Matrix product states, DMRG, and tensor networks

$$\psi^{n_1 n_2} = A^{n_1} A^{n_2}$$

no entanglement local realism independent

$$\psi^{n_1 n_2} = \sum_i A_i^{n_1} A_i^{n_2}$$

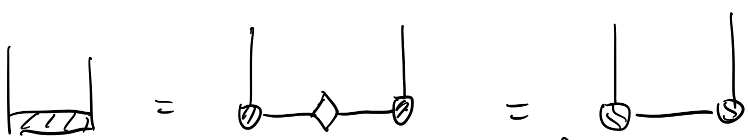
low entanglement in this form means that small number of terms in the sum.

for matrix we do them as

$$\sum_n L_i^{n_1} L_j^{n_2} = \delta_{ij}$$

$$\psi^{n_1 n_2} = \sum_i L_i^{n_1} \sigma_i R_i^{n_2}$$

$$\sum_n R_i^{n_1} R_j^{n_2} = \delta_{ij}$$



gauge free

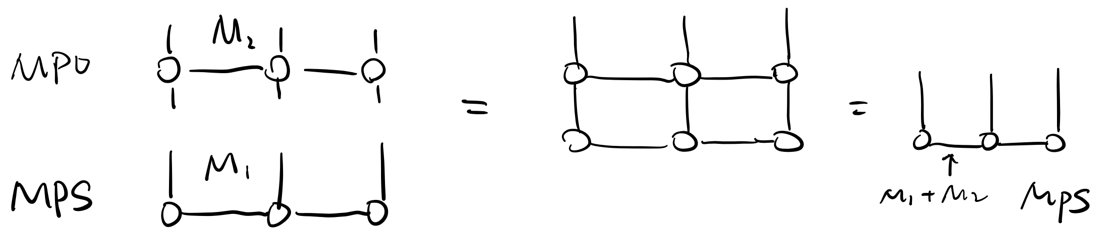
MPO matrix product operator

operator entanglement

1) Four sites system

$$\rightarrow H = \sum_{\{ij\}} S_i \cdot S_j$$

② $$H_L \otimes I_R + I_L \otimes H_R + \sum_{\alpha = x,y,z} S_2^{\alpha} \cdot S_3^{\alpha}$$



M_1 M_2 : bond dimension

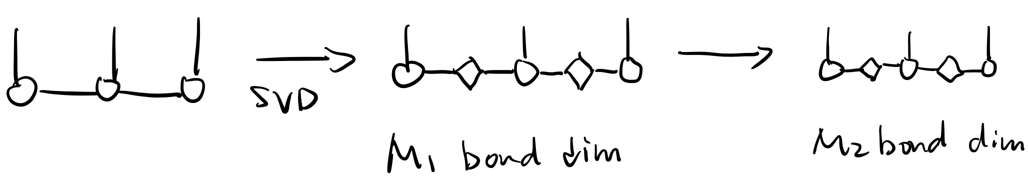
↓ so MPO × MPS increase bond dim

approximate MPS with smaller bond dim :
compression

Compression :

Before :

Arbitrary state ;

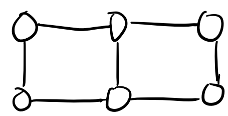


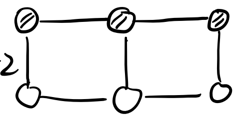
$M_1 > M_2$

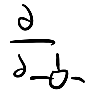
Now variational compression (DMRG)

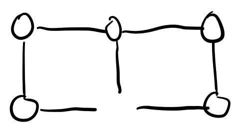
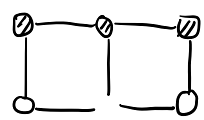
1° $\min_{\Phi} \langle \psi \cdots \Phi | \psi \cdots \Phi \rangle = \min_{\Phi} \left[-2\langle \Phi | \psi \rangle + \langle \Phi | \Phi \rangle \right]$

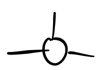
ψ: original

2° 

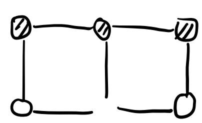
-2  original

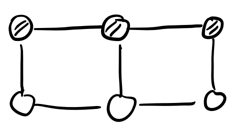
3° 

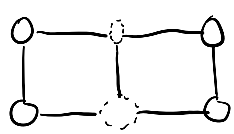
 

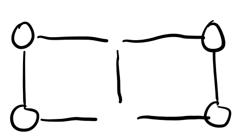
?  $A^{n}_{ij} \equiv \alpha[nij]$ $\alpha[nij] \to \alpha[nij] + \varepsilon g[nij]$

4° consider  as vector $A^{n}_{ij} \triangleq a$

then  is vector b

 $= b^T a$

 $= a^T M a$ where

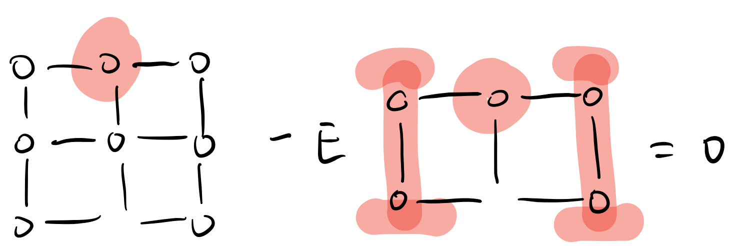
$M =$ 

$$\text{so} \quad \min_a \left(a^T M a - b^T a \right) \Rightarrow Ma = b$$

Energy optimization and ground states. (DMRG)

① $\min_{\psi} \left[\langle \psi | H | \psi \rangle - E \langle \psi | \psi \rangle \right]$

② $\frac{\partial}{\partial} \langle \psi | H | \psi \rangle - E \langle \psi | \psi \rangle = 0$



③ $Ha = Ea$

Time evolution

real time evolution

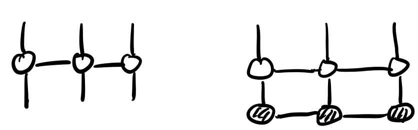
$$|\Phi(\delta t)\rangle = \exp(-iH\delta t)|\Phi(0)\rangle$$

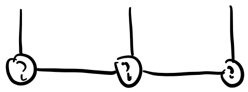
① evolve δt

② compression to reduce entanglement and bond dimension

③ repeat

① $-iH\delta t =$



② 

Short range H: Trotter form

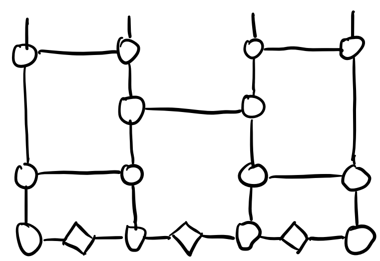
① $e^{-\delta H} = e^{-\delta S_1 \cdot S_2} e^{-\delta S_2 \cdot S_3} \cdots + O(\delta^2)$

② Trotter decomposition

or

△1 Even-odd evolution

time evolution can be broken into even and odd bonds

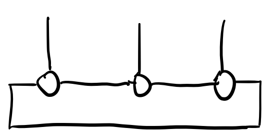


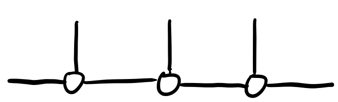
△1 + △2 : Time-evolving block decimation (TEBD)

△2 SVD compression

Periodic and finite MPS

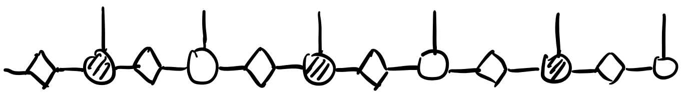
Finite MPS 

Periodic MPS  (satisfies thermodynamic limit)

infinite MPS 

Infinite TEBD

Local algorithms such as TEBD easy to extend to infinite MPS. Unit cell = 2 site infinite MPS



Symmetry (group)

Given global symmetry group, local site basis can be labelled by irreps of group - quantum numbers.

U(1) - site basis labelled by integer n (particle number)
SU(2) - site basis labelled by j, m (spin quanta) $|jm\rangle$

Total state associated with good quantum numbers

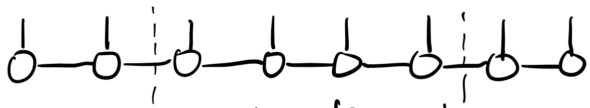
$$|\Psi\rangle = |\Psi(n, j, m)\rangle$$

eg. particle number symmetry

MPS : well defined Abelian symmetry, each tensor fulfils rule $\sum q_{in} = \sum q_{out}$

MPS and PEPS

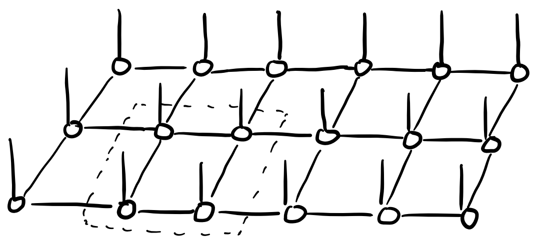
MPS



entanglement

$O(1)$

2D PEPS (projected entangle pair states)



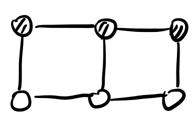
Entanglement : $O(L)$

PEPS generate area law entanglement in any dimension

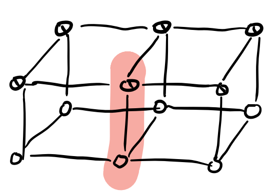
PEPS Contractions (1)

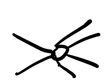
presence of cycles requires approximation

MPS overlap $\langle \Phi | \Psi \rangle$



PEPS overlap $\langle \Phi | \Psi \rangle$



 too much

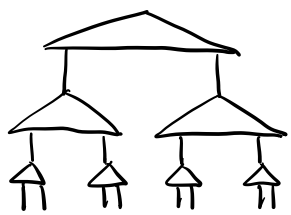
Solution : renomalise (apply isometries)

Tensor networks from RG

RG can be used to construct representations of quantum states

blocking and decimation of quantum states: network of isometry

L sites → | | | | | | | | | | | | | |

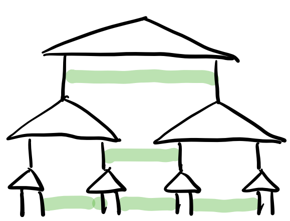


Entanglement RG flow

desirable properties of wavefunction RG flow

Proper RG flow

Construct RG flow restored by inserting disentanglers



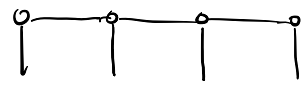
entanglement property:

For region of L site. how much entanglement?

entanglement of 1D area has logarithmic correction

tensor network for critical systems: MERA (gapless)

MERA

① MPS of L :  L $O(L)$

② Multiscale entanglement renormalization ansatz (MERA)

$O(L)$

Applications:

DMRG (MPS) provides almost exact numerical solutions in 1D lattice systems
(PRL 69 (2685) (1992))

DMRG in 2D (quasi-2D system)

PEPS (2D spin)