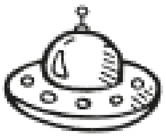
Quantum Teleportation



Lu Wei

USTC

October 25, 2021

Contents

1 Basics 1

1 Basics

- Purpose Alice wants to send the qubit state $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ to Bob which entails passing on information about α and β to Bob.
- Difficulty No-cloning theorem: One can not make an exact copy of unknown quantum state.
- Protocol
  - You need: two classical bits, an entangled qubit pair and a third party to send the pair.
  - step1

    Quantum Teleportation begins with the fact that Alice needs to transmit $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ (a random qubit) to Bob. She doesn't know the state of the qubit. For this, Alice and Bob take the help of a third party (Telamon). Telamon prepares a pair of entangled qubits for Alice and Bob. The entangled qubits could be written in Dirac Notation as:

    $$|e\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

    Alice and Bob each possess one qubit of the entangled pair (denoted as A and B respectively),

    $$|e\rangle = \frac{1}{\sqrt{2}}\left(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\right)$$

    This creates a three qubit quantum system where Alice has the first two qubits and Bob the last one.

    $$\begin{aligned} |\psi\rangle \otimes |e\rangle &= \frac{1}{\sqrt{2}}(\alpha|0\rangle \otimes (|00\rangle + |11\rangle) + \beta|1\rangle \otimes (|00\rangle + |11\rangle)) \\ &= \frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle) \end{aligned}$$

  - step2

    Now according to the protocol Alice applies CNOT gate on her two qubits followed by Hadamard gate on the first qubit. This results in the state:

$$
\begin{aligned}
&(H \otimes I \otimes I)(CNOT \otimes I)(|\psi\rangle \otimes |e\rangle) \\
&= (H \otimes I \otimes I)(CNOT \otimes I)\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|100\rangle + \beta|111\rangle) \\
&= (H \otimes I \otimes I)\frac{1}{\sqrt{2}}(\alpha|000\rangle + \alpha|011\rangle + \beta|110\rangle + \beta|101\rangle) \\
&= \frac{1}{2}(\alpha(|000\rangle + |011\rangle + |100\rangle + |111\rangle) + \beta(|010\rangle + |001\rangle - |110\rangle - |101\rangle))
\end{aligned}
$$

Which can then be separated and written as:

$$
\begin{aligned}
= \frac{1}{2}(&|00\rangle(\alpha|0\rangle + \beta|1\rangle) \\
&+ |01\rangle(\alpha|1\rangle + \beta|0\rangle) \\
&+ |10\rangle(\alpha|0\rangle - \beta|1\rangle) \\
&+ |11\rangle(\alpha|1\rangle - \beta|0\rangle) \quad)
\end{aligned}
$$

– step3

Alice measures the first two qubit (which she owns) and sends them as two classical bits to Bob. The result she obtains is always one of the four standard basis states $|00\rangle, |01\rangle, |10\rangle$, and $|11\rangle$ with equal probability. On the basis of her measurement, Bob's state will be projected to,

$$
\begin{aligned}
|00\rangle &\to (\alpha|0\rangle + \beta|1\rangle) \\
|01\rangle &\to (\alpha|1\rangle + \beta|0\rangle) \\
|10\rangle &\to (\alpha|0\rangle - \beta|1\rangle) \\
|11\rangle &\to (\alpha|1\rangle - \beta|0\rangle)
\end{aligned}
$$

– step4

Bob, on receiving the bits from Alice, knows he can obtain the original state $|\psi\rangle$ by applying appropriate transformations on his qubit that was once part of the entangled pair. The transformations he needs to apply are:

Bob's State	Bits Received	Gate Applied
$(\alpha|0\rangle + \beta|1\rangle)$	00	I
$(\alpha|1\rangle + \beta|0\rangle)$	01	X
$(\alpha|0\rangle - \beta|1\rangle)$	10	Z
$(\alpha|1\rangle - \beta|0\rangle)$	11	ZX

After this step Bob will have successfully reconstructed Alice's state.

– Result: At the end, Bob will have $\psi\rangle$ and Alice won't anymore.