Discrete Form of Black Hole Thermodynamics and Holographic Principle

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Abstract: In this report, I will mainly introduce the related research of black hole thermodynamics and black hole entropy. Because the black hole entropy can be explained by the AdS/CFT correspondence, the research and analysis of AdS/CFT is particularly important. I will mainly explain the Ryu-Takayanagi formula in the AdS/CFT correspondence and its discrete implementation. By stacking and partitioning the internal geometric space, and introducing a local tensor into each partition, we can finally construct a multi-body quantum state. The entanglement entropy of this multi-body quantum state is in good agreement with the Ryu-Takayanagi formula , Thus giving a discrete realization of the Ryu-Takayanagi formula. This is of great significance for the study and analysis of black hole thermodynamics and black hole entropy.

1 Introduction

A black hole is a kind of celestial body that exists widely in the universe. Due to its extremely strong internal gravity, the escape velocity in the horizon is greater than the speed of light, so light cannot escape from its event horizon. The earliest research on black holes can be traced back to the period of Newtonian mechanics. Laplace used the means of classical mechanics[拉普拉斯 (1978)], calculated and predicted the celestial body of the black hole. With the establishment of A. Einstein' s general theory of relativity[Einstein(1915)], K. Schwarzschild first gave a rigorous solution to the Einstein equation[Schwarzschild(1999)]. Subsequent research and solution of the Einstein equation made people realize that black holes are a widespread celestial body. Among them, the Schwarzschild black hole and the Kerr-Newman black hole are two typical examples. By analogy with the four laws of thermodynamics, Bardeen, Carter and Hawking summarized the four laws of black hole thermodynamics in 1973[Bardeen et al.(1973)Bardeen, Carter, and Hawking]. The subsequent proposal of Hawking Radiation[Hawking(1974)] and the establishment of Bekenstein-Hawking entropy[Hawking(1974), Bekenstein(1973)] made people realize that a black hole has a temperature proportional to its surface gravity and the entropy of a black hole is proportional to its event horizon area. From the perspective of modern theoretical physics, Bekenstein-Hawking entropy can be explained by anti-de Sitter/conformal field theory (AdS/CFT) duality[Aharony et al.(2000)Aharony, Gubser, Maldacena, Ooguri, and C Therefore, the study of AdS/CFT duality is extremely important for understanding black holes. AdS/CFT theory is a way to realize the principle of gravitational holography. Specifically, it points out that there is a correspondence between the gravitational theory of the $d+2$-dimensional anti-de Sitter space and the $d+1$-dimensional CFT theory on its surface. This correspondence is very important for the study and understanding of quantum gravity. However, there are very few proof results for strict AdS/CFT correspondence. In recent years, with the research on the correspondence between edges and bodies in quantum systems and the rapid development of quantum information theory, people have gradually realized that it is possible to construct some discrete Model to achieve AdS/CFT correspondence. Because the discrete model is a strictly solvable model, it is very important for us to understand the AdS/CFT correspondence. In this report, I will mainly introduce one of the discrete implementations which is HaPPY code implementation[Pastawski et al.(2015)Pastawski, Yoshida, Harlow, and Preskill]. In this discrete implementation, we can first divide the anti-de Sitter space by stacking, and then we use the method of tensor network to place a local tensor on each stacking piece, thereby constructing a many-body quantum state corresponding to AdS/CFT. This many-body quantum state can realize many properties of AdS/CFT duality which in turn provides enlightenment for understanding black holes and their thermodynamics.

2 Black Hole And Its Thermodynamics

Here, let us first briefly introduce the thermodynamics of black holes. It is a comparison of the four laws of classical thermodynamics by Bardeen, Carter and Hawking in 1973, and summarized the four laws of black hole thermodynamics[Bardeen et al.(1973)Bardeen, Carter, and Hawking]. Specifically, it refers to

theorem 1. The zeroth law of thermodynamics: In the state of thermal equilibrium, the system has the same temperature everywhere.

The zeroth law of black hole thermodynamics: a stationary black hole has the same surface gravitational force on the entire horizon surface.

theorem 2. First law of thermodynamics: $dU = TdS + PdV$.

The first law of black hole thermodynamics: $\delta M = \frac{\kappa}{8\pi G}\delta A + \Omega_H \delta J$. When there is a charge in the black hole, it is also necessary to add a potential termΦdQ. Just as in the first law of thermodynamics, chemical potential can also be added.

theorem 3. The second law of thermodynamics: the entropy of an isolated system never decreases with time:$\delta S \geq 0$.

The second law of black hole thermodynamics: The surface area of the horizon of an isolated black hole never decreases with time:$\delta A \geq 0$.

theorem 4. The third law of thermodynamics: It is impossible to reach absolute zero through any physical process.

The third law of black hole thermodynamics: The surface gravity of a black hole cannot be zero. The third law of black hole thermodynamics is equivalent to denying the existence of naked singularities, which is consistent with the hypothesis of cosmic supervision.

Black hole thermodynamics is a theory produced by applying the basic laws of thermodynamics to the study of black holes in general relativity. Although people still cannot clearly understand this theory, the existence of black hole thermodynamics strongly implies the profound and fundamental connection between general relativity, thermodynamics, and quantum theory. Although it seems to start from the most basic principles of thermodynamics and describe the behavior of black holes restricted by the laws of thermodynamics through classical and semi-classical theories, its significance is far beyond the category of analogy between classical thermodynamics and black holes, and includes the nature of quantum phenomena in the gravitational field.

In black hole thermodynamics, it is very important to analyze black hole entropy, which is similar to the study of entropy in thermodynamics. Bekenstein’s research found that although a black hole is an absolute black body, it can still radiate heat at a specific temperature. This is in contradiction with Plank’s black body radiation law. Because the black body radiation law tells us that the classical thermal system performs thermal radiation because of Matter radiates from the thermal system, but the matter in the black hole cannot escape. This contradiction caused Hawking’s attention. He used the quantum field theory of curved spacetime to rigorously study and analyze the radiation of black holes. His conclusion showed that black holes can also produce radiation, that is to say, black holes are not completely black. This effect is now commonly referred to as Hawking radiation. The surface temperature of the black hole corresponding to Hawking radiation is

$$T_H = \frac{\kappa}{2\pi}, \tag{1}$$

Where κ is the gravity on the surface of the black hole. And the Bekenstein-Hawking entropy of the black hole is

$$S_{BH} = \frac{A}{4G}, \tag{2}$$

Where A is the event horizon area of the black hole. The study of Bekenstein-Hawking entropy led to the discovery of the Ryu-Takayanagi formula in the AdS/CFT correspondence[Ryu et al.(2006)Ryu and Takayanagi]. Specifically, consider a $d+2$-dimensional anti-de Sitter spaceAdS$_{d+2}$and its superficial$d+1$-dimensional conformal field theoryCFT$_{d+1}$, and consider its surface area$\mathcal{A}$and corresponding entropy$S(\mathcal{A})$. We will find correspondence with minimal surface$\gamma_{\mathcal{A}}$ofAdS$_{d+2}$decided by the boundary$\partial\mathcal{A}$of area$\mathcal{A}$:

$$S(\mathcal{A}) = \frac{\mathrm{Area}\left(\gamma_{\mathcal{A}}\right)}{4G_N^{(d+2)}} \tag{3}$$

Because $S(\mathcal{A})$ in CFT$_{d+1}$ measures entanglement between the area $\mathcal{A}$ and its complement $\mathcal{A}^c$ in surface space. This implies to a certain extent that entanglement plays an important role in the entropy of the black hole. Research on black hole information in recent years has also shown this point. Although the results have not been fully proven, researching and analyzing this correspondence from multiple angles is of great significance for understanding black hole entropy. Below we will mainly introduce this corresponding discrete form which is the realization of HaPPY code.

3 The Holographic Principle And The Discrete Form Corresponding To AdS/CFT

In this section, let us introduce the discrete form of AdS/CFT. Usually in order to construct a discrete model of geometric structure, we need

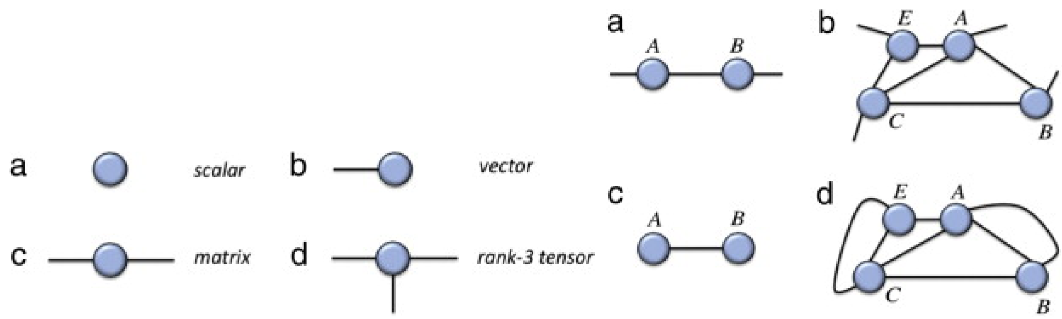

Figure 1: An illustration of a tensor network. The a-d in the left figure represents the tensor of different orders, and the a-d in the right figure represents the contraction between some tensors.

to construct a lattice point model, and then introduce concepts such as metric and curvature into the lattice point model. Because the AdS space is a hyperbolic geometric space, we also need to do similar things. In the HaPPY code model, this is achieved by means of a tensor network, so let's first introduce what a tensor network is.

A tensor is a real or complex number with indicators. It has several indicators called a tensor of several orders, and the number of possible values of the indicator is called the key dimension. For example, a scalar is a number, it has no index, and a vector a_i is a first-order tensor, while a matrix A_{ij} is a second-order tensor. The definition of a higher-order tensor is similar. There are many operations between tensors, and the ones that are important for what we're talking about here are tensor products and contractions. The tensor product of two d_1-order and d_2-order tensors $S_{ij\cdots k}$ and $T_{mn\cdots l}$ can form a new tensor, $X = S \otimes T$. Its weight can be defined as$X_{ij\cdots kmn\cdots l} = S_{ij\cdots k}T_{mn\cdots l}$. Obviously, X is a $d = d_1 d_2$-order tensor. In addition, tensors can be condensed. For example, tensor $S_{ij\cdots k}$ and $T_{mn\cdots l}$, if contactiong i and m index, we'll get a new tensor Y whose component can be defined as $Y_{j\cdots kn\cdots l} = \sum_i S_{ij\cdots k}T_{in\cdots l}$. Similarly, we can shrink more indicators, and shrinking will cause the order of the tensor to decrease. We can also condense the different indicators of a multi-index tensor, and the operation is similar. The reduction between vectors is the inner product

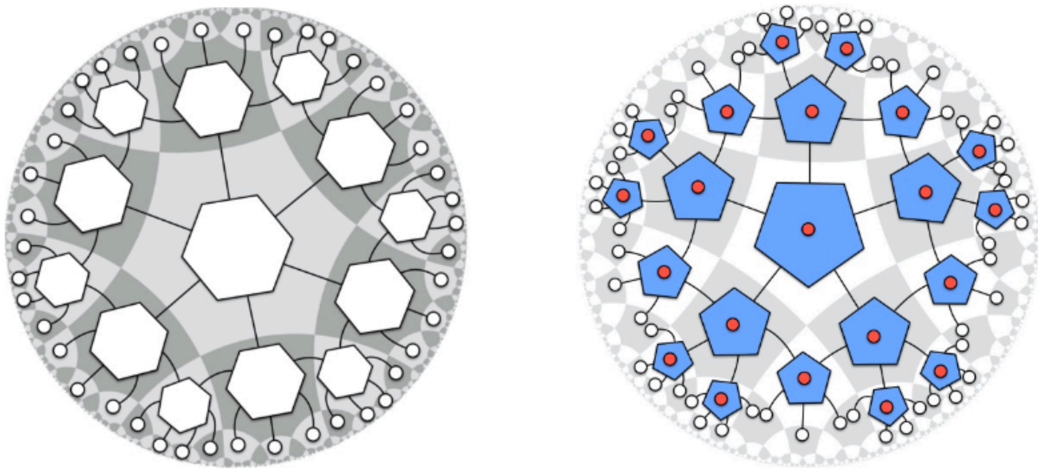

Figure 2: Diagram of HaPPY implementation. The picture on the left is a hexagonal stacking segmentation of AdS space, and the picture on the right is a pentagonal stacking segmentation.

of vectors, and the reduction between matrices is the product of matrices.

Tensors and their shrinkage can be represented by images, which are called tensor networks. This kind of diagram was first proposed by Penrose when he was studying quantum gravity, and is now widely used in physics and computer science. In a tensor network, a tensor is represented by a vertex, and its index is represented by some edges. See picture1。

Tensor networks are widely used in physics, mainly because they can be used to represent quantum states. The component $\psi_{ij\cdots k}$ of a quantum state of N particle under the selected basis can be regarded as a tensor, so we can construct a tensor network with N open edges to represent this quantum state. The contraction part of the corresponding tensor network can be used to encode entanglement information well, which makes it extremely important in the study of quantum information and quantum states of matter. In order to use tensor network quantum states to study AdS/CFT correspondence, we first introduce some special tensors.

- Isometric tensor. We know that the isometric mapping between two Hilbert spaces is a mapping that can maintain the inner product. Under the selected base, the isometric mapping becomes a multi-index

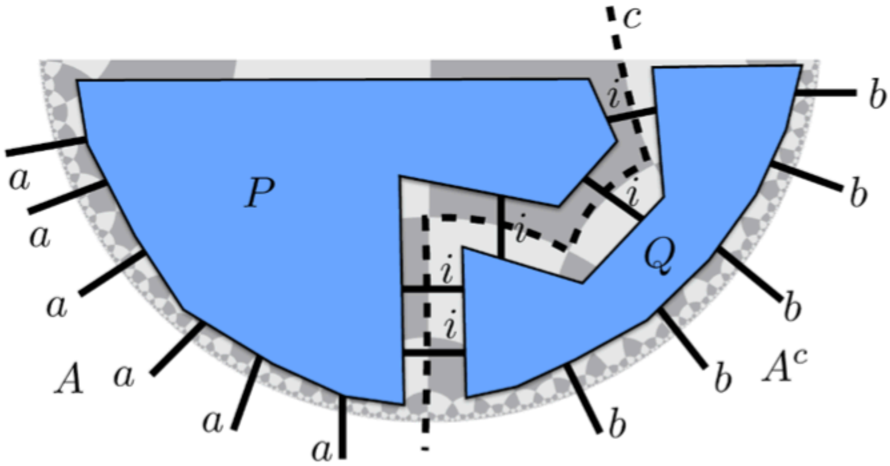

Figure 3: The tensor network realizes the definition diagram of the shortest path corresponding to AdS/CFT.

tensor, and the tensor corresponding to the isometric mapping is called equal Distance tensor.

- Perfect tensor. A tensor with $2n$ indicators $T_{a_1,\cdots a_{2n}}$, if it satisfies the for segmentation $A|A^c$ for any indicator set, the corresponding tensor is the isometric mapping between $\mathcal{H}_A$ and $\mathcal{H}_{A^c}$ of Hilbert space formed by the corresponding indicator, then this tensor is called a perfect tensor.

The most important meaning of a perfect tensor is that the quantum state of the tensor network to which it corresponds to is an absolute maximum entangled state, that is to say, for any two partitions of the system, the entanglement between the two parts of the partition is the maximum entanglement.

With the above preparations, let us now consider the implementation of AdS/CFT tensor network. First of all, we need to divide the AdS space by stacking, which is determined by the geometric nature of the space. Because the AdS space is a hyperbolic space, his stacking segmented stacking pieces are large in the middle, small in the edge, We have given examples of hexagonal and pentagonal stacking in the figure 2. We will mainly use hexagonal stacking for illustration. Each white hexagon in the figure can be regarded as a sixth-order tensor. In the construction of the tensor network AdS/CFT, we choose it as a perfect tensor of order six. The edges connecting different white hexagons can all be regarded as indicators of contraction between tensors. Because the internal indicators have been condensed, there are no more quantum degrees of freedom inside. So in the end, all the degrees of freedom fall on the boundary, because the boundary is a CFT theory, and the inside is the gravitational theory. This structure is in good agreement with the results of AdS/CFT theory.

We now consider how to reconstruct the Ryu-Takayanagi formula. In order to do this, we need to introduce the definition of the internal minimal surface. First of all, because the interior is a two-dimensional disk, the smallest surface is actually the shortest path. At this time, given a region $\mathcal{A}$ (that is, an arc) on the boundary, the boundary of this region is two points. Then we need to introduce the shortest internal path with these two points as the boundary. We can define it in this way: Given two points at the boundary, then in the multipath that connects the two points internally, the path with the least cut edge of the tensor network is the shortest path, and the length of the path can be defined as the number of edges to cut. According to this definition, we can prove

$$S(\mathcal{A}) \leq \text{Area}\,(\gamma_{\mathcal{A}}) \log v, \tag{4}$$

Where v is the key dimension of the tensor network. By choosing an appropriate sixth-order tensor, we can get this upper bound saturatedly. So we reconstructed the Ryu-Takayanagi formula in this way, that is to say, reconstructed the AdS/CFT correspondence. Further research shows that more continuous AdS/CFT corresponding content can be realized under this framework.

4 Results and Discussions

In this report, I summarized the related content of black hole thermodynamics and the tensor network implementation of AdS/CFT. I gave the basic laws of black hole thermodynamics, black hole entropy and its corresponding Ryu-Takayanagi formula under the framework of AdS/CFT. Finally, by introducing the implementation of AdS/CFT tensor network, I give a concrete framework for realizing these corresponding discrete models.

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