On Hitting a Moving Target

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July 18, 2020

Chapter 1

Introduction

Notes on hitting a moving target in three dimensions.

1.1 The Problem

I encountered this problem while investigaing the creation of a video game using the Unity game engine[1]. The game is to be set in space and (in the early stages of development) I required a space ship to be able to shoot an asteroid (to be refered to as the 'target') with a slow projectile.

If the target is stationary we have no issues hitting the target, the 'gun' can simply be aimed at the target, and, after some time (for this scale I wanted the projectile to have to travel for more than one second) the projectile would hit the target. However, if the target is moving (with a constant velocity) by the time the projectile reaches the location of the target, the target has changed position and the projectile misses.

1.2 The Solution

To overcome this problem, instead of aiming directly at the target, we must calculate where the target will be after some time and aim at that location instead. In this case 'some time' is the time our projectile will take to travel to that same location which is depicted in Figure 1.1.
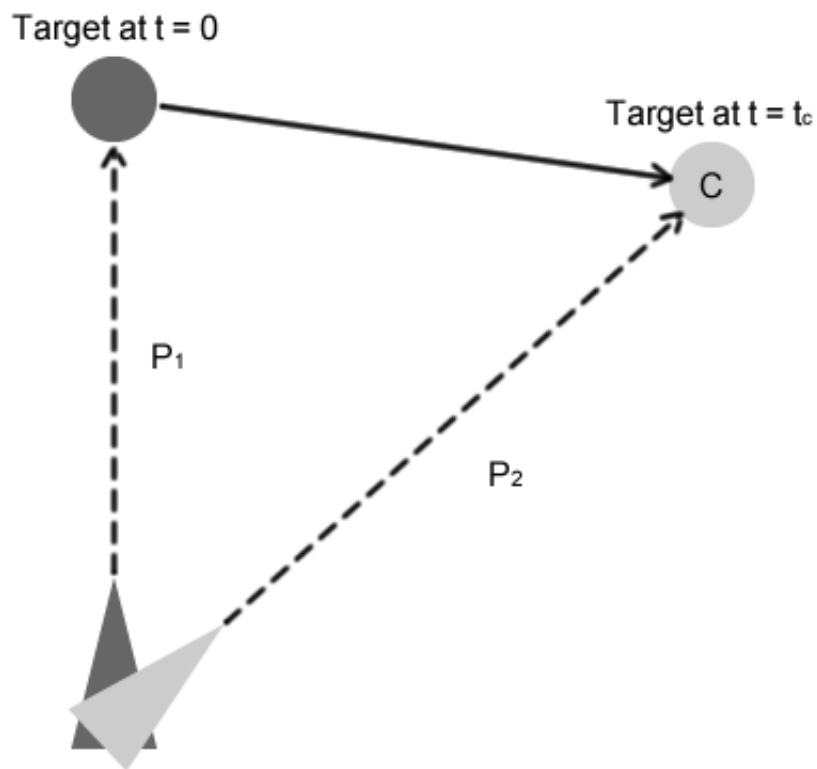
Figure 1.1: A diagram depicting the paths of two projectiles P_1 and P_2, a moving target at $t = 0$ and $t = t_c$ (with $t_c > t$), and the point C where the target and P_2 will collide.

Chapter 2

Calculating the Collision Location for Objects with Constant Velocities

We must first introduce a few definitions; we have a target T_i at the point $\vec{T} = [T_x, T_y, T_z]$ with the velocity $\vec{V} = [V_x, V_y, V_z]$ and a projectile P_i at the point $\vec{P} = [P_x, P_y, P_z]$ with a speed s. We wish to calculate the location $\vec{C}$ that P_i and T_i will collide.

The distance d between $\vec{P}$ and $\vec{C}$ can be calulated with the Pythagoras Theorem

$$d^2 = (C_x - P_x)^2 + (C_y - P_y)^2 + (C_z - P_z)^2 \tag{2.1}$$

and

$$d = st \tag{2.2}$$

where t is the amount of time until the collision will occur. From the equations of linear motion for the target, we also know that

$$C_x = T_x + V_x t \qquad C_y = T_y + V_y t \qquad C_z = T_z + V_z t. \tag{2.3}$$

We can then substitute equation (2.2) into the left hand side of equation (2.1) and the definitions of $C_{x,y,z}$ from equation (2.3) into the right hand side of equation (2.1) to produce the quadratic equation

$$s^2 t^t = (T_x + V_x t - P_x)^2 + (T_y + V_y t - P_y)^2 + (T_z + V_z t - P_z)^2 \tag{2.4}$$

with t as the unknown. Equation (2.4) can be expanded and rearraged to the general form of the quadratic equation

$$0 = \left(V_x^2 + V_y^2 + V_z^2 - s^2\right) t^2 \tag{2.5}$$

$$+ 2\left(V_x\left(T_x - P_x\right) + V_y\left(T_y - P_y\right) + V_z\left(T_z - P_z\right)\right) t \tag{2.6}$$

$$+ T_x^2 + T_y^2 + T_z^2 + P_x^2 + P_y^2 + P_z^2 \tag{2.7}$$

$$- 2\left(T_x P_x + T_y P_y + T_z P_z\right). \tag{2.8}$$

The general quadratic equation $ax^2 + bx + c$ has two solutions given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a},$$ (2.9)

therefore with the definitions

$$x = t$$ (2.10)

$$a = V_x^2 + V_y^2 + V_z^2 - s^2$$ (2.11)

$$b = 2 \left(V_x \left(T_x - P_x \right) + V_y \left(T_y - P_y \right) + V_z \left(T_z - P_z \right) \right)$$ (2.12)

$$c = T_x^2 + T_y^2 + T_z^2 + P_x^2 + P_y^2 + P_z^2 - 2 \left(T_x P_x + T_y P_y + T_z P_z \right)$$ (2.13)

we can calculate t from equation (2.9).

Now that we have the amount of time until the collision will occur t, the location of the collision $\vec{C}$ can simply be calculated by

$$\vec{C} = \vec{T} + \vec{V} t$$ (2.14)

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Chapter 3

Bibliography

[1] https://unity.com/