

## VECTORIAL ANALYSIS APPLIED TO TORIC IOL REALIGNMENT

Herein we expose how vectorial analysis of astigmatism can be employed to guide the postoperative rotation of TIOLs. This adaptation of the double angle method to guide TIOL rotation assumes that the manifest refraction is the difference between the postoperative corneal cylinder and the TIOL cylinder at corneal plane.<sup>7,9</sup> To ease the comparison with the results given by the Berdahl-Hardt calculator, similar nomenclature of the variables was used.

STEP 1. The first step of the process is to transform spectacles refraction to refraction at corneal plane. When distance from spectacles to corneal vertex (d) is expressed in meters, refraction at corneal plane can be defined by the following expression:

$$(1) \text{ Refraction at corneal plane} = \text{spectacles refraction} / [1 - (d \times \text{spectacle refraction})]$$

STEP 2. The second step is to transform the polar astigmatism vectors into quadrangular vectors. Both for the refractive and the TIOL axis measured at the postoperative evaluation. The axis must be entered expressed in radians. The radian expression of the axis can be obtained after multiplying the value in degrees by Pi and dividing it by 180. TIOL cylinder power at corneal plane can be obtained from the TIOL calculator using preoperative biometric information. Refraction must be entered with positive cylinders.

$$(2) \text{ Refraction\_X} = \text{refractive cylinder at corneal plane} \times \cos(2 \times \text{refractive axis})$$

$$(3) \text{ Refraction\_Y} = \text{refractive cylinder at corneal plane} \times \sin(2 \times \text{refractive axis})$$

$$(4) \text{ IOL\_X} = \text{IOL cylinder at corneal plane} \times \cos(2 \times \text{IOL axis})$$

$$(5) \text{ IOL\_Y} = \text{IOL cylinder at corneal plane} \times \sin(2 \times \text{IOL axis})$$

STEP 3. Get the postoperative values of corneal astigmatism from the refraction and IOL data. The perfect TIOL must have the same cylinder astigmatism at corneal plane and the same axis as the patient corneal cylinder. The postoperative refractive cylinder is the difference between the postoperative corneal cylinder and the TIOL cylinder at corneal plane. Thus, we can approximate that:

$$(6) \text{ Corneal\_cylinder\_X} = \text{Refraction\_X} + \text{IOL\_X}$$

$$(7) \text{ Corneal\_cylinder\_Y} = \text{Refraction\_Y} + \text{IOL\_Y}$$

According to the double angle vectorial analysis of astigmatism, cylinder's axis at corneal plane is defined in radians as:

$$(8) \text{ Corneal\_cylinder\_axis} = \arctangent(\text{Corneal\_cylinder\_Y} / \text{Corneal\_cylinder\_X})$$

STEP 4. Get the best TIOL axis. To obtain the best TIOL axis, we must iterate between the possible implant axis (from 0 to 179 degrees) and evaluate the refractive results in each position, and choosing the implant position with the lowest absolute refractive cylinder. This can be performed with programming languages like Python, R or C, or even with a MS Excel Spreadsheet. The residual refractive cylinder of each TIOL position can be predicted from the following expressions:

$$(9) \text{ New\_IOL\_X} = \text{IOL cylinder at corneal plane} \times \cos(2 \times \text{IOL\_new\_axis})$$

$$(10) \text{ New\_IOL\_Y} = \text{IOL cylinder at corneal plane} \times \sin(2 \times \text{IOL\_new\_axis})$$

$$(11) \text{ Residual\_cylinder\_X} = \text{Corneal\_cylinder\_X} - \text{New\_IOL\_X}$$

$$(12) \text{ Residual\_cylinder\_Y} = \text{Corneal\_cylinder\_Y} - \text{New\_IOL\_Y}$$

$$(13) \text{ Residual\_cylinder\_axis} = \arctangent(\text{Residual\_cylinder\_Y} / \text{Residual\_cylinder\_X})$$

IOL\_new\_axis must be expressed in radians. Caution must be taken if the astigmatism axis in radians is negative.

$$(14) \text{If Residual\_cylinder\_axis} < 0 \rightarrow \text{Residual\_cylinder axis} = \text{Residual\_cylinder\_Axis} + \text{Pi}$$

For every value of astigmatism axis, we shall continue as follows:

$$(15) \text{Residual\_cylinder} = \text{Residual\_cylinder\_Y} / \sin(\text{Residual\_cylinder\_axis})$$

STEP 5. Calculate the residual refractive error. The predicted cylinder after rotation is the value of the Residual\_cylinder. The TIOL position is the position in which the minimum value of Residual\_cylinder is obtained. The axis of the residual astigmatism can be obtained after dividing by two the value of Residual\_cylinder\_axis expressed in degrees. The value of residual cylinder axis in degrees can be obtained from the radian expression after multiplying this value by 180 and dividing it by Pi. The residual sphere at corneal plane can be obtained with the following expression:

$$(16) \text{Residual\_sphere} = \text{Postoperative\_Spherical\_Equivalent} - 0.5 \times \text{Residual\_cylinder}$$

The obtained residual refractive is expressed in positive cylinders. To get the residual refraction at spectacles plane, the following expression should be added:

$$(17) \text{Residual refraction at spectacles plane} = \text{refraction at corneal plane} / [1 + (d \times \text{refraction at corneal plane})]$$