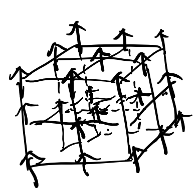
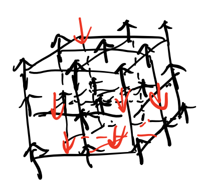
Ising model:

- phase change between ferromagnetic state and paramagnetic state
  describe the magnetism of crystal:
  A magnet above critical T : magnetic disappear
  below : appear again
  } continuous phase transition (second-order phase transition)
  critical phenomenon : Scaling phenomena ; similarity ; long-term correlation
- The alloy's transit between order and disorder

Ising Model

	Low T	High T	Solved
1-D	↑↑↑↑↑↑↑↑↑	↑↑↑↑↑↑↓↓↓↓	Ising - 1925
2-D	↑↑↑↑↑↑↑↑↑ ↑↑↑↑↑↑↑↑↑ ↑↑↑↑↑↑↑↑↑ ↑↑↑↑↑↑↑↑↑↑	↑↑↑↑↑↑↑↓↓↓ ↓↓↓↓↑↑↑↓↓↓ ↑↑↑↓↓↓↑↓↓↓ ↑↑↑↓↑↑↑↑↑↑	Onsager - 1944
3-D			Proven computationally intractable - 2000

magnetism comes from:

- orbital magnetic moment of outer electron
- spin magnetic moment of electron
- nuclear magnetic moment

A. paramagnetism. ↑↑ ↓↑ ↗↑ ↘↖ ↗↙ ↓↑

B. ferromagnetism ↑↑↑↑↑↑↑↑↑↑↑↑

C. antiferromagnetism ↓↑↓↑↓↑↓↑↓↑↓↑

Heisenberg model : $H = \pm J \sum_{\langle i,j \rangle} S_i S_j$

- $-J > 0$: FM state below Curie T
- $J > 0$: AFM state below Neel T
- $T > T_n$: PM

Ising Model : $H = \pm J \sum_{\langle i,j \rangle} \sigma_i \sigma_j + \mu B \sum_i \sigma_i$

$\langle i,j \rangle$: neighbor interaction

Mean Field Approximation :

$H_0 = -\mu \bar{B} \Sigma \sigma_i \qquad \bar{B} = B + \frac{Jz}{\mu} \bar{\sigma}$

1) neighbor interaction replaced by average $\bar{\sigma}_j$

2) suppose $\bar{\sigma}_j = \bar{\sigma}$, irrelated to j ---- shift-invariant ($J > 0$ strictly)

3) z lattice points

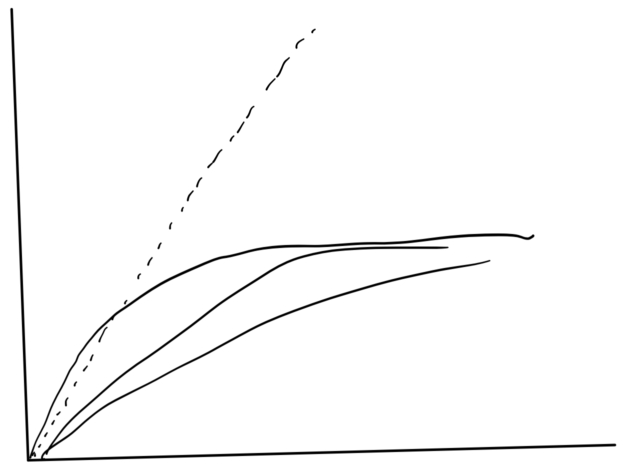
one-dimensional Ising model : $z = 2$

two- : $z = 4$

distribution : $Z_N = [2 \cosh(\beta \mu \bar{B})]^N \quad \bar{\sigma}_j \equiv \bar{\sigma} = \tanh(\beta \mu \bar{B})$

Ferromagnetic phase transition :

$B=0 \quad \bar{\sigma}_j \equiv \bar{\sigma} = \tanh\left(\frac{Jz}{kT}\bar{\sigma}\right)$



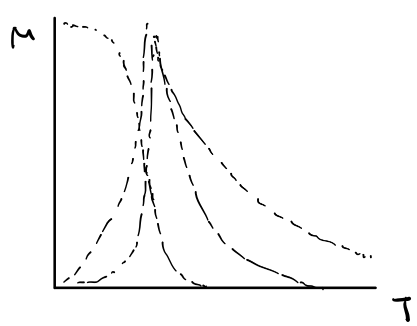
$\frac{Jz}{kT} < 1$, $\bar{\sigma} = 0$, PM

$\frac{Jz}{kT} > 1$, $\bar{\sigma} \neq 0$, FM

so phase-transition $T_c = \frac{Jz}{k}$

only one or two dim , analytical solution exists :

one : $T_c = 0$ average field approximation : $T_c = \frac{2J}{k}$



$T_c = 2.269$

2-dim : $T_c = \frac{2.3J}{k}$; Average field approximation : $T_c = \frac{4J}{k}$

3-dim : $T_c = \frac{4J}{k}$; A : T_c -