Subject : Calculus

Question : How to do Greens theorem?

Answer :

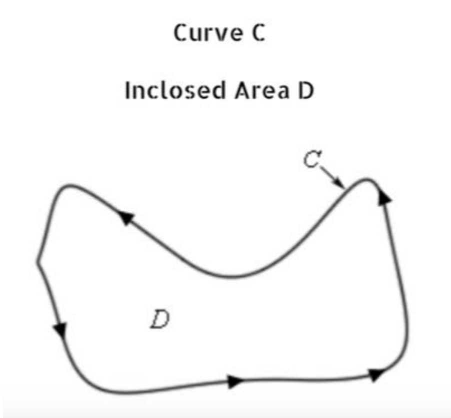
Green's theorem was named after an English mathematician George Green.

Green's theorem gives the relationship between a line integral around a simple closed curve C and a double integral over the plane region D bounded by C.

It is the two-dimensional special case of the Stoke's theorem

Statement of Green's theorem:

Green's theorem states that a closed line integral around the boundary of a plane region D can be computed as a double integral over D. That is if D is a region in the plane and C is the boundary of D, such that D is always on the left-hand side as one goes around C where it is the positive orientation.

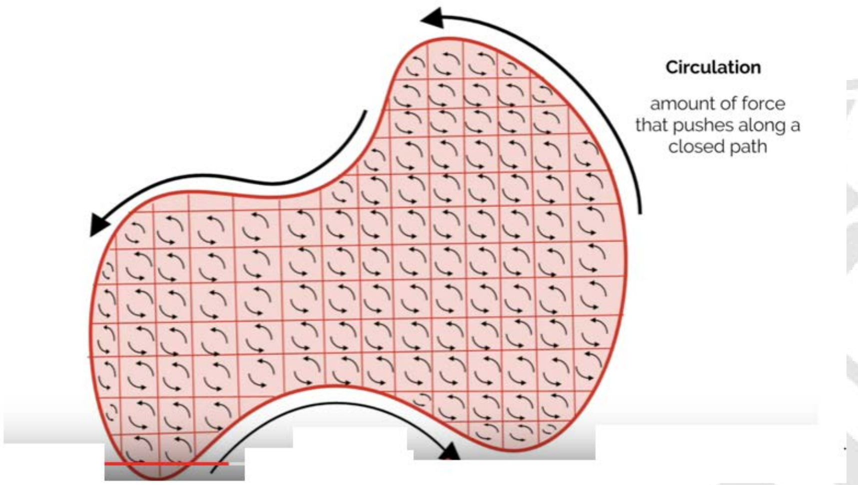


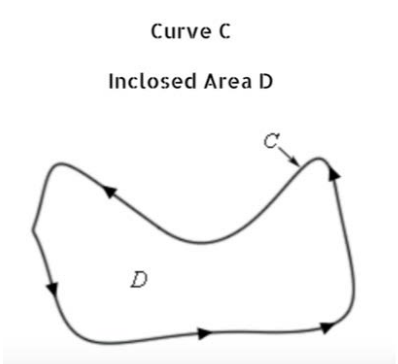
The integral in the below equation represents the circulation around the path in the direction of its orientation.

$$\oint_C F \cdot ds \quad \text{------->1}$$

The circulation can be computed by the line integral directly.

As there are two dimensions that are, F is a two dimensional vector field and C is a closed path we can apply Green's theorem.





Consider the equation one as the macroscopic circulation of vector field F.

$$\oint_C F \cdot ds \text{ ------->1}$$

From the definition of the curl, we can write Green's theorem as, C is a positively oriented boundary of the region D.

$$\oint\limits_{C} F \cdot ds = \iint\limits_{D} \left(\frac{\partial F_2}{\partial x} - \frac{\partial F_1}{\partial y}\right) dA \text{------>2}$$

F2 and F1 are the scalar curl of the two dimensional vector fields.

Where vector, $\vec{F} = F_1 \hat{i} + F_2 \hat{j}$

