

Homework 6

Roman Munoz
CS - 4033

$$1) \min \sum_{i=1}^n \sum_{j=1}^K \Pi_{ij} w_i \|x_i - c_j\|^2$$

$$\Pi_{ij} = \begin{cases} 1 & \text{if } x_i \in C_j \\ 0 & \text{if } x_i \notin C_j \end{cases}$$

w_i is a weight of instance x_i , and K is randomly initialized.

First we fix c_j and optimize Π_{ij} , and step X is not changed.

$$\forall i: \Pi_{i1} w_i \|x_i - c_1\|^2 + \Pi_{i2} w_i \|x_i - c_2\|^2 + \dots + \Pi_{iK} w_i \|x_i - c_K\|^2$$

Second we fix Π_{ij} and optimized c_j .

$$\frac{\partial J(c_t)}{\partial c_t} = \frac{\partial}{\partial c_t} \sum_{i=1}^n \Pi_{it} w_i \|x_i - c_t\|^2$$

$$= \sum_{i=1}^n \Pi_{it} w_i \cdot (x_i - c_t) \cdot 2(-1) = 0$$

thus,

$$c_t = \frac{1}{\sum_{i=1}^n \Pi_{it} \cdot w_i} \cdot \sum_{i=1}^n \Pi_{it} \cdot w_i \cdot x_i$$

In this final step we differentiated w.r.t. c_t , where c_t is a centroid, and w_i is the weight of instance x_i which remained unchanged during this last optimization.

→ Now we apply the algorithm until there is no change to the centroids i.e assignment of data points to clusters is not changing.

Then assign different weights on X for optimal results.