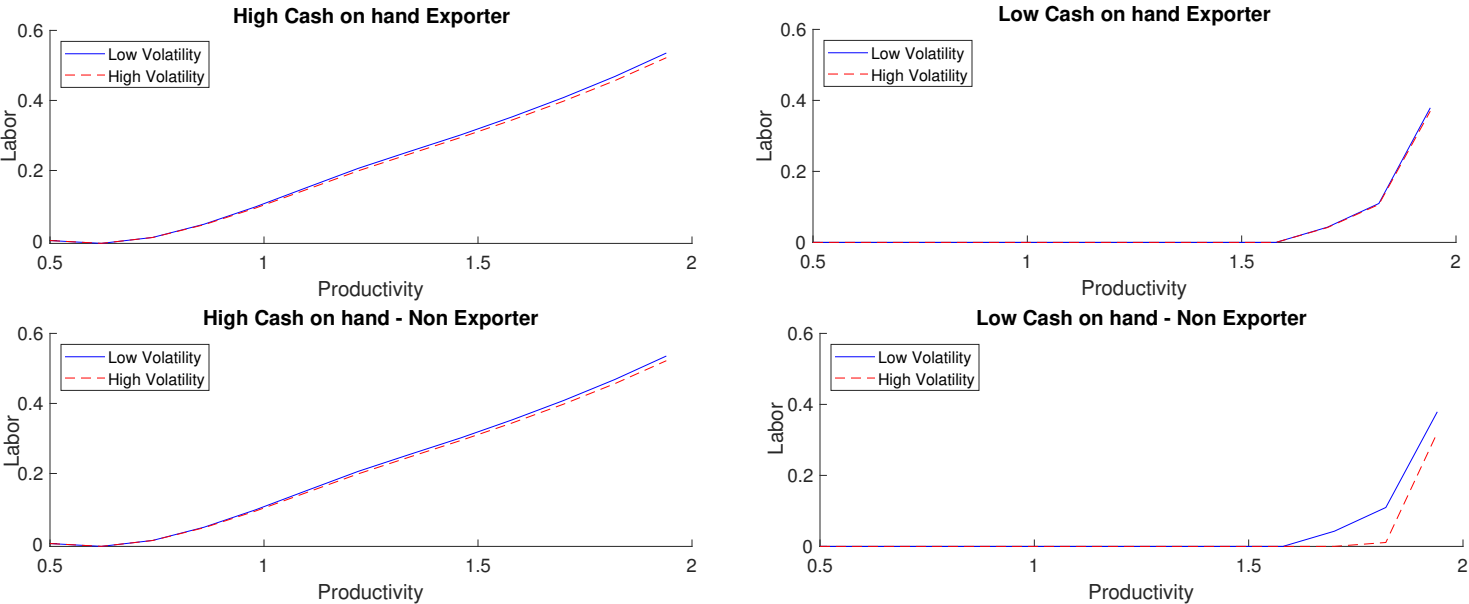
Figure 5: Labour intensity

9   Table

Table 1: Moments of Real exchange rate uncertainty

Variable	All	Developed	Emerging
Average $\tilde{\sigma}_t^2$	4.93	4.11	5.49
$\sigma_{\tilde{\sigma}_t^2}$	2.53	1.11	3.49
σ_{GDP}	2.90	2.24	3.34
Panel A: Standard deviation (SD) relative to output SD			
RER Uncertainty	0.87	0.49	1.04
Consumption	1.06	0.92	1.16
Investment	3.80	3.64	3.91
Exports	2.26	2.45	2.13
Imports	2.99	2.76	3.14
Net exports	2.25	1.90	2.48
Panel B: Correlation with RER uncertainty			
GDP	-0.15	-0.16	-0.15
Consumption	-0.11	-0.12	-0.11
Investment	-0.16	-0.15	-0.2
Exports	-0.14	-0.14	-0.13
Imports	-0.18	-0.18	-0.19
Net exports	0.05	0.01	0.1
Real exchange rate	0.15	0.11	0.15
Nominal Exchange rate with USD	0.15	0.13	0.16
Real exchange rate$_{t-1}$	0.02	0.02	0.01
Nominal Exchange rate with USD$_{t-1}$	0.01	0.01	-0.01

All series are HP filtered in logs with $\lambda = 100$, except for trade balance, that is HP filtered in levels, and the real exchange uncertainty measure that is use in levels. Emerging economies are defined as those with a GDP per capita smaller than 25 thousand USD.Standard deviation of output, and mean expected volatility are in percentage points.* Percentage standard deviation of $\tilde{\sigma}_t$ (not relative to output).

Table 2: RER uncertainty and Aggregate Trade

Panel A: Exports /GDP					
$\tilde{\sigma}_t^2$	-0.09***	-0.11***	-0.11***	-0.11***	-0.08***
	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
R^2	0.15	0.20	0.28	0.28	0.43
Panel B: Imports/ GDP					
$\tilde{\sigma}_t^2$	-0.07**	-0.07**	-0.06**	-0.07**	-0.08**
	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
R^2	0.25	0.29	0.31	0.38	0.37
Panel C: Trade /GDP					
$\tilde{\sigma}_t^2$	-0.08***	-0.08***	-0.08***	-0.08***	-0.06**
	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
R^2	0.24	0.26	0.35	0.35	0.53
Observations	854	840	801	801	801
Country FE	Yes	Yes	Yes	Yes	Yes
Aggregate Ctrl	Yes	Yes	Yes	Yes	Yes
Aggregate Change	No	Yes	Yes	Yes	Yes
Aggregate Change (lag)	No	No	Yes	Yes	Yes
Large Devaluation	No	No	No	Yes	Yes
Year FE	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Country FE: fixed effects by country.2) Aggregate Ctrl: includes as control log of real exchange rate, term of trade, GDP and foreign demand, if country belong or not to WTO or GATT and population. 3) Change: includes the change of the log of the real exchange rate, past value of log real exchange rate, cyclical component of the log of foreign demand, term of trade and GDP (H-P filtered with $\lambda = 100$). 4) Change$_2$: includes lag of change of real exchange rate. 6) Large Devaluation: includes episodes of large devaluation for each country. 5) Year FE: denotes year fixed effects.
Standard errors in brackets. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: RER uncertainty and Bilateral trade

Panel A: Exports$_{i,j}/GDP_i$						
$\tilde{\sigma}_t^2$	-0.50***	-0.14***	-0.15***	-0.13***	-0.11***	-0.11***
	[0.14]	[0.05]	[0.05]	[0.04]	[0.04]	[0.04]
R^2	0.03	0.38	0.39	0.40	0.46	0.46
Observations	60415	42624	42044	41567	41567	41567
Panel B: Imports$_{i,j}/GDP_i$						
$\tilde{\sigma}_t^2$	-0.51***	-0.12**	-0.11***	-0.09**	-0.09**	-0.04
	[0.14]	[0.05]	[0.04]	[0.03]	[0.04]	[0.03]
R^2	0.03	0.37	0.39	0.40	0.40	0.46
Observations	60395	42808	42048	41571	41571	41571
Panel C: Trade$_{i,j}/GDP_i$						
$\tilde{\sigma}_t^2$	-0.48***	-0.12**	-0.11***	-0.09**	-0.09**	-0.06**
	[0.13]	[0.05]	[0.04]	[0.03]	[0.04]	[0.03]
R^2	0.04	0.50	0.52	0.54	0.54	0.61
Observations	60284	42751	42252	41518	41518	41518
Bilateral Fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Gravity Controls	No	No	Yes	Yes	Yes	Yes
Change	No	No	No	Yes	Yes	Yes
Change (lag)	No	No	No	No	Yes	Yes
Large Devaluation	No	No	No	No	Yes	Yes
Year FE	No	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Bilateral FE: fix effects by origin-destination pair.2) Gravity: includes log of real exchange rate of origin country, bilateral exchange rate, term of trade and GDP from origin and destination country, if country belong or not to WTO or GATT(origin and destination), if countries have common currency, if country is EU member, weighted distant, and population (origin and destination). 3) Change: includes growth rate of the bilateral real exchange rate and the lag change of it, GDP growth of both countries,the change in the origin country real exchange rate, past value of log real exchange rate. 4) Change$_2$: includes the lag of the variables included in Change. 5) Large Devaluation: includes a dummy for large devaluations episodes, interacted with origin and distention country
Standard errors in brackets (cluster by origin). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Firm level interest rate and exports

	(1)	(2)	(3)	(4)	(5)
	$es_{l,d,t}$	$es_{l,d,t}$	$es_{l,d,t}$	$es_{l,d,t}$	$es_{l,d,t}$
$\hat{\sigma}_t$	-2.16***	-1.99***	-1.99***	-2.07***	-2.06***
	[0.06]	[0.06]	[0.06]	[0.07]	[0.08]
$r^1_{t-1} \times \hat{\sigma}_t$	-0.13**	-0.12**	-0.12**	-0.14**	-0.11
	[0.06]	[0.06]	[0.06]	[0.06]	[0.08]
$r^2_{t-1} \times \hat{\sigma}_t$	-0.18***	-0.18***	-0.18***	-0.13**	-0.19**
	[0.06]	[0.06]	[0.06]	[0.06]	[0.08]
Observations	131265	131265	131265	107268	107268
R^2	0.78	0.78	0.78	0.79	0.79
Firm, product, destination FE	Yes	Yes	Yes	Yes	Yes
Size FE	Yes	Yes	Yes	Yes	Yes
Gravity$_t$	Yes	Yes	Yes	Yes	Yes
Gravity$_{t-1}$	No	Yes	Yes	Yes	Yes
Δ Gravity$_{t-1}$	No	No	Yes	Yes	Yes
Firm controls	No	No	No	Yes	Yes
Gravity$_t \times r^h_{t-1}$	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and from Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost).5) Gravity$_t \times r^h_{t-1}$: Denotes the interaction between gravity variables and dummy of financial vulnerability
Standard errors in brackets (clustered by exporter). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Firm level data and shipping lags

	(1)	(2)	(3)	(4)	(5)
	$es_{l,d,t}$	$es_{l,d,t}$	$es_{l,d,t}$	$es_{l,d,t}$	$es_{l,d,t}$
$\hat{\sigma}_t$	-2.12***	-1.91***	-1.91***	-1.90***	-1.84***
	[0.06]	[0.25]	[0.07]	[0.07]	[0.08]
Shipping lags$^1_{t-1}$ $\times$ $\hat{\sigma}_t$	-0.08	-0.08	-0.08	-0.21**	-0.21**
	[0.06]	[0.06]	[0.06]	[0.07]	[0.09]
Shipping lags$^2_{t-1}$ $\times$ $\hat{\sigma}_t$	-0.27***	-0.24***	-0.24***	-0.45***	-0.45***
	[0.06]	[0.06]	[0.06]	[0.07]	[0.08]
Observations	131265	131265	131265	107268	107268
R^2	0.78	0.78	0.78	0.79	0.79
Firm, product, destination FE	Yes	Yes	Yes	Yes	Yes
Size FE	Yes	Yes	Yes	Yes	Yes
Gravity$_t$	Yes	Yes	Yes	Yes	Yes
Gravity$_{t-1}$	No	Yes	Yes	Yes	Yes
Δ Gravity$_{t-1}$	No	No	Yes	Yes	Yes
Firm controls	No	No	No	Yes	Yes
Gravity$_t$ $\times$ r^h_{t-1}	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.

Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and from Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost).5) Gravity$_t$ $\times$ r^h_{t-1}: Denotes the interaction between gravity variables and dummy of shipping lags

Standard errors in brackets (clustered by exporter). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Extensive Margin and Real Exchange Rate Uncertainty

Panel A: Entrants			
$\hat{\sigma}_{rer,t}$	-0.08***	-0.13***	-0.12***
	[0.02]	[0.03]	[0.03]
Observations	97539	97453	97453
R^2	0.22	0.22	0.22
Panel B: Stoppers			
$\hat{\sigma}_{,t}$	0.98***	0.87***	0.94***
	[0.01]	[0.06]	[0.05]
Observations	85365	85209	86900
R^2	0.28	0.28	0.28
Firm, Size FE	Yes	Yes	Yes
$Gravity_t$	Yes	Yes	Yes
$Gravity_{t-1}$	No	Yes	Yes
$\Delta\,Gravity_{t-1}$	No	Yes	Yes
Firm controls	No	No	Yes

$I_{i,t}^{Stop}$ equals to one, if a firm exported at t-1 but did not at t. And it is equal to zero if the firm exported at t-1 and t. $I_{i,t}^{Entrant}$ equals to one, if a firm exported at t but did not at t-1. And it is equal to zero if did not exported at t-1 nor at t.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and from Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost), and in the case of panel A, it also includes a dummy denoting if they are re-entrants or not (export at t-1, did not export at and export at t).
Standard errors in brackets (cluster by exporter destination). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Calibration

Parameter	Value	Rationale
σ	3	Standard
β	0.98	Ruhl et al. (2017)
π_d	0.012	Khan et al. (2016)
$\pi_{1,1}^{\sigma}$	0.843	Duration of 19 month
$\pi_{2,2}^{\sigma}$	0.823	Duration of 17 month
σ_L	0.0486	Estimation
σ_H	0.11	Estimation
ρ	0.98	Half life of RER
$\frac{f_s}{f_e}$	2.81	Alessandria et al. (2007)

Parameter	Sunk cost	Default	Low debt	Data
f_e	0.164	0.11	0.12	Exporters Share = 0.195
$\frac{A^d}{A^f}$	261.41	679	679	Exports-sales ratio=0.14
σ_z	0.38	0.32	0.32	Match exporter premia=1.72
π_d^*	-	-	0.006	

Table 8: Moments

Target moments	Sunk Cost	Default	Low debt	Data	Source
Exporters share	0.18	0.19	0.19	0.19	Manuf. survey (2005)[20]
Exports-sales ratio	0.14	0.14	0.15	0.13	Ruhl et al. (2017)
Exporter premia (total sales)	1.75	1.72	1.73	1.72	Bernard et al. (2018)

Non Target moments	Sunk Cost	Default	Low debt	Data	Rationale
Exporter premia (labor)	1.3	1.10	1.12	1.28	Bernard et al. (2018)
Exporter premia (domestic sales)	5.5	4.9	4.2	3.8	Ruhl et al. (2017)

The exports-sales ratio is calculated as the average export-sales ratio among exporters. The exporter premia of domestic shipment is calculated as the ratio of the average value of total shipment of exporter with the average value of total shipment of non exporters, following Ruhl et al. (2017). The exporter premia for total shipment and labor is calculates as the coefficient of a dummy variable with the corresponding dependent variable in logs, following Bernard et al. (2018). This values correspond to a simulation of 1000 firms calculated at the ergodic mean (100 periods after simulating the real exchange rate at its mean with a low volatility).

Table 9: Simulated model

Panel A: Constant RER			
	Sunk cost	Baseline	Low debt
β_1	≈ 0	-0.14	-0.02
	.	[0.009]	[0.003]

Panel B: Time varying RER			
	Sunk cost	Baseline	Low debt
β_1	-0.11	-0.39	-0.20
	[0.009]	[0.013]	[0.009]

Estimation of equation (24) using simulated model. The reported coefficient and p-values, correspond to the average of 200 replication of the model. Each replication simulate 2800 periods, and the regression is done over the last 2500 periods. Row 1 of each panel presents the average coefficient of the regression in level, row 2 presents the average coefficient of the regressions in logs. Column 1 presents the results for an standard sunk cost model while column 2 does it for the extended version. p-values in parenthesis

Table 10: Testing the measures

Measure of RERU	Estimated value
	(1)
Real volatility	-0.42
	(0.021)
Standard deviation (1 year)	-0.18
	(0.020)
Standard deviation (2 years)	-0.21
	(0.027)
Standard deviation (3 years)	-0.22
	(0.033)

Estimation of equation (24) in logs using the data generated by the model. Column (2) adds the following controls: lag of real exchange, change of the real exchange, and the lag of change of the real exchange rate. The reported coefficient and standard errors, correspond to the average among 200 replication of the model, each replication simulate 2800 periods, and regression are done over the last 2500 periods. The first row use as independent variable the actual simulated volatility of real exchange rate, the second one use the propose method using markov swithing estimation, the third, fourth and fifth row use rolling standard deviation over 1, 2 and 3 years respectively. standard errors in parenthesis.

A   Robustness

A.1   Error correction model

To control for the time varying elasticity's with respect to real exchange rate that can generate a biased in the results, I will estimate an error correction model, and expend it with the measure of real exchange rate uncertainty. The estimation is the following:

$$\Delta Exports_{i,t} = \boldsymbol{\beta}\Delta X_{i,t} + \alpha\left(Exports_{i,t-1} - \boldsymbol{\beta^l} X_{i,t}\right) \tag{25}$$

Were X_t is a vector with the following variables real exchange rate, GDP, foreign demand, term of trade and real exchange rate uncertainty. The results are presented in table A.2.I estimate equation (25) for the period 1996-2015, for all the countries in the list except for Argentina,Belgium and Chile since the amount of data available did not allow me to test for co-integration[21] .The results in table A.2 shows that real exchange rate uncertainty have short run effects, but it does not seems to affect the long run value of exports.

A.2   Controls to aggregate estimation

In this section of the appendix, I present the estimation with additional controls, controlling for the correlation with countries in the G7, controlling for the volatility of industrial production to capture volatility of aggregate output, and controls by past trade openness.

The standard deviation if industrial production is an annual measure, constructed as the averages of the rolling standard deviation over 1,2 and 3 years.Similarly the correlation of industrial production between domestic and foreign economies is rolling correlation between domestic economy and the industrial production in the G7 over 3 years.

The estimated equation is the same as equation (3) results are presented in table A.5, columns 1,2-4, and 5-6 for correlation and standard deviation, and for trade openness respectively.

A.3   SVAR for Colombia

Before presenting the evidence at firm level data, It is worth to try to see how real exchange uncertainty affects exports in Colombia at aggregate level. To answer this I estimate an SVAR equation at monthly frequency. I include the following variables in the following order: United States industrial production, United States prime interest rate, real exchange rate uncertainty, Colombian real exchange rate,

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[21] I run the test following Westerlund (2007). The estimation of equation (25), it is done using a dynamic fixed effects estimator.

Colombian central bank interest rate policy, Colombian exports and Colombian industrial production[22].
Following Bloom (2009) I estimate the cyclical component of each variable using a H-P filter with the
smoothing parameter of $\lambda = 129,600$ over the log variables.[23]

Figure 2 presents the exports and industrial production response to a one standard deviation shock
in the real exchange uncertainty, the figure shows that on impact there is not a big reaction on exports,
but after nine months the drop is around 2% and takes more than two year to fully recover (but after
one year the impact is not statically different from zero). Also, as expected, the effect of this shock in
industrial production seems to be zero, or indistinguishable from zero in statically terms.

In the figure A.1, of the appendix C I change the order of the variables and estimate the SVAR in the
following order: United States industrial production, Unites states prime interest rate, Colombian real
exchange rate, Colombian central bank interest rate policy, Colombian exports, Colombian industrial
production, Colombian real exchange rate uncertainty. I find, that the results holds, even after assuming
that the real exchange uncertainty is the least exogenous variable, but the estimated impact is around
1.5%.

A.4   Chilean firm level data

This section presents the results for the Chilean economy using the survey of manufacturing. The ad-
vantages of this data set with respect to the Colombian one, is that it allows me to observe more details
about the firm, like employees, revenues, total production, among other variables. The disadvantages, is
that during the period for which the data is available, there is not a high variation in the measure of real
exchange rate uncertainty, also, I cannot distinguish the destination or product of each exports.

The estimation is as follows:

$$y_{i,t} = \beta_0 + \beta_1 \tilde{\sigma}_t^2 + \beta_h^0 \tilde{\sigma}_t^2 \times \frac{Assets}{Liabilities_{i,t}} + \beta_2 \frac{Assets}{Liabilities_t} + \alpha_i + X_t + \hat{X}_{i,t} + \epsilon_{i,t} \tag{26}$$

Were X_t representes aggregate controls, $\hat{X}_{i,t}$ represents firms controls over time, and α_i represents firm
effect by firm. I estimate productivity following Petrin et al. (2004).The results are presented in table A.4

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[22] This particular ordering is assuming that real exchange rate uncertainty shocks is exogenous to all the the
variables with exception to Industrial production in United states, and the prime interest rate. I include the
interest rates, since it is possible that Colombian central bank reacts to exchange rate volatility shocks, or that
a movement in domestic or foreign interest rate affects exports. **alessandria`export`2014.** found that the
export response to large devaluations depends on the interest rate of each country, implying that inclusion of
the interest rate could relevant.

[23] The lag structure is the one that it is found optimal according the Akaike's information criterion (AIC).

B   Algorithm

Discretize the space Construct discretize space:

1. Discretize the state space.

    (a) Labour and Debt in 135 point linear space greed.

    (b) ξ in a 30 points linear space greed. Using Tauchen method.

    (c) z_i in a 10 points space greed.

    (d) σ_ξ is a two state Markov chain with transition probabilities given by $\pi_{l,l}$ $\pi_{h,h}$

    (e) $m' = \{1, 0\}$

    (f) Cash on hand x in 140 point greed.

Once the state space is constructed I solved the problem in two different loops. First I solve the optimally policy function of the lender, I solve $q(.)$ as a function of $m', l^{f'}, b'$. Then I use $q(.)$ to solve the exporter problem.

Lender's problem Iteration:

1. Guess q^n if n=0.

2. Compute $M(S_t, s_i)$ as following:

$$M^n(z_i, l', b', m', S_t) = \max_{l', b', m'} q^n(z_i, l', b', m', S_t) b'(z_i, l', m', S_t). \tag{27}$$

Denote the arg max of above problem as follows: $\hat{l}', \hat{b}', \hat{m}'$.

3. With $M^n(S_t, s_i)$ I obtained the corresponding default threshold of equation (10) and (11), denoted by κ^n

4. Finally compute q(.) as follows:

$$\beta = q^{n+1}(z_i, l', b', m', S_t)(1 - F(\kappa^n|\xi))$$

(28)

5. If $|q^n - q^{n+1}| \leq \epsilon$ finish, otherwise go to step one, using $q^{n+1} = q^n$

Producer's problem Iteration:

1. Guess $V(z_i, x, m, S)^n$ If n=0 . Fix $V_d = 0$.

2. Define:

   - $V^1(z_i, x, m, S)^n \equiv (z_i, x, 1, S)^n$

   - $V^0(z_i, x, 0, S)^n \equiv (z_i, x, 0, S)^n$

   - $q^{n,1} \equiv q^{n,}(z_i, l', b', 1, S_t)b'(z_i, l', 1, S_t)$

   - $q^{n,0} \equiv q^{n,}(z_i, l', b', 0, S_t)b'(z_i, l', 0, S_t)$

3. Compute optimal decision and value functions conditional on choosing $m' = 1$ and $m' = 0$ as follows:

$$V_c^{n+1,m'}(z_i, x, m, S) = \max_{l',b'} x + q^{n,m'}(l', b', m')b' + (1 - m)m'f_s + Q\mathbf{E}V^{n,m'}(z_i, x', S') \qquad (29)$$

   s.t.
   (6), (10),(11), (12),(13), (22),(21),(23)

4. Update optimal export decision and the value function conditional on not default as follows:

$$V^{cn+1}(z_i, x, m, S) = max\{V^{n+1,m'}(z_i, x, m, S), V^{n+1,m'}(z_i, x, m, S)\} \qquad (30)$$

5. Update the value function:
$$V^{n+1} = max\{V_c^{n+1}, 0\} \qquad (31)$$

6. Iterate until 1-5 until $|V^{t+1} - V^t| \leq \epsilon$

C   Appendix Figures

Figure A.1: SVAR 2. Real exchange uncertainty shocks
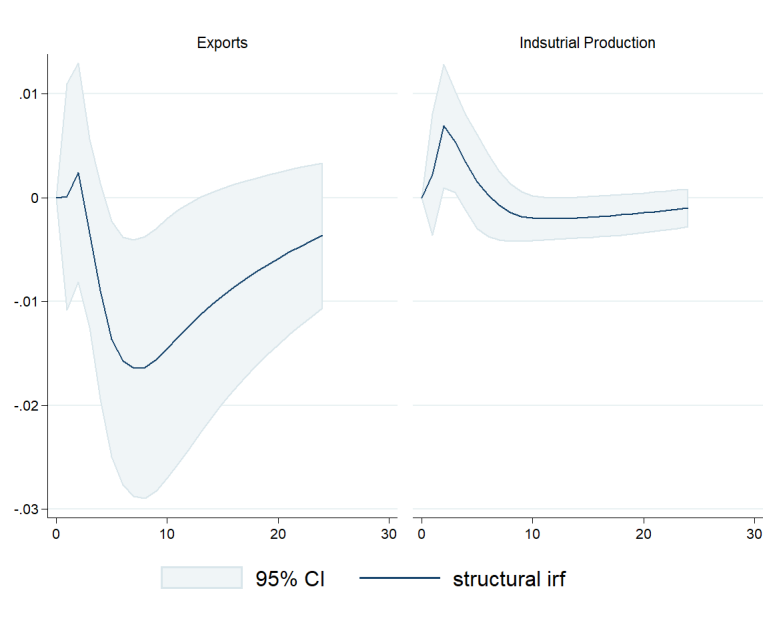
The SVAR estimation is the following: Industrial production of United states, the prime interest rate in USA, Colombia real exchange rate, the interest rate policy of the Colombian central bank, exports, industrial production, real exchange rate uncertainty.All variables are monthly and filter using H-P with parameter=129600, as in Bloom (2009)
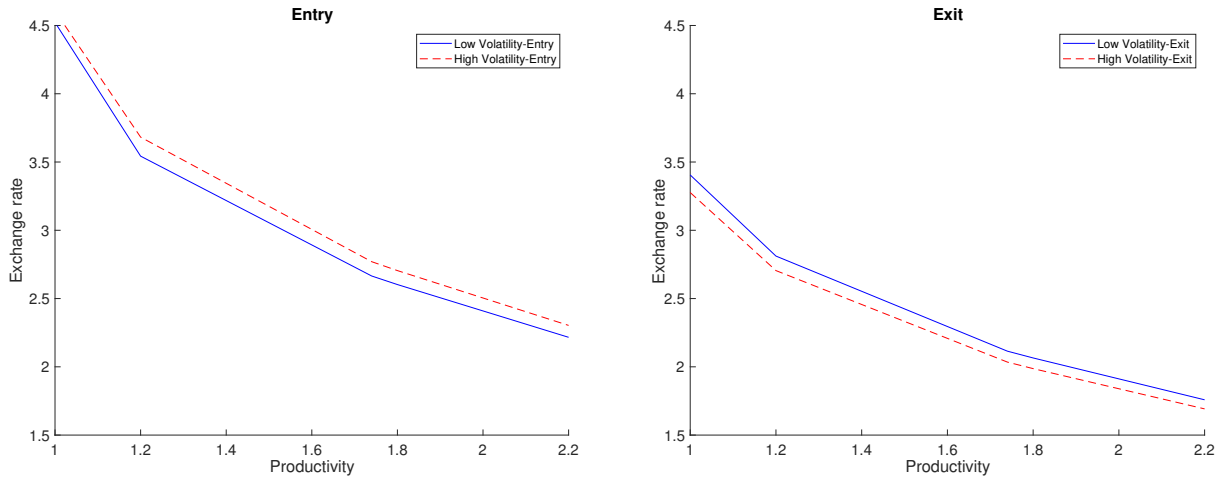
Figure A.2: Extensive Margin in standard model

D   Appendix Tables

D.1   Country list

Table A.1: Country list

Algeria	Finland	Latvia	Slovak Republic
Argentina	France	Lithuania	Slovenia
Australia	Germany	Macedonia, FYR	South Africa
Austria	Greece	Malaysia	Spain
Belgium	Hong Kong	Malta	Sweden
Brazil	Hungary	Mexico	Switzerland
Bulgaria	Iceland	Netherlands	Thailand
Canada	India	New Zealand	Turkey
Chile	Indonesia	Norway	United Kingdom
Croatia	Ireland	Peru	United States
Cyprus	Israel	Philippines	
Czech Republic	Italy	Poland	
Denmark	Japan	Portugal	
Estonia	Korea, Rep.	Singapore	

D.2

Table A.2: Error correction estimation

	Equation 1	Equation 2
Long run relationship		
Real exchange rate$_{t-1}$	0.06	0.13
	[0.19]	[0.18]
GDP$_{t-1}$	0.82***	0.81***
	[0.09]	[0.09]
Foreign demand$_{t-1}$	0.58***	0.57***
	[0.19]	[0.19]
Term of trade$_{t-1}$	-0.12	-0.12
	[0.12]	[0.12]
$\tilde{\sigma}_{t-1}$		-0.13**
		[0.06]
Short run relationship		
α	-0.36***	-0.36***
	[0.03]	[0.03]
ΔReal exchange rate$_t$	1.08***	1.06***
	[0.14]	[0.14]
ΔGDP$_t$	1.40***	1.35***
	[0.09]	[0.10]
ΔForeign demand$_t$	-0.30***	-0.30***
	[0.11]	[0.11]
ΔTerm of trade$_t$	0.12	0.12
	[0.09]	[0.10]
$\Delta\,\tilde{\sigma}_t$		-0.05***
		[0.02]
Observations	728	728
R^2	.09	0.09

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.3: Other measures of firm's financial vulnerability

	(1) $\frac{liabilities}{assets}$	(2) $\frac{interest}{profits}$
$\hat{\sigma}_t$	-2.08***	-2.05***
	[0.08]	[0.08]
$I_{t-1}^1 \times \hat{\sigma}_t$	-0.01	-0.16*
	[0.08]	[0.09]
$I_{t-1}^2 \times \hat{\sigma}_t$	-0.13*	-0.29***
	[0.08]	[0.09]
Observations	111147	111598
R^2	0.79	0.79
Firm, product, destination FE	Yes	Yes
Size FE	Yes	Yes
Gravity$_t$	Yes	Yes
Gravity$_{t-1}$	Yes	Yes
Δ Gravity$_{t-1}$	Yes	Yes
Firm controls	Yes	Yes
Gravity$_t \times r_{t-1}^h$	Yes	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and from Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost).5) Gravity$_t \times r_{t-1}^h$: Denotes the interaction between gravity variables and dummy of financial vulnerability
Standard errors in brackets (clustered by exporter). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.4: Chilena firms

	Exports	Exports	Exports	Exports
$\tilde{\sigma}_t^2$	-5.67***	-11.91*	-13.37*	-13.00**
	[1.85]	[6.49]	[7.20]	[6.59]
$\tilde{\sigma}_t^2 \times \frac{Assets}{Liabilities}{}_t$				0.9164*
				[0.5264]
Observations	6603	5678	5678	5632
R^2	0.02	0.02	0.02	0.02
Firm FE	Yes	Yes	Yes	Yes
Firm controls	Yes	Yes	Yes	Yes
Aggregate Controls	No	Yes	Yes	Yes
RER Change	No	No	Yes	Yes

Standard errors in brackets.* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
1) Firm controls: includes Share of Imports for raw materials, Employment, firm productivity, share of national ownership.2) Aggregate Controls (in logs): Domestic GDP, Foreign Demand, real exchange rate.3) RER Change: includes first, second difference, and the lad and present cyclical component of the Real exchange rate.

Table A.5: Bilateral relation: additional controls

	1	2	3	4	5	6	7
Panel A: Exports /GDP							
$\tilde{\sigma}_t$	-0.06***	-0.06***	-0.054***	-0.05***	-0.04***	-0.04***	-0.04***
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Observations	25075	25075	25075	25075	25075	25075	25075
R^2	0.60	0.60	0.60	0.60	0.62	0.62	0.62
Panel A: Imports /GDP							
$\tilde{\sigma}_t$	-0.04**	-0.04**	-0.04**	-0.04**	-0.04**	-0.04**	-0.04**
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Observations	25062	25062	25062	25062	25062	25062	25062
R^2	0.47	0.47	0.47	0.47	0.47	0.47	0.47
Panel A: Trade /GDP							
$\tilde{\sigma}_t$	-0.04***	-0.04***	-0.03***	-0.03**	-0.03**	-0.03**	-0.03**
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Observations	25061	25061	25061	25061	25061	25061	25061
R^2	0.66	0.66	0.66	0.66	0.67	0.67	0.67
Bilateral FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Gravity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Change	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Change$_2$	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Corr G7	X						
STD$_1$		X					
STD$_2$			X				
STD$_3$				X			
Top$_1$					X	X	X
Top$_2$						X	X
Top$_3$							X

Standard errors in brackets.* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$