# Answers to questions in Lab 1: Filtering operations

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**Instructions**: Complete the lab according to the instructions in the notes and respond to the questions stated below. Keep the answers short and focus on what is essential. Illustrate with figures only when explicitly requested.

Good luck!

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**Question 1**: Repeat this exercise with the coordinates p and q set to (5, 9), (9, 5), (17, 9), (17, 121), (5, 1) and (125, 1) respectively. What do you observe?

Answers:
I observe that different positions of white dots in Fourier domain will directly affect the pattern of sinusoid 'lines' (direction and size) in spatial domain. The angular position of the dot(s) corresponds to the wave direction in spatial domain(perpendicular to the sinusoids 'lines'). The distance between the midpoint and the white dot corresponds to the wavelength of sinusoids in the spatial domain(which represents the signal frequency). Larger distance means higher frequency (and subsequently narrower wavelength), vice versa.

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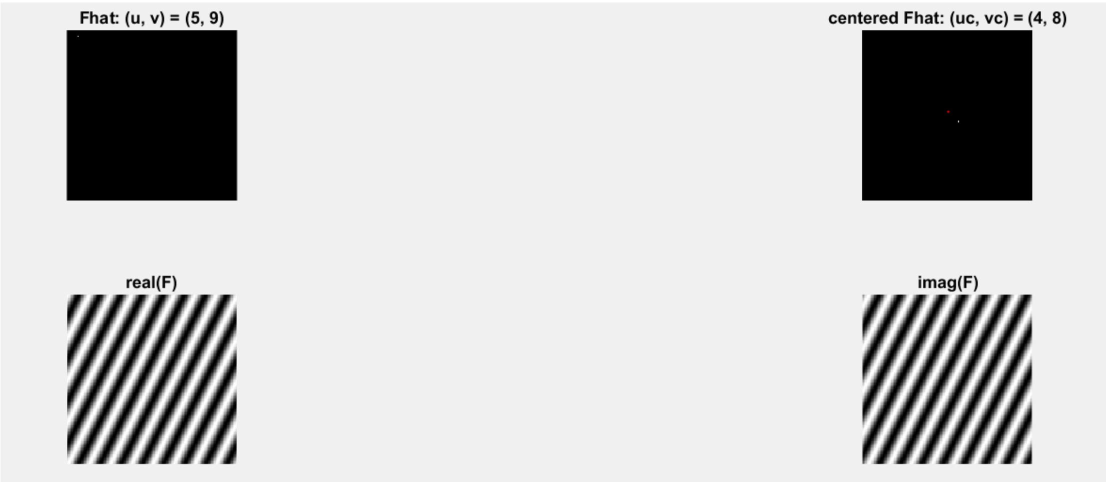
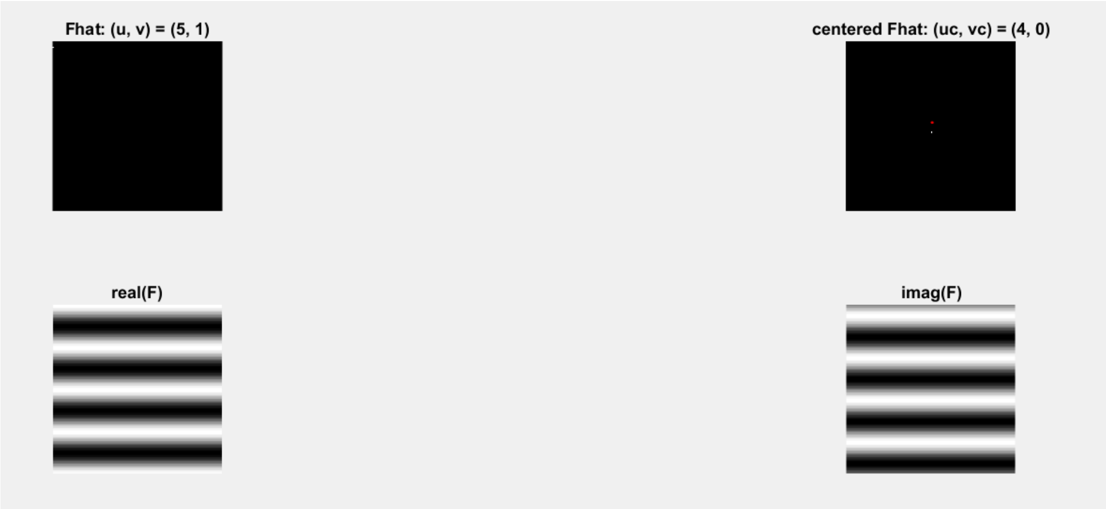
**Question 2**: Explain how a position (p, q) in the Fourier domain will be projected as a sine wave in the spatial domain. Illustrate with a Matlab figure.

Answers:






From the illustrations above, in the 'centered Fhat' figure of Fourier domain we can see that direction and distance from mid point (DC value or image mean) toward white dot(p,q) will influence how the sinusoid wave generated in spatial domain. Shorter distance means lower frequency and higher sine wavelength—represented by longer wavelength. Imaginary line between midpoint and the white dot will have same direction to the sine direction.

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**Question 3**: How large is the amplitude? Write down the expression derived from Equation (4) in the notes. Complement the code (variable amplitude) accordingly.

Answers:

$$A = \frac{1}{N_Y . N_X}$$

Where $N_Y$ is the number of pixels in Y-plane and $N_X$ is the number of pixels in X-plane.

Command to generate amplitude variable is :

```
amplitude = abs(Fhat(u,v));
```

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**Question 4**: How does the direction and length of the sine wave depend on p and q? Write down the explicit expression that can be found in the lecture notes. Complement the code (variable wavelength) accordingly.

Answers:

Direction and distance from mid-point (DC value or image mean) toward white dot(p,q) will influence how the sine wave generated in spatial domain. Shorter distance means lower frequency and higher sine wavelength. The direction of the sine wave will be determined by the white dot position (defined by both u and v) relative to the image mean position.

Wavelength can be expressed by :

$$\lambda = \frac{1}{\sqrt{v^2 + u^2}}$$

Command to generate wavelength variable is :

```
wavelength = abs(1/sqrt((uc)^2 + (vc)^2));
```

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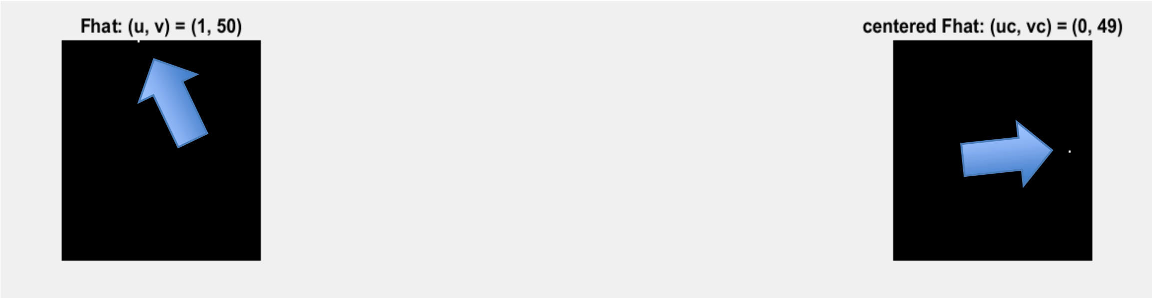
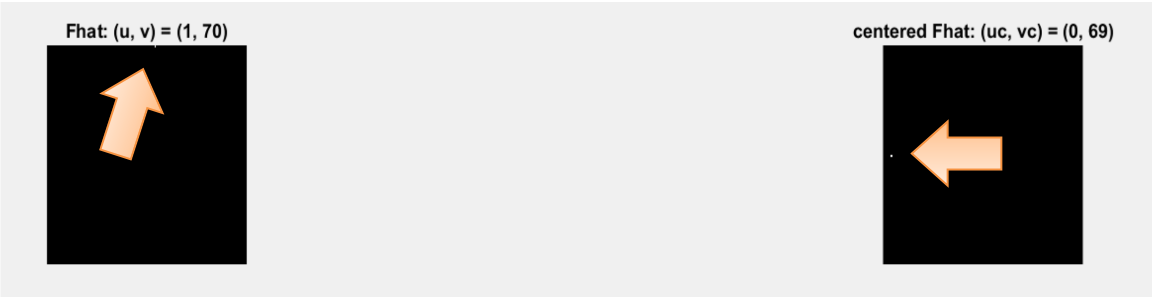
**Question 5**: What happens when we pass the point in the center and either p or q exceeds half the image size? Explain and illustrate graphically with Matlab!

Answers:




From illustrations above, we can see that after ‘q’ or ‘p’ exceeds half of image size (image size is 128x128) in ‘*Fhat*’ figure, the dot is automatically relocated into the left plane of the figure. This will also happen when ‘p’ exceeds the half size of the image—although the relocation of white dot will be in the upper plane.

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**Question 6**: What is the purpose of the instructions following the question *What is done by these instructions?* in the code?

Answers:
The purpose is to prevent the dot from leaving completely from the *centered Fhat* figure and automatically relocated it to the upper plane and/or left plane when p and/or q exceeds half of image size, respectively.

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**Question 7**: Why are these Fourier spectra concentrated to the borders of the images? Can you give a mathematical interpretation? Hint: think of the frequencies in the source image and consider the resulting image as a Fourier transform applied to a 2D function. It might be easier to analyze each dimension separately!

Answers:
In case of F image, the white ‘band’ in spatial domain has the same width all along, and also completely straight along x-direction. The image can be defined as combinations of sine waves along y-direction (with the amplitude exists on the center of the image). The exact

**same combinations** of **straight** sine waves in each y-direction creates very narrow k-space band along y-direction (in frequency domain). Due to relative similarity between image mean and existing sine waves, the Fourier spectra is located at the border, connected to the image mean.

Mathematically,

$$\hat{F}(\mathrm{m,n}) = \sum_{u=56}^{72} e^{-2*pi*i\frac{mu}{N}} * \delta(n)$$

Where non-zero $\hat{F}$ only exists when n = 0. This means the fourier spectra is active at the left border part only.

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**Question 8**: Why is the logarithm function applied?

Answers:
Since most of Fourier spectrum value distribution is on lower range, logarithm function applied to compress huge dynamic range and make fourier spectra details more visible in the frequency domain.

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**Question 9**: What conclusions can be drawn regarding linearity? From your observations can you derive a mathematical expression in the general case?

Answers:
From this exercise we can see that the calculation below:

```
H = F + 2 * G;
Hhat_1 = fft2(H);
```

Can be performed alternatively by:

```
Hhat_2 = fft2(F) + 2*fft2(G);
```

Where finally:
Hhat_1 *equals to* Hhat_2

Linearity of Fourier transform can be expressed as :

$$F\ \{a.r(x) + b.s(y)\} = a.\check{R}(f) + b.\check{S}(f)$$

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**Question 10**: Are there any other ways to compute the last image? Remember what multiplication in Fourier domain equals to in the spatial domain! Perform these alternative computations in practice.

Answers:
Yes, as multiplication in Fourier/frequency domain equals to convolution in the spatial domain and vice versa , we can compute last image alternatively by replacing these commands:

```
showfs(fft2(F .* G));
```

With these commands:

```
showgrey(fftshift(log(20000 + abs(fft2(F)*fft2(G)))));%with
logarithmic adjustment
```

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**Question 11**: What conclusions can be drawn from comparing the results with those in the previous exercise? See how the source images have changed and analyze the effects of scaling.

Answers:
In this Q11 exercise, the source image is a white rectangle with longer side in x-direction, this is different with the previous exercise in which the source image is a white square. The Fourier phase of the spatial domain shows a spectrum of frequency that somehow shows a pattern of rectangles. Expansion of object in x-plane(from square to rectangle) will result in expansion of the fourier spectrum in y-plane. This means compression in certain direction in spatial domain equals compression in its perpendicular direction in Fourier domain, and vice versa.

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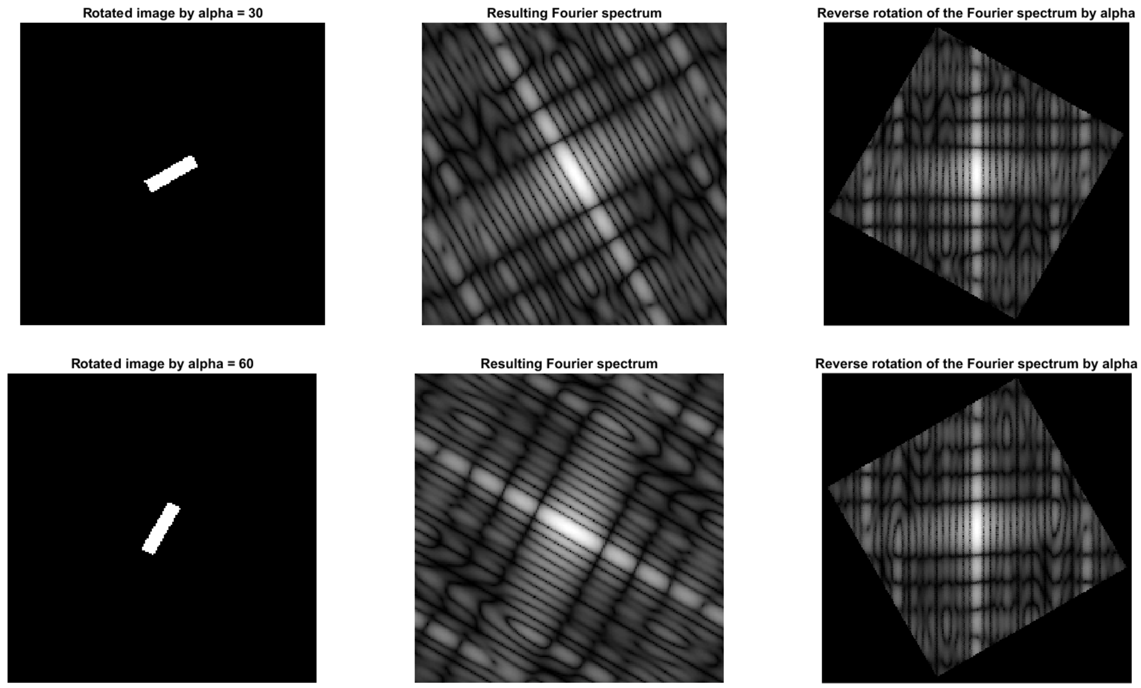
**Question 12**: What can be said about possible similarities and differences? Hint: think of the frequencies and how they are affected by the rotation.

Answers:




Based on the illustration above, we can see that the rotation on the spatial phase (original images) will result in rotation of the spectrum in the frequency domain with the same angle of rotation (i.e. rotating original image by 30 degrees will make the corresponding spectrum in frequency domain to be rotated 30 degrees compared with the spectrum from unrotated original image). The frequency pattern of the Fourier spectrum is not changed by the rotation.

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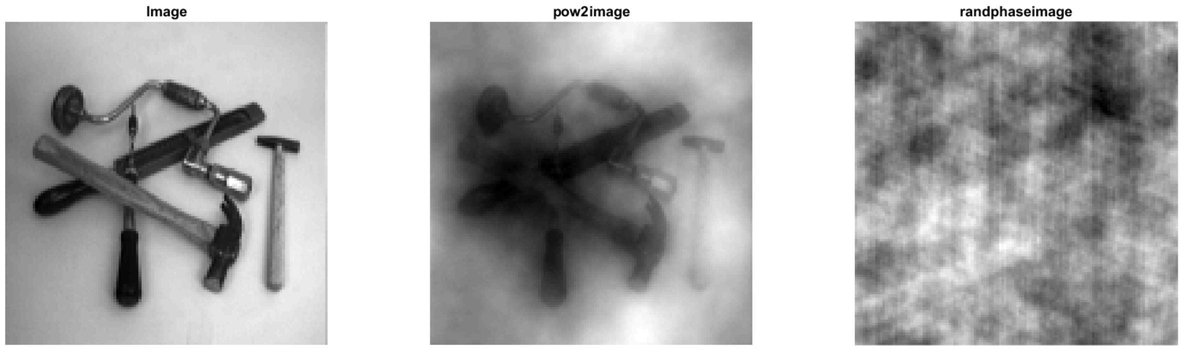
**Question 13**: What information is contained in the phase and in the magnitude of the Fourier transform?

Answers:

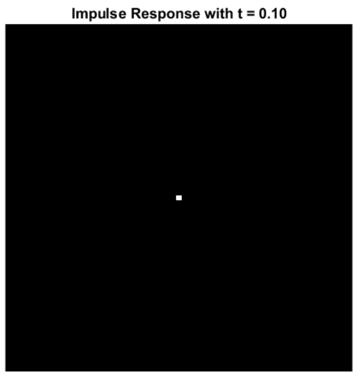
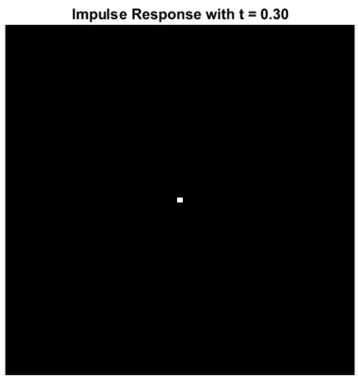
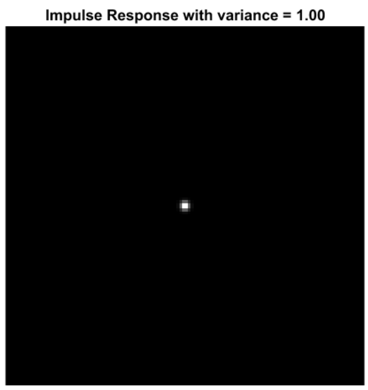



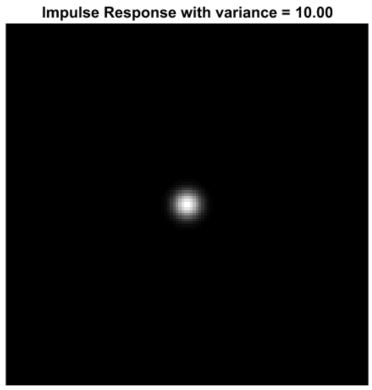
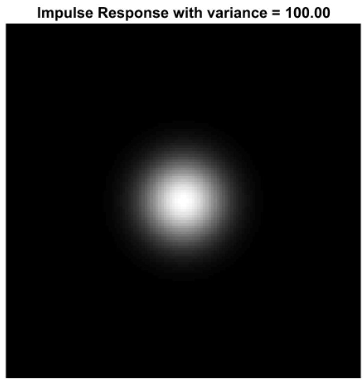

Information contained in the phase of the Fourier transform shows rough distribution of contour from the original image. This is due to the fact that phase determine the frequency distribution of signals in the (spatial) image.

Information contained in the magnitude of the Fourier transform reveals pixel intensity composition from the original image.

**Question 14**: Show the impulse response and variance for the above-mentioned t-values. What are the variances of your discretized Gaussian kernel for t = 0.1, 0.3, 1.0, 10.0 and 100.0?

Answers:

| ***t* value** | **Impulse response** | **variance** |
|---|---|---|
| 0.1 |  | 0.2501 -0.0000<br>-0.0000 0.2501 |
| 0.3 |  | 0.3192 0.0000<br>0.0000 0.3192 |
| 1.0 |  | 1.0000 -0.0000<br>-0.0000 1.0000 |

| | | |
|---|---|---|
| 10.0 |  | 10.0000 -0.0000<br>-0.0000 10.0000 |
| 100.0 |  | 100.0000 -0.0000<br>-0.0000 100.0000 |

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**Question 15**: Are the results different from or similar to the estimated variance? How does the result correspond to the ideal continuous case? Lead: think of the relation between spatial and Fourier domains for different values of t.

Answers:

Results are similar to the estimated variance; however this only holds true when the t value is equal or higher than 1. We found that with t value below 1, the variance matrix will be different compared to the estimated ideal continuous variance. This difference happens due to the discrete nature of the gaussian kernel. When we discretize the gaussian kernel with lower variance, the gaussian kernel will less likely to have gaussian feature distribution.

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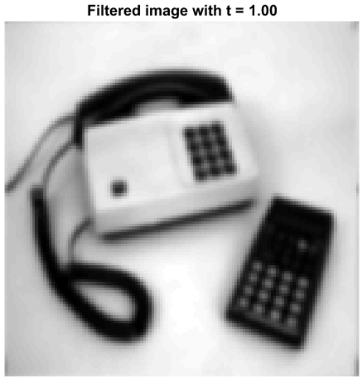
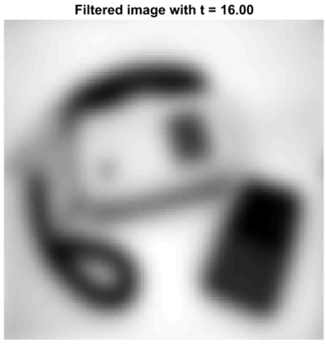
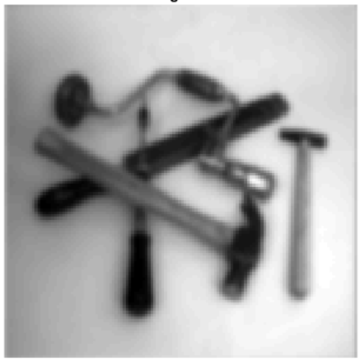
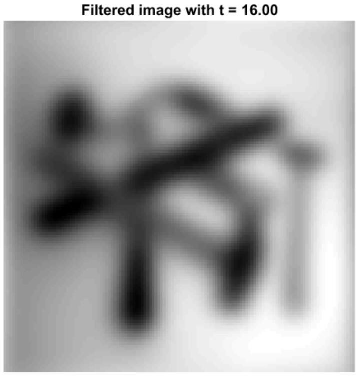
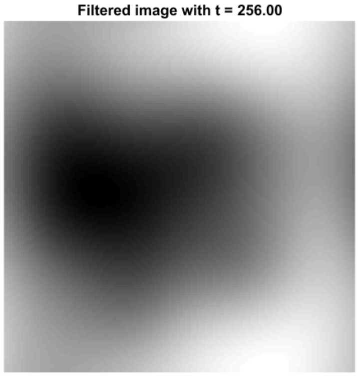
**Question 16**: Convolve a couple of images with Gaussian functions of different variances (like t = 1.0, 4.0, 16.0, 64.0 and 256.0) and present your results. What effects can you observe?

Answers:

| Source Image | t | Results |
|---|---|---|
|  | 1.0 | Filtered image with t = 1.00  |
| | 16.0 | Filtered image with t = 16.00  |
| | 256.0 | Filtered image with t = 256.00  |

| Source Image | t | Results |
|---|---|---|
|  | 1.0 | Filtered image with t = 1.00  |
| | 16.0 | Filtered image with t = 16.00  |
| | 256.0 | Filtered image with t = 256.00  |

From the observation of the table above we can see that lower filtering parameter value (**variance** in this case) can make the original image smoother (lesser high frequency). However, applying variance value for too high can lead to major loss in image details. The higher ranger of frequency in the image can be lowered by using this way.

**Question 17**: What are the positive and negative effects for each type of filter? Describe what you observe and name the effects that you recognize. How do the results depend on the filter parameters? Illustrate with Matlab figure(s).

Answers:

**ADD NOISE**

**Original images with gaussian noise :**

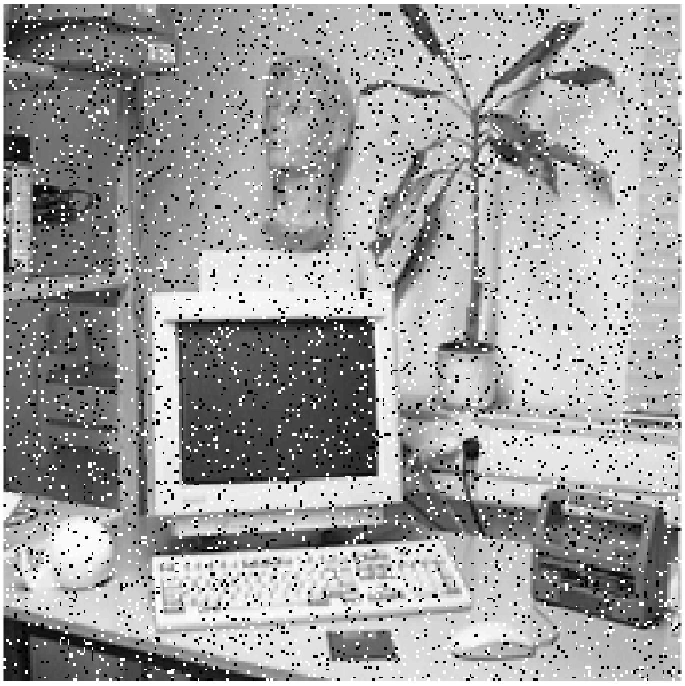
**Gaussian Smoothening Filtering :**

**Median Filtering :**

**Ideal Low-pass Filtering :**

## ADD NOISE

**Original images with salt-and-pepper noise :**



**Gaussian Filtering :**

**Median Filtering :**

**Ideal Low-pass Filtering :**

**Gaussian Smoothening Filter**
(+) Can reduce the gaussian noise well
(-) Cannot reduce the salt-and-pepper noise well

**Median Filter**
(+) Can reduce the gaussian noise well
(+) Can reduce the salt-and-pepper noise very well
(-) May unintendedly reduce some minor detail on the image (painting-like)

**Ideal low-pass filter**
(+) Can reduce the gaussian noise, but there are boxy textures as side-effect
(-) Cannot reduce the salt-and-pepper noise well, the image may even be getting worse after the filtering is applied.

With increasing filter parameter, the noise is more likely to disappear. However, the original details and texture can be sacrificed for this effort. This phenomenon is demonstrated in the figure below:

Low-pass filter, parameter value: 0.40

Low-pass filter, parameter value: **0.25**

Low-pass filter, parameter value: 0.10

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**Question 18**: What conclusions can you draw from comparing the results of the respective methods? Can you give a mathematical interpretation to explain the effects of each filter?

Answers:

Conclusions:

- Median filtering is the most suitable for handling salt-and-pepper noise but risking minor loss in image detail.
- Median filtering and gaussian smoothing filtering are suitable for handling gaussian noise on an image.

- Effort to reduce noise should be performed carefully to prevent overfiltering which may result in loss of image details.
- Manipulation of signal frequency using Fourier transform can be done to reduce the effect of noise.

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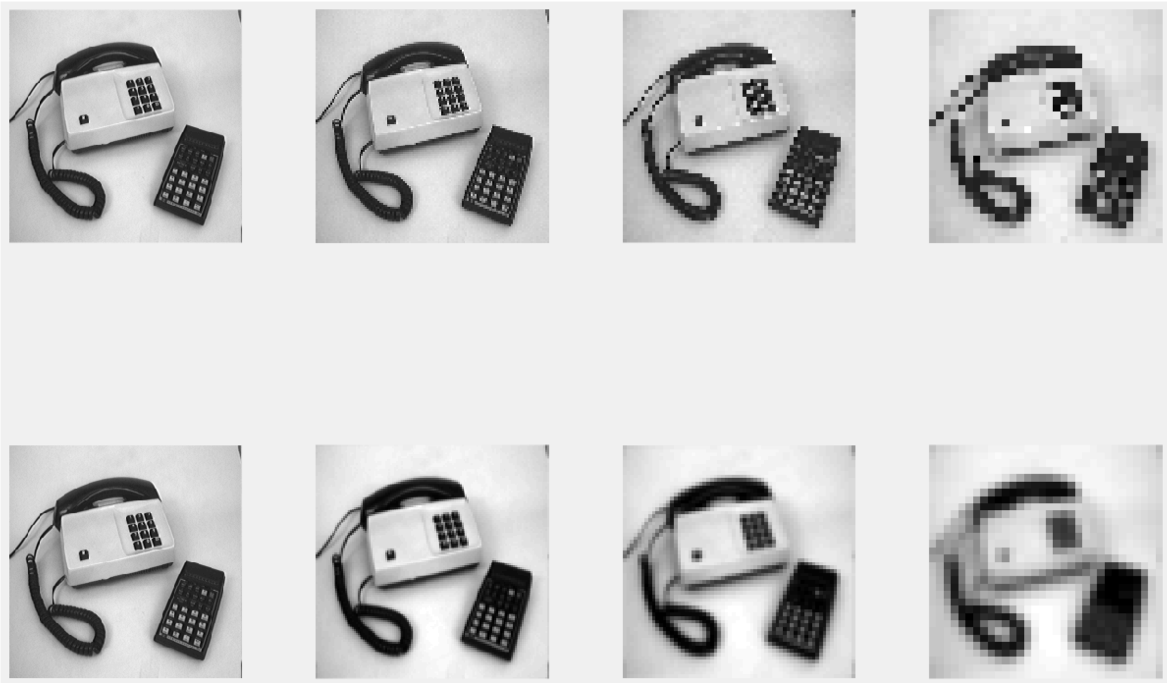
**Question 19**: What effects do you observe when subsampling the original image and the smoothed variants? Illustrate both filters with the best results found for iteration i = 4.

Answers:



Subsampling the original image will decrease the quality of the picture(due to lower resolution). Subsampling the smoothed variants will also decrease the quality. However, the smoothed variants are having less image artifacts. From the 4th iteration of normal subsampling of phonecalc256 image, we can see that there are white dots on the phone and calculator. Those dots artifacts do not exist in the smoothed subsampling.

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**Question 20**: What conclusions can you draw regarding the effects of smoothing when combined with subsampling? Hint: think in terms of frequencies and side effects.

Answers:
In terms of frequency, higher signal frequency in an image makes image artifacts more likely to happen in aliasing case. The smoothing before subsampling image will suppress higher image frequency so that the artifacts will also be reduced.

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