Causal order of quantum correlations

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July 22, 2019, Pengcheng Lab

Main References:

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- Brunner, N., Cavalcanti, D., Pironio, S., Scarani, V., Wehner, S. (2014). Bell nonlocality. Reviews of Modern Physics, 86(2), 419.
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Classification of quantum correlations

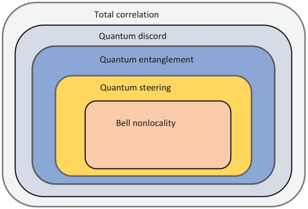
For a given d-dimensional bipartite quantum state ρ, if it exhibits C (Bell nonlocality, entanglement and so on) correlations, it is called a C-correlated state. The set of all C-correlated states can be denoted as $\mathcal{S}_C$ (for example we have $\mathcal{S}_{ent}$ denoting the set of all entangled states).

There are several crucial classes of quantum correlations:

- Total quantum correlation
- Quantum discord
- Quantum entanglement
- Quantum steering
- Bell nonlocality

Hierarchy of quantum correlations

Hierarchy of the state sets which exhibit different correlations.



The most studied form of correlations: quantum entanglement and Bell nonlocality. CHSH inequality $|\langle \mathcal{I}_{AB}^{CHSH} \rangle| = |\langle A_1 B_1 + B_1 A_2 - A_2 B_2 + B_2 A_1 \rangle| \leq 2$, local realistic theory must not violate the inequality. There exist quantum correlations which violate the inequality.

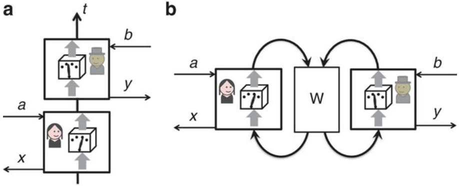
Understanding quantum mechanics from basic assumptions

We still do not know what is the precise basic principles of quantum mechanics.

- Local realistic theory or local hidden variable theory, Bell inequality
- Non-contextual hidden variable, KCBS inequality, etc.
- Macroscopic realistic theory, Leggett-Garg inequality.
- Nontrivial communication complexity, communication games.
- Local orthogonal principle, exclusivity principle, graph theoretical approach.
- Information causality, entropic inequality.

Now a new one, called causal order assumption. Unfortunately, it turns out that quantum mechanics doest not obey the causal order assumption.

Causal order



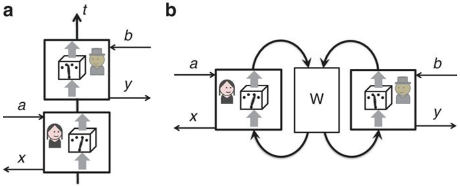
(a) There exists a global background time according to which Alice's actions are strictly before Bob's. she sends her input a to Bob, who can read it out at some later time and give his estimate $y = a$. However, Bob cannot send his bit b to Alice as the system passes through her laboratory at some earlier time. Consequently, she can only make a random guess of Bob's bit.
(b) If the assumption of a definite order is dropped, it is possible to enables a probability of success violate causal inequality.

Causal order

Basic assumption of causal order model (a system entering Alice's/Bob's laboratory, the parties obtaining the bits a, b and b, and producing the guesses x and y):

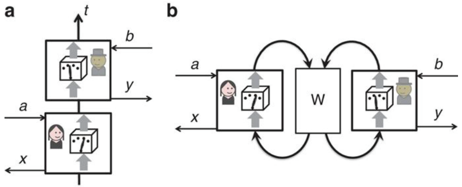
- Causal structure. The main events are localized in a causal structure. A causal structure (such as space-time) is a set of event locations equipped with a partial order $\preceq$ that defines the possible directions of signalling. If $A \preceq B$, we say that A is in the causal past of B. In this case, signalling from A to B is possible, but not from B to A.
- Free choice. Each of the bits can only be correlated with events in its causal future.
- Closed laboratories. Alice's guess x can be correlated with Bob's bit b only if the latter is generated in the causal past of the system entering Alice's laboratory. Analogously for y,a.

Communication game



A communication game by Alice and Bob: After a given party receives the system in her/his lab, she/he will have to toss a coin to obtain a random bit. Denote the bits generated by Alice and Bob in this way by a and b, respectively. In addition, Bob will have to generate another random bit b', whose value, 0 or 1, will specify their goal: if $b' = 0$, Bob will have to communicate the bit b to Alice, whereas if $b' = 1$, he will have to guess the bit a.

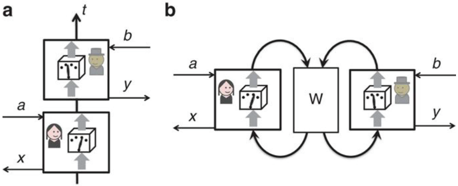
Communication game



A communication game by Alice and Bob: Without loss of generality, we will assume that the parties always produce a guess, denoted by x and y for Alice and Bob, respectively, for the bit of the other (although the guess may not count depending on the value of b). Their goal is to maximize the probability of success:

$$p_s = \frac{1}{2}[p(x = b|b' = 0) + p(y = a|b' = 1)]$$

Causal inequality

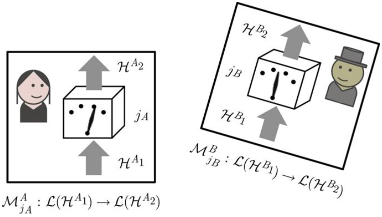


If all events obey causal order, no strategy can allow Alice and Bob to exceed the bound

$$p_s = \frac{1}{2}[p(x = b|b' = 0) + p(y = a|b' = 1)] \leq \frac{3}{4}$$

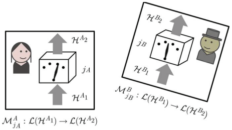
This is called **causal inequality**.

Local quantum experiments with no global causal structure



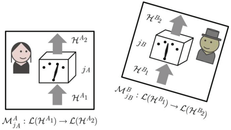
Although the global causal order of events in the two laboratories is not fixed in advance and in general not even definite, the two agents, Alice and Bob, are each certain about the causal order of events in their respective laboratories.

Local operations



- Alice (or Bob) uses a given instrument, she registers one out of a set of possible outcomes, labelled by $j = 1, \cdots, n$. Then for each outcome, there is a corresponding complete positive map $\mathcal{M}_j^A$ for Alice (or $\mathcal{M}_j^B$ for Bob). The probability of output j over state ρ is $p(j|A) = \mathrm{Tr}\mathcal{M}_j^A(\rho)$. And for two parties, we also have the joint probability distribution $p(\mathcal{M}_j^A, \mathcal{M}_i^B)$.
- Each quantum operations $\mathcal{M}_j^A$ (or $\mathcal{M}_j^B$) has it Kraus decomposition $\mathcal{M}_j^A(\rho) = \sum_k E_k^{(j,A)} \rho E_k^{(j,A)\dagger}$

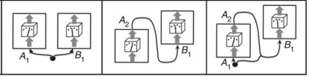
Process matrix



- Consider the Choi-Jamiolkowsky (CJ) isomorphism: A map $\mathcal{M}_j : \mathcal{L}(\mathcal{H}^{A_1}) \to \mathcal{L}(\mathcal{H}^{A_2})$ can be represented by a matrix $M_j = (\mathcal{I} \otimes \mathcal{M}_j(|\phi^+\rangle\langle\phi^+|))^T$ where $|\phi^+\rangle$ is GHZ state.
- For joint probability distribution $p(\mathcal{M}^A_j, \mathcal{M}^B_i)$ we can introduce a process matrix W, such that $p(\mathcal{M}^A_j, \mathcal{M}^B_i) = \mathrm{Tr}(W M^A_j \otimes M^B_i)$. Since $p(\mathcal{M}^A_j, \mathcal{M}^B_i)$ is probability distribution, W should satisfy some conditions: (i) W is positive semidefinite to ensure probability is nonnegative; (ii) probabilities sum up to 1.

Process matrix

$B \npreceq A$	A_1, B_1, A_1B_1	A_2B_1	$A_1A_2B_1$
$A \npreceq B$		A_1B_2	$A_1B_1B_2$
Causal order	States	Channels	Channels with memory



- Under the positivity conditions of process matrix, it can be expanded as $W = \sum_{\mu\nu\lambda\gamma} w_{\mu\nu\lambda\gamma}\sigma_\mu^{A_1} \otimes \sigma_\mu^{A_2} \otimes \sigma_\lambda^{B_1} \otimes \sigma_\gamma^{B_2}$, then by the condition that the probability must sum to 1, we know that the expansion of the process matrix can only contain the term listed in this table. By A_1 type we mean that the term is of the form $\sigma_\mu^{A_1} \otimes I^{rest}$, by A_1B_2 type term we mean $\sigma_\mu^{A_1} \otimes \sigma_\nu^{B_2} \otimes I^{rest}$, etc.

Quantum violation of causal inequality

Consider a special process matrix

$$W = \frac{1}{4}[I + \frac{1}{\sqrt{2}}(\sigma_z^{A_2}\sigma_z^{B_1} + \sigma_z^{A_1}\sigma_x^{B_1}\sigma_z^{B_2})]$$

Alice and Bob both choose to measure the spin z direction $z_{\pm}$. Then we can find that the corresponding success probability is $p_s^{QM} = \frac{2+\sqrt{2}}{4} > \frac{3}{4}$, thus quantum mechanics can present correlations which violated the causal inequality, the causal model description can not applied to quantum mechanics.

- All classical theory have a global causal structure.
- By quantum-to-classical transition, global causal structure can emerge.


thank you
danke
謝謝
gracias
merci
obrigado
dziękuję
bedankt
спасибо
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