

Homework 5
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Task 1.

$$\min_{A, B} \sum_{(i,j) \in O} (A_{i:} B_{:j} - M_{ij})^2 + \lambda_1 \|A\|_F^2 + \lambda_2 \|B\|_F^2$$

$$T = \sum_{i,j \neq k} (A_{i:} B_{:j} - M_{ij})^2 + \sum_{i \in O_k, j=k} (A_{i:} B_{:k} - M_{ik})^2 + \lambda_1 \|A\|_F^2 + \lambda_2 \|B\|_F^2$$

$$\frac{\partial T}{\partial B_{:k}} = 0 + \sum_{i \in O_k, j=k} 2(A_{i:} B_{:k} - M_{ik}) \cdot A_{i:}^T + 2\lambda_2 I \cdot B_{:k}$$

$$\Rightarrow \sum_i 2(A_{i:} B_{:k} - M_{ik}) \cdot A_{i:}^T + \lambda_2 I \cdot B_{:k} = 0$$

$$\Rightarrow \sum_{i \in O_k} A_{i:}^T A_{i:} B_{:k} + \lambda_2 I \cdot B_{:k} = \sum_{i \in O_k} M_{ik} \cdot A_{i:}^T$$

$$\Rightarrow B_{:k} \left(\sum_{i \in O_k} A_{i:}^T A_{i:} + \lambda_2 I \right) = \sum_{i \in O_k} M_{ik} \cdot A_{i:}^T$$

$$\Rightarrow B_{:k} = \left(\sum_{i \in O_k} A_{i:}^T A_{i:} + \lambda_2 I \right)^{-1} \cdot \left(\sum_{i \in O_k} M_{ik} A_{i:}^T \right)$$