Spend it all!

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Problem 1

The University Administration (UA) is given every year an amount of money that is intended for several jobs. Currently, the UA has the following jobs available in its to-do list:

Job	Cost
Lab1WhiteWashing	5200€
Lab2WhiteWashing	7500€
Elevators	3790€
HallRefurbishing	11740€
Gardening	8540€

Job	Cost
Lab1Computers	46950€
Lab2Computers	65730€
Floor1Furniture	62540€
Floor2Furniture	85720€

Job	Cost
CarParkingExtension	57900€
CCTV	22360€
BicycleParking	11350€

We are approaching the end of the year and the UA has still 97.460€ to spend. The UA needs to spend all the money in order to keep getting it next year. If it doesn't, it means that it does not need so much money.

Can you find a subset of jobs to do in order to spend all the money?

Problem 2

Consider now the following generalization of the previous problem:

Each job belongs to one (and only one) category.

Maintenance	Cost
Lab1WhiteWashing	5200€
Lab2WhiteWashing	7500€
Elevators	3790€
HallRefurbishing	11740€
Gardening	8540€

Replacements	Cost
Lab1Computers	46950€
Lab2Computers	65730€
Floor1Furniture	62540€
Floor2Furniture	85720€

New Purchases	Cost
CarParkingExtension	57900€
CCTV	22360€
BicycleParking	11350€

This time the budget is 92.400€.

Can you find a subset of jobs (at least one for each category) to do in order to spend all the money? How many solutions are there?

Problems 3: Spend as much as possible!

If the UA had a budget of 100.000€ only, the previous problem would not be solvable. Indeed, there exists no subset of jobs, with at least one job for each catgory, whose costs sum up to 100.000€.

Considering that the UA can remain with an amount of unspent money of at most 5000€, select a subset of jobs that maximizes the spent money within the budget.

How much money is left?

Problems 4: Spend as much as possible!

Considering that the UA can credit 1000€ beyond the budget, select a subset of jobs that minimizes the spent money beyond the budget. How much money is spent beyond the budget?

Subset Sum

Input. A finite set X of elements where for each $x \in X$, $s(x) \in \mathbb{Z}$, and an integer S.
Output. yes iff there exists $X_1 \subseteq X$ such that $\sum_{x \in X_1} s(x) = S$.

Example $S = 13$

S	s(x)
1	-1
2	-8
3	3
4	-6
5	-5
6	8
7	-7
8	2
9	-9
10	10