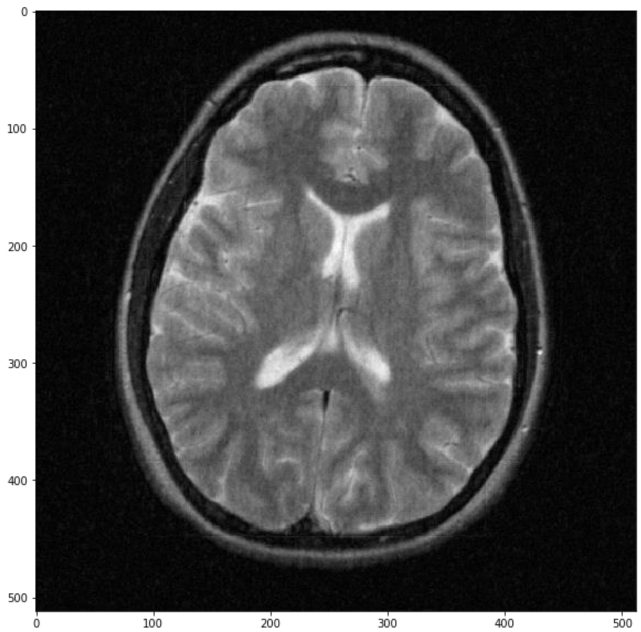

HL2027 PROJECT-1 REPORT

COMPRESSED SENSING

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INTRODUCTION

Magnetic Resonance Imaging (MRI) imaging is a general diagnostic imaging in healthcare. It is used by healthcare industries to assess the condition of patient. MRI use is generally safe, although there are limitation such as unit and maintenance cost, size issue, and acquisition time. MRI requires a relatively long acquisition time for each imaging on a single patient. Reduction of MRI image acquisition time is a technical issue that is actually viable, despite challenges such as patient safety, imaging quality, computation load, etc. By reducing acquisition time, MRI usage can be more efficient and also can decrease the stress on the patient. Compressed sensing (CS) is one of the alternatives to reduce MRI image acquisition duration.

Compressed sensing fundamentally works by undersampling the k-space generated from MRI scanners to reconstruct signals and images which is as similar as possible with signals and images acquired from normal sampling. In other words, measurements are done in significantly fewer repetitions. in This is possible because : a.) medical imagery is naturally compressible by sparse coding. b.) MRI scanners naturally acquire encoded samples, rather than direct pixel samples.

THEORY

This part is a summary from Lustig et al.[1]

The transform sparsity of MRI images can be demonstrated by applying a sparsifying transform to a fully sampled image from a subset of the largest transform coefficients. The sparsity of the image is the percentage of transform coefficients that is sufficient or suitable for diagnostic-grade reconstruction. The challenge is how to reconstruct such image by measuring a limited sample (subset) of the k-space. Fortunately, if an image exhibits transform sparsity and if the k-space undersampling results in incoherent artifacts in that transform domain, the image can be reconstructed from randomly undersampled frequency/ k-space domain.

Incoherent aliasing interference in the sparse transform domain is essential to the CS process. Random point k-space sampling in all directions is deemed impractical, therefore practical incoherent sampling scheme that imitates the interference properties of pure random undersampling as closely as possible but also allows rapid data collection in k-space. In this case, only the sampling will be restricted to only undersampling the phase-encodes and fully sampled the frequency readouts. Non uniform sampling of phase encodes in Cartesian imaging has been proposed in the past as an acceleration method because it produces incoherent artifacts, which is what CS needed. Undersampling phase-encode lines offers pure randomness in the phase-encode dimensions, and scan time which proportional to the undersampling.

To measure the incoherence, the point spread function (PSF) is needed. Let F_u be undersampled Fourier operator,and e_i be the *i*th vector of natural basis, we have :

$$PSF(i;j) = e_j{}^{*}F_u{}^{*}F_u e_i \qquad \text{[Eq.1]}$$

Eq.1 measures the contribution of a unit-intensity pixel at the *i*th position to a pixel at *j*th position.

MR images of interests is often sparse in transform domain rather than usual image domain. In this case incoherence is analyzed by generalizing the notion of PSF to transform point spread function(TPSF) which measures how a single transform coefficient of the underlying object ends up influencing other transform coefficients of the measured undersampled object. Let Ψ be an orthogonal sparsifying transform;

$$TPSF(i;j) = e_j{}^{*}\Psi F_u{}^{*} F_u \Psi^{*} e_i \qquad \text{[Eq.2]}$$

Representation of natural images exhibits a variety of significant non-random structures. First, the energy of most images is mostly located at the center of k-space. Furthermore, using wavelet analysis, one can observe that coarse-scale image components tend to be less sparse than fine-scale components. Thus, undersampling must be done less frequently in near k-space origin. It was discovered that the number of k-space samples should be roughly two to five times the number of sparse coefficients.

PSF AND EXPERIMENTS

The point spread function (PSF) is a natural tool to measure incoherence. PSF(i;j) measures the contribution of a unit-intensity pixel at the i-th position to a pixel at the j-th position.

If Nyquist conditions are met, there is no interference between pixels and for i≠j, PSF(i;j) = 0. However, in the case of undersampling, pixels interfere and it is then possible to measure the incoherence by computing the maximum of the sidelobe-to-peak ratio: max_{i≠j} $\left(\frac{|PSF(i;j)|}{|PSF(i;i)|}\right)$

The MR images of interest are typically sparse in a transform domain rather than the usual image domain. In such a setting, incoherence is analyzed by generalizing the notion of PSF to Transform Point Spread Function (TPSF) which measures how a single transform coefficient of the underlying object ends up influencing other transform coefficients of the measured undersampled object.

In principle, a single point in the transform space at the i-th location is transformed to the image space and then to the Fourier space. The Fourier space is subjected to undersampling, then transformed back to the image space. Finally, a return is made to the transform domain and the j-th location of the result is selected.
The TPSF requires an orthogonal sparsifying transform (Fourier transform above). We want the TPSF of two different pixels to be as small as possible and to have random noise-like statistics.

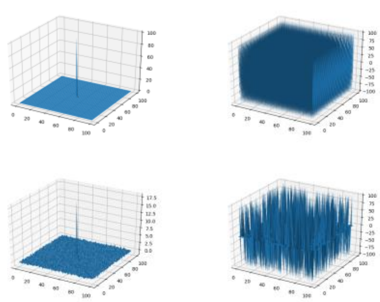
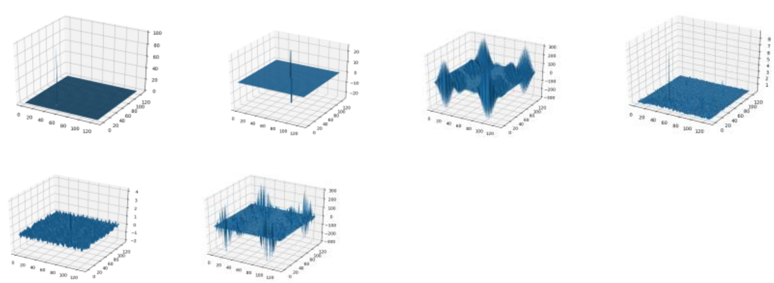
	
FFT -> Random undersampling -> IFFT	IDWT -> FFT -> Random undersampling -> IFFT -> DWT

Figure 1

A few observations in these experiment: the transform seems dampen the values and also add some noise over the 2D Cartesian grid. The PSF of pure 2D random sampling, where samples are chosen at random from a Cartesian grid, offers a standard for comparison. The PSF given i=j indeed looks random as illustrated on the left. And one the right we can observe the TPSF transform using an orthogonal wavelet transform.

IMAGE RECONSTRUCTION AND EXPERIMENTS

Reconstruction algorithm in compressed sensing can be seen in Fig. 2. The reconstruction is divided into two stages : First stage, where the image—which is still noised—is reconstructed from uniformly randomized sampling of k-space. And then, the second stage, acts to denoise the remaining noise from the previous noised image using wavelet denoise operation. The denoising operation is done multiple times based on how many iterations we want. In the experiment we use 15 iterations. After the denoising operation, the image is finally reconstructed.

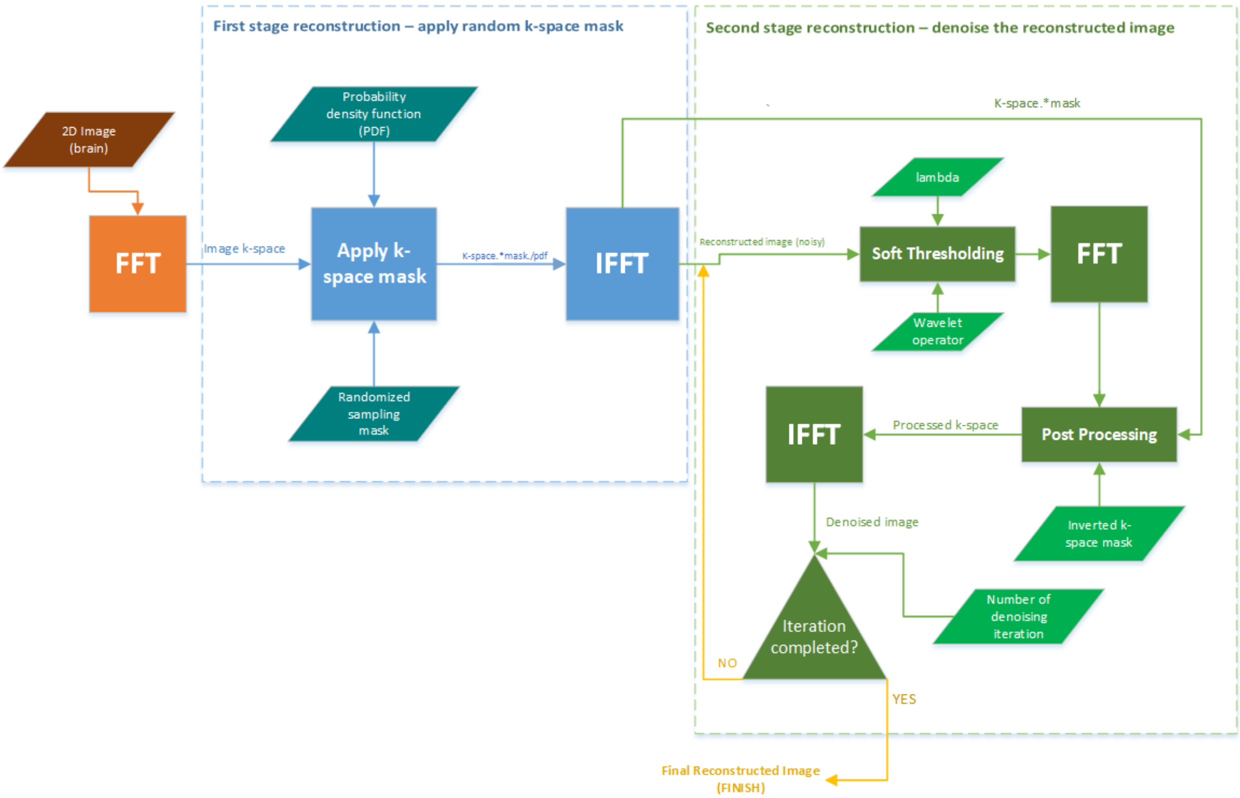
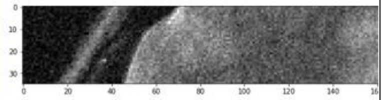
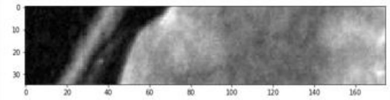
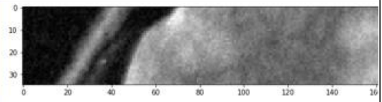
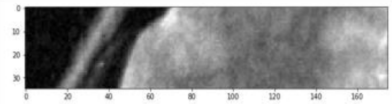
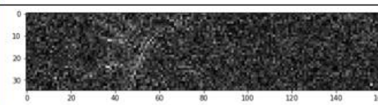
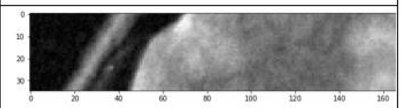
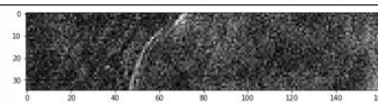
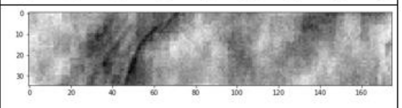
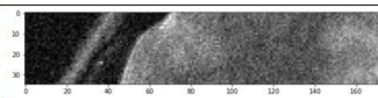
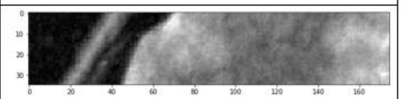


Figure 2. Basic algorithm flowchart of image reconstruction in CS (based on demo4.m file)

To see the how the image reconstruction in CS works, we performed an experiment to reproduce MATLAB demo from demo4.m using Python. In the experiment we varied the PDF shape, sampling pattern, and thresholding mode in wavelet denoising filter. The change or modification on image reconstruction process is performed so that we may see which variable is having the greatest effect on reconstruction process. The result can be seen in Table 1.

Table 1. Result of image reconstruction(modified sampling pattern and PDF are attached)

Condition	PDF	Sampling distribution	Threshold mode	Noised Image (After early reconstruction)	Final Denoised Image (After 15 iterations)
Default	Default	Default	Default (Soft)		
Modified PDF	**Elevated**	Default	Default (Soft)		
	Lowered	Default	Default (Soft)		
Modified Sampling distribution	Default	**Fully Random**	Default (Soft)		
Modified Threshold mode	Default	Default	**Hard**		

By looking at how differing variables could make any impact on final denoised image/final reconstructed image, we quickly judge that the change on sampling distribution made the greatest effect by making the final image difficult to recognize. Moreover, there is an apparent difference in image quality by changing the threshold mode : soft threshold resulted in slightly better final image. Modifying PDF shape gave only very small to non-apparent difference.

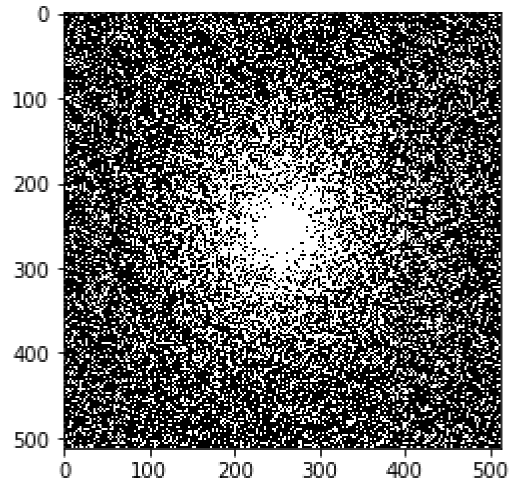
CONCLUSION

a.) PDF does not gives significant change in image reconstruction.
b.) Soft thresholding mode is preferable for better reconstructed image quality.
c.) Sampling pattern is very important in reconstructing the image. The default pattern which is used in the demo gives very good result in reconstructing the image.

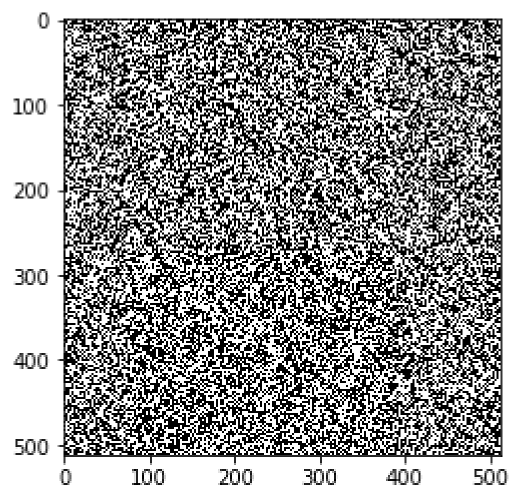
REFERENCES

[1] M. Lustig, D. Donoho and J. M. Pauly, "Sparse MRI: The Application of Compressed Sensing," Magnetic Resonance in Medicine, vol. 58, p. 1182–1195, 2007.
[2] M. Lustig, D. L. Donoho, J. M. Santos and J. M. Pauly, "Compressed Sensing MRI," IEEE Signal Processing Magazine, pp. 72-82, March 2008.

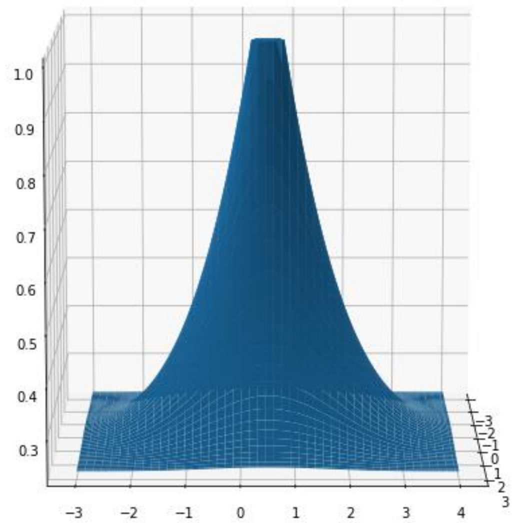
ATTACHMENTS



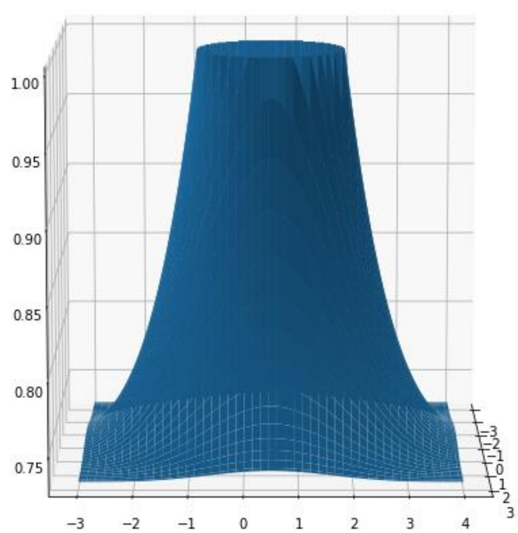
Attachments#1 : Default sampling pattern



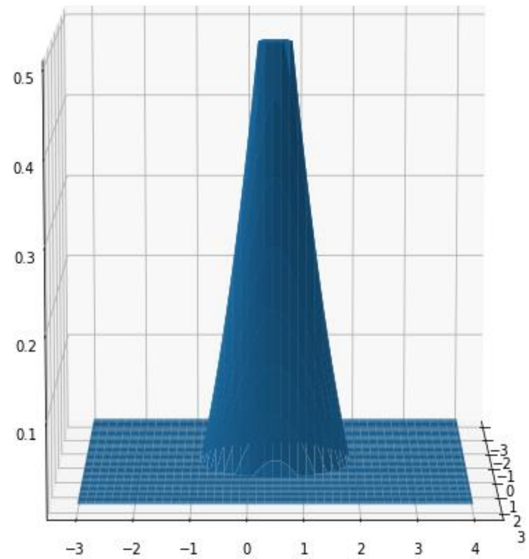
Attachments#2 : Fully random sampling pattern



Attachments#3 : Default PDF



Attachments#4 : Modified PDF (Elevated)



Attachments#5 : Default PDF (Lowered)