Complexity and Approximation

Exercise Sample

1 Multiple choice questions

For each one of the following statements, say whether it is "*True*" or "*False*", or "*Not resolved*". "*Not resolved*" means that, as far as it is known (i.e., studied in class), the statement is neither proved true nor proved false. Si motivi brevemente ogni risposta. Risposte non motivate saranno valutate nulle.

1. If every algorithm for solving SAT has complexity $\Omega(n + n^{\frac{\log \log n}{\log n}})$ then $\mathcal{P} \neq \mathcal{NP}$.
2. 2-SAT can be reduced in polynomial time to SAT
3. There is no approximation algorithm with guarantee 11/10 for the problem of finding the minimum number of colours necessary to *properly colour* a given graph.
4. $\bigcup_{k>0} \mathcal{TIME}(2^{n^k}) \neq \bigcup_{k>0} \mathcal{NTIME}(2^{n^k})$
5. $\mathcal{P} = \mathcal{NP} \Rightarrow \mathcal{APX} \subseteq \mathcal{PTAS}$

2 Open Questions

1. Show that MAX-3-SAT $\leq$ MAX-CLIQUE.

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2. You're designing a tool for showing advertisements (ads) on an e-commerce website. The website has several visitors per day and each visitor $i \in \{1, 2, \ldots, n\}$, has assigned a value v_i representing the expected revenue that can be obtained from this customer. Each visitor i is shown *one* of the m possible ads $A_1, \ldots, A_m$ as they enter the site. Our goal is to make sure that globally *each* ad is seen by a set of customers of reasonably large total value. Thus, given a selection of one advertisement for each customer, we will define the *spread* of this selection to be the minimum, over $j = 1, 2, \ldots, m$, of the total weight of all customers who were shown ad A_j.

   Example. Suppose there are six customers with values $3, 4, 12, 2, 4, 6$, and there are $m = 3$ ads A_1, A_2, A_3. Then, in this instance, one could achieve a spread of 9 by: showing ad A_1 to customer $1, 2, 4$; showing ad A_2 to customer 3; and showing ad A_4 to customer 5 and 6.

   The goal is to find a selection of an ad for each customer that maximizes the spread. Unfortunately, this optimization problem is $\mathcal{NP}$-hard, so we will try to approximate it.

   (a) Show that the spread maximization problem above is $\mathcal{NP}$-hard.

   (b) Give a polynomial-time algorithm that approximates the maximum spread to within a factor of 2. In designing your algorithm you can assume that no single customer has a value which is significantly above the average, and specifically, that for each i, it holds that $v_i \leq \left(\sum_{j=1}^{n} v_j\right)/2m$.

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3. Let $\mathcal{C}$ be the class of decision problems such that for each problem $\mathbb{A}$ in $\mathcal{C}$ there exists a polynomial algorithm $B^{\mathbb{A}}$ such that

   (i) for each *yes* instance $\mathbf{x} \in \mathcal{I}(\mathbb{A})$ there is a certificate $\mathbf{y} \in \{0,1\}^*$, satisfying: $|\mathbf{y}| \leq \log |\mathbf{x}|$ and $B^{\mathbb{A}}(\mathbf{x}, \mathbf{y}) = 1$

   (ii) for each *no* instance $\mathbf{x} \in \mathcal{I}(\mathbb{A})$, for all $\mathbf{y}$ it holds that $B^{\mathbb{A}}(\mathbf{x}, \mathbf{y}) = 0$.

   Show that if INDEPENDENT SET $\in \mathcal{C}$ then $\mathcal{NP} = co\mathcal{NP}$
