Homework 6
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1) $\min \sum_{i=1}^{n} \sum_{j=1}^{k} \Pi_{ij} w_i \| x_i - c_j \|^2$

$\Pi_{ij} = \begin{cases} 1 & \text{if } x_i \in c_j \\ 0 & \text{if } x_i \notin c_j \end{cases}$

w_i is a weight of instance x_i, and k is randomly initialized.

First we fix c_j and optimize Π_{ij}, and step X is not changed

$\forall i: \Pi_{i1} w_1 \| x_i - c_1 \|^2 + \Pi_{i2} w_2 \| x_i - c_2 \|^2 + \ldots + \Pi_{ik} w_k \| x_1 - c_k \|^2$

Second we fix Π_{ij} and optimized c_j.

$$\frac{\partial J(c_t)}{\partial c_t} = \frac{\partial}{c_t} \sum_{i=1}^{n} \Pi_{it} w_t \| x_i - c_t \|^2$$

$$= \sum_{i=1}^{n} \Pi_{it} w_t \cdot (x_i - c_t) \cdot 2(-1) = 0$$

thus,

$$c_t = \frac{1}{\sum_{i=1}^{n} \Pi_{it} \cdot w_t} \cdot \sum_{i=1}^{n} \Pi_{it} \cdot w_t \cdot x_i$$

In this final step we diffenentiated w.r.t. c_t, where c_t is a centroid, and w_t is the weight of instance x_i which remained unchanged during this last optimization.

-> Now we apply the algorithm until there is no change to the centroids i.e assignment of data points to clusters is not changing.
Then assign different weights on X for optimal results.