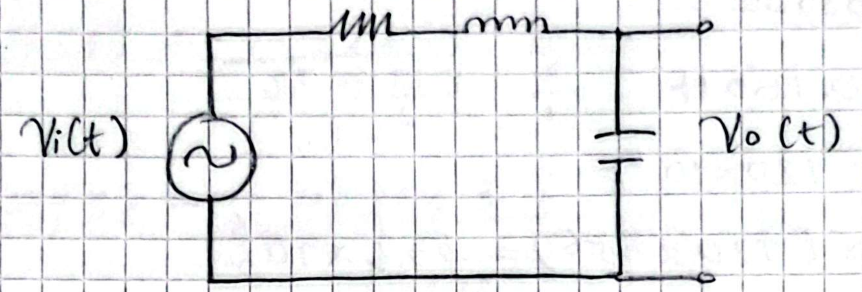
Ejercicio



$R = 1k\Omega$

$L = 180\,mH$

$C = 120\,\mu F$

1. EDO
2. $H(\omega) = \frac{Y(\omega)}{X(\omega)}$ → diagrama de bode [dB]
3. $H(s) = \frac{Y(s)}{X(s)}$ → diagrama polos y ceros
4. $h(t) =$
5. $y(t) = ?$

$$H(s) = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = V_i(s)\left(\frac{Z_C}{Z_R + Z_L + Z_C}\right)$$

$Z_R = R$

$Z_L = sL$

$Z_C = \frac{1}{sC}$

$$H(s) = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{sC}}{R + sL + \frac{1}{sC}}$$

$$H(s) = \frac{1}{sCR + s^2LC + 1}$$

$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

Scribe

Cálculo de $h(t)$

$R = 1\,k\Omega = 1000\,\Omega$

$L = 180\,mH = 0.180\,H$

$C = 120\,\mu F = 120\times10^{-6}\,F$

$LC = (0.180)\times(120\times10^{-6}) = 21.6\times10^{-6}$

$RC = (1000)\times(120\times10^{-6}) = 0.12$

Entonces

$$H(s) = \frac{1}{21.6\times10^{-6}s^2 + 0.12s + 1}$$

$$H(s) = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{46296.3}{s^2 + 0.8666s + 4629.63}$$

Tipo de respuesta

$$H(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

Frecuencia

$\omega_n^2 = \frac{1}{LC} = 4629.63 \qquad \omega_n = \sqrt{4629.63} = 68.04\ rad/s$

Factor

$2\zeta\omega_n = \frac{R}{L} = \frac{1000}{0.180} = 5555.56$

$\frac{RC}{LC} = \frac{R}{L} = \frac{1000}{0.180} = 5555.56$

$$H(s) = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$H(s) = \frac{1}{LCs^2 + RCs + 1}$$

$$H(s) = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

$$H(s) = \frac{46296.3}{s^2 + \frac{0.12}{21.6 \times 10^{-6}}s + 4629.63}$$

$$H(s) = \frac{46296.3}{s^2 + 55555.56 + 4629.63}$$

Comparando con $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$\omega_n^2 = 4629.63 \Rightarrow \omega_n \approx 68.04$ rad/s

$2\zeta\omega_n = 5555.56 \Rightarrow \zeta = \frac{5555.56}{2 \times 68.04} \approx 40.8$

Cálculo de polos y ceros

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 5555.56s + 4629.63 = 0$

$$s_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-5555.56 \pm \sqrt{(5555.56)^2 - 4(4629.63)}}{2}$$

$$s_1 = \frac{-5555.56 + 5553.89}{2} \approx -0.835$$

$$s_2 = \frac{-5555.56 - 5553.89}{2} \approx -5554.725$$

$$H(s) = \frac{46296.3}{(s-s_1)(s-s_2)} = \frac{46296.3}{(s+0.835)(s+5554.725)}$$

Fracciones parciales

$$H(s) = \frac{A}{s + 0.835} + \frac{B}{s + 5554.725}$$

Scribe

$$A = \lim_{s \to -0.835} (s+0.835)H(s) = \frac{46296.3}{-0.835+5554.725} \approx 8.336$$

$$B = \lim_{s \to -5554.725} (s+5554.725)H(s) = \frac{46296.3}{-5554.725+0.835} \approx -8.336$$

$$H(s) = \frac{8.336}{s+0.835} - \frac{8.336}{s+5554.725}$$

Laplace $\mathcal{L}^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}u(t)$

$$h(t) = 8.336\left(e^{-0.835t} - e^{-5554.725t}\right)u(t)$$

Cálculo de $y(t)$

$$v_i(t) = u(t)$$

$$v_i(s) = \mathcal{L}\{u(t)\} = \frac{1}{s}$$

La salida en el dominio de Laplace es

$$Y(s) = H(s)v_i(s)$$

$$Y(s) = H(s)\frac{1}{s} = \frac{1}{s}\left(\frac{8.336}{s+0.835} - \frac{8.336}{s+5554.725}\right)$$

$$Y(s) = \frac{8.336}{s(s+0.835)} - \frac{8.336}{s(s+5554.725)}$$

Fracciones parciales a $\frac{1}{s(s+a)} = \frac{1/a}{s} - \frac{1/a}{s+a}$

Para $a_1 = 0.835$

$$\frac{8.336}{s(s+0.835)} = 8.336\left(\frac{1/0.835}{s} - \frac{1/0.835}{s+0.835}\right) \approx$$

$$8.336\left(\frac{1.197}{s} - \frac{1.197}{s+0.835}\right) \approx \frac{10}{s} - \frac{10}{s+0.835}$$

para $a_2 = 5554.725$

$$\frac{8.336}{s(s+5554.725)} = 8.336\left(\frac{1/5554.725}{s} - \frac{1/5554.725}{s+5554.725}\right) \approx$$

$$8.336\left(\frac{0.0018}{s} - \frac{0.0018}{s+5554.725}\right) \approx \frac{0.015}{s} - \frac{0.015}{s+5554.725}$$

$$Y(s) = \frac{A}{s} + \frac{B}{s+s_1} + \frac{C}{s+s_2}$$

$$Y(s) = \frac{1}{s(LCs^2 + RCs + 1)} = \frac{1}{s(21.6\times10^{-6}s^2 + 0.12s + 1)}$$

$$A = \lim_{s\to0} sY(s) = \frac{1}{1} = 1$$

$$B = \lim_{s\to0.835} (s+0.835)Y(s) = \frac{1}{(-0.835)(21.6\times10^{-6}(-0.835)-0.12+1)}$$

$$\approx -0.998$$

$$C = \lim_{s\to5554.725} (s+5554.725)Y(s) \approx 0$$

$$Y(s) = \frac{1}{s} + \frac{-0.998}{s+0.835} + \frac{-0.002}{s+5554.725}$$

transformada inversa de Laplace

$$y(t) = (1 - 0.998e^{-0.835t} - 0.002e^{+5554.725t})u(t)$$

El segundo término exponencial decae muy rápidamente, entonces

$$y(t) = (1 - e^{-0.835t})u(t)$$