Mathematics for Decisions

Worksheet 1b – COVID Office Planning

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Problem 0: single day, single office

We'd like to focus on the single day assignment of people to the same single office. There are N people who are asked to express how likely they are going to be there (a priority 0 to 3, where 0 means "I'm not coming"). Current COVID-19 health and safety regulations impose that the maximum number of people allowed to share the office simultaneously is $C \leq N$. Consider the pre-filled table on the right.

Person	Priority
Alice	3
Andrea	1
Chiara	2
Elia	0
Fabio	1
Franco	0
Federico	2
Matteo	3
Michele	1
Rossana	1

$C = 6$

1. Can you find an optimal assignment (higher priorities first)?
2. Is it unique?

Problem 1a: single day, multiple offices

Consider Problem 0 with the following extension: There are K offices available in which N people can be assigned. Each office has a maximum capacity of $C_k \leq N$. Any person is given an office (if assigned) and no office (if not assigned). Consider the pre-filled table on the right.

Person	Priority
Alice	3
Andrea	1
Chiara	2
Elia	0
Fabio	1
Franco	0
Federico	2
Matteo	3
Michele	1
Rossana	1

$K = 2, C_1 = 6, C_2 = 3.$

1. Can you find an optimal assignment (higher priorities first)? Is it unique?
2. Can you encode Problem 0 into Problem 1a? What about the vice-versa?

Problem 1b: multiple days, single office

Consider Problem 0 with the following extension: we want to make plans for M (non-necessarily consecutive) days. People express their preferences for each day.

Person	**Mon**	**Tue**	**Wed**	**Thu**	**Fri**
Alice	3	3	3	3	3
Andrea	1	3	1	0	0
Chiara	2	1	3	0	0
Elia	0	0	2	0	0
Fabio	1	0	2	0	1
Franco	0	0	0	3	2
Federico	2	1	1	3	1
Matteo	3	3	3	3	3
Michele	1	0	3	2	0
Rossana	1	0	3	2	0

$C = 6, M = 5$

1. Can you find an optimal assignment for each day? Is it unique?
2. Can you encode Problem 0 into Problem 1b? What about the vice-versa?

Problem 2a: multiple days, multiple offices

Consider Problem 1a and 1b together.

Person	Mon	Tue	Wed	Thu	Fri
Alice	3	3	3	3	3
Andrea	1	3	1	0	0
Chiara	2	1	3	0	0
Elia	0	0	2	0	0
Fabio	1	0	2	0	1
Franco	0	0	0	3	2
Federico	2	1	1	3	1
Matteo	3	3	3	3	3
Michele	1	0	3	2	0
Rossana	1	0	3	2	0

$K = 2, C_1 = 6, C_2 = 3, M = 5$

1. Can you find an optimal assignment for each day? Is it unique?
2. Can you encode Problem 0 into Problem 1b? What about the vice-versa?

Problem 2b: multiple days, multiple offices, same office per user

Consider Problem 1a and 1b together with the following extension: any assigned user keeps the same office for all days.

Person	Mon	Tue	Wed	Thu	Fri
Alice	3	3	3	3	3
Andrea	1	3	1	0	0
Chiara	2	1	3	0	0
Elia	0	0	2	0	0
Fabio	1	0	2	0	1
Franco	0	0	0	3	2
Federico	2	1	1	3	1
Matteo	3	3	3	3	3
Michele	1	0	3	2	0
Rossana	1	0	3	2	0

$K = 2, C_1 = 6, C_2 = 3, M = 5$

1. Can you find an optimal assignment for each day? Is it unique?