Exercises for the course
Machine Learning 2
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# Exercise Sheet 6

### Exercise 1: Convolution Kernel (20 P)

Let $x, x'$ be two univariate real-valued discrete time series. We define the convolution kernel

$$k(x, x') = \|x * x'\|^2$$

that measures the similarity between them, where

$$[x * x']_t = \sum_{\tau=-\infty}^{\infty} x(\tau) \cdot x'(t - \tau)$$

is a discrete convolution over infinite-dimensional signals.

(a) *Write* a test in Python that verifies empirically that the kernel is positive semi-definite and *run* it.

(b) *Show* that the convolution kernel is positive semi-definite, that is, show that

$$\sum_{i=1}^{K} \sum_{j=1}^{K} c_i c_j k(x_i, x_j) \geq 0$$

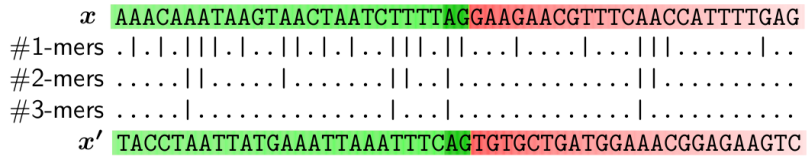
for all inputs $x_1, \ldots, x_K$ and choice of real numbers $c_1, \ldots, c_K$, and *find* an explicit feature map for this kernel.

### Exercise 2: Weighted Degree Kernels (20 P)

We would like to implement a classifier for genes sequences (a sequence of symbols $\{A, C, T, G\}$). The weighted degree kernel is proposed for such task and is defined as:

$$k(x, x') = \sum_{m=1}^{M} \beta_m \sum_{n=1}^{N-m+1} I(u_{m,n}(x) = u_{m,n}(x')).$$

where $u_{m,n}(x)$ is a string of length $m$ which starts at position $n$ in sequence $x$, and $\beta_m \geq 0$. The symbol $I(.)$ denotes the indicator function which returns 1 if the input argument is true and 0 otherwise.




(a) *Show* that $k$ is a positive semi-definite kernel. That is, show that

$$\sum_{i=1}^{K} \sum_{j=1}^{K} c_i c_j k(x_i, x_j) \geq 0$$

for all inputs $x_1, \ldots, x_K$ and choice of real numbers $c_1, \ldots, c_K$.

(b) *Give* a feature map associated to this kernel for the special case $M = 1$.

(c) *Give* a feature map associated to this kernel for the special case $M = 2$ with $\beta_1 = 0$ and $\beta_2 = 1$.

### Exercise 3: Programming (60 P)

Download the programming files on ISIS and follow the instructions.