Topics in Applied Optimization

Assignment - 5

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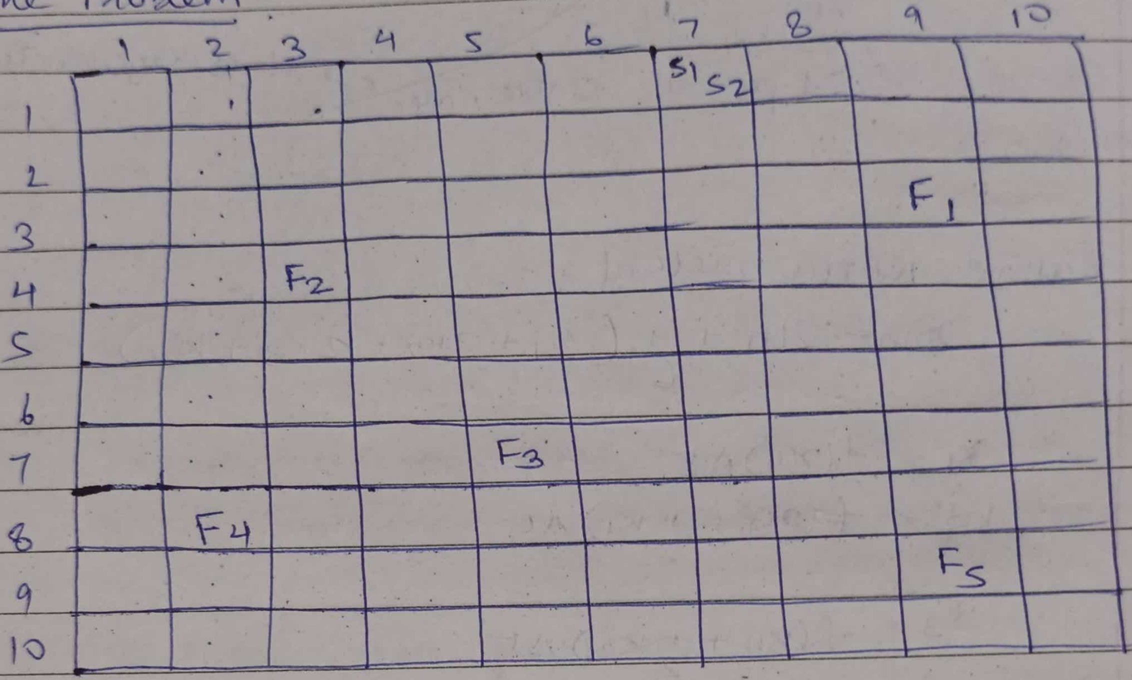
2 The Problem



Here, no. of flies = 5 → n

no. of spiders = 2 → m

The flies are in positions

(3, 9) → F_1

(4, 3) → F_2

(7, 5) → F_3

(8, 2) → F_4

(9, 9) → F_5

Spider's initial locations:

(1, 7) → S_1

(1, 7) → S_2

Method 1: Base Policy method.

This policy suggests that movement of spiders is independent of each other. The method suggests that spider moves a square closer towards the nearest fly considering where it starts from.

Also, ordinary rollout says that both the spiders will move in same manner.

let us calculate the manhattan distance :
where $d = |x_1 - x_2| + |y_1 - y_2|$ for two points (x_1, y_1) & (x_2, y_2)

spider 1 → S_1 (1, 7) Spider 2 → S_2(1, 7)
For fly 1 : at (3, 9) : $d = |1-7| + |7-9| = 4$
For fly 2 at (4, 3) : $d = |1-4| + |7-3| = 7$
for fly 3 at (7, 5) : $d = |1-7| + |7-5| = 8$
For fly 4 at (8, 2) : $d = |1-8| + |7-2| = 12$
for fly 5 at (9, 9) : $d = |1-9| + |7-9| = 10$

→ step 1 :

	Spider 1	Spider 2
current position	(1, 7)	(1, 7)
Nearest fly	(3, 9)	(3, 9)
Manhattan distance	4	4
Chosen move	right	right
update position	(1, 8)	(1, 8)

Thus, the α factor of both the spiders is 4.
Since, fly 1 has smallest manhattan distance, spiders will move towards its location i.e (3, 9). following the path (1, 8) → (1, 9) → (2, 9) → (3, 9)
Thus for both the spiders, the steps they take is 4.

→ Step 2 :

After step 1, both the spiders have now moved to location (3, 9) following the path in step 1.

	Spider 1	Spider 2
Current Position	(3,9)	(3,9)
Nearest fly	(9,9)	(9,9)
Manhattan distance	6	6
Chosen move	down	down
Update position	(4,9)	(4,9)

Thus, a factor is 6 for both spiders.
~~Thus~~ The spiders will move towards the nearest fly i.e (9,9) {F_5} following the path:

$(4,9) \rightarrow (5,9) \rightarrow (6,9) \rightarrow (7,9) \rightarrow (8,9) \rightarrow (9,9)$

No. of steps taken by both spiders is = 6

We chose to go to F_5 location as that ~~as~~ ~~it~~ has the smallest manhattan distance from the current location of spiders which was (3,9).

→ **Step 3:**

From step 2, we know that the updated location of spiders is F_5 i.e (9,9)

Let's calculate the next move by chosing the smallest manhattan distance b/w (9,9) and remaining flies.

$F_2 \rightarrow d = |4-9| + |3-9| = 11$

$F_3 \rightarrow d = |7-9| + |5-9| = 6$

$F_4 \rightarrow d = |8-9| + |2-9| = 8$

Clearly, spiders will go to F_3 now located at (7,5)

	Spider 1	Spider 2
Current position	(9, 9)	(9,9)
Nearest fly	(7,5)	(7,5)
manhattan distance	6	6
chosen move	left	left
updated position	(9,8)	(9,8)

Q factor for both spiders is 6.
Spiders move from (9,9) to (~~9,5~~ 7,5) following path:

(9,8) → (9,7) → (9,6) → (9,5) → (8,5) → (7,5)
total steps taken by both spiders is 6.

step 4:
from step 3, we know that the updated location of spiders is F_3 i.e (7,5).

Let's calculate the next move by choosing the smallest manhattan distance b/w (7,5) and remaining flies.

$F_2 \rightarrow d = |7-4| + |5-3| = 5$
$F_4 \rightarrow d = |8-7| + |5-2| = 4$

Clearly, spiders will go to F_4 now located at (8,2).

	Spider 1 & Spider 2
nearest distance	(7,5)
nearest fly	(8,2)
manhattan distance	4
chosen move	left
updated position	(7,4)

a factor for both the spiders is 4.
spiders move from (7,5) to (8,2) following path:
(7,4) → (7,3) → (7,2) → (8,2).

Thus, total steps taken by both spiders = 4

- step 5:

from step 4, we know the updated location of spiders that is F_4 (8,2).

The only spider left now is F_2 at (4,3) so which is at a manhattan distance of : $|8-4| + |2-3| = 5$

	spider 1 & 2
current position	(8,2)
nearest fly	(4,3)
manhattan distance	5
chosen move	right
updated position	(8,3)

a factor for both the spiders is 5
spiders move from (8,2) to (4,3) following path:
(8,3) → (7,3) → (6,3) → (5,3) → (4,3)

Thus, total no of steps taken by both spiders = 5

Thus, the total number of steps for orinary rollout =
Total number of steps taken by both spiders

$= 2(\#step1 + \#step2 + \#step3 + \#step4 + \#step5)$
$= 2(4+6+6+4+5)$
$= 50.$

Date:

method 2 : Simultaneous minimization, agent by-agent (using ordinary rollout)

Here,

Initialization : spiders 1 and 2 start at the fourth square from the right at the top row.

Now, there has to be simultaneous move selection by both the spiders.
The spiders will start simultaneously, and chose the move that minimizes the overall a factor (we see the joint movement of both).
where,
a factor → sum of manhattan distances of both spiders to its nearest fly.

Mathematically,

$$\tilde{\mu}(x) \in \arg\min_{u^1 \in U^1(x)} E_w\Big[g(x, u^1, \mu^2(x), \ldots, \mu^m(x), \omega) + \alpha J_\mu\big(f(x, u^1, \mu^2(x), \ldots, \mu^m(x), \omega)\big)$$

Now, for $\tilde{\mu}$ to be optimal, J_μ should be optimal.

This occurs when one spider moves left and one spider moves right using base policy $\bar{g}$.
Out of the 16 possible moves, This was chosen.

This algorithm is repeated until all the flies are caught by spiders.

The directions for both the spiders ~~at~~ to achieve this is shown, where L → left, R → Right, U → up, D → Down

1) LR
2) LL
3) LL
4) LR
5) DL
6) DR
7) DR
8) LR
9) DD
10) DD
11) DD
12) DD
13) RD
14) RD
15) RD
16) UD

Total time steps taken to catch all the flies = 16

{at each step the computations taken are a little less than 16}.

Method 3: Sequential minimization (using multi-agent rollout)

Here,

Initialization: Spiders 1 and 2 start at the fourth square from the right of at the top row.

Here, there is sequential move selection by both the spiders.

1) Starting with spider 1:
- compute the manhattan distance between its position and all other flies.
- choose the move such that it minimizes the manhattan distance calculated (i.e move towards nearest fly). Choose horizontal direction in case there's a tie for the smallest distance.
- Then finally broadcast the move taken to all the spiders.

Date :

2) For spider 2 :

- Observe the move taken by the 1st spider and also assume the future moves by it.
- Choose the move such that it minimizes the manhattan distance for the remaining flies (i.e move towards the nearest surviving fly).
- Then, finally broadcast the move to all the spiders.

Here, base policy $\tilde{\mu} = (\mu^1, \mu^2)$

$$\tilde{\mu}^1(x) \in \arg\min E_w \{ g(x, u^1, \mu^2(x), w) + \alpha J_\mu (f(x, u^1, \mu^2(x), w)) \}$$

$$\tilde{\mu}^2(x) \in \arg\min E_w \{ g(x, \tilde{\mu}^1(x), u^2, w) + \alpha J_\mu (f(x, \tilde{\mu}^1(x), u^2, w)) \}$$

In every stage, it takes $O(mn)$ computations

where $n \rightarrow$ no of flies $= 5$

$m \rightarrow$ no of spiders $= 2$

The directions for both the spiders to acheeive this is shown, where L → left, R → Right, U → Up, D → Down

1) LR	9) DD
2) LL	10) DD
3) LL	11) DD
4) LR	12) DD
5) DL	13) RD
6) DR	14) RD
7) DR	15) RD
8) LR	16) UD

Total time steps taken to catch all the flies = 16

{each step computations done is $(n \times m) = 5 \times 2 = 10$}

Two real-life examples/ problems where this model can be used are:

1) Supply Chain Optimization: Apply the Spider-and-flies model to optimize the movement of goods in a supply chain. Here, spiders represent autonomous vehicles and flies represent delivery locations. This model helps to optimize routes and resource allocation for efficient supply chain management.

2) Emergency Response Coordination: Adapt the model for coordinating emergency response efforts, where spiders are ~~autonomo~~ autonomous robots or response teams and flies represent areas of interest / incidents. The model helps in deploying resources efficiently by planning optimal routes and enhances overall operations of emergency response scenarios.

Optional question code is submitted along.