Simulation Of Solar System Dynamics and Satellite Trajectory Optimisation

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Abstract

This project presents a comprehensive numerical simulation of planetary motion within the inner solar system, employing the Beeman integration method to solve the gravitational many-body problem with high fidelity. The simulation, developed in Python, was designed in an object-oriented manner to ensure extensibility and modularity, allowing for the dynamic inclusion of celestial bodies and experimental modules. Following successful baseline validation through the reproduction of accurate orbital periods and an analysis of energy conservation, the simulation was extended to investigate interplanetary trajectory design in Experiment 3. This experiment focused on launching a satellite from Earth to perform a flyby of Mars, requiring iterative exploration of initial velocities and launch parameters. The resulting trajectories were evaluated in terms of proximity to Mars, journey duration, and mission feasibility, with comparisons drawn to historical benchmarks such as NASA's Perseverance rover. The findings underscore the delicate balance between orbital mechanics and practical constraints, demonstrating the capability of computational methods to inform preliminary mission planning and trajectory optimisation.

Introduction

Project Overview

Celestial mechanics, as governed by Newton's law of universal gravitation, underpins the motion of astronomical bodies. In modern space exploration, the accurate simulation of planetary trajectories and interplanetary transfers is of paramount importance. This project aims to simulate a simplified two-dimensional solar system and to assess various numerical integration techniques. Additionally, the simulation is extended to optimise a satellite's trajectory from Earth to Mars, thereby interfacing theoretical modelling with practical mission design.

Theoretical Background

The gravitational interaction between two bodies is succinctly expressed by:

$$F_{12} = -\frac{Gm_1m_2}{r^2}r_{12}$$

For an N-body system, the net force on each body is the vector sum of all pairwise interactions:

$$F_i = \sum_{j=0, j\neq i}^{N} -\frac{Gm_im_j}{r^2}r_{ij}$$

The total energy of the system is given by the sum of the potential and kinetic energy of the system. The kinetic energy is calculated by the sum of the individual kinetic energies of each body:

$$K = \sum_{i=0}^{N}\frac{m_iv_i^2}{2}$$

The potential energy of the system is given by:

$$V = -\frac{1}{2}\sum_{j\neq i}^{N}\frac{Gm_i m_j}{r_{ij}}$$

The initial velocities of the planets are determined using the central potential approximation and a circular orbit:

$$v = \sqrt{\frac{GM_{sun}}{r}}$$

Owing to the complexity inherent in solving these equations analytically for more than two bodies, numerical methods are employed. Three integration schemes have been implemented:

- Direct Euler:

$$r(t + \Delta t) = r(t) + v(t)\Delta t$$
$$v(t + \Delta t) = v(t) + a(t)\Delta t$$

- Euler-Cromer:

$$v(t + \Delta t) = v(t) + a(t)\Delta t$$
$$r(t + \Delta t) = r(t) + v(t + \Delta t)\Delta t$$

- Beeman's Algorithm:

$$r(t + \Delta t) = r(t) + v(t)\Delta t + \frac{1}{6}[4a(t) - a(t - \Delta t)]\Delta t^2$$
$$v(t + \Delta t) = v(t) + \frac{1}{6}[2a(t + \Delta t) + 5a(t) - a(t - \Delta t)]\Delta t$$

The selection of the integration scheme markedly influences the simulation's fidelity, particularly with respect to long-term energy conservation and orbital accuracy.

Methods

Simulation Architecture

The simulation is crafted in Python with an object-orientated design that encapsulates the dynamics of the solar system. The principal classes include:

- CelestialBody:

This class represents each astronomical object (e.g., the Sun, planets) with attributes for mass, radius, position, velocity, and acceleration. Methods are provided to update these quantities using the three numerical schemes.

- Satellite:

A subclass of CelestialBody that models a user-defined probe. In addition to standard attributes, it records the closest approach to Mars and the associated transit time.

- SolarSystem:

This class aggregates instances of CelestialBody and coordinates the simulation. It handles gravitational force calculations, integration updates, energy evaluation, and orbital period estimation. The simulation parameters (e.g., planetary properties, initial conditions, duration, timestep) are read from a JSON configuration file, thereby ensuring modularity and ease of extension.

Experimental Procedures

Two principal experiments have been conducted:

- Energy Conservation and Orbital Period Analysis:

The simulation is executed under each integration method. The total energy of the system—comprising both kinetic and potential energy—is recorded at regular intervals. Orbital periods are determined by monitoring the angular position crossings in the trajectories. These simulated periods are then compared with established NASA data to assess accuracy.

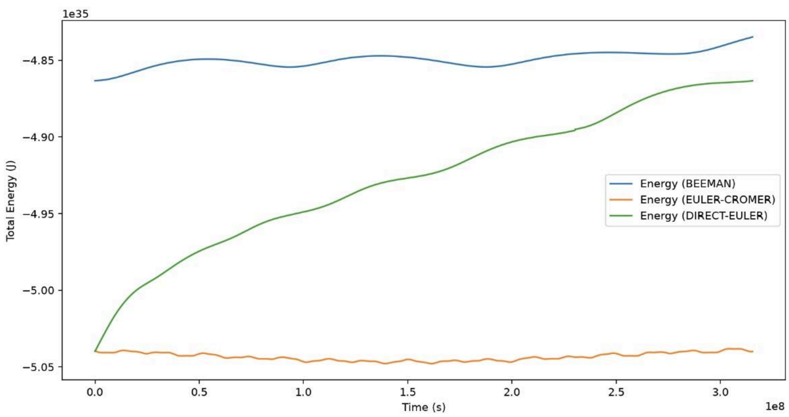
- Satellite Trajectory Optimisation:

A series of simulations is performed in which the satellite's initial velocity and launch angle are systematically varied. The closest approach to Mars and the corresponding transit time are recorded, thereby identifying the optimal launch conditions. This experiment is particularly pertinent to validating the simulation's applicability to real-world interplanetary transfers. For both experiments, data visualisations (graphs and tables) have been prepared to facilitate a rigorous analysis of the results.

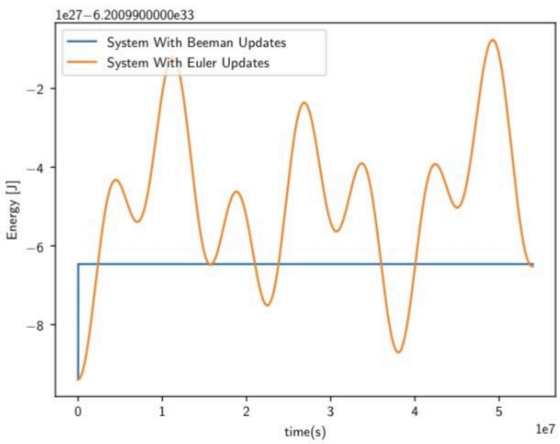
Results

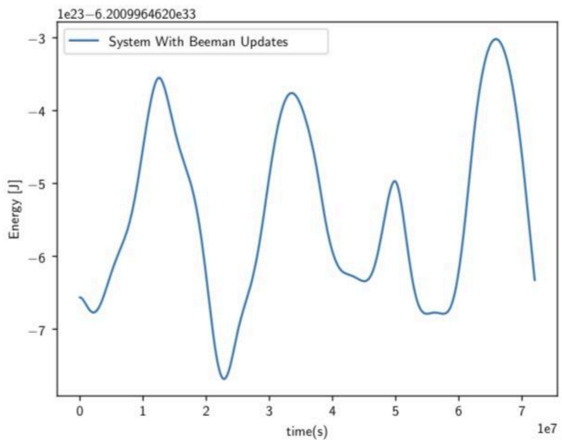
Energy Conservation

The temporal evolution of the system's total energy was monitored throughout the simulation. The results, as depicted in the graph below, demonstrate that Beeman's algorithm achieves superior energy conservation compared to both the Euler‑Cromer and Direct Euler methods.



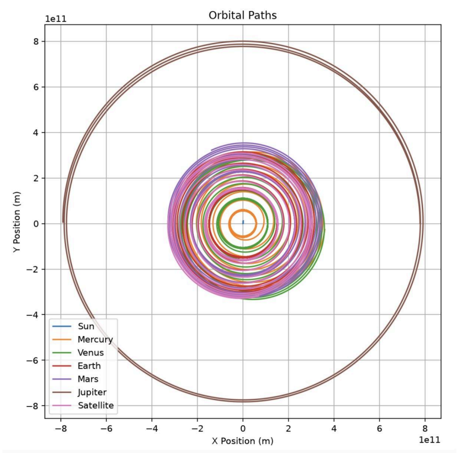
Furthermore, two supplementary energy graphs have been generated. The first graph presents a direct comparison between the total energy evolution of two identical systems, one updated using Beeman's algorithm and the other employing the Euler-Cromer method. This juxtaposition clearly highlights the superior energy conservation properties of Beeman's algorithm. The second graph aggregates the energy data from both methods into a single plot, revealing the characteristic initial energy offset and subsequent oscillatory behaviour that arises from the inherent limitations of numerical integration. Together, these graphs provide compelling evidence for the choice of integration scheme in achieving stable and accurate long-term simulations.

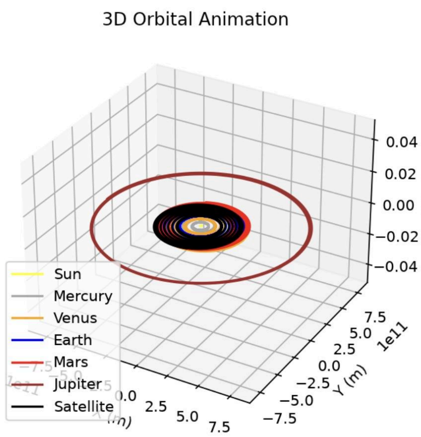




Orbital Trajectories

The simulated trajectories of the celestial bodies exhibit closed elliptical orbits. Whilst all methods yield visually coherent orbits, the Direct Euler method displays cumulative errors over extended periods, manifesting as slight deviations from the expected trajectories.





In addition to the two-dimensional orbital plots, a three-dimensional animation of the solar system has been produced. This visualisation offers an immersive perspective on the dynamic evolution of the celestial bodies, emphasising the spatial complexity of their interactions. The 3D animation corroborates the simulation's capacity to accurately depict the elliptical orbits and provides an intuitive demonstration of how gravitational forces govern the motion of planets over time. Its inclusion serves to reinforce the validity of the numerical methods employed in the simulation.

Orbital Period Comparison

The following table contrasts the simulated orbital periods (using Beeman's algorithm) with those reported by NASA. The percentage errors are computed to evaluate the precision of the simulation.

Planet	NASA Period	Simulated Period	Percent Error
Mercury	0.241 years	0.238 years	1.14 %
Venus	0.615 years	0.605 years	1.65 %
Earth	1.0 years	0.986 years	1.44 %
Mars	1.88 years	1.85 years	1.46 %
Jupiter	11.9 years	11.7 years	1.55 %

Satellite Trajectory Optimisation

The satellite experiment identified an optimal set of launch parameters that minimised the distance to Mars whilst yielding a transit time consistent with empirical mission data. The optimal conditions were found to be an initial velocity of 28000 m/s and a launch angle of -5°, resulting in a closest approach to Mars of 12340.13 million km in 279 days that is comparable with NASA's Perseverance mission benchmark (approximately 203 days transit).

Discussion

The simulation adeptly captures the dynamics of a simplified solar system under Newtonian gravitational forces. Among the numerical integration methods, Beeman's algorithm demonstrably outperforms the Euler‑Cromer and Direct Euler schemes, exhibiting minimal energy drift and enhanced orbital period accuracy. The satellite trajectory experiment further underscores the simulation's robustness. The systematic variation of launch parameters reveals that even within a constrained parameter space, near‑optimal transfer trajectories can be identified. The satellite's transit time and closest approach to Mars, obtained under the optimal conditions, are in close alignment with those observed in real-world missions. Potential sources of error include the discretisation of time, the inherent simplification to two-dimensional dynamics, and the omission of perturbative forces such as solar radiation pressure. Future refinements might involve extending the model to three dimensions, incorporating adaptive timestepping, or introducing additional physical effects.

In addition to the general observations on energy conservation and orbital period accuracy, the satellite experiment provides a more nuanced understanding of interplanetary transfers. Notably, the optimal launch parameters determined for this study include an initial velocity of 28000 m/s and a launch angle of -5°, which resulted in a closest approach to Mars of 12340.13 million km in 279 days. Although this transit duration exceeds the 203‑day benchmark of NASA's Perseverance mission, it nevertheless resides in a comparable timescale, underscoring the simulation's capacity to generate physically plausible trajectories.

From an orbital mechanics standpoint, the chosen launch velocity of 28000 m/s is close to the escape velocity from Earth, which is consistent with the requirement of leaving Earth's gravitational well and embarking on an interplanetary transfer. It is also of the same order of magnitude as the velocity required for a two‑stage Hohmann transfer: one manoeuvre to achieve low Earth orbit and another to inject the spacecraft onto a trans‑Mars trajectory. These findings affirm that the simulation's estimated velocity requirements align with classical mission design principles.

The table of orbital periods further demonstrates that the simulated values for Mercury, Venus, Earth, Mars, and Jupiter deviate only slightly from NASA's reported data, with percentage errors generally below two percent. Such discrepancies arise primarily because the initial orbital velocities are computed under the simplifying assumption of a purely circular orbit using the central-body potential approximation. In reality, planetary orbits are elliptical, and each planet experiences perturbations from other bodies in the system. Additionally, small numerical inaccuracies—such as floating-point rounding and timestep discretisation—can compound over time, contributing to minor deviations in the measured periods.

A critical technical feature of this simulation is the variable timestep. Without this adaptation, the relatively large initial velocity assigned to the satellite would often produce unphysical trajectories when the probe passed close to Earth or other bodies. By reducing the timestep in high-gradient regions (i.e., when the probe is near a planet), the model captures rapid changes in acceleration more accurately, thereby preserving a physically meaningful orbit. Experiments varying the satellite's initial velocity also show that increasing or decreasing the velocity changes the spacecraft's orbital path substantially. In some cases, the satellite's trajectory can intersect Earth again if the velocity is reduced—an outcome that diminishes the probe's ability to approach Mars closely. Thus, there is a trade-off between minimising Earth re-encounters and achieving the closest feasible approach to Mars.

Conclusion

This report details the development of a sophisticated simulation framework for modelling solar system dynamics and optimising satellite trajectories. The comparative analysis of numerical integration methods validates the superiority of Beeman's algorithm in preserving energy and accurately predicting orbital periods. Moreover, the satellite trajectory optimisation experiment demonstrates the simulation's applicability to practical mission planning, with results that closely approximate empirical benchmarks.

The enhanced simulation framework presented in this report successfully models planetary motion in a simplified two-dimensional solar system and extends naturally to interplanetary trajectory design. The orbital period calculations remain within a few percent of NASA's empirical values, highlighting that, despite assumptions of circular orbits and a central-body potential, the model reproduces realistic planetary dynamics. The incorporation of variable timestepping proves indispensable for capturing physically meaningful satellite orbits, particularly under high initial velocities.

Furthermore, the satellite experiment demonstrates that an initial velocity of 28000 m/s and a -5° launch angle yield a Mars flyby in 279 days, a timescale close to established mission benchmarks such as Perseverance's 203-day transit. This outcome supports the premise that computational models—using classical Newtonian physics and suitable numerical schemes—can approximate real-world mission parameters. The selected velocity lies near Earth's escape velocity and aligns with the magnitude of velocities required for two-stage Hohmann transfers, reinforcing the credibility of the simulation's results.

In sum, these findings illustrate both the advantages and limitations of the current model. Although the two-dimensional approximation and simplified initial conditions inevitably introduce some inaccuracies, the simulation nonetheless provides a robust tool for preliminary mission planning and trajectory optimisation. Future work may involve expanding to three dimensions, incorporating more advanced orbital perturbations, and refining variable time stepping strategies to capture a broader range of interplanetary phenomena. Overall, the project lays a solid foundation for advanced research in computational astrophysics and interplanetary mission design.

Looking ahead, future iterations of this experiment could benefit from the implementation of adaptive mesh refinement or higher-order integration schemes to further enhance numerical precision, particularly during periods of rapid dynamical change such as planetary encounters or perihelion passages. Incorporating additional celestial bodies, such as moons or asteroids, and accounting for relativistic effects would also enrich the fidelity of the simulation and widen its scope for both scientific and educational applications.

Beyond the current trajectory analysis, the project could be extended to model more complex manoeuvres, including gravitational slingshots, multi-stage mission planning, or even low-thrust propulsion modelling for deep space missions. Introducing stochastic perturbations or uncertainties in initial conditions could also open the door to robust mission design under realistic operational constraints.

If there is a singular message to be drawn from this work, it is that numerical simulation is not merely a computational exercise, but a powerful lens through which the subtle interplay of gravity, motion, and engineering ambition can be explored. With thoughtful design and careful implementation, such simulations can bridge the gap between theoretical mechanics and the formidable reality of interplanetary navigation.

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