Transient response in RC series circuit having D.C. excitation

Equation of current for an RC CIRCUIT when it is supplied by a DC source

Let a d.c. voltage V be applied (at t= 0) by closing a switch‘s’ in a series R-C circuit. The current at t>0 being ‘i', application of KVL leads to

$$Ri + \frac{1}{C}\int i\,dt = V \qquad \ldots(1)$$

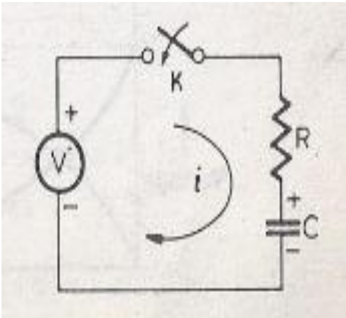

Fig.1 Series RC circuit

Differentiation of equation (1) results,

$$R\frac{di}{dt} + \frac{i}{C} = 0 \; or \left(p + \frac{1}{RC}\right)i = 0 \qquad \ldots(2)$$

Equation (2) is a homogeneous differential equation whose solution will contain only complementary function, the particular function being zero.

$$\therefore \qquad i = i_C = Ke^{-t/RC} \qquad \ldots(3)$$

With application of voltage and assuming no initial charge across the capacitor, the capacitor will not produce any voltage across it but acts as a short circuit causing the circuit current to be (V/R).

i.e., at $$t = 0^+, i\left(0^+\right) = \frac{V}{R}$$

Hence, from equation (3), at $t = 0^+$

$$\frac{V}{R} = K$$

Finally, we obtain, $i = \frac{V}{R}e^{-t/RC}A \qquad \ldots(4)$

It may be observed that the charging current is a decaying function. The plot is as shown in Fig. 2. As the capacitor is getting charged, the charging current dies out.

The corresponding voltage drops across the resistor and the capacitor can be obtained as follows:

$$v_R = iR = V_e^{-t/RC} \qquad \ldots(5)$$

and
$$v_C = \frac{1}{C}\int i\,dt = \frac{1}{C}\int \frac{V}{R} e^{-t/RC}$$

$$= V\left(1 - e^{-t/RC}\right) \qquad \ldots(6)$$

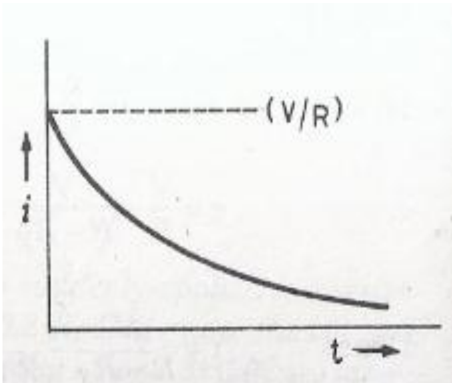

Fig.2, Profile of current in RC charging circuit

Observing equations (5) and (6) reveals that v_R is a decaying function while v_C is an exponentially rising function (profiles of v_R and v_C are shown in Fig3). The steady state voltage across capacitor is V volts.

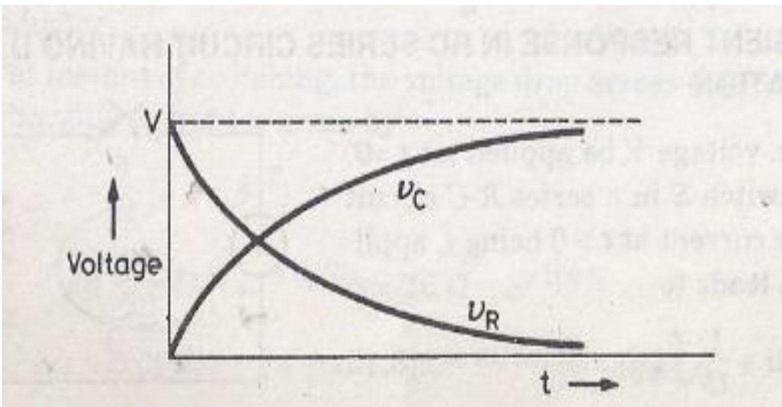

Fig.3. Profile of v_R and v_C in RC charging circuit

Time constant for RC circuit excited by DC source

Time constant is obtained by putting t = RC which gives $v_C = V\left(1-0.368\right) = 0.632V$ i.e., the time by which the capacitor attains 63.2% of steady state voltage.

The instantaneous power are given by

$$PR = i\,v_R = \frac{V^2}{R} e^{-2}\,\frac{t}{RC} W$$

and $pC = i\,v_C = \frac{V^2}{R}\left(e^{-t/RC} - e^{-2t/RC}\right)W$

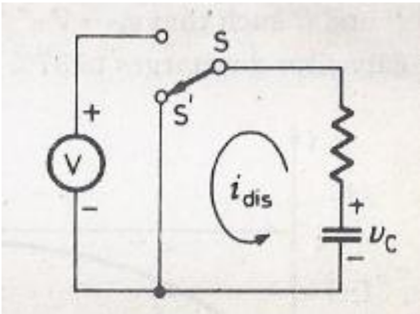

Fig.4: Discharging in RC series circuit

Let us now study the discharging case when the switch S is thrown to a contact S' such that the R-C circuit is shorted and the voltage source is withdrawn (fig4)

Application of KVL yields

$$Ri + \frac{1}{C}\int i\,dt = 0 \qquad \ldots(7)$$

Differentiating (7), we get

$$R\frac{di}{dt} + \frac{i}{C} = 0 \; or \left(p + \frac{1}{RC}\right) i = 0 \qquad \ldots(8)$$

The equation (8), being a homogeneous differential equation, its solution reveals

$i = i_C$ (Complementary function)

$$= K' e^{-t/RC} \qquad \ldots.(9)$$

However, at $t = 0^+$, the voltage across the capacitor will start discharging the current through the resistor which is opposite to the original current direction (shown by i_{dis} in fig.8). Hence, the direction of I during discharge is negative and its magnitude is given by (V/R).

Thus, $i\left(0^+\right) = -\frac{V}{R}$

Hence, from equation (9), we get

$$-\frac{V}{R} = K'\left(at\ t = 0^+\right)$$

The complete solution is then

$$i = -\frac{V}{R} e^{-t/RC} A$$

The deny transient is plotted in the fig. 5.

The corresponding transient voltage is given by

v_R (Voltage drop across R) $= iR = -Ve^{-t/RC}$

and v_C (voltage drop across C) $= \frac{1}{C}\int i\,dt = V\,e^{-t/RC}$

Obviously, $v_R + v_C = 0$

Fig. 6. represents the profiles of v_R and v_C with t.

In the discharging circuit, the time constant is given by the product of R and C such that $v_C = Ve^{-1} = 0.369V = 0.37V$ i.e., the time by which the capacitor discharges to 37% of its initial voltage.

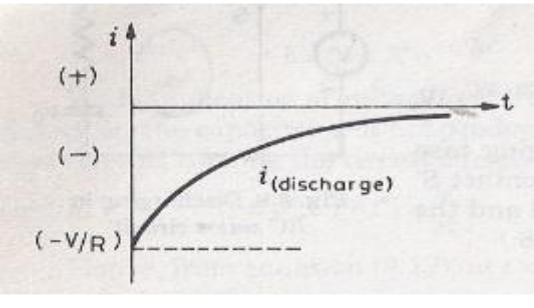

Fig.5: Current decay transient in RC discharging circuit

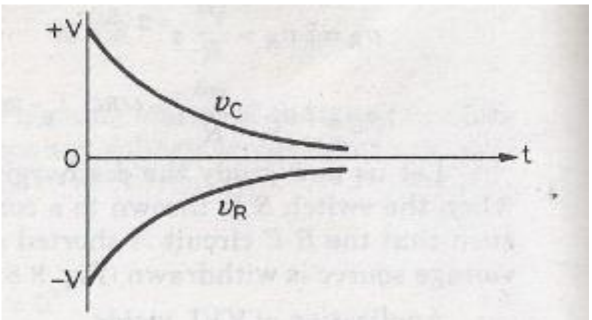

Fig.6. v_R and v_C in RC discharging circuit

The instantaneous power is given by

$$p_R = v_R i = \frac{V^2}{R} e^{-2t/RC} W$$

and $p_C = v_C i = -\frac{V^2}{R} e^{-2t/RC} W$

The charge stored in the capacitor during charging is given by

$$q = Cv_C = CV\left(1 - e^{-t/RC}\right) or\, q = Q\left(1 - e^{-t/\mathrm{RC}}\right)$$

While that during discharging is given by

$q = C\, v_C = CVe^{-t/\mathrm{RC}}$ coulombs.

Or,

$q = Qe^{-t/RC}$coulombs.