Mathematics for Decisions

Worksheet 1c – AMPL Exercises

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Overview

Sports and workouts

2-2. (a) You have been advised by your doctor to get more exercise, specifically, to burn off at least 2000 extra calories per week by some combination of walking, jogging, swimming, exercise-machine, collaborative indoor recreation, and pushing yourself away from the table at mealtimes. You have a limited tolerance for each activity in hours/week; each expends a certain number of calories per hour, as shown below:

	walking	jogging	swimming	machine	indoor	pushback
Calories	100	200	300	150	300	500
Tolerance	5	2	3	3.5	3	0.5

How should you divide your exercising among these activities to minimize the amount of time you spend?

Hint: his problem is similar to the Diet one.

Sports and workouts – More variety

(b) Suppose that you should also have some variety in your exercise — you must do at least one hour of each of the first four exercises, but no more than four hours total of walking, jogging, and exercise-machine. Solve the problem in this form.

Fast-food

2-5. A chain of fast-food restaurants operates 7 days a week, and requires the following minimum number of kitchen employees from Monday through Sunday: 45, 45, 40, 50, 65, 35, 35. Each employee is scheduled to work one weekend day (Saturday or Sunday) and four other days in a week. The management wants to know the minimum total number of employees needed to satisfy the requirements on every day.

(a) Set up and solve this problem as a linear program.

For instance, consider a Monday. Employees working that day will have one of the following schedules:

1. Monday-Tuesday-Wednesday-Thursday-Saturday
2. Monday-Tuesday-Wednesday-Thursday-Sunday
3. Monday-Tuesday-Wednesday-Friday-Saturday
4. Monday-Tuesday-Wednesday-Friday-Sunday
5. Monday-Tuesday-Thursday-Friday-Saturday
6. Monday-Tuesday-Thursday-Friday-Sunday
7. Monday-Wednesday-Thursday-Friday-Saturday
8. Monday-Wednesday-Thursday-Friday-Sunday

Set Covering

The Ministry of Health wants to build some orthopedic-specialized hospitals, able to serve, within the range of 200 kms, Latina, Lecce, Matera, Napoli, Potenza, Salerno e Roma. Here follows, for every place, the list of other cities far less then 200 kms:

- Latina: Latina, Napoli, Roma;
- Lecce: Lecce, Matera;
- Matera: Lecce, Matera, Potenza;
- Napoli: Latina, Napoli, Potenza, Salerno;
- Potenza: Matera, Napoli, Potenza, Salerno;
- Salerno: Napoli, Potenza, Salerno;
- Roma: Latina, Roma.

For example, if an hospital is built in Napoli, it would be able to serve Latina, Potenza and Salerno too, that are less than 200 kms far.
We want to decide in which of the seven places to build the hospitals, such that every city is served at least by one hospital, far less than 200 kms, and considering that two hospitals cannot be built in the same city.

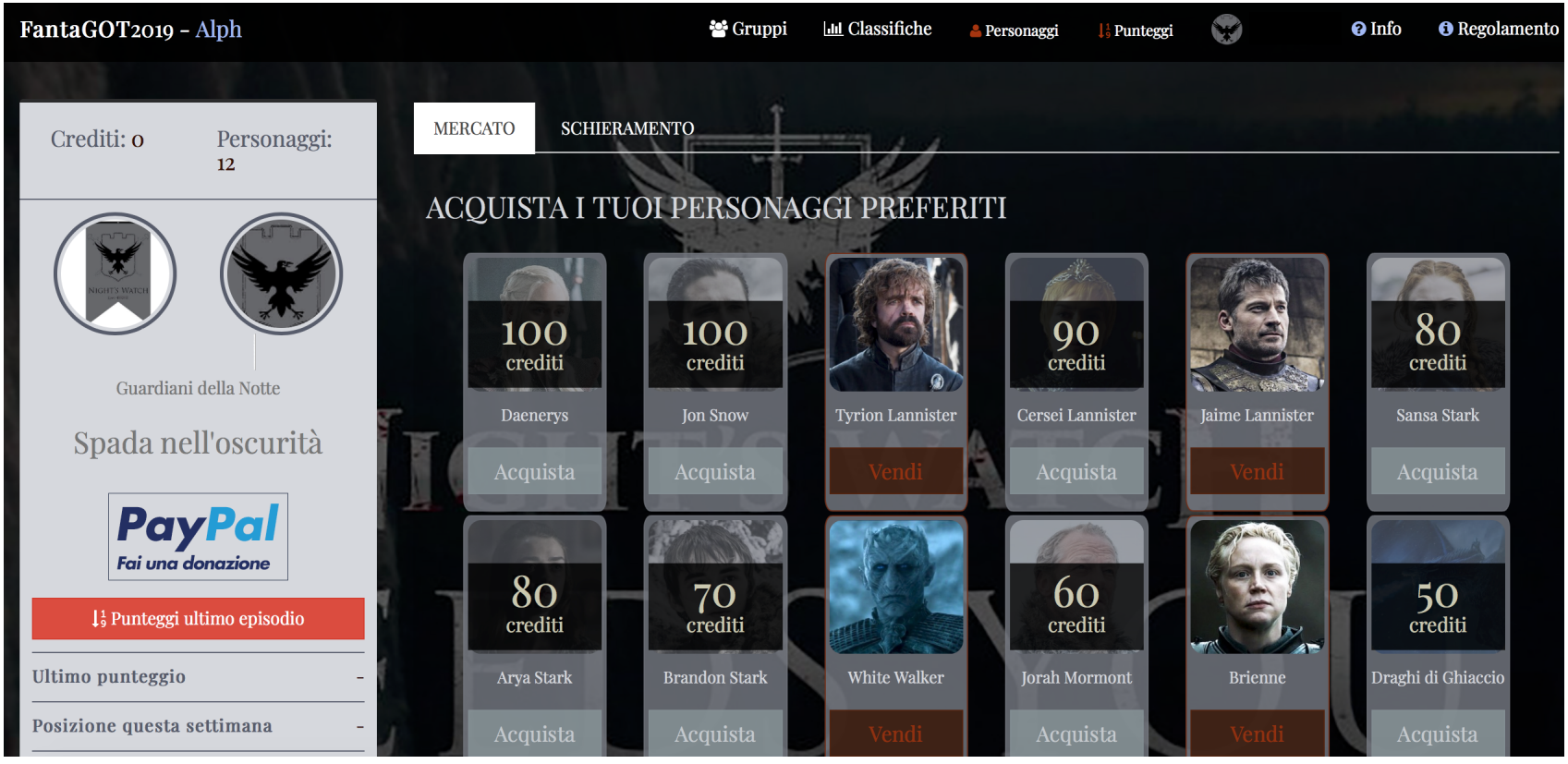
Production / Lot Sizing

A company produces cubic meter glass fiber and would like to plan the production for the following six weeks. The production capacity is limited and this limit varies according to the considered week. Weekly demand is already known for the whole period. Production and warehousing costs also vary according to the week. All values are shown in the following table:

Week	Capacity	Demand	Prod. Cost	Warehousing Cost
1	140	100	5	0.20
2	100	120	8	0.30
3	110	100	6	0.20
4	100	90	6	0.25
5	120	120	7	0.30
6	100	110	6	0.40

Plan the production minimizing production and warehousing costs.

Fantasy GoT


FantaGOT2019 - Alph
Gruppi
Classifiche
Personaggi
Punteggi
Info
Regolamento
Crediti: 0
Personaggi: 12
Guardiani della Notte
Spada nell'oscurità
PayPal
Fai una donazione
Punteggi ultimo episodio
Ultimo punteggio -
Posizione questa settimana -
MERCATO
SCHIERAMENTO
ACQUISTA I TUOI PERSONAGGI PREFERITI
100 crediti
Daenerys
Acquista
100 crediti
Jon Snow
Acquista
Tyrion Lannister
Vendi
90 crediti
Cersei Lannister
Acquista
Jaime Lannister
Vendi
80 crediti
Sansa Stark
Acquista
80 crediti
Arya Stark
Acquista
70 crediti
Brandon Stark
Acquista
White Walker
Vendi
60 crediti
Jorah Mormont
Acquista
Brienne
Vendi
50 crediti
Draghi di Ghiaccio
Acquista

Fantasy GoT (I)

The new season of Game of Thrones is about to begin. Why not enjoy it more, playing with friends and relatives to a *Fantasy GoT*?

Once signed in to its website, **Fantasy GoT** allows you to build your own team, choosing twelve characters among the ones in the series of *A Song of Ice and Fire*.

Every character is assigned with a *price*, more or less proportional to its relevance in the series, and with a *value*, usually established according to his/her behavior in previous seasons.

Fantasy GoT (II)

CHARACTERS:	PRICE	VALUE :=
DaenerysTargaryen	100	9
JonSnow	100	10
TyrionLannister	90	10
CerseiLannister	90	8
JaimeLannister	80	10
SansaStark	80	7
AryaStark	80	8
BrandonStark	70	7
WhiteWalker	60	10
JorahMormont	60	5
Brienne	60	8
IceDragons	50	7
Davos	50	8
Dragons	50	8
TheHound	50	7
TheMountain	40	4
Tormund	40	7
TheonGreyjoy	40	7
Gendry	40	8
SamwellTarly	40	8
BericDondarrion	40	5
Bronn	40	5
EuronGreyjoy	40	4
GreyWorm	40	5
Edd	30	5
Varys	30	6
Oyburn	30	4
YaraGreyjoy	30	5
Melisandre	30	6
Missandei	20	4
Podrick	20	4
Gilly	20	6

Fantasy GoT (III)

Obviously, as in every fantasy game you can find online, you cannot buy all the strongest characters because of a *limited budget*, for example 600 credits.

How to build your team? There can be several goals:

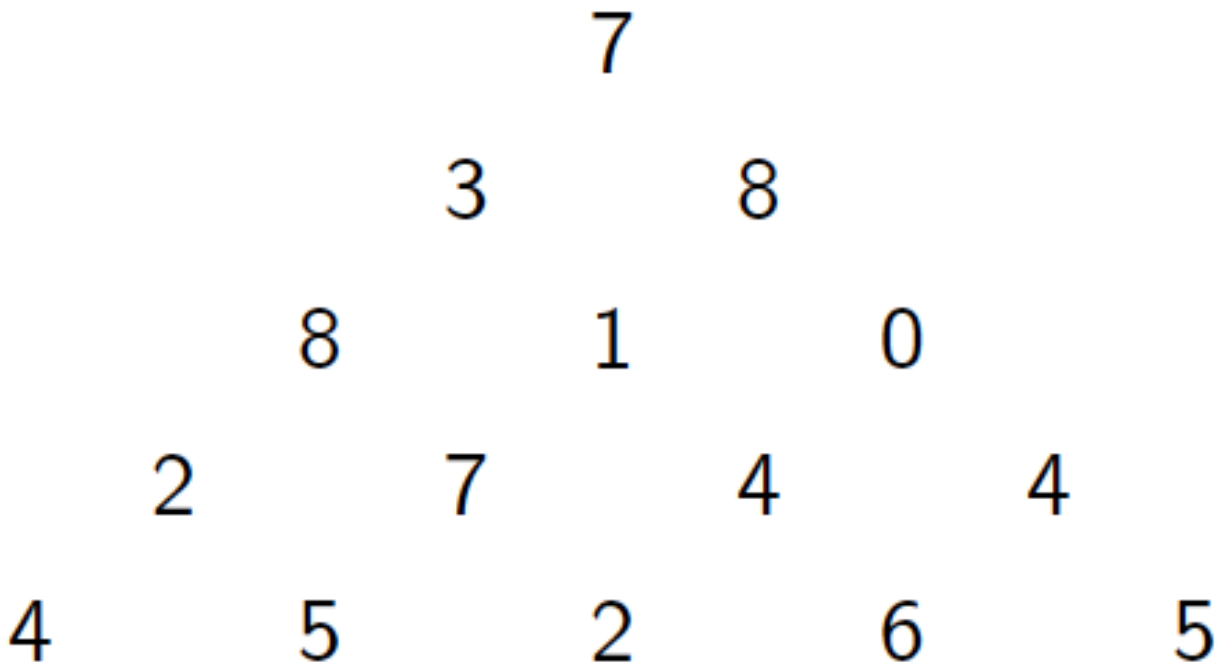
- Maximizing the total value of chosen characters;
- Using as much as you can the available budget.

Sudoku

8								
		3	6					
	7			9		2		
	5				7			
				4	5	7		
			1				3	
		1					6	8
		8	5				1	
	9					4		

How to model this game as
an Integer Linear Programming problem?

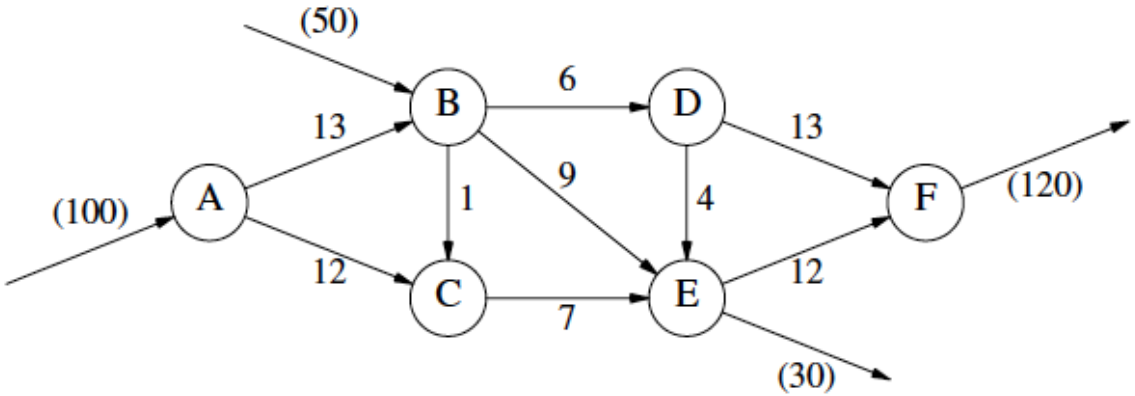
Triangle



Modelling the problem to compute the biggest sum of the numbers, you obtain a path from the top of the triangle until its basis. At each step, you can diagonally go down to the right or to the left, picking only one number.

Network Transshipment

15-1. The following diagram can be interpreted as representing a network transshipment problem:



The arrows into nodes A and B represent supply in the indicated amounts, 100 and 50; the arrows out of nodes E and F similarly represent demand in the amounts 30 and 120. The remaining arrows indicate shipment possibilities, and the numbers on them are the unit shipping costs. There is a capacity of 80 on every arc.

Can you find the minimum-cost flow that satisfies the demands exiting from nodes E and F?

Steel

Consider the following AMPL problem:

```
set PROD;  # products

param rate {PROD} > 0;      # tons produced per hour
param avail >= 0;           # hours available in week

param profit {PROD};        # profit per ton
param market {PROD} >= 0;   # limit on tons sold in week

var Make {p in PROD} >= 0, <= market[p]; # tons produced

maximize Total_Profit: sum {p in PROD} profit[p] * Make[p];

               # Objective: total profits from all products

subject to Time: sum {p in PROD} (1/rate[p]) * Make[p] <= avail;

               # Constraint: total of hours used by all
               # products may not exceed hours available
```

Figure 1-4a: Steel production model (`steel.mod`).

```
set PROD := bands coils;

param:    rate  profit  market :=
  bands    200    25     6000
  coils    140    30     4000 ;

param avail := 40;
```

Figure 1-4b: Data for steel production model (`steel.dat`).

Steel T – Multiperiod (I)

Now, suppose we want to consider T periods. How should we modify the model and the data file?

Steel T – Sensitivity analysis

```
ampl: model steelT.mod; data steelT.dat;
ampl: option solver cplex;
ampl: option cplex_options 'sensitivity';

ampl: solve;
CPLEX 8.0.0: sensitivity
CPLEX 8.0.0: optimal solution; objective 515033
16 dual simplex iterations (0 in phase I)

suffix up OUT;
suffix down OUT;
suffix current OUT;
```

Steel T – Variables

The .current suffix indicates the objective function coefficient in the current problem, while .down and .up give the smallest and the largest values of the objective coefficient, respectively, for which the current LP basis remains optimal.

```
ampl: display Sell.down, Sell.current, Sell.up;
:        Sell.down Sell.current    Sell.up       :=
bands 1    23.3           25        1e+20
bands 2    25.4           26        1e+20
bands 3    24.9           27           27.5
bands 4    10             27           29.1
coils 1    29.2857        30           30.8571
coils 2    33             35        1e+20
coils 3    35.2857        37        1e+20
coils 4    35.2857        39        1e+20
;
```

Note: values of -1e+20 and 1e+20 in the .down and .up column correspond to what CPLEX calls *-infinity* and *+infinity* in its tables.

Steel T – Constraints

The interpretation is similar except that it applies to a constraint's constant term (i.e., the right-hand-side value)

```
ampl: display Time.down, Time.current, Time.up;
: Time.down Time.current   Time.up     :=
1    37.8071        40      66.3786
2    37.8071        40      47.8571
3    25             32      45
4    30             40      62.5
;
```

Cocoa Purchase Planning

Our chocolate factory buys different kind of cocoa beans from three suppliers and then sells chocolate bars to retailers.
For next winter, we'll produce four new kind of chocolate bars and we have already received orders to be filled, as given in the following table:

Kind	**Orders**	**S1**		**S2**		**S3**	
		Capacity	**Price**	**Capacity**	**Price**	**Capacity**	**Price**
Milk	80	40	5	45	6	30	4
White	70	25	4	30	5	25	6
Dark	120	70	8	75	7	30	10
Ruby	90	50	10	30	15	30	12
Total		150		160		90	

Determine the optimal cocoa purchase planning minimizing total costs.

Advertisement

(a) You are in charge of an advertising campaign for a new product, with a budget of $1 million. You can advertise on TV or in magazines. One minute of TV time costs $20,000 and reaches 1.8 million potential customers; a magazine page costs $10,000 and reaches 1 million. You must sign up for at least 10 minutes of TV time. How should you spend your budget to maximize your audience? Formulate the problem in AMPL and solve it.

(b) It takes creative talent to create effective advertising; in your organization, it takes three person-weeks to create a magazine page, and one person-week to create a TV minute. You have only 100 person-weeks available. Add this constraint to the model and determine how you should now spend your budget.

(c) Radio advertising reaches a quarter million people per minute, costs $2,000 per minute, and requires only 1 person-day of time. How does this medium affect your solutions?

(d) How does the solution change if you have to sign up for at least two magazine pages? A maximum of 120 minutes of radio?