Subject : Electrical-network analysis

Question : Equation of current for an RC CIRCUIT when it is supplied by a DC source

Answer :

Transient response in RC series circuit having D.C. excitation

Let a d.c. voltage V be applied (at t= 0) by closing a switch‘s’ in a series R-C circuit. The current at t>0 being ‘i', application of KVL leads to

$$Ri + \frac{1}{C}\int i\,dt = V \qquad \ldots(1)$$

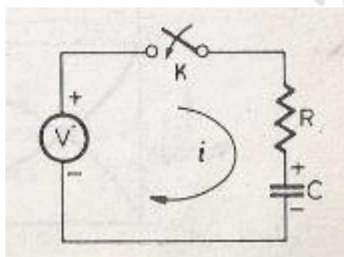

Fig.1 Series RC circuit

Differentiation of equation (1) results,

$$R\frac{di}{dt} + \frac{i}{C} = 0 \; or \left(p + \frac{1}{RC}\right)i = 0 \qquad \ldots(2)$$

Equation (2) is a homogeneous differential equation whose solution will contain only complementary function, the particular function being zero.

$$\therefore \qquad i = i_C = Ke^{-t/RC} \qquad \ldots(3)$$

With application of voltage and assuming no initial charge across the capacitor, the capacitor will not produce any voltage across it but acts as a short circuit causing the circuit current to be (V/R).

i.e., at $$t = 0^+, i\left(0^+\right) = \frac{V}{R}$$

Hence, from equation (3), at $t = 0^+$

$$\frac{V}{R} = K$$

Finally, we obtain, $$i = \frac{V}{R}e^{-t/RC} A \qquad \ldots(4)$$

It may be observed that the charging current is a decaying function. The plot is as shown in Fig. 2. As the capacitor is getting charged, the charging current dies out.

The corresponding voltage drops across the resistor and the capacitor can be obtained as follows:

$$v_R = iR = V_e^{-t/RC} \qquad \ldots(5)$$

and $$v_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R}e^{-t/RC}$$

$$= V\left(1 - e^{-t/RC}\right) \qquad \ldots(6)$$

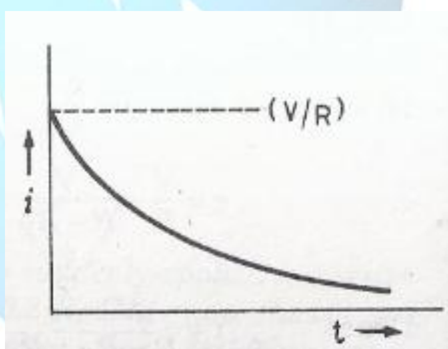

Fig.2, Profile of current in RC charging circuit

Observing equations (5) and (6) reveals that v_R is a decaying function while v_C

is an exponentially rising function (profiles of v_R and v_C are shown in Fig3). The steady state voltage across capacitor is V volts.

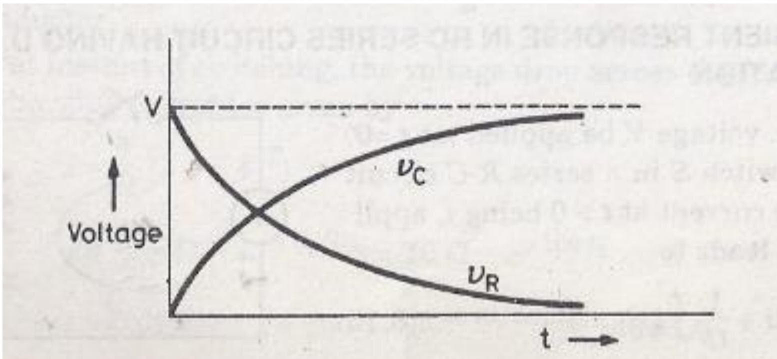

Fig.3. Profile of v_R and v_C in RC charging circuit

