

Provas Algébricas

Demonstre que $\overline{A \cap B} = \bar{A} \cap \bar{B}$

$$\begin{aligned}\overline{A \cap B} &= \{x \in U : x \notin A \cap B\} \\ &= \{x \in U : \neg(x \in A \cap B)\} \\ &= \{x \in U : \neg(x \in A \wedge x \in B)\} \\ &= \{x \in U : \neg(x \in A) \vee \neg(x \in B)\} \\ &= \{x \in U : x \notin A \vee x \notin B\} \\ &= \{x \in U : x \in \bar{A} \vee x \in \bar{B}\} \\ &= \{x \in U : x \in \bar{A} \cup \bar{B}\} \\ \overline{A \cap B} &= \overline{A \cup B}\end{aligned}$$

Apresente que $A \cap (\bar{B} \cap \bar{C}) \subseteq A \cap (\bar{B} \cup \bar{C})$

PIV(13)

$$\bar{B} \cap \bar{C} \subseteq \bar{B} \quad \text{e} \quad \bar{B} \cap \bar{C} \subseteq \bar{C}$$

X

$$B \subseteq \bar{B} \cup C \quad \text{e} \quad C \subseteq \bar{B} \cup C \quad \text{PIV(14)}$$

X

$$\bar{B} \cap \bar{C} \subseteq \bar{B} \subseteq \bar{B} \cup C \quad \text{transitividade}$$

$$\boxed{\bar{B} \cap \bar{C} \subseteq \bar{B} \cup C}$$

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A	B	$A \cap B$	$A \cap \bar{B}$	$\bar{A} \cap B$	$\bar{A} \cap \bar{B}$	$\overline{A \cap B} \leftrightarrow \bar{A} \cap \bar{B}$
1	1	1	0	0	0	V
1	0	0	0	0	1	V
0	1	0	0	1	0	V
0	0	0	1	1	1	V

Tautologia

Apresente uma prova Algébrica de que $\overline{A \cup (B \cap C)} = \bar{A} \cap (\bar{B} \cap \bar{C})$ PIV(8)

$$= \bar{A} \cap (\bar{B} \cup \bar{C}) \quad \text{PIV(8)}$$

$$= (\bar{B} \cup \bar{C}) \cap \bar{A} \quad \text{PIV(2)}$$

$$= (\bar{C} \cup \bar{B}) \cap \bar{A} \quad \text{PIV(2)}$$

Apresente uma prova Algébrica de que $(A - B) - C = A \cap (\bar{B} \cap \bar{C})$

$$= (A - B) \cap \bar{C} \quad \text{PIV(16)}$$

$$= (A \cap \bar{B}) \cap \bar{C} \quad \text{PIV(16)}$$

$$= A \cap (\bar{B} \cap \bar{C}) \quad \text{PIV(1)}$$

Apresente uma prova Algébrica de que $A - (B - C) = A \cap (\bar{B} \cup C)$

$$A - (B \cap \bar{C}) \quad \text{PIV(16)}$$

$$A \cap \overline{(B \cap \bar{C})} \quad \text{PIV(16)}$$

$$A \cap (\bar{B} \cup C) \quad \text{PIV(8)}$$

$$A \cap (\bar{B} \cup C) \quad \text{PIV(7)}$$

PIV = Propriedade Intuitivamente Válidas

$$\begin{aligned}
 1. \quad A &= A \cap U && \text{PIV}(4) \\
 &A \cap (B \cup \bar{B}) && \text{PIV}(10) \\
 &(A \cap B) \cup (A \cap \bar{B}) && \text{PIV}(6)
 \end{aligned}$$

$$\begin{aligned}
 2. \quad A - B &= A \cap \bar{B} && \text{PIV}(16) \\
 &\subseteq A && \text{PIV}(13)
 \end{aligned}$$

$$\begin{aligned}
 3. \quad (A - B) \cap B &= (A \cap \bar{B}) \cap B && \text{PIV}(16) \\
 &= A \cap (\bar{B} \cap B) && \text{PIV}(7) \\
 &= A \cap \emptyset && \text{PIV}(10) \\
 &= \emptyset && \text{PIV}(5)
 \end{aligned}$$

$$\begin{aligned}
 4. \quad (A \cup B) \cap \bar{A} &= (A \cap \bar{A}) \cup (B \cap \bar{A}) && \text{PIV}(6) \\
 &= \emptyset \cup (B \cap \bar{A}) && \text{PIV}(10) \\
 &= B \cap \bar{A} && \text{PIV}(4)
 \end{aligned}$$

$$\begin{aligned}
 5. \quad (A \cup B) \cap \bar{B} &= (A \cap \bar{B}) \cup (B \cap \bar{B}) && \text{PIV}(6) \\
 &= (A \cap \bar{B}) \cup \emptyset && \text{PIV}(10) \\
 &= A \cap \bar{B} && \text{PIV}(4)
 \end{aligned}$$