Real Exchange Rate Uncertainty Matters

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Abstract I introduce a novel forward-looking measure of real exchange rate uncertainty (RERU) and examine its effects on international trade. I show that rising RERU triggers a precautionary response from exporters: export intensity and entry to new markets decline, while export prices and exit increase. My empirical results show that these microeconomic adjustments are primarily driven by exporters facing higher interest rates. To capture these dynamic responses, I develop a sunk-cost trade model with heterogeneous firms and firm-specific default risk. Increased RERU exposes exporters to greater financial vulnerability, who hedge against this risk by increasing markups or exiting the export market, which explains my documented facts. Quantitatively, the model predicts a standard deviation increase in RERU reduces total exports by 6%.

JEL Classifications: F10, F31, F40

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1 Introduction

It has been more than 40 years since the collapse of Bretton Woods, and there is no clear consensus about the consequences that foreign exchange rate uncertainty has on international trade flows. After this episode, it has been widely documented that exchange rates became particularly difficult to predict and highly volatile, creating what is known as currency risk. While the problems associated with currency risk are generally well recognized by firms engaging in international activities and by policymakers, the literature was not able to find any meaningful relationship between exchange rate uncertainty and international trade. Understanding the relevance and channels through which foreign exchange rate uncertainty affects international trade is imperative in an increasingly connected world.

In this paper, I solve this problem by proposing a new forward-looking and model-consistent measure of real exchange rate uncertainty (henceforth RERU). Using this measure of RERU, I then document 3 firm-level facts on how firms respond to these shocks consistent with the existence of a "precautionary margin of international trade". In particular, when real exchange rate uncertainty increases, exporters 1) reduce their export intensity, 2) are more likely to stop exporting, and 3) are less likely to start exporting to new markets. I show that these results are mainly explained by those exporters facing higher paying interest rates, suggesting that financial conditions are key dimensions to understanding these facts. I then extend an otherwise standard dynamic model of international trade to incorporate these features. In particular, I model financial imperfection as friction on financial contracts where firms are able to default on their debt. I then show that the existence of debt default is essential to generate firms that behave consistently with the empirical results. Finally, I calibrate this model to firm-level data from Colombia and show that RERU has large quantitative effects on international trade.

The new measure of RERU follows from a regime-switching estimation method to estimate the RERU that agents in an economy face when making decisions. This way of proceeding allows me to clearly distinguish between shocks to the level of the real exchange rate and its expected volatility, as it allows me to pin down a time-varying expected uncertainty over the real exchange rate conditional on information available at a particular time.

At the aggregate level, I find a negative relationship between total international trade and RERU. This result follows from the results of a standard gravity equation expanded with the RERU measure. In particular, I find that a one standard deviation increase in RERU is associated with a drop in total trade over GDP of 5%. To give some context, this drop represents nearly one-third of the drop in trade experienced during the trade collapse that occurred in 2008. But what is behind this negative relationship in the data?

According to standard sunk cost models of international trade, as in Dixit (1989a), Dixit (1989b), and Alessandria et al. (2007), when real exchange rate uncertainty increases, all adjustment in export markets is due to changes in the extensive margin of trade. In these models, firms have to pay a sunk cost to start exporting, after which they face smaller continuation costs to keep exporting. When profits become more uncertain, as in any sunk-cost investment model, firms find it optimal to delay their investment decision. This delay in investment takes a particular form in these frameworks as fewer firms are willing to enter new export markets, and fewer exporters are willing to stop exporting. These changes have offsetting effects. When RERU increases, on the one hand, there will be fewer firms entering new markets, reducing aggregate export; on the other hand, exporters will be less willing to stop exporting, increasing aggregate exports.

I test these predictions using Colombian firm-level data. Contrary to the predictions of the standard sunk cost model, the results show that changes in RERU generate trade responses through both intensive and extensive margins. With respect to the intensive margin, when real exchange rate uncertainty increases, exporters reduce their export intensity. This reaction is mostly explained by exporters paying higher interest rates. The adjustment through the extensive margin shows that a higher RERU reduces firms' willingness to export. In particular, fewer firms have started exporting to new markets, and more exporters have stopped exporting. These empirical results are robust to a different range of specifications and to the addition of different types of controls. I argue that these results are consistent with what I call the "precautionary margin of international trade".

The existence of this precautionary margin of international trade implies that standard sunk cost type models will underestimate the impact that real exchange uncertainty has on international trade, mainly due to two reasons. The first one is the lack of response in the intensive margin predicted by this model. The second is due to the prediction that fewer firms will quit exporting when RERU is high. These predictions are contrary to the empirical patterns documented at the firm level.

I overcome these problems by developing a new dynamic model of international trade based on Dixit (1989b) that incorporates the existence of financial risk. I follow Arellano et al. (2018) in modeling financial imperfections as friction on financial contracts where firms are able to default on their debt and borrow from a risk-neutral lender. Once the financial frictions are added, the model results become consistent with the documented firm-level facts.

The existence of debt default is essential to motivate firms to behave consistently with the empirical pattern. The intuition is as follows. While increases in real exchange rate uncertainty leave the expected value of the exchange rate intact, the probability that firms assigned to end up in a financially vulnerable situation increases. This leads exporters to engage in precautionary practices, increasing markups or quitting the export market to reduce the risk they face. These precautionary practices lead to a drop in aggregate exports through the extensive and intensive margin of trade, consistent with the empirical patterns in the data. To estimate the quantitative relevance of these mechanisms, I estimate the extended model to match the key moments of exporter behavior. I find that a one standard deviation change in RERU can reduce total exports by 6%.

Literature This paper is contained in the literature that studies how uncertainty affects real allocations. For example, Bloom (2009) and Arellano et al. (2018) argue that uncertainty played a significant role in explaining the 2008 crisis due to capital adjustment costs in the former work or due to financial frictions in the latter. Also, recent papers have proposed new measures for aggregate volatility to analyze the effect of uncertainty shocks on different aggregate economic variables as in Jurado et al. (2015), Fernandez-Villaverde et al. (2010), and Fernández-Villaverde et al. (2011). Furthermore, there is an increasing interest in understanding how uncertainty can affect international trade. For example, Novy et al. (2014) document that incorporating aggregate volatility shocks into a model with inventories helps to explain the high volatility of international trade flows. In contrast, Alessandria et al. (2015) discussed the direction of the causality between idiosyncratic uncertainty and aggregate shocks and found that in a sunk cost model of exports, idiosyncratic uncertainty shocks generate a counterfactual increase in exports, and Fernández-Villaverde et al. (2011) studies how volatility shocks to the interest rate can affect the macroeconomic performance of small open economies.

In particular, this paper is more closely related to two branches of literature: the literature that studies how real exchange rate volatility affects international trade and the literature that studies how financial frictions can affect international trade. The former is a literature that started in the early seventies and is summarized by McKenzie (1999), Clark et al. (2004) and Bahmani-Oskooee et al. (2007). The main conclusion is that the literature did not find any meaningful relationship between exchange rate volatility and trade variables at the aggregate level. Recently, some papers have studied the sectoral effects that real exchange uncertainty has on trade as in Lin et al. (2018), Héricourt et al. (2015), and Héricourt et al. (2016), which find that firms that belong to financially vulnerable sectors tend to respond negatively to exchange rate volatility.

Concerning the second literature, several papers suggest that financial imperfections can affect trade in different ways. Manova (2013) shows that financially developed countries have comparative advantages in sectors that are more financially vulnerable, Kohn et al. (2016) show that financial imperfections can help to understand export dynamics during large devaluations, and Kohn et al. (2016) show that financial imperfections can help to explain new exporter dynamics. More recently, Brooks et al. (2019) found that the relevance of financial imperfections to understand the gains from a trade reform episode depends on the way financial imperfections are modeled and that for Colombia, the data seems to point out that a standard collateral constraint can be a misleading interpretation of how trade responds to trade reforms.

This paper makes four main contributions to these three strands of the literature. First, I propose a new method to measure real exchange rate uncertainty that can be easily applied to a wide range of countries and that can distinguish first-moment shocks to the real exchange rate from second-moment shocks, which is not possible to achieve using rolling standard deviations over the changes in the real exchange rate or GARCH/ARCH type methods. I find that using this measure, the aggregate relation between real exchange uncertainty and international trade is meaningful not only statistically but also economically, something that the former literature was not able to find at the aggregate level. Second, I show that both the intensive and the extensive margin of trade respond to changes in RERU and that these changes are related to a firm's financial situation and the shipping lags it faces. Third, I propose a new model building on Alessandria et al. (2007) and Arellano et al. (2018) that can replicate the negative relationship between real exchange uncertainty and trade and the firm level responses at both the intensive and the extensive margin by incorporating debt default and shipping lags as a novel mechanism. Fourth, I test the relevance of the new measure using simulated data from the model and compare it with other measures used by the literature. The lack of empirical relevance at the aggregate level found in the literature examining real exchange volatility and trade is likely due to the measure of real exchange uncertainty used in the past.

The structure of the paper is as follows. Section 2 presents a simplified model to highlight the core mechanism. Section 3 briefly describes the data. Section 4 discusses how I construct the measure of real exchange uncertainty and its relationship with other aggregate variables. Section 5 documents the facts relating to trade and real exchange uncertainty. Section 6 develops the model. Section 7 presents the quantitative exercises using the model, and section 7 concludes.

2 The mechanism in a simple example

Let's start with a simplified model highlighting the core mechanism of this precaution mechanism in international trade. In this setup, firms live two periods; they have an outstanding debt, b, which will be paid in the second period, after which they will get an additional value $\kappa > 0$ and die. To sell abroad, firms make a contract invoiced in foreign currency where the buyer and the seller agree on the first period over the quantities and prices to be delivered in the following period. In the second period, the buyer gets the products and pays them back to the firm. The firm transforms this income at the spot exchange rate e and uses it to pay back its employees at the agreed wage and its outstanding debt and to issue dividends if there is a rest; all these payments are made in local currency. If the firm's income flow is insufficient to pay back its total debt (wages and debt), it goes bankrupt and can't collect its scrap value κ.

Given the debt b, a set price p, an exchange rate e, wages, w, labor l, and a quantity sold q, the firms' condition for not going bankrupt is given by:

$$p_i q_i e - w l_i - b_i \geq 0 \tag{1}$$

This implies that there is a break-even exchange rate, e^*, above which the firm will not go bankrupt and below which it will go bankrupt, given by:

$$e^* = \frac{(w l_i + b_i + f_e)}{p_i q_i} \geq 0 \tag{2}$$

Higher debt, wages, or work hired, ceteris paribus, increases the break-even exchange rate value for the firm. Higher nominal sales conditional on the previous elements reduce the break-even exchange rate. Given the above condition, the exporter's problem is then given by:

$$\max_{p,l} \beta \mathbb{E}\{\mathbb{I}_{e \geq e^*}[p_i q_i e - w l_i - b_i + V]\} \tag{3}$$

subject to $q_i = p^{-\theta}$, $l = \frac{q}{z_i}$, and (2). Hence, the exporter's problem can be re-write as:

$$\max_{l} \beta \int_{e^*(l)}^{\infty} \{(zl)_i^{\frac{1-\theta}{\theta}} e - wl - b_i - f_e + V\} f(e) de \tag{4}$$

Consequently, after applying the Leibniz integral rule, the first-order conditions are given by:

$$z_i^{\frac{1-\theta}{\theta}} l^{-1/\theta} \int_{e^*(l)}^{\infty} e f(e) de - w(1 - F(e^*) - \frac{\partial e^*}{\partial l} f(e^*) V = 0$$

note that unlike the standard case, the marginal cost of hiring more labor is given by two components: the usual marginal cost of labor, w, and the additional risks of ending up in a financially vulnerable situation attached to hiring more labor $\frac{\partial e^*}{\partial l} f(e^*) V$. By using the first order condition, we get that the average firms' ex-ante prices in domestic currency are given by:

$$E\{pe|_{e \geq e^*}\} = \frac{\theta}{\theta - 1} \frac{w}{z} \left[1 + \underbrace{\frac{\partial e^*}{\partial l} \frac{f(e^*)}{1 - F(e^*)} \frac{V}{w}}_{\text{risk cost}} \right] \tag{5}$$

In this case, the expected marginal cost of selling an additional unit contemplates two components: the usual marginal cost of increasing production, $\frac{w}{z}$; and also the changes in the probability of losing the future firm value, as shown in the second term of the equation (5).

The additional "risk cost" associated with increasing production distorts the chance of the firm's usual first-order condition and creates a wedge between the expected returns of selling abroad and the associated technological costs. To see this more clearly, imagine the case where firms priced their products after knowing the realization of the exchange rate and hence face no exchange rate risks. In this case, the price in domestic currency, ceteris paribus, will be given by $e\hat{p}_i = \frac{\theta}{\theta-1} \frac{w}{z}$, and hence the expected observed domestic price - for those firms not going out of the market - will be given by:

$$E\{\hat{p}e|_{e \geq e^*}\} = \frac{\theta}{\theta - 1} \frac{w}{z} \tag{6}$$

By comparing equation (5) to (6), we can draw at least two important conclusions regarding the relevance of exchange rate risk for international trade flows. The first one is that the existence of liquidity risks under the existence of rate uncertainty increases prices on average and hence decreases quantities exported. The second is that the exchange rate risk induces firms to price foreign sales higher than domestic sales, all else equal, and hence, exchange rate risks work as a relatively higher trade cost to export, or in other terms, it is similar to the existence of an iceberg cost to export.

The above implications are of extreme importance for papers studying the consequences and causes of trade shocks, as in Alessandria et al. (2014), for international trade and macroeconomic dynamics. It suggests that if exporters price in foreign currency and face liquidity risks, variation in the exchange rate uncertainty can be captured as a time-varying cost to exports - relative to domestic sales- in models without these frictions. Hence, it provides a new explanation and a micro foundation for the observed time variation in iceberg costs that models need to match the data. Since, in general, hazard rates ($\frac{f(e^*)}{1-F(e^*)}$) are increasing in volatility, this implies periods of higher exchange rate uncertainty will be associated with smaller exports, which standard models without these features will then capture in time-varying iceberg costs.[1]

3 Data

In this section, I present the data used in the empirical analysis. The empirical analysis is divided into two parts. First, I analyze the relationship between time-varying real exchange rate uncertainty and trade openness at the aggregate level for several countries. Then, I focus on how exporters react to changes in real exchange rate uncertainty. For the aggregate analysis, I use panel data composed of 58 countries listed in Table A.1 of the appendix. The panel goes from 1995-2015. The availability of the real exchange rate series for some countries mainly restricts the sample period. I use this panel to aggregate the relevance of

[1]See Arellano et al. (2018) for how increased uncertainty affects hazard rates.

real exchange rate uncertainty shocks over total trade, exports, and imports. The sources for the aggregate data are:

– Bank of International Settlements: Monthly effective real exchange rate (RER) indices.
– Penn World Tables and World Development Indicators: Aggregate variables like exports, imports, GDP, terms of trade, and price indices.
– CEPII: Gravity equations variables like distance, common language, trade agreements, colonial relationships, and entry cost (in monetary terms and in time).

Firm level data The firm-level database is a panel from 2006 to 2015 constructed using two sources. The main source is Colombian customs data, which reports all international exports at the firm-destination-product level at a monthly frequency. The second source comes from the "Superintendencia de Sociedades". This data is reported at the firm level at an annual frequency and covers around 20 thousand firms that represent more than 85% of Colombian GDP, according to the organization. I use this data to construct the financial variables at the firm level. The data can be merged for 2006-2015, which allows me to construct a panel at the annual frequency for the mentioned period.

In the appendix A.4, I present some results using firm-level data from Chile. The data comes from the manufacturing survey, which is a panel that goes from 1997-2006. I use this dataset as a robustness check since it allows me to control for variables at the firm level that I cannot observe using the Colombian firm-level data, like employment and estimated total factor productivity (TFP) at the firm level, and to show that these results are not unique to Colombia.

4 Measuring Real exchange rate uncertainty

This section develops a new way to measure time-varying real exchange uncertainty, motivated by the recent work of Jurado et al. (2015) and Fernandez-Villaverde et al. (2010). The proposed measure of real exchange rate uncertainty relies on a two-step procedure: the computation of the forecast error of the real exchange rate and the estimation of the expected volatility for every period.

I follow Jurado et al. (2015) in considering uncertainty as the inability of agents to make accurate predictions of the variables. This implies that the absolute value of the error forecast should be consistently low during low uncertainty periods and consistently high during high uncertainty periods.

To measure the error forecast of the agents, I follow Meese et al. (1983) and Kilian et al. (2003). I assume that agents forecast the real exchange rate as if it behaves as a random walk[2].

Once I obtain a series of the forecast error, I estimate the expected real exchange rate uncertainty by assuming that the process for the forecast error is characterized by a Markov regime-switching process in variances using the method developed by Hamilton (1989) [3]. Using the Markov regime-switching in variances allows me to distinguish between periods characterized by low and high uncertainty and compute the future expected volatility of agents, conditional on the information available at a particular moment.

Construction of the measure

I proceed in two steps to construct the time-varying measure of real exchange rate uncertainty for each country. In the first step, I compute the forecast error. In the second one, I estimate the Markov-switching process of the error forecast to compute the expected volatility at any moment in time.

1. **Error forecast computation**: Compute forecast Error of h months ahead, μ_t^h:

$$\mu_{t+h}^h = y_{t+h} - E\left[y_{t+h}|I_t\right]$$

[2]Meese et al. (1983), Engel (1994), and Kilian et al. (2003) have shown that different models used to predict the real exchange rate cannot improve the out of sample prediction of assuming that the real exchange rate behaves as a random walk.

[3]Fernandez-Villaverde et al. (2010) presents a discussion of different methods to estimate time-varying variances. I assume the Markov switching behavior to be consistent with the model specification.

Where I_t is available information at time t. y_t represents the natural logarithm of the real exchange rate index at period t. As mentioned, I assume that agents predict the RER as a random walk. This implies that:

$$E\left[y_t|I_t\right] = y_t$$

2. **Uncertainty computation**: In this step, we need to compute the expected variance of the error forecast $\tilde{\sigma}^{2h,j}_t = E_t\left[\sigma^2_{\mu^h}|I_t\right]$. To do so, I proceed in several sub-steps:
   (a) *Choose number of states*: Let τ be the amount of states of the underlying process. Fix $\tau = j\ for j \in \{0;1;2;3\}$.
   (b) *Estimation of the process given* τ: Estimate the process for $\mu^{h,\tau}_t$.

$$\mu^{h,\tau}_t = \theta^{s,\tau}_t + \epsilon^{s,\tau}_t$$

   With:

$$\epsilon^{s,\tau}_t \sim N(0, \sigma^2_{s,\tau})$$

   And the transition probability matrix, Π, with dimensions $\tau \times \tau$, wher s denotes the possible states of the economy, $s\ \in \{s_1;..;s_\tau\}$
   (c) *Computation of probabilities*: Compute the j step ahead forecast probability of $s_{t+j} = s_i$ given information I_t:

$$P^{j,\tau}_{i,t} = prob(s^\tau_{t+j} = s^\tau_i|I_t)\text{for i } \in \{1, 2, ..\tau\} \tag{7}$$

   (d) *Optimal* τ: Using the likelihood test, for each country, compute the optimal τ. Let the optimal amount of states be defined as τ^*.
   (e) *Computation of uncertainty*: Compute $\tilde{\sigma}^{2h,j}_t = E\left[\sigma^2_{s,\tau^*,t+h}|I_t\right]$ using $P^{j,\tau^*}_{i,t}$ as estimated in sub step (c).

There are several advantages to using this method. First, it relies only on a real exchange rate series to be constructed, which is available for several countries after 1995. Second, it has a clear quantitative mapping to economic models; third, it is easy to compute. However, these advantages come at the cost of moving the real exchange rate to capture the uncertainty measure. For example, if the real exchange rate has been constant for over five years, but some agents change their perception of the real exchange rate uncertainty during this period, the measure cannot capture this change. A problem is shared with all the measures used by the literature.

Additionally, the measure is completely agnostic of the sources generating the changes in the regime or what determines the probability of change from one regime to another. By assumption, these changes are exogenously given [4]. This implies that the empirical results should not be interpreted as causal.

Finally, as stated before, this measure of uncertainty has the advantage that it does not directly depend on first-moment shocks to the real exchange rate as do common measures used in the literature, such as GARCH types of estimations or the moving average of the standard deviation of the exchange rate that assume that changes in the real exchange rate generate changes in the volatility [5]. I will use $h = 6$ and $j = 1$ to construct the monthly real exchange rate uncertainty measure. I will take the average monthly value over the corresponding year to get annual estimates of the real exchange rate uncertainty.

Cyclical features of real exchange rate uncertainty. To understand how real exchange rate uncertainty is related to other aggregate variables, Table 1 presents the correlation of the real exchange uncertainty measure with the cyclical component of other aggregate variables. Panel A shows the relative volatility of each variable with respect to output, while Panel B presents the correlation of each variable with the RERU. The table shows that RERU is almost half as volatile as GDP for developed economies and 80% as volatile as GDP for emerging economies. Similar to the literature on macroeconomic uncertainty, I find that the uncertainty in the real exchange rate is counter-cyclical. The measure of uncertainty is negatively correlated with GDP, consumption, investment, exports, and imports, while the

[4] This is why controlling not only for standard gravity equation determinants but for changes in foreign demand, real exchange rate, and GDP is important.

[5] See Fernandez-Villaverde et al. (2010) for a discussion about this point.

correlation with net exports does not seem to be significantly different from zero. Finally, the real exchange rate uncertainty is positively correlated with the real exchange rate and the nominal exchange rate between the domestic country and the USD. This implies that not controlling for changes in the real exchange rate could bias the results downward since movements in the real exchange rate tend to increase exports and reduce imports.

5 Real exchange rate uncertainty and international trade

This section explores the relationship between real exchange rate uncertainty and international trade. First, I focus on the aggregate relation between international trade and real exchange rate uncertainty. Then, I use firm-level Colombian data to explore how real exchange rate uncertainty affects export behavior at both the extensive and the intensive margin. To the best of my knowledge, I present three new firm-level facts about the relationship between exporters and real exchange rate uncertainty. These firm-level facts are key to understanding why a standard dynamic model of trade would under-predict the effects of real exchange rate uncertainty in international trade and why we need theory able to generate the precautionary behavior at the intensive and extensive margins of trade. [6]

Aggregate Facts

To estimate the relationship between trade and real exchange uncertainty, I estimate a gravity equation expanded with the real exchange rate uncertainty measure. For country i at time t I estimate the following equation:

$$\ln(y_{i,t}) = \beta_0 + \beta_1 \ln(\tilde{\sigma}^2_{i,t}) + X_{i,t} + \alpha_i + \gamma_t + \epsilon_{i,t} \tag{8}$$

where $y_{i,t}$ represents exports, imports, or total trade over GDP, depending on the case. $X_{i,t}$,

[6] In the appendix I estimate an SVAR in the tradition of Sims (1980) and **Bloom˙09** Similar results using Chilean data are presented in the appendix A.4.

α_i,γ_t represent aggregate controls for each country, fixed effects by country, and time-fixed effects. In particular, I control for changes in GDP, real exchange rate, terms of trade, past changes and lagged values, and episodes associated with large devaluations.

Aggregate Fact 1: Negative relationship between RERU and trade openness. Table 2 presents the results; only the parameter of interest β_1 is presented. The estimation implies that a one standard deviation change (relative to the mean change) in real exchange rate uncertainty is associated with:

1. A drop in exports to GDP between 4% and 5.5%.
2. A drop in imports to GDP between 3% and 4%.
3. A drop in total trade to GDP between 3% and 4%.

To put the results in perspective, according to the World Bank, during the 2008 crisis, the drop in exports to GDP for the whole world was, on average, about 13%. Table 2 shows that controlling for the change of the aggregate variables (column 2) or for their past changes (column 3) does not change the results for exports and total trade over GDP. Once I control for year-fixed effects (column 4), the results remain the same.

One possible objection to this result is that exchange rate elasticity could change over time as it responds to changes in the real exchange rate. If the elasticity to changes in the real exchange changes over time, as found in Alessandria et al. (2014), and since the real exchange uncertainty is correlated with changes in the real exchange rate, it could be that the observed negative relationship is just due to misspecification. To overcome this problem, I estimate an error correction model to capture the possible reactions between the long and short-run effects of the real exchange rate in exports. The estimation results are presented in Table A.2 of the appendix A.1. In this case, the negative relationship still holds.[7].

Bilateral Trade. I also estimate a similar equation as before but for the bilateral relationship across countries. This allows me to control for variables that could affect total and bilateral trade and that are not included in the first case, like bilateral changes in the real

[7]The estimated equation for the error correction model, (31) is presented in the appendix A.1

exchange rate. For country i and j at time t the estimated equation is given by:

$$\ln(y_{i,j,t}) = \beta_0 + \beta_1 \ln(\tilde{\sigma}^2_{i,t}) + \tilde{X}_{i,t} + X_{i,t} + X_{i,j,t} + \alpha_i + \gamma_t + \epsilon_{i,t} \tag{9}$$

Where $y_{i,j,t}$ represents bilateral exports, imports, or total trade over GDP from country i to country j. $X_{i,j,t}$ represents controls at the bilateral relationship level and the standard gravity controls. $X_{i,t}$ and $X_{j,t}$ represent aggregate controls at country level. $\alpha_{i,j}$ and γ_t represent fixed effects by bilateral relationship and year fixed effects, respectively.

The estimation results of the equation (9) are presented in Table 3. These results are consistent with the ones found in the previous estimation. Controlling for bilateral variables does not seem to change the results. In this case, a standard deviation change in real exchange rate uncertainty is associated with an average drop of 5% in bilateral exports over GDP, 2.5% in bilateral imports over GDP, and 3.1% in total bilateral trade over GDP.

As before, there can be several concerns about some omitted variable or reverse causality problems with estimation (9). It could be that the real exchange rate uncertainty captures changes in the uncertainty or volatility of a country's GDP or that the changes in real exchange uncertainty reflect changes in the co-movement of the domestic economy with foreign countries. To control for this, in Table A.5 in appendix A.2 I present the same estimation as in (9), but controlling for these variables. In column 1, I include as a control a moving average correlation over the last three years of the industrial production between the domestic country and the industrial production of all G7 countries. In columns 2-4, I include as a control the rolling standard deviation (over 1, 2, and 3 years) of the log changes in the industrial production for both the domestic and foreign economy. In all these cases, the results hold.

Finally, another possible objection is that the real exchange uncertainty shocks could reflect the fact that an economy is more closed in terms of trade, implying the existence of reverse causality [8]. To control for this, I estimate the same equation as in (9), including lags in the trade openness of the domestic economy. Results are presented in Table A.5 of

[8] In a standard two-country model, the real exchange rate would be more volatile if there is less trade among the two countries. Even though the scope of the paper is about changes in real exchange uncertainty and not the average volatility

appendix A.2. The estimation using these controls shows that the main results still hold, but for exports and total trade, the estimated coefficient shrinks up to 33% in some cases.

Firm level facts

This section focuses on how exporters respond to changes in real exchange uncertainty. I present the estimations using firm-level data to identify possible mechanisms that help explain the aggregate patterns. I divide the analyses between the intensive margin of trade and the extensive margin of trade. I document that when real exchange rate uncertainty increases: 1) firms that paid higher interest rates and/or faced higher shipping lags reduce their export intensity by more; 2) firms are more likely to stop exporting; and 3) firms are less likely to start exporting to new markets. These firm-level results contradict the predictions of standard dynamic models of trade. More importantly, these results show that using these types of models to understand how real exchange rate uncertainty affects international trade would lead to underestimation of its effects. The underestimation is due to the inability of these types of models to generate facts 1) and 2).

Precautionary motive and the intensive margin of trade

Motivated by a wide variety of work showing that international trade is intensive in time and financial requirements as documented by Manova (2013), Kohn et al. (2016), Kohn et al. (2016), Fillat et al. (2015), and Leibovici et al. (2019) among others, and works such as Arellano et al. (2018), Khan et al. (2016), showing that the existence of default risk at firm level can induce further adjustment in firm production, I focus on two main mechanisms that are important to understand how exporters react to uncertainty, financial vulnerability and shipping lags.

To construct a measure that denotes the financial vulnerabilities that firms face, I follow the theoretical predictions in Arellano et al. (2018) and use the interest rate that firms pay

of the real exchange rate, it can be imagined that changes in total trade over GDP could be generating the movements in the real exchange rate volatility.

as a measure of financial vulnerability. Using the Colombian data described in section 2, I construct the interest rate that firms pay as the interest rate that the firm paid divided by the total liabilities the firm had over a year[9]. Once I have an interest rate measure for each firm, I group each firm in different groups according to their percentile of the distribution. I group firms in different percentiles each year. I construct a dummy for each percentile, denoting if the firm belonged or not to that particular group that year. I use this dummy as a measure of default risk that firms face in the main regression. I use other measures such as robustness checks, such as leverage or the ratio of interest payments over total profits, and group them in several ways. Lastly, I use a lagged dummy interacted with the measure of RERU to see how different firms react to RERU depending on their financial vulnerability.

To construct a measure of shipping lags, I proceed similarly. First, using the "Doing Business" survey of the World Bank, I obtain the reported time for each country to process an import, and I use it as a proxy for the total shipping lags that Colombian exporters face. I group each destination according to this measure in different percentiles. Then I use the corresponding dummy interacted with the RERU measure to see how different firms react to RERU depending on the shipping lags they face. For firm i, that exports product l to country j, I estimate the following equations:

$$\begin{aligned}\ln\left(es_{i,l,j,t}\right) = \beta_0 + \beta_1 \ln(\tilde{\sigma}_t^2) + \sum_{h=0}^{3} \beta_h^0 \ln(\tilde{\sigma}_t^2) \times \ I_{i,t-1}^h \\ + \sum_{h=0}^{3} \beta_h^1 \ \times \ I_{i,t-1}^h + \alpha_{i,l,j} + X_{i,l,j,t} + \hat{X}_{i,t} + \epsilon_{i,j,h,t}\end{aligned} \tag{10}$$

where the dependent variable $es_{i,l,j,t}$ is the export intensity of the product-destination export and is constructed as the export value in pesos of the product-destination divided by the total income of the firm in pesos. $\tilde{\sigma}_t^2$ represents the measure of real exchange rate uncertainty, and $I_{i,t-1}^h$ represents a dummy variable that is one if the firm belonged the percentile h in the previous period, and zero otherwise, depending on the case, representing the

[9]Ideally I will use the marginal interest rates since this data is not available I use the average interest rate as a proxy for the marginal interest rate firms pay.

financial vulnerability of each firm or the shipping lags that firm faces. $\alpha_{i,l,j}$ represents fixed effects by firm, product, and destination. $X_{i,j,t}$ represents standard gravity controls, bilateral exchange rates, multilateral real exchange rates, domestic and foreign absorption, terms of trade, aggregate productivity, and the changes of these variables (lag and log difference). $\hat{X}_{i,t}$ represents firm-level controls: number of destinations by firm and by firm-product pair, age of a firm exporting to a market, actual and lag profits, and import share (total imports divided by operational costs).

Fact 2: export intensity drops during high RERU Table 4 presents the estimations of equation (10) using the interest rate to construct the financial vulnerability dummy. Two main results are striking: exporters reduce their export intensity when real exchange rate uncertainty is high (first row), and those exporters facing a financially vulnerable situation reduce their export intensity between 9% and 6% more (second and third row). The first column presents the results when controlling for the standard gravity considerations, fixed effects at firm, product, and destination levels, and size. The third to the fifth column sequentially aggregates further controls as described by the table. Results remain mostly unchanged as we add these additional controls at firm, home, and destination levels.

Table A.3 in the appendix presents the estimations of equation (10) using in column one the leverage of the firms (measured as total liabilities over total assets) and in column two using interest payments over total profits as two different measures of financial vulnerability. Results remain unchanged.

Table 5 presents the results for shipping lags; the estimated equation is the same as in (10) but using the shipping lag dummy. Row one shows the average reaction of export intensity to RERU, and rows 2 and 3 show the differential reactions for those firms facing higher shipping lags (in the second and third tercile for the distribution). In particular, firms facing higher shipping lags drop their export share between 10% and 25% more. These results are consistent with other papers, as Leibovici et al. (2019) that find that shipping lags are relevant to explain trade flows.

Precautionary motive and the extensive margin of trade

The above estimations reflect how the intensive margin at the firm level is related to exchange rate uncertainty. Now, I analyze how real exchange rate uncertainty affects the extensive margin. Standard sunk cost models predict that when uncertainty increases, firms are less likely to enter export markets, and exporters are less likely to stop exporting, as I will show later. I test this prediction and I find that while the former prediction holds in the data, the latter one is contrary to my empirical results. I create a dummy variable $I_{i,t}^{Stop}$ that equals one if a firm exported at period t-1 and did not export at t, and is equal to zero if a firm exported at t-1 and at t [10]. Similarly, I create a dummy $I_{i,t}^{Entrant}$ that equals one if the firm exported to a given market at t but did not export to that market at t-1.[11] I estimate the following linear probability model:

$$I_{i,t}^{h} = \beta_0 + \beta_1 \ln(\tilde{\sigma}_{t-1}^2) + \xi_{i,l,d} + X_{i,d,l,t} + X_{i,t} + \epsilon_{i,l,d,t} \tag{11}$$

Facts 3 and 4: The precautionary extensive margin The results are presented in Table 6. In this case, a standard deviation increase with respect to the mean generates between a 5% and an 8% percentage point decrease in the probability of a firm entering a new export market and an increase in the probability of an exporter stopping exporting of between 10% and 20% percentage points.

In conclusion, I have presented four facts about real exchange uncertainty and exporter behavior. The first fact shows a negative association between international trade and real exchange rate uncertainty. When looking at firm-level data, fact 2 shows that exporters tend to reduce their export share when real exchange rate uncertainty is high and also that those facing higher shipping lags or that are more financially vulnerable tend to reduce their export intensity by more. Facts 3 and 4 show that when RERU is high, exporters are more likely to stop exporting and less likely to enter new markets.

[10] A period t means a year.

[11] Note that in this case, the extensive margin is referring to each destination which the firm is engaging with.

While the latter fact (less likely entry to new markets) is predicted by standard dynamic models of international trade with sunk costs, facts 2 and 4 are not. These models do not predict any change in the intensive margin and, contrary to the empirical results, predict that firms are less likely to exit. These results imply that using a standard dynamic model of international trade would underestimate the effects that RERU has on international trade flows and that to capture this relationship properly, we need a model that can properly capture firm-level facts. The next section develops a model consistent with all the documented firm-level facts.

6 Model

Motivated by facts 2, 3, and 4, I extend the standard sunk cost model of Alessandria et al. (2007) to incorporate two additional frictions, the existences of shipping lags and financial friction as modeled in Arellano et al. (2018). [12][13]

To focus on the relevance of each mechanism, I develop a partial equilibrium model in a small open economy with two types of shocks, a nominal exchange rate shock and a shock to volatility [14]. In the model, the economy is populated by two types of agents: producers of varieties and lenders. The producers of varieties sell their products in competitive monopolistic markets and can issue debt to a risk-neutral lender. The lender is modeled as a representative agent that is risk-neutral and lends and borrows money to/from firms.

Production of varieties. There is a measure μ_{fi} of firms that produce goods. A firm in the model is a producer of one variety that can hold debt and can sell to the domestic market, denoted as d, and the foreign market, denoted as f. At the moment each firm is born, the firm draws a productivity z_i from a log-normal random variable Z, characterized by a mean μ_z and a variance σ_Z^2. The firm's productivity is fixed over the firm's lifetime. These firms

[12] This way of modeling financial frictions is also in line with Arellano et al. (2012) and Khan et al. (2016), and is motivated by fact number 2.

[13] The motivation for shipping lags is related and trying to capture in a simple way fact number 3.

[14] Itskhoki et al. (2017) shows that incorporating shocks to the demand for foreign assets, that generate changes in the nominal exchange rate, can help to explain several of the puzzles related to the real exchange rate

are monopolistically competitive and use labor l in a constant returns production function to produce output $y = lz_i$.

There are two main differences between the domestic and foreign markets: the timing of production and the currency in which prices are set. In the domestic market, at time t, a firm decides how much to produce and sell in the domestic currency. The firm pays the labor costs and receives the profits at time t. When selling to the foreign market, the firm faces shipping lags, so it has to decide production at period t and sell it at t+1 when it receives the revenues in foreign currency and pays its workers. [15]

As in Alessandria et al. (2007), to enter the export market, each firm has to pay a sunk cost f_s, and once they are in the export market, they have to pay a continuation fixed cost f_e. The sunk cost is paid at period t by firms (if they did not export at t-1) to export at t+1. The fixed cost, f_e, is paid once the exporting is done at t+1. This generates the option value of exports, making the entry and exit decision a dynamic one, as in Dixit (1989b) and Alessandria et al. (2007). Furthermore, as noted by Fillat et al. (2015), this will generate the risk of exporting since firms will be willing to stay in the export market even if they expect negative profits, not to forgo the export option value.

Finally, on the financial side, each firm can issue debt denominated in national currency, b_t, to a risk-neutral lender at a price q_t. The main difference with standard export models is that firms are now allowed to default on debt, implying they will not issue negative dividends as in Arellano et al. (2018). If the firm has to issue equity, it will be forced to leave the market and default on debt. This assumption, together with shipping lags, generates firms that are averse to risk. As the volatility of the exchange rate increases, the probability of facing a lower exchange rate realization will increase, and firms will decide to reduce the risk to which they are exposed. This reduction of risk can be made in two ways: reducing the labor they hire and hence generating a decrease in exports through the intensive margin or leaving the export market.

[15] See Gopinath (2015) for more details about invoicing. According to Dian data, at least 2/3 of the firms invoice their export prices in U.S. dollars between 2011 and 2018.

Domestic and foreign demand. The demand that a firm faces is exogenously given by:

$$y_i^{D,j} = A^j (p_i^j)^{-\sigma} \tag{12}$$

for $j \in \{d, f\}$. Since the demand does not fluctuate, the sales to the domestic market do not vary over time. In this setup, the model captures the additional riskiness that the exchange rate generates over the nominal demand in the foreign market. The domestic market is a risk-free market to which firms will always sell the same quantities.[16]

Financial imperfection. As stated before, firms cannot issue negative dividends. Each firm pays its equity holder its revenues net of production costs and net payments of debt. Equity payments are not allowed to be negative by the non-negative equity payout condition:

$$d_t = p_t^d y_t^d - w l_t^d + m_t\{e^{\xi_t} p_t^f y_t^f - w l_t^f - f_e\} - m_{t+1}(1-m_t) f_s + q_t b_{t+1} - b_t \geq 0 \tag{13}$$

The first two terms denote the domestic profits, and the third term is the profits of exporting if the firm had decided to export ($m_t = 1$) in the previous period. $m_{t+1}(1-m_t)f_s$ is the sunk cost payment the firm has to make if the firm decides to export the following period but does not export in period t. Finally, $q_t b_{t+1} - b_t$ is the net payments of debt.

The price of the bond $q_t = q_t(z_i, b_{t+1}, l_{t+1}^f, \xi_t, \sigma_{\xi,t})$ reflects the compensation that a risk-neutral lender will receive for the loss it will incur in case the firm decides to default. It depends on the aggregate state $S_t = \{\xi_t, \sigma_\xi\}$, the productivity of the firm, and the firm decision $l_{t+1}^f, m_{t+1}, b_{t+1}$.

To characterize the default decision of a firm, first define the maximal borrowing that a firm can do as:

$$M_b(z_i, S_t) = \max_{l_{t+1}^f, b_{t+1}, m_{t+1}} q(z_i, b_{t+1}, l_{t+1}^f, S_t) b_{t+1} \tag{14}$$

[16] In a general equilibrium environment, this will not hold. The aggregate prices and wages would fluctuate with the exchange rate, making domestic demand vary.

And let $\bar{l}^f_{t+1}, \bar{b}_{t+1}, \bar{m}_{t+1}$, be the decisions that maximize the issuance of new debt. Now, define the exporter's liquidity needs as follows:

$$LN(l^f_{t+1}, z_i, b_{t+1}) = p^d_{t+1}y^d_{t+1} - wl^d_{t+1} - wl^f_{t+1} - f_e + M_b(z_i, S_{t+2}) - b_{t+1} \tag{15}$$

Which denotes how much liquidity from the export market a firm that exports at t+1 will need to cover all the expenses net of the domestic profits it will get. If $LN(.)$ is positive, then domestic profits are enough to cover all the expenses net of debt payments.

For an exporter, define ξ^*_{t+1} as the minimum exchange rate level at which a firm that is exporting will not default:

$$e^{\xi^*_{t+1}} = \begin{cases} -\frac{LN(l^f_{t+1}, z_i, b_{t+1})}{p^f_{t+1}y^f_{t+1}} & \text{if } LN(l^f_{t+1}, z_i, b_{t+1}) < 0 \text{ and } m_{t+1} = 1 \\ 0 & \text{if } LN(l^f_{t+1}, z_i, b_{t+1}) > 0 \text{ and } m_{t+1} = 1 \end{cases} \tag{16}$$

Equation (16) characterizes the threshold level of the exchange rate at which an exporter will default. If the LN(.) is positive, there is no value of $e^{\xi^*_{t+1}}$ at which the firm will default. In this case, domestic profits at t+1 are big enough to pay all the costs of net debt payments.

The default decision for a firm that does not export does not depend on the level of the exchange rate but on the amount of debt it has and its productivity.
Define d_{net} as the indicator variable that indicates when a firm will default.

$$d_{ne,t+1} = \begin{cases} 1 & \text{if } b_{t+1} > p^d_{t+1}y^d_{t+1} - \bar{m}_{t+2}f_s - wl^d_{t+1} + M_b(z_i, S_{t+2}) \text{ and } m_{t+1} = 0 \\ 0 & \text{if } b_{t+1} < p^d_{t+1}y^d_{t+1} - \bar{m}_{t+2}f_s - wl^d_{t+1} + M_b(z_i, S_{t+2}) \text{ and } m_{t+1} = 0 \end{cases} \tag{17}$$

Lender's problem. The lender is assumed to be risk-neutral. Given a free risk interest rate, r, and the associated discount rate β, the lender will lend to the firm at prices $q_t(z_i, b_{t+1}, l^f_{t+1}, m_{t+1}, \xi_t, \sigma_{\xi,t})$. For a firm that has decided to export the following period, the bond price will be given by:

$$q_t(z_i, b_{t+1}, l_{t+1}^f, 1, \xi_t, \sigma_{\xi,t}) = \frac{1}{1+r}(1 - F(\xi_t^* | \xi_t, \sigma_{\xi,t})) \tag{18}$$

where $F(.|\xi, \sigma_\xi)$ is the cumulative distribution function of the exchange rate conditional on the values ξ, σ_ξ. And for a firm that decided not to export, the bond price will be given by:

$$q_t(z_i, b_{t+1}, l_{t+1}^f, 0, \xi_t, \sigma_{\xi,t}) = \frac{1}{1+r} d_{ne,t+1} \tag{19}$$

Cash on hand. Similar to Arellano et al. (2018), the problem of the firm can be expressed using cash on hand, denoted by x, as a state variable. This simplifies the problem by reducing the number of state variables to 5, making the solution of the model easier in computational terms. The cash on hand of each firm in this case is given by:

$$x_t = p_t^d y_t^d - w l_t^d + m_t \{ e^{\xi_t} p_t^f y_t^f - w l_t^f - f_e \} - b_t \tag{20}$$

This allows me to simplify the state space to $\{z_i, x_t, m_t, \xi, \sigma_\xi\}$. Given an exchange rate realization, the dividends of the firm will be given by:

$$d_t = x_t - (1 - m_t) m_{t+1} f_s + q_t b_{t+1} \tag{21}$$

Now, the decision rules for labor, debt, and entry and exit from the export market can be expressed as a function of the cash on hand, export status, firm productivity, and the aggregate state of the economy.

Exchange rate process. The exchange rate process is assumed to follow an AR(1) process with a time-varying standard deviation. The time-varying standard deviation evolves according to a Markov chain with only two states denoted by σ_H and σ_L. The process for the nominal exchange rate, ξ_t is then given by:

$$\log(\xi_t) = \mu_s^\xi + \rho \log(\xi_{t-1}) + \sigma_s \epsilon_t \tag{22}$$

where μ_s^ξ has the standard convex correction, such that:

$$\mu_s^\xi = \left(\mu^\xi - \frac{\sigma^2}{2(1-\rho^2)}\right)(1-\rho) \tag{23}$$

And the matrix of transition probabilities between states is given by:

$$\Pi = \begin{bmatrix} \pi_{L,L}^s & \pi_{L,H}^s \\ \pi_{H,L}^s & \pi_{H,H}^s \end{bmatrix} \tag{24}$$

Firm's Recursive problem. Each firm is characterized by its productivity z_i, the cash on hand x_t, and the export status m_t. Following Khan et al. (2016), I assume that firms face a probability of dying given by π_d; this is relevant to make firms willing to issue debt and to be financially constrained. Given the amount of cash on hand, the firm decides how much new debt to issue, its export status tomorrow, and the production for the foreign market, provided that it does not default. To simplify the notation, I will drop the time subscript.

At the beginning of each period, firms decide to default or not. Formally, the problem that the firm solves is as follows:

$$V(z,x,b,\xi,\sigma_\xi) = max\{V^c(z,x,m,\xi,\sigma_\xi), V^d\} \tag{25}$$

V^c denotes the continuation value, and V^d is a constant value that the firm will get if it decides to default, normalized to zero[17]. If the firm decides to continue, it has to solve the following problem:

$$V^c(z,x,m,\xi,\sigma_\xi) = \max_{l^d,l^{f\prime},m',b'} d(z,x,m,\xi,\sigma_\xi) + \mathbb{E}\{QV(z,x',m',\xi',\sigma_\xi')|\sigma_\xi,\xi\} \tag{26}$$

s.t.

$$d = x - m'(1-m)f_s + q(z,l^{f\prime},m',b')b' \geq 0 \tag{27}$$

[17]This implies it will default only when dividends are negative.

$$x' = p^{d\prime} y^{d\prime} - w l^{d\prime}_t + m' \{ e^{\xi'} p^{f\prime} y^{f\prime} - w l^{f\prime}_t - f_e \} - b' \tag{28}$$

$$y^d + y^f m = z_i (l^d + l^f m) \tag{29}$$

and (12), (16),(17), (18),(19)

Similar to Dixit (1989b) and Alessandria et al. (2007), on top of the default threshold, there will be two thresholds that characterize the export decision, the entry and the exit thresholds. The main difference with their setups is that the threshold will not only depend on the aggregate variable (the exchange rate level and the volatility in this case) and firm productivity but also on the cash-on-hand level that each firm has. This generates the possibility of the extensive margin of trade reacting to both the aggregate and the individual risk.

Equilibrium. Let $S_t = (\xi, \sigma_\xi)$ and $s_i = (z_i, x_i, m_i)$ denote the aggregate and the idiosyncratic state variables. The equilibrium for this economy is a set of policy functions $\{l^d(s_i, S), l^f(s_i, S), m'(s_i, S), b'((s_i, S))\}$, a value function $V(s_i, S)$, and a set of prices $\{q(s_i, S),$ $p^d(s_i, S), p^f(s_i, S)\}$ such that given the parameter values, the aggregate state S, and the individual states s_i:

1. The optimal solution for each firm coincides with the policy and value functions.
2. The bond prices are given by (16) and (17).
3. Individual markets for each variety clear.

The partial equilibrium nature of the model implies that firms do not need to keep track of the distribution of cash on hand and export status over the whole economy. This assumption simplifies the computational burden of the problem, but at the cost that the results of the model can likely change once the same exercises are done in general equilibrium. Most likely, the partial equilibrium nature of the exercise will imply that the model will overestimate the impacts of real exchange uncertainty shocks due to the fact that once prices and wages adjust to real exchange rate shocks, firms will not decrease labor as under the partial equilibrium set up. In section B, I present the algorithm I use to solve the firm's and lender's problems.

Optimal decisions. Before I present the main quantitative results of the model, I discuss the optimal policy functions related to the amount of trade in the economy, i.e., the labor used for the production of exports, the extensive margins of firms, and the amount of debt issued by firms that decide to export in the following period. The results presented in this section use the parameter values presented in Table 7.

Figure 3 plots the minimum exchange rate level at which each firm is willing to enter or exit the export market. The top graphs present the entry and exit decisions for firms with high levels of cash on hand, while the bottom graphs show the entry and exit decision for firms with low levels of cash on hand. The red dotted lines represent the entry and exit decisions for states with high volatility of the exchange rate, and the blue line is the decision under low volatility states. Similarly, Figure A.2 presents the entry and exit decisions for a standard sunk model without either shipping lags or default.

Figure 3 displays two results. First, as is standard in the sunk models, the threshold to enter the export market is higher than the one to exit it. This is due to the option value of exports originated by the existence of the sunk cost to export as shown in Dixit (1989b) and Alessandria et al. (2007). Second, firms require a smaller exchange rate level to enter or exit the export market when they hold large amounts of cash on hand.

Figure 3, also displays how adding shipping lags and default originates a new force that counteracts the standard effects generated by the existence of the option value of exports. When firms face more risk due to the increase in the exchange rate uncertainty or low cash on hand, firms are more willing to exit. I call this the precautionary quitting motive. The bottom panel shows that when firms have low cash on hand, the existence of additional risk reduces the differences between the entry and the exit thresholds.

The top right panel of 3 shows that when uncertainty increases, the precautionary quitting motive and the option value of exports work in opposite directions. When exchange rate volatility is higher, the least productive firms are more willing to leave the market, and the most productive are morere are willing to stay. In this case, the precautionary quitting motive is stronger than the option value of exports for low-productivity firms. Firms that are more productive face smaller risks in the model because they receive higher profits from the domestic market, and the fixed cost they pay as a share of expected profits is smaller. When uncertainty increases, the increase in the probability of default is not big enough to compensate for the effect that uncertainty has on the option value of exporting.

When firms have high cash on hand, high productivity firms face minimal risk, making the quitting decision behave as in the standard model. However, as the risk starts increasing, the motive of precautionary quitting kicks in, reducing the willingness of firms to stay in the export market. [18]

In conclusion, the quitting decision is driven by two opposing forces in the model; on the one hand, the option value of exports makes firms willing to delay quitting the export market, and on the other hand, the risk of being in the export market makes firms more willing to quit. In the model, the risk a firm faces increases with exchange rate uncertainty and decreases with the amount of cash on hand. For periods of low uncertainty and high cash on hand, the value of exporting is higher, but it decreases as we increase the uncertainty of the real exchange rate.

To further understand the entry and exit decisions, Figure 4 presents debt issuance for firms that had decided to export in the following period. The top panel presents the amount of debt each firm issues conditional on the level of cash on hand and the export status a firm has. The bottom panel shows that when an exporter holds high levels of cash on hand, they decide to reduce their dividends today to increase their saving. This result helps us to understand why firms with high cash on hand are more willing to enter the export market or to stay in it. When firms have high cash on hand, they decide to save more, reducing their exposure to risk in the next period. The reduction in risk through savings is particularly important for low productivity firms since the cash flows originated in the domestic market are smaller. As it can be appreciated in the third graph of figure 4, firms with low cash

[18] In Figure A.2 of the appendix C, I present the policy functions for entry and exit for a standard sunk model in partial equilibrium. In this case, the uncertainty generates a delay in entry and exit as in Dixit (1989b) and Caballero (1992). The increase in uncertainty delays entry, making firms require a higher exchange rate to enter the export market. On the other hand, it decreases the exchange rate level at which they are indifferent to continuing exporting or quitting the export market.

on hand will not be able to save as much due to the liability constraint, which does not allow the firm to reduce the risk as expected. This inability to save increases the exposure to risk, reducing entry, increasing exit, and reducing trade through its intensive margin. Figure 5 presents the labor decision for firms with different productivities and different levels of uncertainty at the mean level of exchange rate. The top panel shows the labor decision for firms that have exported in the current period, while the bottom panel shows the labor decision for firms that have not exported in this period. It can be seen that besides the effects that higher volatility has on the extensive margin, it also affects the intensive margin. On average, firms reduce the amount of labor they hire by 3% when uncertainty about the real exchange rate is high. The intuition here is similar to in Khan et al. (2016) and Arellano et al. (2018); the existence of financial imperfections makes firms willing to reduce the labor they hire during highly volatile environments to reduce the exposure to bad shocks.

7 Quantitative results

Now, I use the model to perform two quantitative exercises intended to respond to the following two questions:

1. Can the proposed mechanism help us to explain the negative relationship between real exchange rate uncertainty shocks and aggregate exports?
2. How different is the proposed method to measure real exchange uncertainty with respect to the other methods used in the literature?

To answer these questions, I calibrate the model to match relevant moments of international trade. Once the calibration is done, to answer the first question, I simulate a model with and without shocks to the real exchange rate and regress the simulated volatility of the real exchange rate against aggregate exports.

To answer the second question, I present a second simulation. In this case, I estimate the proposed measure of real exchange uncertainty and the rolling standard deviation to test if this method can properly capture the real exchange uncertainty and to what magnitude.

Calibration

I calibrate three different models. One is a standard sunk cost model without either shipping lags or default, which I call the sunk cost model. Then I calibrate the extended version, which I call the shipping lags model. Finally, I calibrate the same version presented in the previous section but reducing the death probability of firms by half, and I call this version Low π_d. Table 7 presents the calibrated parameters for the three versions; the second column presents the value of each parameter, and the third column presents the rationale behind it. The model is calibrated at a quarterly frequency, and the only parameters estimated within the model are the ratio $\frac{A^{cl}}{A}$, the dispersion, and the mean of the firm productivity $\mu_z,\ \sigma_z$, and the level of the fixed cost f_e. These parameters are estimated to match the export share, the exports-sales ratio, and the exporter premium measure of the ratio of total shipments between exporters and non-exporters. The parameters associated with the real exchange rate are estimated outside the model for the Colombian case.

In Table 8, I present the values of the targeted moments and the prediction of the model. I decide to target these moments because they are important for the mechanism stated in the model. The exporter premium and the share of exporters help to discipline the model in terms of the relevance of the extensive margin against the intensive margin since the magnitudes of the impact of real exchange uncertainty could be different depending on which effect is more important. While the ratio of domestic sales over foreign sales is important in the model, since firms face no uncertainty in the domestic market, if a firm's total income depends heavily on export income, it can overreact to uncertainty shocks since the overall risk would be artificially large.

Real exchange uncertainty shocks: The relevance of the mechanism.

I use the model to test how large the response of aggregate exports to uncertainty shocks is. I do this in two ways, first I simulate the model with uncertainty shocks and real exchange rate shocks, and then I simulate the model under a pure uncertainty shock, and compare the results between the standard sunk cost model, the proposed extension that I call shipping lags, and the proposed extension using a lower value for the death probability, called low π_d.

In the standard sunk cost model, the timing for entry and exit and the timing for payment of f_e and f_s are the same as in the shipping lags model. The difference is that firms will never default or face shipping lags. Firms are allowed to produce after they observe the exchange rate shocks, and dividends will be equal to current profits. The third case, called low π_d, is exactly the same as the proposed model, but with $\pi_d = 0.005$. The comparison between the proposed extension and the standard sunk cost model will point out the relevance of the two proposed mechanisms while lowering π_d allows me to test for the relevance of the financial constraint since a lower π_d will reduce the probability of firms facing financial problems.

I simulate the model over 2800 periods. During the first 300 periods, I assume the volatility shock is always in the low state and that the real exchange rate is at the mean value. From period 300 to 2800, I simulate the exchange rate following the process described in the model section. I replicate each simulation 200 times. In each replication, I regress the aggregate exports against the real exchange and the volatility value (remember, volatility only takes two values representing high or low volatility states).

For each replication I estimate the following regression over the generated data:

$$y_t = \beta_0 + \beta_1 \tilde{\sigma}_{\xi,t} + \beta_2 rer_t + \epsilon_t \tag{30}$$

Table 9 presents the average coefficient for the estimation of equation (30). Row 1 of each panel of Table 9 runs the specification (30) in levels, while row 2 of each panel presents the results of the same specification but in logs. Column 1 corresponds to the standard sunk cost model, column 2 presents the coefficient for the proposed extension of it, and column three presents the estimation for the Low π_d version. The results show that in both cases, the prediction of the model implies a negative relationship between aggregate exports and volatility. While the standard sunk cost model predicts near zero response to pure exchange rate shocks, the proposed model predicts that a standard deviation change (with respect to the mean) in real exchange volatility generates a drop in aggregate exports of 4%, while the version with low π_d implies a drop of 0.6%. When I simulate the uncertainty shocks with shocks to the level of the exchange rate, I find that the standard sunk cost model predicts a drop in aggregate exports of 3.3%, while the estimation for the proposed model is 10% and 6% for the case with the low probability of death.

Real exchange rate uncertainty shocks: The relevance of the estimation method

The last exercise for which I will use the model is to test the different measures used to measure real exchange uncertainty. I will simulate the model exactly in the same way as in the first case, but I will make 150 replications.

Since the model is calibrated at a quarterly frequency, I will simulate the real exchange data and the exports at a quarterly frequency. For each generated series of the real exchange rate, I will generate five different measures of the uncertainty of the exchange rate shocks. The first series is the original (the same as the one used to feed the model), the second measure of uncertainty is the proposed measure of real exchange uncertainty using Markov switching regimes, and the other three measures of uncertainty are going to be rolling standard deviation of the log differences of the real exchange rate over the last 4, 8 and 12 quarters, as used in the literature. The idea of this exercise is to use the model to test how well these measures of uncertainty are capturing the underlying relationships. Ideally, we should expect the coefficient using the first series (the original) to be the same as the one using the other measures.

I estimate equation (30) in logs. The results are presented in Table 10. As before, I present the average values of the estimated coefficient and the average standard error. The results show that when using the proposed measure of real exchange uncertainty, the estimated coefficient is near the true coefficient since the latter lies inside the 95% percent confidence interval of the former. While using the rolling standard deviation measure, the coefficient is about half of the true effect.

These results can help us understand why, when using the proposed measure of real exchange uncertainty, I find significant responses at the aggregate level, while previous papers in the literature did not find a meaningful economic relationship.

8 Conclusion

In this paper, I study how real exchange rate uncertainty affects international trade. In order to answer this question, I propose a new method based on regime switching estimation to estimate a measure of real exchange rate uncertainty. Using this measure, I document four facts. First, at the aggregate level, I find that real exchange uncertainty is negatively related to international trade. Then, using firm-level data, I document three new firm-level facts relating to exporter behavior and real exchange rate uncertainty. I find that when real exchange rate uncertainty increases, 1) firms reduce their export share, and those that pay higher interest rates and/or face higher shipping lags reduce it by more; 2) firms are more likely to stop exporting; and 3) firms are less likely to start exporting to new markets. These results show the mechanism through which real exchange rate uncertainty affects aggregate international trade and also show that standard dynamic models of trade are ill-suited to understand how real exchange rate uncertainty affects international trade. These models do not predict movements in the intensive margin of trade and have predictions that are opposite to fact 2. This generates such models to under-predict the effects that real exchange uncertainty has on international trade.

Based on this result, I built a partial equilibrium sunk cost model extended with shipping lags and endogenous default. I find that this extension can replicate all the firm-level empirical findings. I estimate the model to match Colombian exporter data, and I find that a one standard deviation increase in real exchange rate uncertainty can generate a drop in aggregate exports between 5% and 10%. Then, I use the model to test the ability of different empirical measures to capture the effects that real exchange rate uncertainty has on trade. I show that the standard measure used by the literature can capture at most half of the effects predicted by the model.

The results I find empirically can also be used to revisit some long-lasting questions in international macroeconomics. These results can help to explain why exchange rate volatility seems to be unrelated or ”disconnected” from other fundamentals as discussed in Itskhoki et al. (2017). To the extent that real exchange uncertainty is high, this can imply that international trade flows react less to exchange rate movements, implying that exchange rate movements need to be larger to push back an economy to its equilibrium levels.

Finally, this model can be easily extended to incorporate second-moment shocks to aggregate productivity or foreign demand. For example, an interesting avenue for future research could be to use this model to understand how foreign uncertainty shocks propagate to different countries and the relevance of international trade as a propagation mechanism.

References

Alessandria, G. & Choi, H. (2007). Do sunk costs of exporting matter for net export dynamics? *Quarterly Journal of Economics*, 48.

Alessandria, G., Pratap, S., & Yue, V. (2014). *Export dynamics in large devaluations ;y.*

Alessandria, G., Choi, H., Kaboski, J. P., & Midrigan, V. (2015). Microeconomic uncertainty, international trade, and aggregate fluctuations. *Journal of Monetary Economics*, *69*, 20–38.

Arellano, C., Bai, Y., & Zhang, J. (2012). Firm dynamics and financial development. *Journal of Monetary Economics*, *59*(6), 533–549.

Arellano, C., Bai, Y., & Kehoe, P. (2018). Financial frictions and fluctuations in volatility. *Journal of Political Economy*.

Bahmani-Oskooee, M. & Hegerty, S. W. (2007). Exchange rate volatility and trade flows: a review article. *Journal of Economic Studies*, *34*(3), 211–255.

Bernard, A. B., Jensen, J. B., Redding, S. J., & Schott, P. K. (2018). Global firms. *Journal of Economic Literature*, *56*(2), 565–619.

Bloom, N. (2009). The impact of uncertainty shocks. *Econometrica*, *77*(3), 623–685.

Brooks, W. & Dovis, A. (2019). Credit market frictions and trade liberalizations. *Journal of Monetary Economics*.

Caballero, R. J. (1992). Uncertainty, investment, and industry evolution. *NBER WORKING PAPER SERIES*, 31.

Clark, P. P., Tamirisa, N., & Wei, S.-J. (2004). *Exchange rate volatility and trade flows - some new evidence*. International Monetary Fund.

Dixit, A. (1989a). Entry and exit decisions under uncertainty. *Journal of Political Economy*, *97*(3), 620–638.

Dixit, A. (1989b). Hysteresis, import penetration, and exchange rate pass-through. *The Quarterly Journal of Economics*, *104*(2), 205.

Engel, C. (1994). Can the markov switching model forecast exchange rates? *Journal of International Economics*, *36*, 151–165.

Fernandez-Villaverde, J. & Rubio-Ramirez, J. (2010). *Macroeconomics and volatility: data, models, and estimation* (Working paper No. w16618). National Bureau of Economic Research.

Fernández-Villaverde, J., Guerrón-Quintana, P., Rubio-Ramírez, J. F., & Uribe, M. (2011). Risk matters: the real effects of volatility shocks. *American Economic Review*, *101*(6), 2530–2561.

Fillat, J. L. & Garetto, S. (2015). Risk, returns, and multinational production. *The Quarterly Journal of Economics*, *130*(4), 2027–2073.

Gopinath, G. (2015). *The international price system* (No. w21646). National Bureau of Economic Research. Cambridge, MA.

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica*, *57*(2), 357.

Héricourt, J. & Poncet, S. (2015). Exchange rate volatility, financial constraints, and trade: empirical evidence from chinese firms. *The World Bank Economic Review*, *29*(3), 550–578.

Héricourt, J. & Nedoncelle, C. (2016). How multi-destination firms shape the effect of exchange rate volatility on trade: micro evidence and aggregate implications, 46.

Itskhoki, O. & Mukhin, D. (2017). *Exchange rate disconnect in general equilibrium* (No. w23401). National Bureau of Economic Research. Cambridge, MA.

Jurado, K., Ludvigson, S. C., & Ng, S. (2015). Measuring uncertainty. *American Economic Review*, *105*(3), 1177–1216.

Khan, A., Senga, T., & Thomas, J. K. (2016). *Default risk and aggregate fluctuations in an economy with production heterogeneity.*

Kilian, L. & Taylor, M. P. (2003). Why is it so difficult to beat the random walk forecast of exchange rates? *Journal of International Economics*, *60*(1), 85–107.

Kohn, D., Leibovici, F., & Szkup, M. (2016). Financial frictions and new exporter dynamics. *International Economic Review*, *57*(2), 453–486.

Leibovici, F. & Waugh, M. E. (2019). International trade and intertemporal substitution. *Journal of International Economics*, *117*, 158–174.

Lin, S., Shi, K., & Ye, H. (2018). Exchange rate volatility and trade: the role of credit constraints. *Review of Economic Dynamics*, *30*, 203–222.

Manova, K. (2013). Credit constraints, heterogeneous firms, and international trade. *The Review of Economic Studies*, *80*(2), 711–744.

McKenzie, M. D. (1999). The impact of exchange rate volatility on international trade flows. *Journal of Economic Surveys*, *13*(1), 71–106.

Meese, R. A. & Rogoff, K. (1983). Empirical exchange rate models of the seventies. *Journal of International Economics*, *14*(1), 3–24.

Novy, D. & Taylor, A. (2014). *Trade and uncertainty* (No. w19941). National Bureau of Economic Research. Cambridge, MA.

Petrin, A., Poi, B. P., & Levinsohn, J. (2004). Production function estimation in stata using inputs to control for unobservables. *The Stata Journal: Promoting communications on statistics and Stata*, *4*(2), 113–123.

Ruhl, K. J. & Willis, J. L. (2017). New exporter dynamics. *International Economic Review*, *58*(3), 703–726.

Sims, C. A. (1980). Macroeconomics and reality. *Econometrica*, *48*(1), 1.

Westerlund, J. (2007). Testing for error correction in panel data. *Oxford Bulletin of Economics and Statistics*, *69*(6), 709–748.

9 Figures

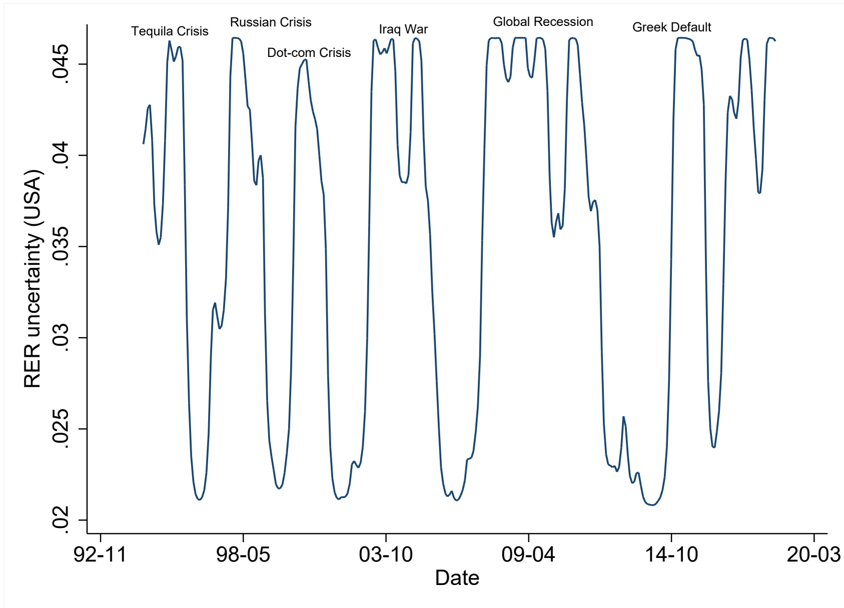


Figure 1: Real Exchange rate uncertainty USA

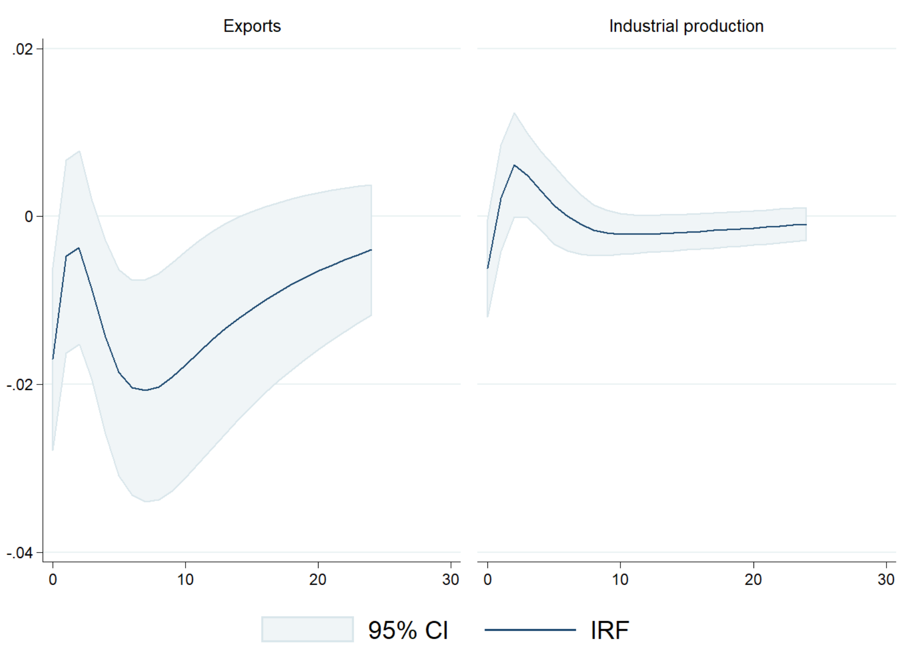


Figure 2: IRF to real exchange uncertainty shock

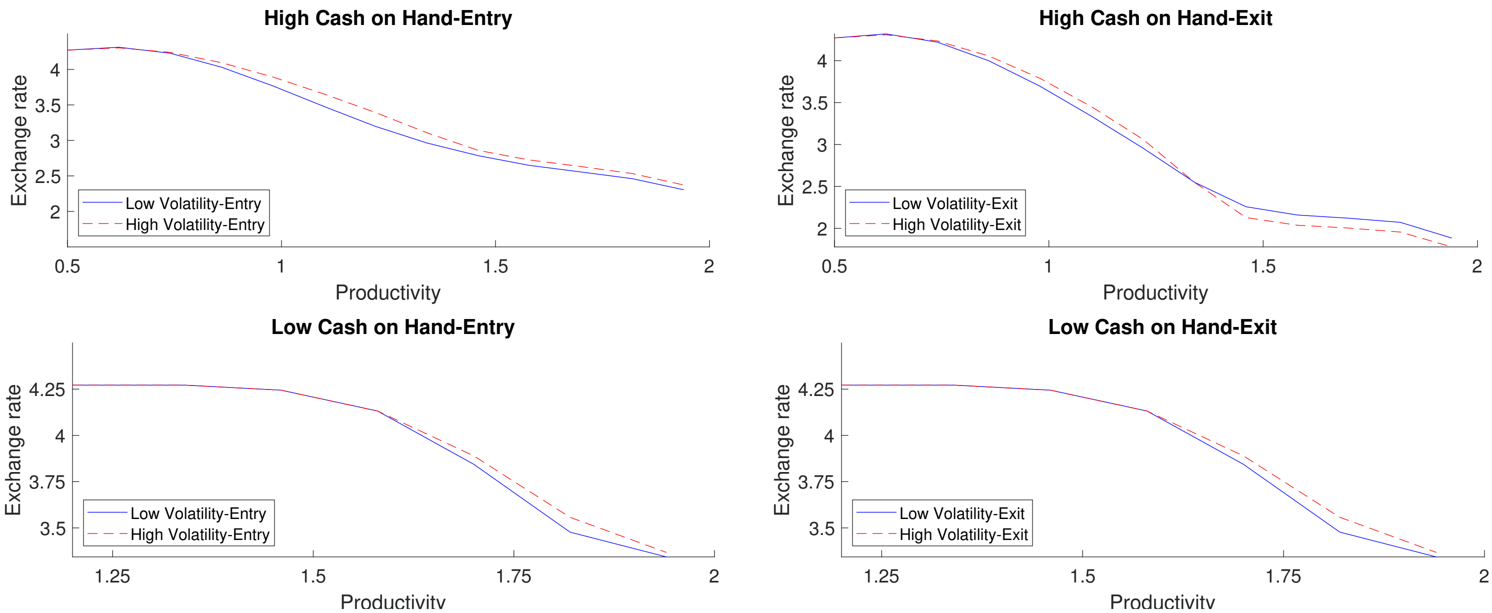


Figure 3: Extensive Margin

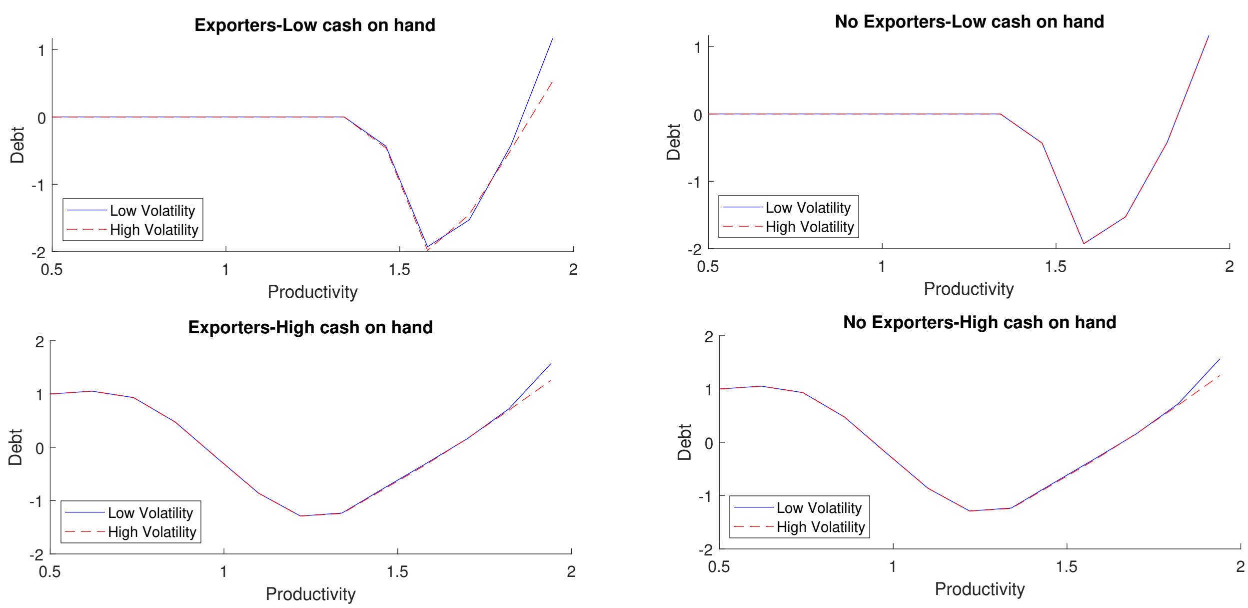


Figure 4: Debt decision

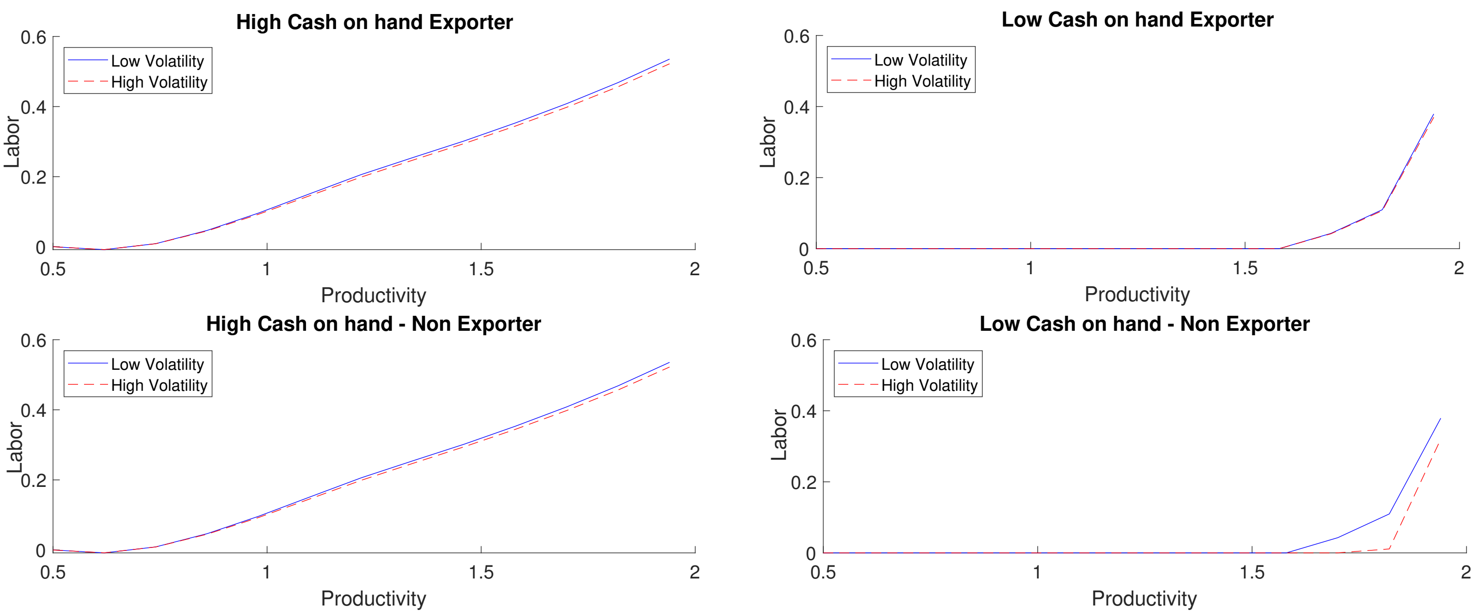


Figure 5: labor intensity

10 Table

Table 1: Moments of Real exchange rate uncertainty

Variable	All	Developed	Emerging
Average $\tilde{\sigma}_t^2$	4.93	4.11	5.49
$\sigma_{\tilde{\sigma}_t^2}$	2.53	1.11	3.49
σ_{GDP}	2.90	2.24	3.34
Panel A: Standard deviation (SD) relative to output SD			
RER Uncertainty	0.87	0.49	1.04
Consumption	1.06	0.92	1.16
Investment	3.80	3.64	3.91
Exports	2.26	2.45	2.13
Imports	2.99	2.76	3.14
Net exports	2.25	1.90	2.48
Panel B: Correlation with RER uncertainty			
GDP	-0.15	-0.16	-0.15
Consumption	-0.11	-0.12	-0.11
Investment	-0.16	-0.15	-0.2
Exports	-0.14	-0.14	-0.13
Imports	-0.18	-0.18	-0.19
Net exports	0.05	0.01	0.1
Real exchange rate	0.15	0.11	0.15
Nominal Exchange rate with USD	0.15	0.13	0.16
Real exchange rate$_{t-1}$	0.02	0.02	0.01
Nominal Exchange rate with USD$_{t-1}$	0.01	0.01	-0.01

All series are HP filtered in logs with $\lambda = 100$, except for trade balance, that is HP filtered in levels, and the real exchange uncertainty measure that is use in levels. Emerging economies are defined as those with a GDP per capita smaller than 25 thousand USD.Standard deviation of output and mean expected volatility are in percentage points.* Percentage standard deviation of $\tilde{\sigma}_t$ (not relative to output).

Table 2: RER uncertainty and Aggregate Trade

Panel A: Exports /GDP					
$\tilde{\sigma}_t^2$	-0.09***	-0.11***	-0.11***	-0.11***	-0.08***
	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
R^2	0.15	0.20	0.28	0.28	0.43
Panel B: Imports/ GDP					
$\tilde{\sigma}_t^2$	-0.07**	-0.07**	-0.06**	-0.07**	-0.08**
	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
R^2	0.25	0.29	0.31	0.38	0.37
Panel C: Trade /GDP					
$\tilde{\sigma}_t^2$	-0.08***	-0.08***	-0.08***	-0.08***	-0.06**
	[0.03]	[0.03]	[0.03]	[0.03]	[0.03]
R^2	0.24	0.26	0.35	0.35	0.53
Observations	854	840	801	801	801
Country FE	Yes	Yes	Yes	Yes	Yes
Aggregate Ctrl	Yes	Yes	Yes	Yes	Yes
Aggregate Change	No	Yes	Yes	Yes	Yes
Aggregate Change (lag)	No	No	Yes	Yes	Yes
Large Devaluation	No	No	No	Yes	Yes
Year FE	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Country FE: fixed effects by country.2) Aggregate Ctrl: includes as control log of real exchange rate, term of trade, GDP and foreign demand, if country belong or not to WTO or GATT and population. 3) Change: includes the change of the log of the real exchange rate, past value of log real exchange rate, cyclical component of the log of foreign demand, term of trade and GDP (H-P filtered with $\lambda = 100$). 4) Change$_2$: includes lag of change of real exchange rate. 6) Large Devaluation: includes episodes of large devaluation for each country. 5) Year FE: denotes year fixed effects.
Standard errors in brackets. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 3: RER uncertainty and Bilateral trade

Panel A: Exports$_{i,j}/GDP_i$						
$\tilde{\sigma}_t^2$	-0.50***	-0.14***	-0.15***	-0.13***	-0.11***	-0.11***
	[0.14]	[0.05]	[0.05]	[0.04]	[0.04]	[0.04]
R^2	0.03	0.38	0.39	0.40	0.46	0.46
Observations	60415	42624	42044	41567	41567	41567
Panel B: Imports$_{i,j}/GDP_i$						
$\tilde{\sigma}_t^2$	-0.51***	-0.12**	-0.11***	-0.09**	-0.09**	-0.04
	[0.14]	[0.05]	[0.04]	[0.03]	[0.04]	[0.03]
R^2	0.03	0.37	0.39	0.40	0.40	0.46
Observations	60395	42808	42048	41571	41571	41571
Panel C: Trade$_{i,j}/GDP_i$						
$\tilde{\sigma}_t^2$	-0.48***	-0.12**	-0.11***	-0.09**	-0.09**	-0.06**
	[0.13]	[0.05]	[0.04]	[0.03]	[0.04]	[0.03]
R^2	0.04	0.50	0.52	0.54	0.54	0.61
Observations	60284	42751	42252	41518	41518	41518
Bilateral Fixed effects	Yes	Yes	Yes	Yes	Yes	Yes
Gravity Controls	No	No	Yes	Yes	Yes	Yes
Change	No	No	No	Yes	Yes	Yes
Change (lag)	No	No	No	No	Yes	Yes
Large Devaluation	No	No	No	No	Yes	Yes
Year FE	No	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Bilateral FE: fix effects by origin-destination pair.2) Gravity: includes log of real exchange rate of origin country, bilateral exchange rate, term of trade and GDP from origin and destination country, if country belong or not to WTO or GATT(origin and destination), if countries have common currency, if country is EU member, weighted distant, and population (origin and destination). 3) Change: includes growth rate of the bilateral real exchange rate and the lag change of it, GDP growth of both countries,the change in the origin country real exchange rate, past value of log real exchange rate. 4) Change$_2$: includes the lag of the variables included in Change. 5) Large Devaluation: includes a dummy for large devaluations episodes, interacted with origin and distention country
Standard errors in brackets (cluster by origin). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 4: Firm level interest rate and exports

	(1)	(2)	(3)	(4)	(5)
	$\text{es}_{l,d,t}$	$\text{es}_{l,d,t}$	$\text{es}_{l,d,t}$	$\text{es}_{l,d,t}$	$\text{es}_{l,d,t}$
$\hat{\sigma}_t$	-2.16***	-1.99***	-1.99***	-2.07***	-2.06***
	[0.06]	[0.06]	[0.06]	[0.07]	[0.08]
$r^1_{t-1} \times \hat{\sigma}_t$	-0.13**	-0.12**	-0.12**	-0.14**	-0.11
	[0.06]	[0.06]	[0.06]	[0.06]	[0.08]
$r^2_{t-1} \times \hat{\sigma}_t$	-0.18***	-0.18***	-0.18***	-0.13**	-0.19**
	[0.06]	[0.06]	[0.06]	[0.06]	[0.08]
Observations	131265	131265	131265	107268	107268
R^2	0.78	0.78	0.78	0.79	0.79
Firm, product, destination FE	Yes	Yes	Yes	Yes	Yes
Size FE	Yes	Yes	Yes	Yes	Yes
Gravity_t	Yes	Yes	Yes	Yes	Yes
Gravity_{t-1}	No	Yes	Yes	Yes	Yes
$\Delta\ Gravity_{t-1}$	No	No	Yes	Yes	Yes
Firm controls	No	No	No	Yes	Yes
$\text{Gravity}_t \times r^h_{t-1}$	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost).5) $\text{Gravity}_t \times r^h_{t-1}$: Denotes the interaction between gravity variables and dummy of financial vulnerability
Standard errors in brackets (clustered by exporter). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 5: Firm level data and shipping lags

	(1)	(2)	(3)	(4)	(5)
	$\mathrm{es}_{l,d,t}$	$\mathrm{es}_{l,d,t}$	$\mathrm{es}_{l,d,t}$	$\mathrm{es}_{l,d,t}$	$\mathrm{es}_{l,d,t}$
$\hat{\sigma}_t$	-2.12***	-1.91***	-1.91***	-1.90***	-1.84***
	[0.06]	[0.25]	[0.07]	[0.07]	[0.08]
Shipping $\text{lags}^1_{t-1} \times \hat{\sigma}_t$	-0.08	-0.08	-0.08	-0.21**	-0.21**
	[0.06]	[0.06]	[0.06]	[0.07]	[0.09]
Shipping $\text{lags}^2_{t-1} \times \hat{\sigma}_t$	-0.27***	-0.24***	-0.24***	-0.45***	-0.45***
	[0.06]	[0.06]	[0.06]	[0.07]	[0.08]
Observations	131265	131265	131265	107268	107268
R^2	0.78	0.78	0.78	0.79	0.79
Firm, product, destination FE	Yes	Yes	Yes	Yes	Yes
Size FE	Yes	Yes	Yes	Yes	Yes
Gravity_t	Yes	Yes	Yes	Yes	Yes
Gravity_{t-1}	No	Yes	Yes	Yes	Yes
$\Delta\ Gravity_{t-1}$	No	No	Yes	Yes	Yes
Firm controls	No	No	No	Yes	Yes
$\text{Gravity}_t \times r^h_{t-1}$	No	No	No	No	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost).5) $\text{Gravity}_t \times r^h_{t-1}$: Denotes the interaction between gravity variables and dummy of shipping lags
Standard errors in brackets (clustered by exporter). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 6: Extensive Margin and Real Exchange Rate Uncertainty

Panel A: Entrants			
$\hat{\sigma}_{rer,t}$	-0.08***	-0.13***	-0.12***
	[0.02]	[0.03]	[0.03]
Observations	97539	97453	97453
Panel B: Stoppers			
$\hat{\sigma}_{,t}$	0.98***	0.87***	0.94***
	[0.01]	[0.06]	[0.05]
Observations	85365	85209	85209
Firm, Size FE	Yes	Yes	Yes
Gravity$_t$	Yes	Yes	Yes
Gravity$_{t-1}$	No	Yes	Yes
$\Delta\ Gravity_{t-1}$	No	Yes	Yes
Firm controls	No	No	Yes

$I_{i,t}^{Stop}$ equals to one, if a firm exported at t-1 but did not at t. And it is equal to zero if the firm exported at t-1 and t. $I_{i,t}^{Entrant}$ equals to one, if a firm exported at t but did not at t-1. And it is equal to zero if did not exported at t-or at t.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost), and in the case of panel A, it also includes a dummy denoting if they are re-entrants or not (export at t-1, did not export at and export at t).
Standard errors in brackets (cluster by exporter destination). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table 7: Calibration

Parameter	Value	Rationale
σ	3	Standard
β	0.98	Ruhl et al. (2017)
π_d	0.012	Khan et al. (2016)
$\pi^{\sigma}_{1,1}$	0.843	Duration of 19 month
$\pi^{\sigma}_{2,2}$	0.823	Duration of 17 month
σ_L	0.0486	Estimation
σ_H	0.11	Estimation
ρ	0.98	Half life of RER
$\frac{f_s}{f_e}$	2.81	Alessandria et al. (2007)

Parameter	Sunk cost	Default	Low debt	Data
f_e	0.164	0.11	0.12	Exporters Share = 0.195
$\frac{A^d}{A^f}$	261.41	679	679	Exports-sales ratio=0.14
σ_z	0.38	0.32	0.32	Match exporter premia=1.72
π^*_d	-	-	0.006	

Table 8: Moments

Target moments	Sunk Cost	Default	Low debt	Data	Source
Exporters share	0.18	0.19	0.19	0.19	Manuf. survey (2005)[19]
Exports-sales ratio	0.14	0.14	0.15	0.13	Ruhl et al. (2017)
Exporter premia (total sales)	1.75	1.72	1.73	1.72	Bernard et al. (2018)

Non Target moments	Sunk Cost	Default	Low debt	Data	Rationale
Exporter premia (labor)	1.3	1.10	1.12	1.28	Bernard et al. (2018)
Exporter premia (domestic sales)	5.5	4.9	4.2	3.8	Ruhl et al. (2017)

The export-sales ratio is calculated as the average export-sales ratio among exporters. The exporter premia of domestic shipment is calculated as the ratio of the average value of total shipment of exporter with the average value of total shipment of non exporters, following Ruhl et al. (2017). The exporter premia for total shipment and labor is calculates as the coefficient of a dummy variable with the corresponding dependent variable in logs, following Bernard et al. (2018). This values correspond to a simulation of 1000 firms calculated at the ergodic mean (100 periods after simulating the real exchange rate at its mean with a low volatility).

Table 9: Simulated model

Panel A: Constant RER			
	Sunk cost	Baseline	Low debt
β_1	≈ 0	-0.14	-0.02
	.	[0.009]	[0.003]

Panel B: Time varying RER			
	Sunk cost	Baseline	Low debt
β_1	-0.11	-0.39	-0.20
	[0.009]	[0.013]	[0.009]

Estimation of equation (30) using simulated model. The reported coefficient and p-values, correspond to the average of 200 replication of the model. Each replication simulate 2800 periods, and the regression is done over the last 2500 periods. Row 1 of each panel presents the average coefficient of the regression in level; row 2 presents the average coefficient of the regressions in logs. Column 1 presents the results for an standard sunk cost model while column 2 does it for the extended version. p-values in parenthesis

Table 10: Testing the measures

Measure of RERU	Estimated value
	(1)
Real volatility	-0.42
	(0.021)
Standard deviation (1 year)	-0.18
	(0.020)
Standard deviation (2 years)	-0.21
	(0.027)
Standard deviation (3 years)	-0.22
	(0.033)

Estimation of equation (30) in logs using the data generated by the model. Column (2) adds the following controls: lag of real exchange, change of the real exchange, and the lag of change of the real exchange rate. The reported coefficient and standard errors, correspond to the average among 200 replication of the model, each replication simulate 2800 periods, and regression are done over the last 2500 periods. The first row use as independent variable the actual simulated volatility of real exchange rate, the second one use the propose method using markov swithing estimation, the third, fourth and fifth row use rolling standard deviation over 1, 2 and 3 years respectively. Standard errors in parenthesis.

A Robustness

A.1 Error correction model

To control for the time-varying elasticity with respect to real exchange rate that can generate a bias in the results, I will estimate an error correction model and expend it with the measure of real exchange rate uncertainty. The estimation is the following:

$$\Delta Exports_{i,t} = \boldsymbol{\beta} \Delta X_{i,t} + \alpha \left(Exports_{i,t-1} - \boldsymbol{\beta^l} X_{i,t} \right) \tag{31}$$

Where X_t is a vector with the following variables: real exchange rate, GDP, foreign demand, term of trade, and real exchange rate uncertainty. The results are presented in table A.2. I estimate equation (31) for the period 1996-2015, for all the countries in the list except for Argentina, Belgium, and Chile since the amount of data available did not allow me to test for co-integration[20].The results in table A.2 show that real exchange rate uncertainty has short-run effects, but it does not seem to affect the long-run value of exports.

A.2 Controls to aggregate estimation

In this section of the appendix, I present the estimation with additional controls, controlling for the correlation with countries in the G7, controlling for the volatility of industrial production to capture the volatility of aggregate output, and controls by past trade openness.

The standard deviation of industrial production is an annual measure constructed as the averages of the rolling standard deviation over 1,2 and 3 years. Similarly the correlation of industrial production between domestic and foreign economies is rolling correlation between domestic economy and the industrial production in the G7 over three years.

The estimated equation is the same as equation (9) results are presented in table A.5, columns 1,2-4, and 5-6 for correlation and standard deviation and for trade openness, respectively.

[20] I run the test following Westerlund (2007). The estimation of equation (31), it is done using a dynamic fixed effects estimator.

A.3 SVAR for Colombia

Before presenting the evidence at firm-level data, It is worth trying to see how real exchange uncertainty affects exports in Colombia at the aggregate level. To answer this, I estimate an SVAR equation at monthly frequency. I include the following variables in the following order: United States industrial production, United States prime interest rate, real exchange rate uncertainty, Colombian real exchange rate, Colombian central bank interest rate policy, Colombian exports, and Colombian industrial production[21]. Following Bloom (2009), I estimate the cyclical component of each variable using an H-P filter with the smoothing parameter of $\lambda = 129,600$ over the log variables.[22].

Figure 2 presents the exports and industrial production response to a one standard deviation shock in the real exchange uncertainty; the figure shows that on impact, there is not a big reaction on exports, but after nine months the drop is around 2% and takes more than two years to fully recover (but after one year the impact is not statically different from zero). Also, as expected, the effect of this shock in industrial production seems to be zero, or indistinguishable from zero in statistical terms.

In the figure A.1, of the appendix C, I change the order of the variables and estimate the SVAR in the following order: United States industrial production, United States prime interest rate, Colombian real exchange rate, Colombian central bank interest rate policy, Colombian exports, Colombian industrial production, Colombian real exchange rate uncertainty. I find that the results hold, even after assuming that the real exchange uncertainty is the least exogenous variable, but the estimated impact is around 1.5%.

A.4 Chilean firm-level data

This section presents the results for the Chilean economy using a manufacturing survey. The advantage of this data set, with respect to the Colombian one, is that it allows me to

[21] This particular ordering is assuming that real exchange rate uncertainty shocks are exogenous to all the variables with exception to Industrial production in United States, and the prime interest rate. I include the interest rates since it is possible that the Colombian central bank reacts to exchange rate volatility shocks or that a movement in domestic or foreign interest rates affects exports. **alessandria˙export˙2014.** found that the export response to large devaluations depends on the interest rate of each country, implying that inclusion of the interest rate could be relevant.

[22] The lag structure is the one that it is found optimal according to the Akaike's information criterion (AIC).

observe more details about the firm, like employees, revenues, and total production, among other variables. The disadvantages, is that during the period for which the data is available, there is not a high variation in the measure of real exchange rate uncertainty, also, I cannot distinguish the destination or product of each exports.

The estimation is as follows:

$$y_{i,t} = \beta_0 + \beta_1 \tilde{\sigma}_t^2 + \beta_h^0 \tilde{\sigma}_t^2 \times \frac{Assets}{Liabilities}_{i,t} + \beta_2 \frac{Assets}{Liabilities}_t + \alpha_i + X_t + \hat{X}_{i,t} + \epsilon_{i,t} \quad (32)$$

Where X_t represents aggregate controls, $\hat{X}_{i,t}$ represents firms controls over time, and α_i represents firm effect by firm. I estimate productivity following Petrin et al. (2004).The results are presented in table A.4

B Algorithm

Discretize the space Construct discretize space:

1. Discretize the state space.
   (a) labor and Debt in 135 points linear space greed.
   (b) ξ in a 30-point linear space greed. Using Tauchen method.
   (c) z_i in a 10 points space greed.
   (d) σ_ξ is a two-state Markov chain with transition probabilities given by $\pi_{l,l}$ $\pi_{h,h}$
   (e) $m' = \{1, 0\}$
   (f) Cash on hand x in 140 point greed.

Once the state space was constructed, I solved the problem in two different loops. First I solve the optimally policy function of the lender, I solve $q(.)$ as a function of $m', l^{f\prime}, b',$. Then I use $q(.)$ to solve the exporter problem.

Lender's problem Iteration:

1. Guess q^n if n=0.
2. Compute $M(S_t, s_i)$ as following:

$$M^n(z_i, l', b', m', S_t) = \max_{l', b', m'} q^n(z_i, l', b', m', S_t) b'(z_i, l', m', S_t). \tag{33}$$

   Denote the arg max of above problem as follows: $\hat{l}', \hat{b}', \hat{m}'$.

3. With $M^n(S_t, s_i)$ I obtained the corresponding default threshold of equation (16) and (17), denoted by κ^n
4. Finally compute q(.) as follows:

$$\beta = q^{n+1}(z_i, l', b', m', S_t)(1 - F(\kappa^n | \xi))$$

(34)

5. If $|q^n - q^{n+1}| \leq \epsilon$ finish, otherwise go to step one, using $q^{n+1} = q^n$

Producer's problem Iteration:

1. Guess $V(z_i, x, m, S)^n$ If n=0 . Fix $V_d = 0$.

2. Define:
   - $V^1(z_i, x, m, S)^n \equiv (z_i, x, 1, S)^n$
   - $V^0(z_i, x, 0, S)^n \equiv (z_i, x, 0, S)^n$
   - $q^{n,1} \equiv q^{n,}(z_i, l', b', 1, S_t)b'(z_i, l', 1, S_t)$
   - $q^{n,0} \equiv q^{n,}(z_i, l', b', 0, S_t)b'(z_i, l', 0, S_t)$

3. Compute optimal decision and value functions conditional on choosing $m' = 1$ and $m' = 0$ as follows:

$$V_c^{n+1,m'}(z_i, x, m, S) = \max_{l',b'} x + q^{n,m'}(l', b', m')b' + (1-m)m'f_s + Q\mathbf{E}V^{n,m'}(z_i, x', S') \tag{35}$$

   s.t.

   (12), (16),(17), (18),(19), (28),(27),(29)

4. Update optimal export decision and the value function conditional on not default as follows:

$$V^{cn+1}(z_i, x, m, S) = max\{V^{n+1,m'}(z_i, x, m, S), V^{n+1,m'}(z_i, x, m, S)\} \tag{36}$$

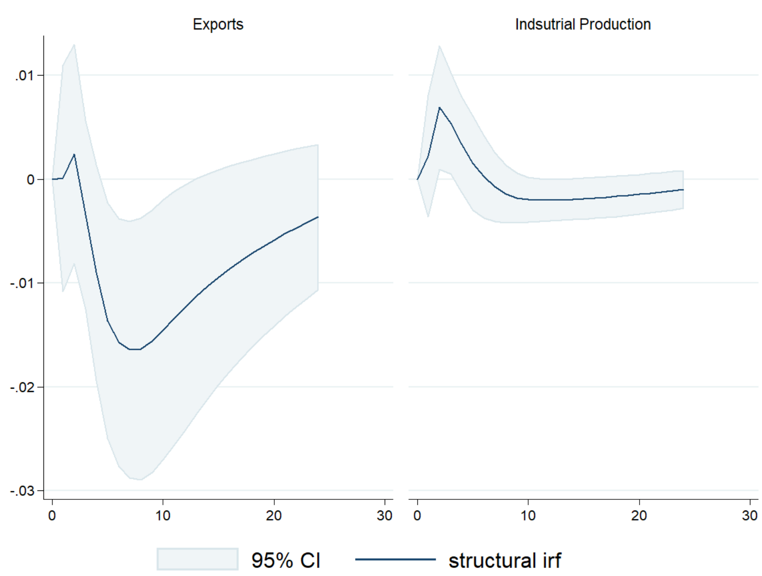
5. Update the value function:

$$V^{n+1} = max\{V_c^{n+1}, 0\} \tag{37}$$

6. Iterate until 1-5 until $|V^{t+1} - V^t| \leq \epsilon$

C Appendix Figures

Figure A.1: SVAR 2. Real exchange uncertainty shocks



The SVAR estimation is the following: Industrial production of United states, the prime interest rate in USA, Colombia real exchange rate, the interest rate policy of the Colombian central bank, exports, industrial production, real exchange rate uncertainty.All variables are monthly and filter using H-P with parameter=129600, as in Bloom (2009)

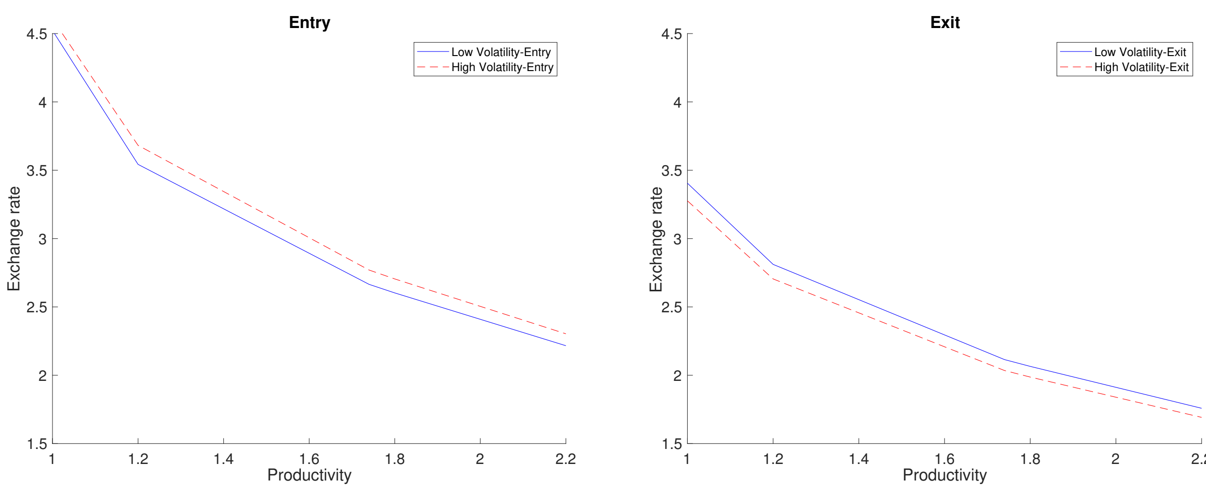


Figure A.2: Extensive Margin in standard model

D Appendix Tables

D.1 Country list

Table A.1: Country list

Algeria	Finland	Latvia	Slovak Republic
Argentina	France	Lithuania	Slovenia
Australia	Germany	Macedonia, FYR	South Africa
Austria	Greece	Malaysia	Spain
Belgium	Hong Kong	Malta	Sweden
Brazil	Hungary	Mexico	Switzerland
Bulgaria	Iceland	Netherlands	Thailand
Canada	India	New Zealand	Turkey
Chile	Indonesia	Norway	United Kingdom
Croatia	Ireland	Peru	United States
Cyprus	Israel	Philippines	
Czech Republic	Italy	Poland	
Denmark	Japan	Portugal	
Estonia	Korea, Rep.	Singapore	

D.2

Table A.2: Error correction estimation

	Equation 1	Equation 2
Long run relationship		
Real exchange rate$_{t-1}$	0.06	0.13
	[0.19]	[0.18]
GDP$_{t-1}$	0.82***	0.81***
	[0.09]	[0.09]
Foreign demand$_{t-1}$	0.58***	0.57***
	[0.19]	[0.19]
Term of trade$_{t-1}$	-0.12	-0.12
	[0.12]	[0.12]
$\tilde{\sigma}_{t-1}$		-0.13**
		[0.06]
Short run relationship		
α	-0.36***	-0.36***
	[0.03]	[0.03]
ΔReal exchange rate$_t$	1.08***	1.06***
	[0.14]	[0.14]
ΔGDP$_t$	1.40***	1.35***
	[0.09]	[0.10]
ΔForeign demand$_t$	-0.30***	-0.30***
	[0.11]	[0.11]
ΔTerm of trade$_t$	0.12	0.12
	[0.09]	[0.10]
$\Delta\ \tilde{\sigma}_t$		-0.05***
		[0.02]
Observations	728	728
R^2	.09	0.09

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.3: Other measures of firm's financial vulnerability

	(1)	(2)
	$\frac{liabilities}{assets}$	$\frac{interest}{profits}$
$\hat{\sigma}_t$	-2.08***	-2.05***
	[0.08]	[0.08]
$I^1_{t-1} \times \hat{\sigma}_t$	-0.01	-0.16*
	[0.08]	[0.09]
$I^2_{t-1} \times \hat{\sigma}_t$	-0.13*	-0.29***
	[0.08]	[0.09]
Observations	111147	111598
R^2	0.79	0.79
Firm, product, destination FE	Yes	Yes
Size FE	Yes	Yes
Gravity$_t$	Yes	Yes
Gravity$_{t-1}$	Yes	Yes
$\Delta\ Gravity_{t-1}$	Yes	Yes
Firm controls	Yes	Yes
Gravity$_t$ $\times$ r^h_{t-1}	Yes	Yes

$\tilde{\sigma}_t$ is the measure of real exchange rate uncertainty.
Controls: 1) Firm, product, destination FE: denotes fix effects for each firm, product, destination.2) size FE: represents two dummy variables, according to the size of the firm with respect to total sales, and another with respect to the amount of assets. Each dummy group firms in three group with respect to the relative size of the firm in each year. 3) Gravity: includes the multilateral real exchange from Colombia, and from Colombia and each destination the bilateral real exchange rate, term of trade, total absorption, aggregate tfp, population, entry .3) $\Delta Gravity$: Represents the log difference of all gravity variable between t and t-1. 4) Firm controls: includes actual and past profits and previous year import share (total imports over operational cost).5) Gravity$_t$ $\times$ r^h_{t-1}: Denotes the interaction between gravity variables and dummy of financial vulnerability
Standard errors in brackets (clustered by exporter). * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table A.4: Chilena firms

	Exports	Exports	Exports	Exports
$\tilde{\sigma}_t^2$	-5.67***	-11.91*	-13.37*	-13.00**
	[1.85]	[6.49]	[7.20]	[6.59]
$\tilde{\sigma}_t^2 \times \frac{Assets}{Liabilities}_t$				0.9164*
				[0.5264]
Observations	6603	5678	5678	5632
R^2	0.02	0.02	0.02	0.02
Firm FE	Yes	Yes	Yes	Yes
Firm controls	Yes	Yes	Yes	Yes
Aggregate Controls	No	Yes	Yes	Yes
RER Change	No	No	Yes	Yes

Standard errors in brackets.* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$
1) Firm controls: includes Share of Imports for raw materials, Employment, firm productivity, share of national ownership.2) Aggregate Controls (in logs): Domestic GDP, Foreign Demand, real exchange rate.3) RER Change: includes first, second difference, and the lad and present cyclical component of the Real exchange rate.

Table A.5: Bilateral relation: additional controls

	1	2	3	4	5	6	7
Panel A: Exports /GDP							
$\tilde{\sigma}_t$	-0.06***	-0.06***	-0.054***	-0.05***	-0.04***	-0.04***	-0.04***
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Observations	25075	25075	25075	25075	25075	25075	25075
R^2	0.60	0.60	0.60	0.60	0.62	0.62	0.62
Panel A: Imports /GDP							
$\tilde{\sigma}_t$	-0.04**	-0.04**	-0.04**	-0.04**	-0.04**	-0.04**	-0.04**
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Observations	25062	25062	25062	25062	25062	25062	25062
R^2	0.47	0.47	0.47	0.47	0.47	0.47	0.47
Panel A: Trade /GDP							
$\tilde{\sigma}_t$	-0.04***	-0.04***	-0.03***	-0.03**	-0.03**	-0.03**	-0.03**
	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]	[0.01]
Observations	25061	25061	25061	25061	25061	25061	25061
R^2	0.66	0.66	0.66	0.66	0.67	0.67	0.67
Bilateral FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Gravity	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Change	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Change_2	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Year FE	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Corr G7	X						
STD_1		X					
STD_2			X				
STD_3				X			
Top_1					X	X	X
Top_2						X	X
Top_3							X

Standard errors in brackets.* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$