Homework 3

CS - 4033

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1)

- $\alpha = [\alpha_1, \dots, \alpha_m]^T$
- $Y = [y_1, \dots, y_n]^T$
- $\tilde{K}$ is an $n \times m$ matrix, with $\tilde{K}_{ij} = K(x_i, x_j)$

From AKRR:

$$\tilde{\beta} = \sum_{i=1}^{m} \alpha_i \phi(x_i)$$

Now to solve:

$$\min_{\beta} \sum_{i=1}^{n} (\phi(x_i)^T \tilde{\beta} - y_i)^2 + \lambda \tilde{\beta}^T \tilde{\beta}$$

$$J(\beta) = \sum_{i=1}^{n} \left[\phi(x_i)^T \sum_{j=1}^{m} \alpha_j \phi(x_j) - y_i \right]^2 + \lambda \sum_{i=1}^{p} \alpha_i \phi(x_i)^T \cdot \sum_{j=1}^{p} \alpha_j \phi(x_j)$$

$$= \sum_{i=1}^{n} \left[\sum_{j=1}^{m} \alpha_j \phi(x_i)^T \cdot \phi(x_j) - y_i \right]^2 + \lambda \sum_{i=1}^{p} \sum_{j=1}^{p} \alpha_i \alpha_j \cdot \phi(x_i)^T \cdot \phi(x_j)$$

From Kernel trick: $\phi(x_i)^T(x_j) = K(x_i, x_j)$, thus,

$$J(\beta) = \sum_{i=1}^{n} \left[\sum_{j=1}^{m} \alpha_j \cdot K(x_i, x_j) \right]^2 + \lambda \sum_{i=1}^{p} \sum_{j=1}^{p} \alpha_i \alpha_j \cdot K(x_i, x_j)$$

And since $\tilde{K}_{ij} = K(x_i, x_j)$:

$$J(\beta) = \sum_{i=1}^{n} (\tilde{K}_{1ij} \alpha - y_i)^2 + \lambda \sum_{ij} \alpha_i \alpha_j \cdot \tilde{K}_{2ij}$$

Matrix form:

$$J(\beta) = (\tilde{K}_1 \cdot \alpha - Y)^T (\tilde{K}_1 \cdot \alpha - Y) + \lambda \alpha^T \tilde{K}_2 \alpha$$

$$= \alpha^T \tilde{K}_1^T \tilde{K}_1 \cdot \alpha - 2\alpha^T \tilde{K}_1^T Y + Y^T Y + \lambda \alpha^T \tilde{K}_2 \alpha$$

Now if we take derivative:

$$\frac{\partial J(\beta)}{\partial \beta} = 2 \tilde{K}_1^T K_1 \cdot \alpha - 2 \tilde{K}_1^T Y + 2\lambda \tilde{K}_2 \alpha$$

$$\alpha = (\tilde{K}_1^T \tilde{K}_1 + \lambda \tilde{K}_2)^{-1} \cdot \tilde{K}_1 Y$$

2. From previous result, we can see that it takes t.c. $O(nmp)$ for $\tilde{K}_1^T$ and it takes $O(m^2 n)$ for $\tilde{K}_1^T \tilde{K}_1$ since $\tilde{K}_1^T$ is an $m \times n$ matrix and $\tilde{K}_1$ is an $n \times m$ matrix. Additionally it takes $O(m^3)$ to take the inverse.

$\therefore$ the total t.c. $= O(nmp) + O(m^2 n) + O(m^3)$, and since $m \le d$ then, t.c. $= O(nmp) + O(m^2 n)$