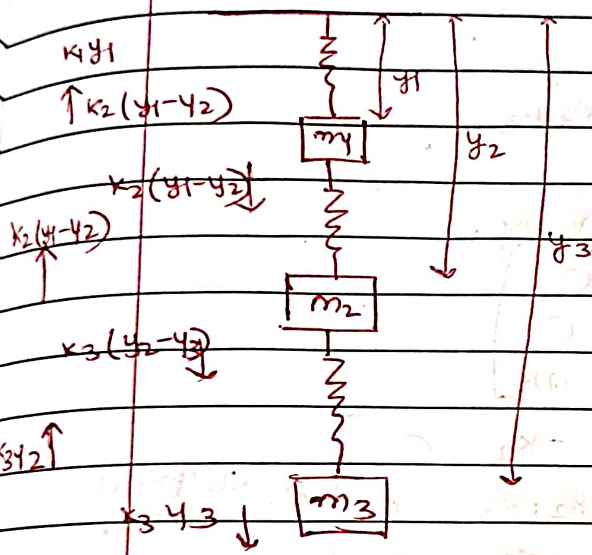
Q2- In the given ques,
we have a system of springs and masses m_1, m_2 and m_3 at vertical displacements y_1, y_2 and y_3 having spring constants k_1, k_2 and k_3 respectively.



We need to write the eq^n of motion for this system.
Suppose a_1, a_2, a_3 are the accelerations of masses m_1, m_2 and m_3 respectively.

$$m_1 a_1 = -k_1 y_1 - k_2(y_1 - y_2) = 0$$
$$\Rightarrow m_1 a_1 + k_1 y_1 + k_2(y_1 - y_2) = 0$$
$$\Rightarrow m_1 a_1 + (k_1 + k_2) y_1 - k_2 y_2 = 0 \quad -(1)$$

$$m_2 a_2 = k_2(y_1 - y_2) - k_3(y_2 - y_3) = 0$$
$$\Rightarrow m_2 a_2 = k_2(y_1 - y_2) + k_3(y_2 - y_3) = 0$$
$$\Rightarrow m_2 a_2 - k_2 y_1 + (k_2 + k_3) y_2 - k_3 y_3 = 0 \quad -(2)$$

$$m_3 a_3 = k_3 y_2 - k_3 y_3$$
$$\Rightarrow m_3 a_3 - k_3 y_2 + k_3 y_3 = 0 \quad -(3)$$

By eq^n (1),

$$-m_1 a_1 = (k_1+k_2)y_1 - k_2 y_2$$

By eq^n (2),

$$-m_2 a_2 = -k_2 y_1 + (k_2+k_3)y_2 - k_3 y_3$$

By eq^n (3),

$$-m_3 a_3 = -k_3 y_2 + k_3 y_3$$

The mass matrix M is given by:

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$

$$A = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} \qquad K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix} \Rightarrow \text{stiffness matrix}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

Let A_1, A_2, A_3 be amplitudes of motion of m_1, m_2 and m_3 respectively.

$$y_1(t) = A_1 e^{i\omega t}$$
$$y_2(t) = A_2 e^{i\omega t}$$
$$y_3(t) = A_3 e^{i\omega t}$$ where $\omega \Rightarrow$ natural freq.

$$a_1 = \frac{d^2(y_1(t))}{dt^2} = -A_1 \omega^2 e^{i\omega t}$$

$$a_2 = -A_2 \omega^2 e^{i\omega t} \qquad a_3 = -A_3 \omega^2 e^{i\omega t}$$

$$Y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} A_1 e^{i\omega t} \\ A_2 e^{i\omega t} \\ A_3 e^{i\omega t} \end{bmatrix}$$

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$$A = \begin{bmatrix} -a_1 \\ -a_2 \\ -a_3 \end{bmatrix} = \begin{bmatrix} A_1\omega^2 e^{i\omega t} \\ A_2\omega^2 e^{i\omega t} \\ A_3\omega^2 e^{i\omega t} \end{bmatrix}$$

$$MA = Ky$$

$$M\omega^2 y = Ky \quad (\because A = \omega^2 y)$$

$$\Rightarrow (K - \omega^2 M)y = 0$$

Thus eq^n of motion of this system is :

$$(K - \omega^2 M)y = 0$$

where $K = \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$ $M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$

and $y = \begin{bmatrix} y_1(t) \\ y_2(t) \\ y_3(t) \end{bmatrix}$

$$K - \omega^2 M = \begin{bmatrix} k_1+k_2-\omega^2 m_1 & -k_2 & 0 \\ -k_2 & k_2+k_3-\omega^2 M_2 & -1 \\ 0 & -k_3 & k_3-\omega^2 m_3 \end{bmatrix}$$

$k_1 = k_2 = k_3 = 1$; $m_1 = 2$; $m_2 = 2$; $m_3 = 4$

$$K - \omega^2 M = \begin{bmatrix} 2-2\omega^2 & -1 & 0 \\ -1 & 2-2\omega^2 & -1 \\ 0 & -1 & 1-4\omega^2 \end{bmatrix}$$

$$\det(K - \omega^2 M) = 0$$

$$\Rightarrow (2-2\omega^2)\left[(2-2\omega^2)(1-4\omega^2) - 1\right] + \left[4\omega^2 - 1\right] = 0$$

$$(2-2\omega^2)\left[8\omega^4 - 10\omega^2 + 1\right] + 4\omega^2 - 1 = 0$$

$$16\omega^6 - 36\omega^4 + 18\omega^2 - 1 = 0$$

$\Rightarrow \quad \omega_1 = 0.2517$ rad/s

$\omega_2 = 0.7946$ rad/s

$\omega_3 = 1.2451$ rad/s