TOPOLOGY COMPREHENSIVE EXAM: PHD

Duration: 3hrs.
Total: 100 points

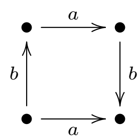
(1) *(20 points)*

Let $i : A \subset X$ be a closed embedding of topological spaces. Recall that a retraction of X to A is a map $r : X \to A$ such that $r \circ i = 1_A$. Show that A is not a retract of X in the following situations.

(a) $X = D^2$, $A = S^1$ embedded in X as the boundary
(b) $X = \mathbb{R}^3$, $A =$ any closed subspace of X homeomorphic to S^1
(c) $X = S^1 \times D^2$ (solid torus), $A = S^1 \times S^1$(torus) embedded in X as the boundary
(d) $X=$ Mobius band, A is the boundary circle

(2) *(20 points)* The Klein bottle K is the quotient space of $S^1 \times I$ obtained by identifying $(z, 0)$ with $(z^{-1}, 1)$ for $z \in S^1$.

Show that K has a cell complex decomposition $X_0 \subset X_1 \subset X_2 = K$ with one 0-cell, two 1-cells and one 2-cells. The 1-skeleton X_1 is $S^1 \vee S^1$ and X_2 is obtained by attaching a 2-cell to X_1 via the attaching map $S^1 \to S^1 \vee S^1$ given by



Use this to compute $\pi_1(K)$.

(3) *(20 points)*

Determine the fundamental groups of the following spaces:

(a) The direct product $\mathbb{R}P^2 \times MB$ of the projective plane and a Mobius band.
(b) Two 2-spheres with their equatorial circles identified.