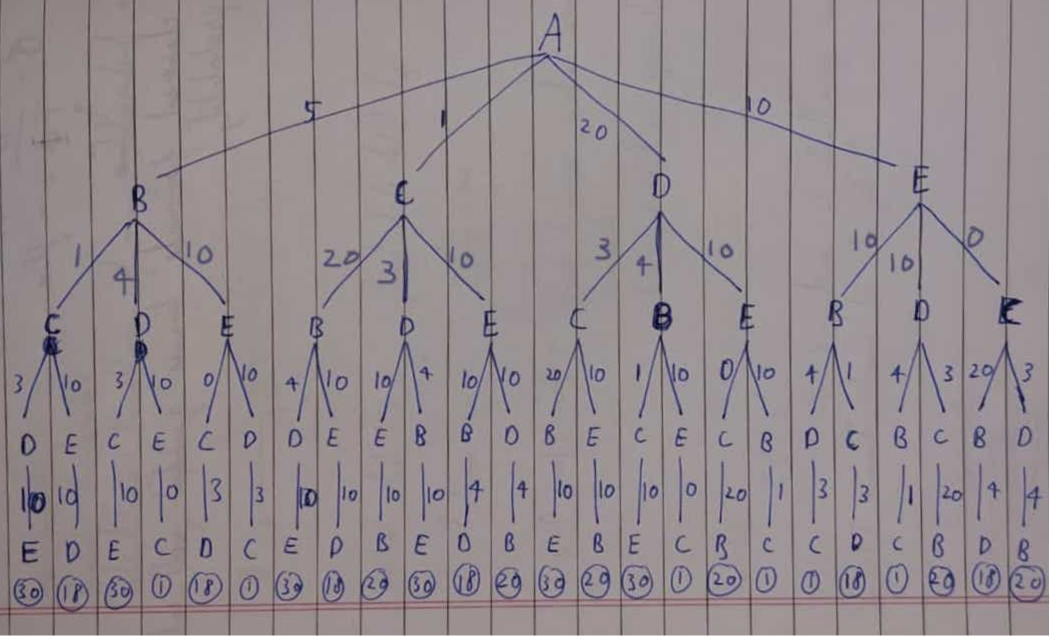
TAO

Assignment - 4

Sriteja Reddy Pashya
(2021111019)

Romica Raisinghani
(2021101053)

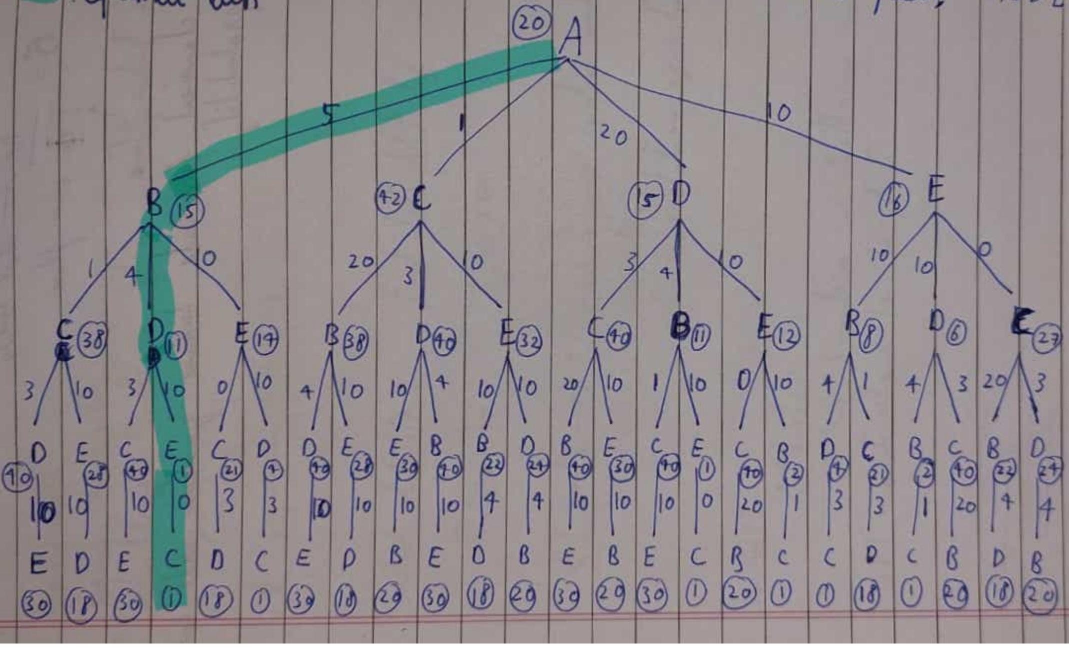
Total Cost = 20



Total Cost = 20

: Optimal Path

Costs written inside O are exact DP costs for backward pass. ABDECA is optimal



$$\text{cost\_matrix} = \begin{array}{c} A \\ B \\ C \\ D \\ E \\ \\ \end{array} \begin{array}{c} \left[\begin{array}{ccccc} - & 5 & 1 & 20 & 10 \\ 20 & - & 1 & 4 & 10 \\ 1 & 20 & - & 3 & 10 \\ 18 & 4 & 3 & - & 10 \\ 30 & 10 & 0 & 10 & - \end{array}\right] \\ \begin{array}{ccccc} A & B & C & D & E \end{array} \end{array}$$

2) To verify that the nearest neighbour heuristic starting with city A generates the tour ACDBEA with cost 48, we can follow these steps :

1) Start at city A

2) Find the nearest unvisited city, which is C (cost 1)

3) Add city C to the tour and mark it as visited

4) Repeat steps 2 and 3 until all cities have been visited and the tour returns to city A

This can be shown as : * Initially visited = {A}

1) From A, optimal choice is C { cost = 1 }
visited = {A, C}

2) From C, optimal choice is D { cost = 1 + 3 }
visited = {A, C, D}

3) From D, optimal choice is B { cost = 1+3+4 }
visited = {A, C, D, B}

4) From B, optimal choice is E { cost = 1+3+4+10 }
visited = {A, C, D, B, E}

5) From E, since all cities have been visited, hence return to start city A { cost = 1+3+4+10+30 }
= 48

Thus, the optimal path starting from A is ACDBEA with total cost 48.

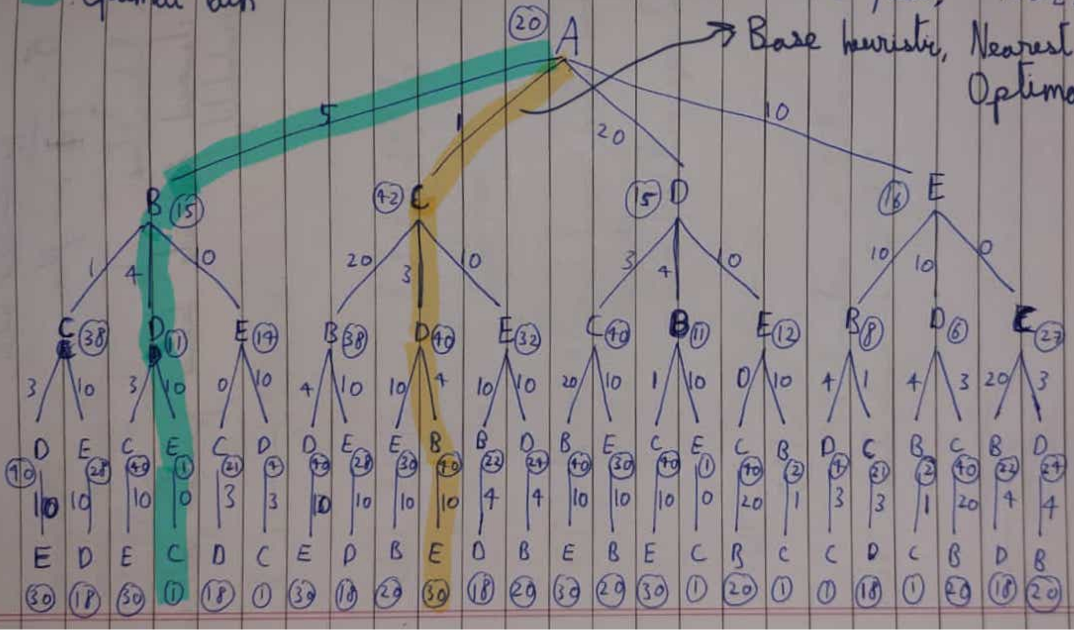
Hence, proved

Total Cost = 20

Optimal Path

Costs written inside O are exact DP costs for backward pass, ABDECA is optimal

Base heuristic, Nearest Neighbour Optimal path



3) In this part, we need to verify that rollout with one-step lookahead minimization, using nearest neighbour as the base heuristic will generate tour AECDBA with cost 37.

Now, for rollout with one-step lookahead, the control u_k^* must be such that:

$$u_k^*(x_k) = \underset{u_k \in U_k(x)}{\arg\min} \; g(x_k, u_k) + H_{k+1}(f(x_k, u_k))$$

where $H_{k+1}(f(x_k, u_k))$ is the heuristic cost

We start at city A,
The choices are:

B → 49 (5 + 44) path AB
C → 48 (1 + 47)
D → 63 (20 + 43)
E → 37 (10 + 27)

The minimum amongst these is 37
so, path becomes A → E

Now, from E,
The choices are:

B → 32 (10 + 22)
D → 53 (10 + 43)
C → ~~24~~ 27 (0 + 27)

The minimum amongst these is 27
So, path becomes A → E → C

Now, from C,
The choices are:

D → 27 (3+24)
B → 42 (20+22)

The minimum amongst them is 27
Hence, path becomes A → E → C → D.

From D,
Only choice left is:

B = 24 (4+20)

Thus, the final path is A → F → C → D → B → A
with a cost of 37.

* Also,
The cost with rollout method ≤ the cost without rollout method.

Hence, proved

4) In this part, we need to apply rollout with two-step lookahead minimization
So, here the control u_k^* must be such that:

$$u_k^*(x_k) = \arg\min_{u_k \in U_k(x_k)} g(x_k, u_k) + g(f(x_k, u_k), u_{k+1}) + H_{k+2}(f(x_{k+1}, u_{k+1}))$$

where,

$g(x_k, u_k) + g(f(x_k, u_k), u_{k+1}) \rightarrow$ two step cost

and,

$H_{k+2}\left(f(x_{k+1}, u_{k+1})\right) \rightarrow$ Heuristic cost

We start the tour at A,
The choices are:

Path ABCDEA $\{BC \rightarrow 5+1+43\}$ = 49
Path ABDCEA $\{BD \rightarrow 5+4+43\}$ = 52
Path ABECDA $\{BE \rightarrow 5+10+21\}$ = 36

Path ACDBEA $\{CD \rightarrow 1+3+44\}$ = 48
Path ACBDEA $\{CB \rightarrow 1+20+44\}$ = 65
Path ACEBDA $\{CE \rightarrow 1+10+32\}$ = 43

Path ADBCEA $\{DB \rightarrow 20+4+41\}$ = 65
Path ADCEBA $\{DC \rightarrow 20+3+40\}$ = 63
Path ADECBA $\{DE \rightarrow 20+10+40\}$ = 70

Path AEBCDA $\{EB \rightarrow 10+10+22\}$ = 42
Path AEDCBA $\{ED \rightarrow 10+10+43\}$ = 63
Path AECDBA $\{EC \rightarrow 10+0+27\}$ = 37

Thus, the optimal choice {with min. cost} is BE with cost 36.
Hence, path becomes $A \rightarrow B$

Now, from B,
The choices are:

Path ABCDEA $\{CD \rightarrow 1+3+40\}$ = 44
Path ABCEDA $\{CE \rightarrow 1+10+28\}$ = 39
Path ABDCEA $\{DC \rightarrow 4+3+10\}$ = 17
Path ABDECA $\{DE \rightarrow 4+10+1\}$ = 15
Path ABECDA $\{EC \rightarrow 10+0+21\}$ = 31

Page No.:
Date:

Path ~~ABECA~~ ABEDCA {ED → 10+10+4} = 24

Thus, the optimal choice is DE with cost 15

Hence, path becomes A → B → D

Now, from D;
The choices are:
Path ABDCEA {CE → 3+10+30} = 33
Path ABDECA {EC → 10+0+1} = 11

Thus, the optimal choice is EC with cost 11

Hence, path becomes A → B → D → E

Now, from E only one choice to go to C and then back to A.
Thus, the optimal path is A → B → D → E → C → A
with total cost 5 + 4 + 10 + 0 + 1 = 20

Hence, proved.

5) Complexity of computations:

a) For exact DP:
We see that since A is the starting city of the tour:
No. of arcs in the tree till to node:
$= 4 + 4 \times 3 + 4 \times 3 \times 2 + 4 \times 3 \times 2 \times 1$
$= 4 + 12 + 24 + 24$
$= 64$

Thus, $(64+k)$ computations are req. for finding the optimal cost. {Here, $k \to$ constant}
This is equivalent to:

$$N + N(N-1) + N(N-1)(N-2) - - - - + N!$$

$$\approx \underline{\underline{O(N^N)}} \text{ for a general case } N$$

b) For nearest neighbour heuristic:
Since A is given as the starting city of tour, the no. of arcs for comparison are:

$$= 4 + 3 + 2 + 1 + 1$$
$$= 11$$

Thus, the computations required are equivalent to:
$$N + (N-1) + (N-2) + - - + 1$$
$$\approx \underline{\underline{O(N^2)}} \text{ for a general case } N$$

c) For rollout with one-step lookahead minimization:

The no of comparisons req. can be done by ~~gouing~~ going one step to next stage node & then applying heuristic

$$= (3+2+1) + (3+2+1) + (3+2+1) + (3+2+1) + 1$$
{additional 1 for comparison}

$$\approx 4(3+2+1)$$

Now, for the second step, similarly we can say $\approx 3(3+2+1)$ and so on for next steps

Date :

Thus,

for a general case,

$= N((N-1) + (N-2) + N-3 + \cdots) + (N-1)((N-2) + (N+3) + \cdots)$

$+ \cdots$

$\approx \underline{\underline{O(N^3)}}$

d) for rollout with two-step lookahead minimization:-

We check two steps further and after that the nearest neighbour heuristic.

In part c, no. of comparisons in 1^{st} step $= 4(3 \times 2 \times 1)$

Here, no. of comparisons in 1^{st} step $= (4+3)(2+1)$

for 2nd step : $(3 \times 2)(1)$ & so on ...

Thus,

for a general case

$= N(N-1)[(N-2) + (N-3) + \cdots] + (N-1)(N-2)[(N-3) + (N-4) + \cdots]$

$\cdots$

$\approx \underline{\underline{O(N^4)}}$

* We observe that in rollout (one-step, two-step, ...) vary in polynomial degrees in their computational complexity from the base heuristic (nearest neighbour)